Localization of Relative-Position of Two Atoms Induced by Spontaneous Emission

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(Dated: January 14, 2005)

We revisit the back-action of emitted photons on the motion of the relative position of two cold atoms. We show that photon recoil resulting from the spontaneous emission can induce the localization of the relative position of the two atoms through the entanglement between the spatial motion of individual atoms and their emitted photons. The result provides a more realistic model for the analysis of the environment-induced localization of a macroscopic object.

PACS numbers: 03.65.-w, 32.80.-t, 42.50.-p

I. INTRODUCTION

To understand the transition from the quantum world to the classical world, one of the central issues is to consider how the macroscopic object is localized in a certain spatial domain [1, 2]. Superposition and its many particle version-the entanglement are the essential features of quantum physics that permits macroscopic objects to spread across the whole space, while classical physics is based on a local realism and thus a macroscopic object in classical world has well-defined position. The theory of quantum decoherence is a successful description about the quantum-classical transition and it is explained that the loss of quantum coherence of macroscopic objects is due to their coupling with the environments. Therefore a perfect knowledge of the mechanism of decoherence is crucial for the understanding of the quantum-classical transition since the de-localization usually results from quantum coherence. On the other hand in the science of quantum information, the information is mainly processed by using the quantum coherence. Knowing how decoherence destroys the wave nature of the matter in the wave-particle duality make it possible to find decoherence-free states as the computation space [3].

Theoretical studies in this context have concerned a variety of modelled systems [4, 5, 6, 7, 8, 9]. The corresponding experiments have also been done in the last years[10, 11, 12] to demonstrate the dynamic process of decoherence, the collapse and revival of quantum coherence. We studied the phenomenon of quantum decoherence of a macroscopic object by introducing a novel concept, the adiabatic quantum entanglement between collective states (such as that of the center-of-mass (C.M)) and inner states [9, 13]. In the adiabatic separation of slow and fast variables of a macroscopic object, its wave function can be written as an entangled state with correlation between adiabatic inner states and quasi-classical motional configuration of the C.M. Since the adiabatic inner states are factorized with respect to the composing parts of the macroscopic object [14], this adiabatic separation can induce quantum decoherence for the collective motion. This observation thus provides us with a possible solution to the Schrödinger cat paradox at least in the model level. Here, quantum-classical transition is just characterized by the localization of macroscopic object. When this idea was generalized to a triple system (the measured system together with the pointer state of Schrödinger-cat-like mater and its inner variables) similar to that by Zurek [5, 6], a consistent approach for quantum measurement was present by Zhang, Liu and Sun [8].

Now a next-step is naturally to investigate the actual physical systems (such as atoms interacting with the vacuum) illustrating the essence of such environment-induced decoherence. Here, to focus on the essence of problem we need not to consider the realistic macroscopic object consisting of too many particles. In principle the localization phenomenon of the relative coordinate of two atoms induced by environment is sufficient to account for the fundamental conception behind such quantum decoherence problem.

Actually, people have studied a more realistic model involving a sequence of external scattering interactions with a system of two particles considering neither the inter-particle interaction [15] nor the inner structure of particle. It shows that the scattering interactions progressively entangles two particles and decohers their relative phase, naturally leading to the localization of the particles in relative-position space. More profound result for the measurement-induced localization has been discovered and the phenomenon of phase entanglement is defined first in ref. [16]. It is found that there is phase entanglement only in the coordinate-space, which can interpret spatial localization phenomenon of atom. They also make a Schmidt-mode analysis of the entanglement between the emitting atom and the emitted photon generated in the process of the spontaneous emission and show that the localization of phonon can be controlled by measuring the atom state[17]. Newly a model of two entangled atoms located inside two spatially separated cavities has also been investigated [18]. It is found that

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the local decoherence takes an infinite time and the disentanglement due to spontaneous emission may take a finite time. For the application in quantum computing, You has investigated decoherence effects due to motional degrees of freedom of trapped electronically coded atomic or ionic qubits[19].

