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Scheduling Deteriorating Jobs with a Common Due Window on a Single Machine

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Abstract: We consider the problem of scheduling deteriorating jobs and common due window location on a single machine. The actual processing time of a job is a linear increasing function of its starting time. The problem is to determine the optimal earliest due date, the due window size and the job schedule simultaneously to minimize costs for earliness, tardiness, earliest due date assignment and due window size penalties. A polynomial time optimal algorithm is presented to solve the problem.

Key words: Single machine, deteriorating jobs, due window, algorithm

INTRODUCTION

For most scheduling problems, it is assumed that the processing times of jobs are fixed parameters (Pinedo, 2002). However, this assumption is not appropriate for the modeling of many modern industrial processes where the processing time of a job may deteriorate while waiting to be processed. Examples can be found in maintenance scheduling, steel production, fire fighting and resource allocation, etc, in which any delay in processing a job may result in an increasing effort to accomplish the job. Such situations may also occur when the machines gradually lose efficiency in the course of processing jobs, so that jobs processed later require a longer processing time. The reader is referred to Alidaee and Womer (1999) and Cheng *et al*. (2004) for more practical motivations to model job deterioration in such a manner.

In the class of scheduling problems with due window, a time interval needs to be determined such that jobs completed within this interval are not penalized. Jobs completed prior to or after the due window are penalized according to their earliness/tardiness values. It is clear that a late and wide due window increases the production flexibility and the delivery options of the supplier. On the other hand, in this case his competitiveness is reduced. The main question is, therefore, when to schedule the due-window (in addition to the standard job sequencing decisions). Until now, plentiful research has been conducted on these issues under different scheduling environments. Liman *et al*. (1998) studied the single-machine common due window scheduling problem. The objective is to find the optimal size and location of the due-window to minimize a total cost function. Mosheiov and Sarig (2008) extended the problem proposed by Liman *et al*. (1998) to the case of time-dependent job processing times. They assumed that the processing time of a job is a function of its position in the sequence. Yang *et al*. (2010) further extended their result to the case where a machine maintenance may occur during the process.

PROBLEM FORMULATION AND PRELIMINARIES

We are given a single machine and a set $J = \{J_1, J_2,..., J_n\}$ of n independent and non-preemptive jobs which are available at time 0 for processing. The job processing times deteriorate linearly as an increasing function of their starting times. Let a_j denote the basic processing time of job J_j, j = 1, 2,..., n. The actual processing time p_j of job J_j starting at time t is $p_j = a_j+bt$, where, b>0 is the job-independent deteriorating rate. All jobs have a common due window defined by the earliest due date d and the due window size D. In our problem, both d and D are to be determined. Hence, an optimal solution to the problem will consist of the job sequence, the actual job starting times, the earliest due date and the window size. Let $C_j(\pi)$ denote that completion time of job J_j in a feasible schedule π. If there is no ambiguity, we omit π and use C_j to denote $C_j(\pi)$. Our objective is to find an optimal schedule π* to minimize a cost function that includes earliness, tardiness, earliest due date assignment and due window size penalties, given by the following equation:

$$f = \sum_{j=1}^{n}\left(\alpha E_j + \beta T_j + \gamma d + \delta D\right) \quad (1)$$

where, E_j = max {0, d-C} is the earliness of job J_j; T_j = max $\{0,C_j-(d+D)\}$ is the tardiness of job J_j; α, β, γ and δ are non-negative parameters representing the cost of one unit of earliness, tardiness, due date and due window

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penalty, respectively. Let "CDW" denote that the studied problem is a common due window scheduling problem. Using the three-field notation of Graham *et al*. (1979), the problem can be denoted as:

$$1|CDW|\sum_{j=1}^{n}\left(\alpha E_j+\beta T_j+\gamma d+\delta D\right)$$

For simplicity, let $J_{[j]}$ denote the job that occupies the jth position in a schedule and let $p_{[j]}$, $a_{[j]}$, $C_{[j]}$, $E_{[j]}$ and $d_{[j]}$ be defined correspondingly. The following two lemmas which are very useful for our subsequent analysis were obtained by Kuo and Yang (2008).

