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String Theory Origins of Supersymmetry[1]

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Abstract

The string theory introduced in early 1971 by Ramond, Neveu, and myself has two-dimensional world-sheet supersymmetry. This theory, developed at about the same time that Golfand and Likhtman constructed the four-dimensional super-Poincaré algebra, motivated Wess and Zumino to construct supersymmetric field theories in four dimensions. Gliozzi, Scherk, and Olive conjectured the spacetime supersymmetry of the string theory in 1976, a fact that was proved five years later by Green and myself.

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1 S-Matrix Theory, Duality, and the Bootstrap

In the late 1960s there were two parallel trends in particle physics. On the one hand, many hadron resonances were discovered, making it quite clear that hadrons are not elementary particles. In fact, they were found, to good approximation, to lie on linear parallel Regge trajectories, which supported the notion that they are composite. Moreover, high energy scattering data displayed Regge asymptotic behavior that could be explained by the extrapolation of the same Regge trajectories, as well as one with vacuum quantum numbers called the *Pomeron*. This set of developments was the focus of the S-Matrix Theory community of theorists. The intellectual leader of this community was Geoffrey Chew at UC Berkeley. One popular idea espoused by Chew and followers was "nuclear democracy" – that all hadrons can be regarded as being equally fundamental. A more specific idea was the "bootstrap", that the forces arising from hadron exchanges are responsible for binding the hadrons, as composites of one another, in a more or less unique self-consistent manner.

The second major trend in the late 1960s grew out of the famous SLAC experiments on deep inelastic electron scattering. These gave clear evidence for point-like constituents (quarks and gluons) inside the proton. This led to Feynman's "parton" model, which was also an active area of research in those days, and eventually to QCD.

String theory, which is the subject I want to focus on here, grew out of the S-Matrix approach to hadronic physics. The bootstrap idea got fleshed out in the late 1960s with the notion of a duality relating s-channel and t-channel processes that went by the name of "finite energy sum rules" [1] - [7]. Another influential development was the introduction of "duality diagrams", which keep track of how quark quantum numbers flow in various processes [8, 9]. Later, duality diagrams would be reinterpreted as string world-sheets, with the quark lines defining boundaries. A related development that aroused considerable interest was the observation that the bootstrap idea requires a density of states that increases exponentially with mass, and that this implies the existence of a critical temperature, called the Hagedorn temperature [10] - [13].

2 The Dual Resonance Model

The bootstrap/duality program got a real shot in the arm in 1968 when Veneziano found a specific mathematical function that explicitly exhibits the features that people had been discussing in the abstract [14]. The function, an Euler beta function, was proposed as a phenomenological description of the reaction $\pi + \omega \to \pi + \pi$ in the narrow resonance approximation. This was known to be a good approximation, because near linearity of

Regge trajectories implies that the poles should be close to the real axis. A little later, Lovelace and Shapiro proposed a similar formula to describe the reaction $\pi + \pi \to \pi + \pi$ [15, 16]. Chan and Paton explained how to incorporate "isospin" quantum numbers in accord with the Harari–Rosner rules [17]. Also, within a matter of months Virasoro found an alternative formula with many of the same duality and Regge properties that required full s-t-u symmetry [18]. Later it would be understood that whereas Veneziano's formula describes scattering of open-string ground states, Virasoro's describes scattering of closed-string ground states.

In 1969 several groups independently discovered N-particle generalizations of the Veneziano four-particle amplitude [19] - [23]. The N-point generalization of Virasoro's four-point amplitude was constructed by Shapiro [24]. In short order Fubini and Veneziano, and also Bardakci and Mandelstam, showed that the Veneziano N-particle amplitudes could be consistently factorized in terms of a spectrum of single-particle states described by an infinite collection of harmonic oscillators [25] - [28]. This was a striking development, because it suggested that these formulas could be viewed as more than just an approximate phenomenological descriptions of hadronic scattering. Rather, they could be regarded as the tree approximation to a full-fledged quantum theory. I don't think that anyone had anticipated such a possibility one year earlier. It certainly came as a surprise to me.