In this article, we continue the studies of the environment-induced decoherence for the motion of the C.M of a pair atoms induced by the back-action of light emitted from the atomic inner states. Our investigation will emphasize the reality of physical model examining the localization of relative position due to such spontaneous emission. Under the second order approximation we study in detail the time evolution of the C.M relevant state by considering the realistic environment formed by the photons in spontaneous emission, which causes the atomic recoils. The corresponding localization phenomena is characterized by the spatial reduced density matrix in the real space as the vanishing of the off-diagonal elements. This paper is organized as follows. In section II, by neglecting multi-photon processes as the higher order approximation we present a simplified model to study the time evolution of the spatial states of the two atom system under the back-action of emitted photons. In section III the spatial decoherence induced by atomic spontaneous emission is studied by the caculation of the reduced density matrix. Section IV demonstrates the localization of a macroscopic object resulting from the spatial decoherence by two simple examples, and finally conclusions are given in section V.

II. MOTION OF C.M OF TWO ATOMS INFLUENCED BY VACUUM ELECTROMAGNETIC FIELD

Our system consists of a pair of noninteracting two-level atoms of same mass m and same transition frequency ω_0 placed in the vacuum electromagnetic field. Here and infra we use black body text to denote vector quantities for convenient. The atoms are spatially separated in the positions $\mathbf{r}_A$and $\mathbf{r}_B$ respectively and the corresponding momentums are $\mathbf{p}_A$and $\mathbf{p}_B$ (as illustrated in FIG.1). We denote the C.M and relative momentums respectively by$\mathbf{P} = \mathbf{p}_A + \mathbf{p}_B$and $\mathbf{p} = (\mathbf{p}_A - \mathbf{p}_B)/2$, and similarly the C.M and relative positions can be denoted respectively by $\mathbf{X}$ and $\mathbf{r}$.

Under the rotating wave approximation, the Hamiltonian of our system reads:

$$\begin{aligned} H &= \frac{\mathbf{P}^2}{2M} + \frac{\mathbf{p}^2}{2\mu} + \frac{1}{2}\hbar\omega_0\left(\sigma_z^{(A)} + \sigma_z^{(B)}\right) \\ &\quad + \sum_{\mathbf{k}} \hbar\omega_k a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + \hbar \sum_{\mathbf{k}} g(\mathbf{k})[(\sigma_+^{(A)} e^{i\mathbf{k}\cdot(\mathbf{X}+\frac{\mathbf{r}}{2})} \\ &\quad + \sigma_+^{(B)} e^{i\mathbf{k}\cdot(\mathbf{X}-\frac{\mathbf{r}}{2})}) a_{\mathbf{k}} + h.c], \end{aligned} \tag{1}$$

where $M = 2m$ and $\mu = m/2$. The atomic transition operator are denoted by $\sigma_+^{(i)} = |e_i\rangle\langle g_i|$ and $\sigma_-^{(i)} = |g_i\rangle\langle e_i|$ $(i = A, B)$ with respect to the excited states $|e\rangle$ and the ground states $|g\rangle$ of each atom. $a_{\mathbf{k}}^\dagger$ and $a_{\mathbf{k}}$ are the annihilation and creation operators of the vacuum electromagnetic fields mode $\mathbf{k}$ with frequency $\omega_k = ck$ $(k = |\mathbf{k}|)$. The coupling constant

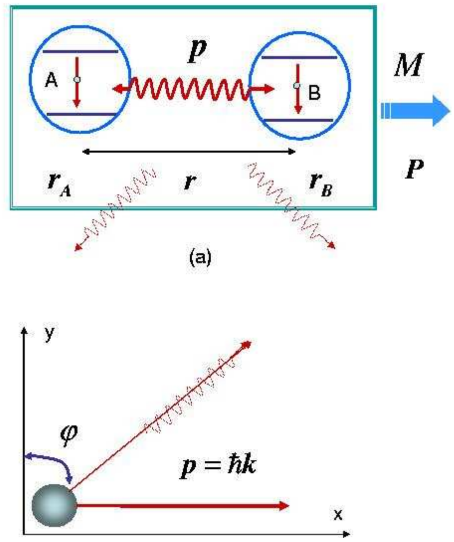

FIG. 1: *(a). Schematic illustration of two atoms A and B in the positions $\mathbf{r}_A$ and $\mathbf{r}_B$ interacting with vacuum electromagnetic field coherently. (b). The vacuum light scattering in a two-dimensional space: the atoms move only along the x axis while the photon can be emitted along the angle φ between the wave vector $\mathbf{k}$ of the emitted photon and the y axis. The momentum kick given by the emitted photon to the atom is $p_\varphi = \hbar k \sin\varphi$.*

$$\hbar g(\mathbf{k}) = \sqrt{\frac{\hbar\omega_k}{2\varepsilon_0 V}}\boldsymbol{\varepsilon}_{\mathbf{k}} \cdot \mathbf{d} \tag{2}$$

depends on the effective mode volume V, the polarization vector of the field vector $\boldsymbol{\varepsilon}_{\mathbf{k}}$and the transition dipole moment of the atom $\mathbf{d}$.