Lemma 1: For $1|p_j = a_j+bt|C_{max}$, if the job sequence is $\pi = (J_{[1]}, J_{[2]}, ..., J_{[n]})$ and the starting time of the first job is 0, then the makespan of π is:

$$C_{max}=C_{[n]}=\sum_{j=1}^{n}a_{[j]}(1+b)^{n-j} \tag{2}$$

Lemma 2: For $1|p_j = a_j+bt|\sum C_j$, if the job sequence is $\pi = (J_{[1]}, J_{[2]},..., J_{[n]})$ and the starting time of the first job is 0, then the total completion time of π is:

$$\sum_{j=1}^{n}C_j=\sum_{j=1}^{n}a_{[j]}\sum_{i=0}^{n-j}(1+b)^i \tag{3}$$

Moreover, the following lemma, given by Hardy *et al*. (1967), is crucial to our subsequent analysis for the determination of job sequence.

Lemma 3: (HLP rule): Given two sequences of numbers $\{x_j\}$ and $\{y_j\}$ of the same length. The sum $\sum_j x_jy_j$ of products of the corresponding elements is minimized if the sequences are monotonic in the opposite sense.

PROPERTIES OF OPTIMAL SOLUTION

Here, optimal properties of optimal solutions for our considered problem are established. Clearly, the objective function Eq. 1 can be alternatively expressed as follows:

$$f=\alpha\sum_{j=1}^{n}E_j+\beta\sum_{j=1}^{n}T_j+\gamma nd+\delta nD \tag{4}$$

Property 1: There exists an optimal schedule in which the machine is not idle between the processing of the jobs and the first job starts at time 0.

Proof: It can be easily proved by the job shifting argument.

Property 2: There exists an optimal schedule in which the due-window's starting time (i.e., the earliest due date) d and the due-window's completion time (d+D) coincide with job completions. Furthermore, the index of the job completed at the due-window's starting time is K = [n (δ-γ)/α] and the index of the job completed at the due window's finishing time is K = [n (β-δ)/β].

Proof: For a given job processing sequence, from Lemma 1 and Property 1, each job's starting time and completion time can be easily determined. Note that the result has been proved by Liman *et al*. (1998) when the job processing times are all fixed constants. In their proof, it is immaterial whether processing times are time-dependent, it is only concerned with how many jobs are completed before and after the due window. Hence, the result holds.

Write K = [n (δ-γ)/α] and L^* = [n (β-δ)/β]. In the next lemma, we introduce the positional weights (as in Liman *et al*. (1998)) and present the total cost as a function of these weights.

Property 3: The total cost can be written as:

$$f=\sum_{j=1}^{n}W_ja_{[j]} \tag{5}$$

where,

$$W_j=\begin{cases}(\alpha K^*+\gamma n-\delta n)(1+b)^{K^*-j}\\+(\beta L^*+\delta n-\beta n)(1+b)^{L^*-j}\\-\alpha\sum_{i=0}^{K^*-j}(1+b)^i\\+\beta\sum_{i=L^*+1}^{n}(1+b)^{i-j},\ j=1,\cdots,K^*\\(\beta L^*+\delta n-\beta n)(1+b)^{L^*-j}\\+\beta\sum_{i=L^*+1}^{n}(1+b)^{i-j},\ j=K^*+1,\cdots,L^*\\\beta\sum_{i=0}^{n-j}(1+b)^i,\ j=L^*+1,\cdots,n\end{cases} \tag{6}$$

Proof: Given K^* and L^*, by Properties 3.1 and 3.2, the total cost f can be expressed as follows:

$$\begin{aligned}f&=\alpha\sum_{j=1}^{n}E_j+\beta\sum_{j=1}^{n}T_j+\gamma nd+\delta nD=\alpha\sum_{j=1}^{K^*}\left(d-C_{[j]}\right)+\\&\beta\sum_{j=L^*+1}^{n}\left(C_{[j]}-d-D\right)+\gamma nd+\delta nD\\&=\alpha K^*d-\alpha\sum_{j=1}^{K^*}C_{[j]}+\beta\sum_{j=L^*+1}^{n}C_{[j]}\\&-\beta\left(n-L^*\right)(d+D)+\gamma nd+\delta nD\\&=\left(\alpha K^*+\gamma n-\delta n\right)d+\left(\beta L^*+\delta n-\beta n\right)(d+D)-\alpha\sum_{j=1}^{K^*}C_{[j]}+\beta\sum_{j=L^*+1}^{n}C_{[j]}.\end{aligned} \tag{7}$$