One problem that was immediately apparent was that since the oscillators transformed as Lorentz vectors, the time components would give rise to negative-norm ghost states. Everyone knew that such states would violate unitarity and causality. Virasoro came to the rescue by identifying an infinite set of subsidiary conditions, which plausibly could eliminate the negative-norm states from the spectrum [29]. These subsidiary conditions are defined by a set of operators, which form the famous Virasoro algebra [30]. One price for eliminating ghosts in the way suggested by Virasoro was that the leading open-string Regge trajectory had to have unit intercept, and hence, in addition to a massless vector, it contributes a tachyonic ground state to the spectrum.

Once it was clear that we were dealing with a system with a rich spectrum of internal excitations, and not just a bunch of phenomenological formulas, it was natural to ask for a physical interpretation. The history of who did what and when is a little tricky to sort out. As best I can tell, the right answer – a one-dimensional extended object (or "string") – was discovered independently by three people: Nambu, Susskind, and Nielsen [31]–[37]. The string interpretation of the dual resonance model was not very influential in the development of the subject until the appearance of the 1973 paper by Goddard, Goldstone, Rebbi, and Thorn [38]. It explained in detail how the string action could be quantized in light-cone gauge. Subsequently Mandelstam extended this approach to the interacting theory [39].

3 The RNS Model and World-Sheet Supersymmetry

The original dual resonance model (bosonic string theory), developed in the period 1968–70, suffered from several unphysical features: the absence of fermions, the presence of a tachyon, and the need for 26-dimensional spacetime. These facts motivated the search for a more realistic string theory. The first important success was achieved in January 1971 by Pierre Ramond, who had the inspiration of constructing a string analog of the Dirac equation [40]. A bosonic string $X^\mu(\sigma,\tau)$ with $0 \leq \sigma \leq 2\pi$ has a momentum density $P^\mu(\sigma,\tau) = \frac{\partial}{\partial\tau} X^\mu(\sigma,\tau)$, whose zero mode

$$p^\mu = \frac{1}{2\pi}\int_0^{2\pi} P^\mu(\sigma,\tau)\, d\sigma$$

is the total momentum of the string. Ramond suggested introducing an analogous density $\Gamma^\mu(\sigma,\tau)$, whose zero mode

$$\gamma^\mu = \frac{1}{2\pi}\int_0^{2\pi} \Gamma^\mu(\sigma,\tau)\, d\sigma$$

is the usual Dirac matrix. He then defined Fourier modes of the product $\Gamma \cdot P$

$$F_n = \frac{1}{2\pi}\int_0^{2\pi} e^{-in\sigma}\Gamma \cdot P\, d\sigma. \quad n \in \mathbf{Z}$$

The zero mode,

$$F_0 = \gamma \cdot p + \text{oscillator terms}$$

is an obvious generalization of the Dirac operator, suggesting a wave equation of the form

$$F_0|\psi >= 0$$

for a free fermionic string.[2]

By postulating the usual commutation relations for X^μ and P^μ, as well as

$$\{\Gamma^\mu(\sigma,\tau), \Gamma^\nu(\sigma',\tau)\} = 4\pi\eta^{\mu\nu}\delta(\sigma-\sigma'),$$

he discovered the super-Virasoro (or $N = 1$ superconformal) algebra[3]

$$\{F_m, F_n\} = 2L_{m+n} + \frac{c}{3}\left(m^2 - \frac{1}{4}\right)\delta_{m+n,0}$$

$$[L_m, F_n] = \left(\frac{m}{2} - n\right) F_{m+n}$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0},$$

[2]Ramond included an additional mass parameter, so that the equation he gave was $(F_0 - m)|\psi >= 0$, but it was later shown that consistency requires $m = 0$ [41].

[3]Ramond did not give the central terms.

extending the well-known Virasoro algebra (given by the L_n's alone).