For simplicity, we consider the problem in a two-dimensional space. We assume the atoms move only along the x axis while photons can be emitted along any direction into the field. We also suppose φ is the angle between the wave vector $\mathbf{k}$ of the emitted photon and the y axis. The momentum kick given by the emitted photon to the atom is $p_\varphi = \hbar k \sin\varphi$. At $t = 0$ the relative momentum of the atoms is undefined and its state is a linear combination (superposition) of the eigenstates $|p\rangle$ of relative momentum operator $\mathbf{p}$. The total momentum state is $|P\rangle$ corresponding to total momentum $\mathbf{P}$ and the two atoms are both in excited states initially. The initial wave function of the system can be written as a product state,

$$|\Psi(0)\rangle = |P\rangle \otimes \int d^3p C_p |p\rangle \otimes |e_1\rangle \otimes |e_2\rangle \otimes |0\rangle \tag{3}$$

where C_p is the distribution function corresponding to the relative momentum eigenstate $|p\rangle$ and satisfies the normalization condition $\int_{-\infty}^{\infty}|C_p|^2 dp = 1$, $|e_1\rangle$ and $|e_2\rangle$ represent the excited states of the atom A and B respectively. $|0\rangle$ means the vacuum state of the electromagnetic filed. The time evolution of the state $|\Psi(t)\rangle$ is described by the Schrödinger equation.

To the second order approximation about the weak coupling characterized by $g(\mathbf{k})$, the state vector $|\Psi(t)\rangle$ can be calculated as

$$\begin{aligned} |\Psi(t)\rangle &= \exp(-i\omega_0 t)[\int dpA(p,t)\,|P,p,e_1,e_2,0\rangle + \\ &\sum_{\mathbf{k}}\int dpB_{\mathbf{k},p}(t)\left[\left|P-p_\varphi, p-\frac{1}{2}p_\varphi, g_1, e_2, 1_{\mathbf{k}}\right\rangle\right. \\ &\left.+\left|P-p_\varphi, p+\frac{1}{2}p_\varphi, e_1, g_2, 1_{\mathbf{k}}\right\rangle\right] \\ &+\sum_{\mathbf{k},\mathbf{k}'}\int dpD_{\mathbf{k},\mathbf{k}',p}(t)\,|P-p_\varphi-p'_\varphi\rangle \\ &\otimes\left|p-\frac{1}{2}(p_\varphi-p'_\varphi)\right\rangle\otimes|g_1,g_2\rangle\otimes|1_{\mathbf{k}}1_{\mathbf{k}'}\rangle] \end{aligned} \quad (4)$$

with $p'_\varphi = \hbar k'\sin\varphi'$ and $\int dp$ roughly denotes the definite integral of $\int_{-\infty}^{\infty} dp$. We notice that $\exp(-i\omega_0 t)$ is a common phase factor, and the first term in Eq. (4) means that both atoms are in the excited states while the field is in the state of vacuum. The second term denotes one of the atoms decays to the ground state $|g_i\rangle$ $(i = 1,2)$ from the excited states $|e_i\rangle$ with a photon of momentum $\hbar\mathbf{k}$ emitted simultaneously. The last term describes the situation when both atoms jump down to the ground states emitting two photons with momentum $\hbar\mathbf{k}$ and $\hbar\mathbf{k}'$ respectively.

The time-dependent coefficients $A(p,t)$, $B_{\mathbf{k},p}(t)$ and $D_{\mathbf{k},\mathbf{k}',p}(t)$ can be calculated by directly solving the Schrödinger equation. The obtained system of equations are

$$\dot{A}_p + i(\omega_A - \omega_0)A_p = -2i\sum_{\mathbf{k}} B_{\mathbf{k},p}(t)\,g(\mathbf{k}), \quad (5)$$

$$\begin{aligned} &\dot{B}_{\mathbf{k},p} + [i\omega_B(\mathbf{k}) - \omega_0]B_{\mathbf{k},p} = \\ &-iA_p g(\mathbf{k}) - i\sum_{\mathbf{k}'} D_{\mathbf{k},\mathbf{k}',p}\, g(\mathbf{k}') \end{aligned} \quad (6)$$