Note that $C_{K^*} = d$, $C_{L^*} = d$, by Lemmas 1 and 2 Eq. 7 can be expressed as:

$$
\begin{aligned}
f &= (\alpha K^* + \gamma n - \delta n)C_{K^*} + (\beta L^* + \delta n - \beta n)C_{L^*} - \alpha\sum_{j=1}^{K^*} C_{[j]} + \beta\sum_{j=L^*+1}^{n} C_{[j]} \\
&= (\alpha K^* + \gamma n - \delta n)\sum_{i=1}^{K^*} a_{[i]}(1+b)^{K^*-i} \\
&+ (\beta L^* + \delta n - \beta n)\sum_{i=1}^{L^*} a_{[i]}(1+b)^{L^*-i} \\
&- \alpha\sum_{i=1}^{K^*} a_{[i]}\left(\sum_{j=0}^{K^*-i}(1+b)^j\right) \\
&+ \beta\sum_{j=L^*+1}^{n}\sum_{i=1}^{j} a_{[i]}(1+b)^{j-i} = \sum_{j=1}^{K^*}\{(\alpha K^* + \gamma n - \delta n)(1+b)^{K^*-j} \\
&+ (\beta L^* + \delta n - \beta n)(1+b)^{L^*-j} \\
&- \alpha\sum_{i=0}^{K^*-j}(1+b)^i + \beta\sum_{i=L^*+1}^{n}(1+b)^{i-j}\} a_{[j]} \\
&+ \sum_{j=K^*+1}^{L^*}\left\{\begin{matrix}(\beta L^* + \delta n - \beta n)(1+b)^{L^*-j} \\ +\beta\sum_{i=L^*+1}^{n}(1+b)^{i-j}\end{matrix}\right\} a_{[j]} \\
&+ \sum_{L^*+1}^{n}\{\beta\sum_{i=0}^{n-j}(1+b)^i\} a_{[j]} \\
&= \sum_{j=1}^{n} W_j a_{[j]}
\end{aligned}
\tag{8}
$$

Where:

$$
W_j = \begin{cases}
(\alpha K^* + \gamma n - \delta n)(1+b)^{K^*-j} \\
+(\beta L^* + \delta n - \beta n)(1+b)^{L^*-j} \\
-\alpha\sum_{i=0}^{K^*-j}(1+b)^i \\
+\beta\sum_{i=L^*+1}^{n}(1+b)^{i-j}, \ j = 1, \cdots, K^* \\
(\beta L^* + \delta n - \beta n)(1+b)^{L^*-j} \\
+\beta\sum_{i=L^*+1}^{n}(1+b)^{i-j}, \ j = K^*+1, \cdots, L^* \\
\beta\sum_{i=0}^{n-j}(1+b)^i, \ j = L^*+1, \cdots, n
\end{cases}
$$

Property 4: There exists an optimal schedule such that the processing order of the jobs can be obtained by assigning jobs to positions according to the HLP rule, i.e., by matching the elements of W_j with a_j in opposite orders.

Proof: Note that the term W_j defined in Eq. 6 can be viewed as a positional, job-independent penalty for any job scheduled in the jth position. Therefore, by Lemma 3, the total cost defined by Eq. 5 is minimized by applying the HLP rule to the positional weight and the basic processing time.

AN OPTIMAL ALGORITHM

Here, summarizing the above discussion, we present the following optimal algorithm.

Algorithm CDW:

Step 1: Set $K^* = [n(\delta-\gamma)/\alpha]$, $L^* = [n(\beta-\delta)/\beta]$

Step 2: Calculate W_j according to Eq. 6, for j = 1, 2, ..., n

Step 3: Determine the processing order of the jobs according to the HLP rule. Let π be the obtained job sequence. If necessary, renumber the jobs such that π (1, 2,..., n)

Step 4: Calculate the earliest due date as:

$$d = \sum_{j=1}^{K^*} a_j(1+b)^{K^*-j},$$

the due window size as:

$$D = \sum_{j=1}^{L^*} a_j(1+b)^{L^*-j} - \sum_{j=1}^{K^*} a_j(1+b)^{K^*-j}$$

and the total cost as:

$$f = \sum_{j=1}^{n} W_j a_j$$

Theorem 1: Algorithm CDW solves the problem

$$1|DW|\sum_{j=1}^{n}(\alpha E_j + \beta T_j + \gamma d + \delta D)$$

in O (n log n) time.

Proof: The correctness of Algorithm CDW follows from Properties 1-4. To determine the computational complexity of the algorithm, note that Step 1 can be performed in constant time; Steps 2 and 4 require O (n) time; while Step 3 can be done in O (n log n) time. Hence, the time complexity of the Algorithm CDW is O (n log n).

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