At the same time, Neveu and I were working together at Princeton on the development of a new bosonic string theory containing a field $H^\mu(\sigma,\tau)$ satisfying the same anticommutation relations as $\Gamma^\mu(\sigma,\tau)$, but with boundary conditions that give rise to half-integral modes. A very similar super-Virasoro algebra arises, but with half-integrally moded operators

$$G_r = \frac{1}{2\pi}\int_0^{2\pi} e^{-ir\sigma} H\cdot P\, d\sigma, \quad r \in \mathbf{Z}+1/2$$

replacing the F_n's. In our first paper (in February 1971) we introduced an interacting bosonic string theory based on these operators [42]. However, that paper simply appended additional structure onto the Veneziano model. One month later, we presented a better scheme that does not contain the Veneziano model tachyon at $M^2 = -1$ [43]. However, it contained a new tachyon at $M^2 = -1/2$ that we identified as a slightly misplaced "pion". We thought that our theory came quite close to giving a realistic description of nonstrange mesons, so we called it the "dual pion model." This identification arose because only amplitudes with an even number of pions were nonzero. Thus we could identify a G-parity quantum number for which the "pions" were odd. It was obvious that one could truncate the theory to the even G-parity sector, and then it would be tachyon-free. However, we did not emphasize this fact, because we wanted to keep the pions. Our hope at the time was that a mechanism could be found that would shift the tachyonic pion and the massless rho to their desired masses.

In April 1971, André Neveu visited Berkeley, where he presented our results. He received an enthusiastic reception there because Bardakçi, Halpern, and Mandelstam had all tried previously to incorporate fermionic fields in string theory. The person who got most deeply involved, however, was Mandelstam's student Charles Thorn. He, André, and I figured out how the G_r operators act as subsidiary gauge conditions in the interacting theory [44]. The key step was to redefine the vacuum by a 'picture-changing' operator. We called the original string Fock space F_1 and the new one F_2. Only in the F_2 picture were the super-Virasoro conditions realized in a straightforward way.

In the same April-May period we began to appreciate the formal similarity between our construction and the previous work of Ramond. This led us to conjecture that our model could be extended to include Ramond's fermions.[4] Neveu and I succeeded in finding a vertex operator describing the emission of a 'pion' from a fermionic string. We used it to construct amplitudes for two fermions and any number of pions [45]. Two weeks later Charles Thorn presented a paper containing the same results [46]. He also obtained the first explicit formulas for fermion emission.

[4] Neveu's recollection of the sequence of events is slightly different from mine.

Let me now turn to the question of what all this has to do with supersymmetry. First of all, it is now understood that the Virasoro algebra describes two-dimensional conformal transformations, which can be regarded as analytic mappings of a Riemann surface. The infinitesimal generator $L_n \sim -z^{n+1}\frac{d}{dz}$ corresponds to $z \rightarrow z + \epsilon z^{n+1}$. The super-Virasoro (or superconformal) algebra can be regarded as a generalization to 'super-analytic' mappings of a 'super Riemann surface,' with local coordinates z and θ, where θ is a Grassmann number.

In August 1971, Gervais and Sakita presented a paper proposing an interpretation of the various operators in terms of a two-dimensional world-sheet action principle [47]. (See also [48, 49] for related work.) Specifically, they took the $X^\mu(\sigma,\tau)$, which transform as scalars in the world-sheet theory, together with free Majorana (2-component) fermions $\psi^\mu(\sigma,\tau)$. The action is

$$S = \frac{1}{2\pi}\int d\sigma d\tau \left\{\partial_\alpha X^\mu \partial^\alpha X_\mu - i\overline{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu\right\},$$

where ∂_α are world-sheet derivatives $(\frac{\partial}{\partial\tau}, \frac{\partial}{\partial\sigma})$ and ρ^α are two-dimensional Dirac matrices. They noted that this has a global fermionic symmetry: The action S is invariant under the supersymmetry transformation

$$\delta X^\mu = \bar{\epsilon}\psi^\mu$$

$$\delta\psi^\mu = -i\rho^\alpha \epsilon\, \partial_\alpha X^\mu,$$

where ϵ is a constant infinitesimal Majorana spinor. So this demonstrated that the theory has global world-sheet supersymmetry. I think that this was the first consistent supersymmetric action to be identified. However, it did not occur to us at that time to explore whether the corresponding string theory could also have spacetime supersymmetry. Perhaps the presence of the tachyonic "pion" in the spectrum prevented us from considering the possibility. A few years later, this theory was also explored by Zumino [50], a fact which I think was historically important in setting the stage for his subsequent work with Wess [51] on supersymmetric field theory in four dimensions.