and

$$\begin{aligned} &\dot{D}_{\mathbf{k},\mathbf{k}',p} + [i\omega_D(\mathbf{k},\mathbf{k}') - \omega_0]D_{\mathbf{k},\mathbf{k}',p} = \\ &-iB_{\mathbf{k},p}\, g(\mathbf{k}') - iB_{\mathbf{k}',p}\, g(\mathbf{k}), \end{aligned} \quad (7)$$

where the coefficients

$$\hbar\omega_A = P^2/2M + p^2/2\mu + \hbar\omega_0, \quad (8)$$

$$\hbar\omega_B(\mathbf{k}) = \frac{(P-p_\varphi)^2}{2M} + \frac{(p-\frac{1}{2}p_\varphi)^2}{2\mu} + \hbar\omega_k \quad (9)$$

describe the scattering processes with photon recoil while

$$\begin{aligned} \hbar\omega_D(\mathbf{k},\mathbf{k}') &= \frac{\left(P-p_\varphi-p'_\varphi\right)^2}{2M} \\ &+\frac{\left(p-\frac{1}{2}p_\varphi+\frac{1}{2}p'_\varphi\right)^2}{2\mu} \\ &+\hbar\omega_k + \hbar\omega_{k'} - \hbar\omega_0 \end{aligned} \quad (10)$$

means that the two photon scattering will induce the momentum transfer. Notice that in the above calculation we have ignored the higher order multi-photon processes.

Starting with the initial conditions $A(p,0) = C_p$, $B_{\mathbf{k},p}(0) = 0$ and $D_{\mathbf{k},\mathbf{k}',p}(0) = 0$, the Laplace transformation about the above system of equations can give the explicit solutions to $A_p(p,t)$, $B_{\mathbf{k},p}(t)$ and $D_{\mathbf{k},\mathbf{k}',p}(t)$ in the Weisskopf-Wigner approximation [20] (see Appendix for detailed calculations). For the purpose of this paper we need not write down them here for arbitrary time t. In the limit $t\to\infty$, we have $A_p(\infty)\to 0$, $B_{\mathbf{k},p}(\infty)\to 0$ and

$$\begin{aligned} D_{\mathbf{k},\mathbf{k}',p}(\infty) &\to \frac{g(\mathbf{k}')g(\mathbf{k})C_p}{\left(i\left(\omega_B(\mathbf{k}) - \omega_D(\mathbf{k},\mathbf{k}')\right) + \frac{\Gamma}{2}\right)} \\ &\times\frac{e^{-i(\omega_D(\mathbf{k},\mathbf{k}')-\omega_0)t}}{\left(i\left(\omega_B(\mathbf{k}') - \omega_D(\mathbf{k},\mathbf{k}')\right) + \frac{\Gamma}{2}\right)} \end{aligned} \quad (11)$$

where Γ is the decay rate of an atom from state $|e\rangle$ to state $|g\rangle$. Neglecting the small recoil energies, we have

$$\begin{aligned} D_{\mathbf{k},\mathbf{k}',p}(\infty) &\to \frac{g(\mathbf{k}')g(\mathbf{k})C_p}{i\left(\omega_0 - \omega_{k'} - \left(\frac{p}{2\mu}-\frac{P}{M}\right)\frac{p'_\varphi}{\hbar}\right) + \frac{\Gamma}{2}} \\ &\times\frac{e^{-i(\omega_D(\mathbf{k},\mathbf{k}')-\omega_0)t}}{i\left(\omega_0 - \omega_k - \left(\frac{p}{2\mu}-\frac{P}{M}\right)\frac{p_\varphi}{\hbar}\right) + \frac{\Gamma}{2}} \end{aligned} \quad (12)$$

In general the C.M momentum of a hot atom is very large and so its momentum exchange with the electromagnetic field can be neglected. In this sense the electromagnetic field does not influence its C.M state nearly. However, it is not the case for the ultracold atoms because their C.M momentums are very small. The influence of the interaction between the atoms and the electromagnetic field on the spatial motion of the atoms becomes very important. Therefore it is an crucial issue about how the spatial states of the ultracold atoms are affected by their electromagnetic field environments. In the following section, we will go on to study how the spontaneous emission affects the distribution of the atomic relative position.