When ψ^μ has periodic boundary conditions (and hence integrally-labeled Fourier modes), or the corresponding open string boundary conditions, it corresponds to Ramond's $\Gamma^\mu(\sigma,\tau)$. When it is taken to be antiperiodic, it corresponds to the $H^\mu(\sigma,\tau)$ introduced by Neveu and me. The operators F_n or G_r correspond to Fourier modes of the supersymmetry Noether current, just as L_n corresponds to modes of the two-dimensional energy–momentum tensor. A deeper understanding of the significance of the super-Virasoro gauge conditions became possible following the development of supergravity. That theory involved making space-time supersymmetry local, so it became natural to attempt the same for world-sheet supersymmetry. This was achieved by introducing a two-dimensional 'zweibein' field e_α^a that describes the geometry of the world sheet and a Rarita-Schwinger field χ_α, which is a gauge field for world-sheet supersymmetry [52, 53]. The superconformal symmetry arises as constraint conditions after the local symmetries are used to choose a covariant gauge (rather like Gauss's law in electrodynamics).

Having identified the subsidiary constraint conditions, it became plausible that one could prove that the spectrum of physical propagating degrees of freedom is ghost-free. The counting looked encouraging, because the constraints were in one-to-one correspondence with the time components of the oscillators ($\alpha_n^0 \leftrightarrow L_n$ and $b_r^0 \leftrightarrow G_r$). Indeed, inspection of some low-lying levels supported this conjecture. But it was not known for sure whether this was true for the entire spectrum.

The proof of the no-ghost theorem began with the construction of a set of vertex operators that create physical excitations satisfying the Virasoro constraints by Del Giudice, Di Vecchia, and Fubini [54]. Shortly afterwards, Brower and Thorn worked out the algebra of these operators [55] and Brower used these results to prove that the bosonic string spectrum is ghost-free for $D \leq 26$ [56]. (There are ghosts for $D > 26$.) A somewhat different proof was obtained by Goddard and Thorn, who also showed that the dual pion model is ghost-free for $D \leq 10$ [57]. The no-ghost theorem for the dual pion model was also established using Brower's methods by myself [58] as well as by Brower and Friedman [59]. Other related results were subsequently obtained by Gervais and Sakita [60], Olive and Scherk [61], and Corrigan and Goddard [62].

4 Spacetime Supersymmetry

In the NS (bosonic) sector the mass formula is $M^2 = N - \frac{1}{2}$, where

$$N = \sum_{n>0} \alpha_{-n} \cdot \alpha_n + \sum_{r>0} r b_{-r} \cdot b_r,$$

which is to be compared with the formula $M^2 = N - 1$ of the bosonic string theory. This time the number operator N has contributions from the b oscillators as well as the α oscillators.[5] Thus the ground state, which has $N = 0$, is now a tachyon with $M^2 = -1/2$.

This is where things stood until the 1976 work of Gliozzi, Scherk, and Olive [63]. They noted that the spectrum admits a consistent truncation (called the GSO projection), which is necessary for the consistency of the interacting theory. In the NS sector, the GSO projection keeps states with an odd number of b-oscillator excitations, and removes states with an even number of b-oscillator excitations. (This corresponds to projecting onto the even G-parity

[5] The reason that the normal-ordering constant is $-1/2$ instead of -1 works as follows: Each transverse α oscillator contributes $-1/24$ and each transverse b oscillator contributes $-1/48$. The result follows since the bosonic theory has 24 transverse directions and the superstring theory has 8 transverse directions.

sector of the dual pion model.) Once this rule is implemented the only possible values of N are half integers, and thus the spectrum of allowed masses is integral

$$M^2 = 0, 1, 2, \ldots .$$

In particular, the bosonic ground state is now massless. So the spectrum no longer contains a tachyon. The GSO projection also acts on the R sector, where there is an analogous restriction on the d oscillators. This amounts to imposing a chirality projection on the spinors.

Let us look at the massless spectrum of the GSO-projected theory. The ground state boson is now a massless vector, represented by the state $\zeta_\mu b^\mu_{-1/2}|0;p\rangle$, which has $d-2=8$ physical polarizations. The ground state fermion is a massless Majorana–Weyl fermion which has $\frac{1}{4}\cdot 2^{d/2}=8$ physical polarizations. Thus there are an equal number of massless bosons and fermions, as is required for a theory with spacetime supersymmetry. In fact, this is the pair of fields that enter into ten-dimensional super Yang–Mills theory. The claim is that the complete theory now has spacetime supersymmetry.

If there is spacetime supersymmetry, then there should be an equal number of bosons and fermions at every mass level. Let us denote the number of bosonic states with $M^2=n$ by $d_{NS}(n)$ and the number of fermionic states with $M^2=n$ by $d_R(n)$. Then we can encode these numbers in generating functions

$$f_{NS}(w) = \sum_{n=0}^{\infty} d_{NS}(n) w^n$$

$$= \frac{1}{2\sqrt{w}} \left(\prod_{m=1}^{\infty} \left(\frac{1+w^{m-1/2}}{1-w^m} \right)^8 - \prod_{m=1}^{\infty} \left(\frac{1-w^{m-1/2}}{1-w^m} \right)^8 \right)$$

and

$$f_R(w) = \sum_{n=0}^{\infty} d_R(n) w^n = 8 \prod_{m=1}^{\infty} \left(\frac{1+w^m}{1-w^m} \right)^8 .$$

The 8's in the exponents refer to the number of transverse directions in ten dimensions. The effect of the GSO projection is the subtraction of the second term in f_{NS} and reduction of coefficient in f_R from 16 to 8. In 1829, Jacobi discovered the formula[6]

$$f_R(w) = f_{NS}(w).$$

For him this relation was an obscure curiosity, but we now see that it provides strong evidence for supersymmetry of the GSO-projected string theory in ten dimensions. A complete proof of supersymmetry for the interacting theory was constructed by Green and me five years

[6]He used a different notation, of course.

after the GSO paper [64]. We developed an alternative world-sheet theory to describe the GSO-projected theory [65]. This formulation has as the basic world-sheet fields X^μ and θ^a, representing ten-dimensional superspace. Thus the formulas can be interpreted as describing the embedding of the world-sheet in superspace.

5 Concluding Remarks

Compared to the older RNS formulation, The GS formulation has a number of advantages and disadvantages. The main advantage is that it makes the spacetime supersymmetry manifest, whereas that symmetry is an extremely obscure in the RNS formalism, as we have just seen. A significant disadvantage of the GS formalism is that it involves a subtle combination of first-class and second-class constraints that cannot be disentangled covariantly. As a result, covariant quantization becomes a real problem. Green and I showed that the quantization is very simple and straightforward in light-cone gauge and used it to carry various tree and one-loop computations. However, from a more fundamental point of view, the lack of a satisfactory covariant quantization procedure is a serious shortcoming.

There have been many attempts over the years to address the problem of covariant quantization. As far as I know, all proposals prior to one this year by Berkovits have severe problems. Berkovits has introduced a new version of the GS formalism, which involves the use of a "pure spinor" λ [66]. Even though λ is a spacetime Majorana spinor, it is a commuting world-sheet field that satisfies the constraints $\lambda\gamma^\mu\lambda = 0$. It is conceivable that Berkovits proposal will be successful.

In addition to making supersymmetry obscure, the RNS formalism has a second drawback. Namely, it is not well-suited to incorporating background fields belonging to the Ramond–Ramond sector. In the GS formalism, on the other hand, there is no special difficulty associated to RR backgrounds. This issue becomes important in the context of the AdS/CFT duality that relates N = 4 gauge theory to type IIB string theory in an $\mathrm{AdS}_5 \times S^5$ background. Here the background also includes an RR five-form field strength. Because of the current inability to handle such backgrounds, most studies have focused on the supergravity approximation to the string theory, which is sufficient only in a certain limit. Various authors have attempted to use the GS formalism to handle the RR background, but then the problem of quantization arises. I think it might be possible to overcome this difficulty by using Berkovits' formalism, if one can figure out how to apply it in this context. This problem is important, because some modification of this configuration might lead someday to a scheme that describes QCD. So, if these issues can be sorted out, the original dream of string theory – to compute the properties of hadrons – could still be realized!

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