III. THE SPATIAL DECOHERENCE INDUCED BY INCOHERENT SPONTANEOUS EMISSION

According to the above analysis, after a sufficiently long time $t >> 1/\Gamma$, the state of the system becomes

$$|\Psi\rangle \rightarrow \sum_{\mathbf{k},\mathbf{k}'} \int dp D_{\mathbf{k},\mathbf{k}',p}(\infty) \left|P - p_\varphi - p'_\varphi\right\rangle \otimes \left|p - \frac{1}{2}(p_\varphi - p'_\varphi)\right\rangle \otimes |g_1, g_2\rangle \otimes |1_\mathbf{k} 1_{\mathbf{k}'}\rangle . \quad (13)$$

Supposing that the modes of field are closely spaced in frequency domain, we can replace $\sum_{\mathbf{k},\mathbf{k}'}$ by the integral of

$$\frac{V^2}{(2\pi)^4} \int_0^\infty k dk \int_0^{2\pi} d\varphi \int_0^\infty k' dk' \int_0^{2\pi} d\varphi' .$$

Considering that the velocity of a realistic atom is far smaller than the light velocity in vacuum, we can further simplify Eq. (13) in the representation of the C.M-relative coordinates (X and x):

$$|\Psi\rangle = N \int d\xi e^{\frac{i}{\hbar}px} e^{-\frac{i}{2\hbar}x(p_\varphi - p'_\varphi)} |X, x, g_1, g_2, 1_{k,\varphi} 1_{k',\varphi'}\rangle \times \frac{C_p e^{\frac{i}{\hbar}PX} e^{-\frac{i}{\hbar}X(p'_\varphi + p_\varphi)} e^{-i(\omega_D(\mathbf{k},\mathbf{k}') - \omega_0)t}}{\left[i(\omega_0 - \omega_{k'}) + \frac{\Gamma}{2}\right]\left[i(\omega_0 - \omega_k) + \frac{\Gamma}{2}\right]} \quad (14)$$

where we use $\int d\xi$ roughly denotes the definite multi-integral of

$$\int_0^\infty dk \int_0^{2\pi} d\varphi \int_0^\infty dk' \int_0^{2\pi} d\varphi' \int_{-\infty}^\infty dx \int_{-\infty}^\infty dX \int_{-\infty}^\infty dp$$

and N is a normalization factor including the slowly varying $g(\mathbf{k}')$ and $g(\mathbf{k})$.

Tracing over the variables of the electromagnetic field, the inner states of the atoms and the C.M motion, one can obtain the reduced density matrix

$$\begin{aligned} &\rho(x, x', t) = \\ &N' \int d\varphi d\varphi' e^{-i\frac{\omega_0}{2c}(x - x')\sin\varphi} e^{i\frac{\omega_0}{2c}(x - x')\sin\varphi'} \\ &\times \psi\left(x + \frac{\hbar\omega_0 t(\sin\varphi + \sin\varphi')}{2\mu c}, t\right) \\ &\times \psi^*\left(x' + \frac{\hbar\omega_0 t(\sin\varphi + \sin\varphi')}{2\mu c}, t\right) \\ \approx\ & N'\psi(x, t)\psi^*(x', t) J_0\left[\frac{\omega_0}{2c}(x - x')\right]^2 \end{aligned} \quad (15)$$

where N' is the normalization factor and

$$\psi(x, t) = \int_{-\infty}^\infty C_p \exp(\frac{i}{\hbar}\left(px - \frac{p^2}{2\mu}t\right))dp. \quad (16)$$

In Eq.(15), considering that the term of $\hbar\omega_0 t(\sin\varphi + \sin\varphi')/(2\mu c)$ means the small offset induced by atomic spontaneous emission of the relative position between the two atoms, we have expanded $\psi(x + \hbar\omega_0 t(\sin\varphi + \sin\varphi')/(2\mu c), t)$ around x to the first order approximation and also supposed $\omega_0 >> \Gamma$. $J_0(z)$ is the Bessel function of the first kind. Here we have used the following integral formula (a is real number):

$$\int_0^{2\pi} \cos(a\sin(z))dz = 2\pi J_0(a),$$

Now we can define the decoherence factor $F(x, x')$ as

$$F(x, x') = J_0^2(\frac{\pi}{\lambda}(x - x')). \quad (17)$$

Here $\lambda = 2\pi c/\omega_0$ is the wave length of the atomic radiation. The elements of the reduced density matrix can be rewritten as

$$\rho(x, x', t) = N'\psi(x, t)\psi^*(x', t) F(x, x'). \quad (18)$$

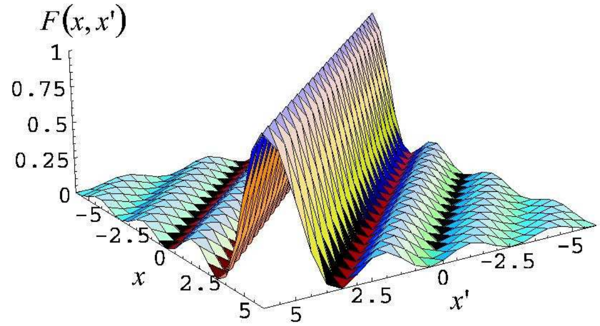

FIG. 2: Schematics of the decoherence factor $F(x, x')$. When $\frac{\pi}{\lambda}(x - x') < e^{-1}$, there exists a perfect quantum interference.When $F(x, x') > e^{-3}$, the quantum coherence disappears.

In FIG. 2, we draw the schematic curve of $F(x, x')$. It is illustrated that the off-diagonal elements of the reduced density matrix decline to zero and thus the quantum coherence of system is lost. And the diagonal elements of the reduced density matrix are suppressed with the ultimate breadth $\lambda/(\pi e)$. The result also shows that the quantum interference becomes more clear as the wave length of photon emitted become larger or the distance of the atoms becomes smaller.

IV. FROM DECOHERENCE TO LOCALIZATION OF MACROSCOPIC OBJECT

Now we take a simple example to illustrate how the above discussed decoherence can result in the localization of a macroscopic object. We take the initial state as a superposition

$$\Psi(0) = \psi(x) = \frac{1}{\sqrt{2}}\left(G_-(x) + G_+(x)\right) \tag{19}$$

of two Gaussian wave packets

$$G_\pm(x) = \frac{1}{\sqrt[4]{2\pi d^2}}\exp[-\frac{(x\pm a)^2}{4d^2}]. \tag{20}$$

As pointed out in ref.[1], the models with this initial state may arise in the double slit experiment. Now we study the dynamical evolution of the wave packets when $t >> 1/\Gamma$. In the coordinate picture the elements of the corresponding reduced density matrix can be expressed as

$$\begin{aligned}\rho(x, x\prime, t) &= N'\psi(x,t)\,\psi^*(x',t)\,F(x,x') = \\ &\quad N' J_0(\frac{\pi}{\lambda}(x-x'))^2\left\{G_-(x,t)G_-(x',t)\right. \\ &\quad +G_-(x,t)G_+(x',t) + G_+(x,t)G_-(x',t) \\ &\quad \left.+G_+(x,t)G_+(x',t)\right\}, \end{aligned} \tag{21}$$

where

$$G_+(x,t) = \frac{1}{(2\pi)^{1/4}\sqrt{d + it\hbar/(2\mu d)}} \times\exp\left(-\frac{\left(1-\frac{i\hbar}{2\mu d^2}t\right)(x+a)^2}{4d^2 + \frac{t^2\hbar^2}{\mu^2 d^2}}\right). \tag{22}$$

In fact, the spreading speed of the wave packets may be faster than the speed of the decay of the atoms when the distance of the two atoms is far smaller because the spreading speed is fast in the case. The wave packets have overlapped before the atoms decay. We will discuss the case in the following and here we suppose the distance of the atoms is large enough so that we can say the two packets have not overlapped when we study it at some time $t >> \frac{1}{\Gamma}$. We draw three groups of two-dimensional schematics for the reduced density matrix at different times in FIG.3. The first figure in each group is corresponding to the case when there is no spontaneous emission and the other figure is corresponding to the case when the influence of the spontaneous emission is considered. It demonstrates the evolution of the localization of the two initial Gaussian wave packets.

Actually, about ten years ago[21], we have generally studied a quite simple dynamical problem: the motion of the wave packet for a 'free' particle of one dimension in present a dissipative environment. An interesting result is that the dissipation suppresses the spreading of the wave packet if the breadth of initial wave packet is so wide that the effect of Brownian motion can be ignored. However, for the case with dissipation, there appears to be a significant difference about the wave packet spreading. This suppression of the wave packet spreading by dissipation possibly provides a mechanism to localize the macroscopic object. It might be of interest to note that the finite value of the width of the damped particle wave packet for $t \to \infty$ leads to exactly the same final value for the uncertainty product of the damped free particle, also found by Schuch et al. using a nonlinear Schrödinger equation[22]. In the following, we will demonstrate such localization in our present realistic model.

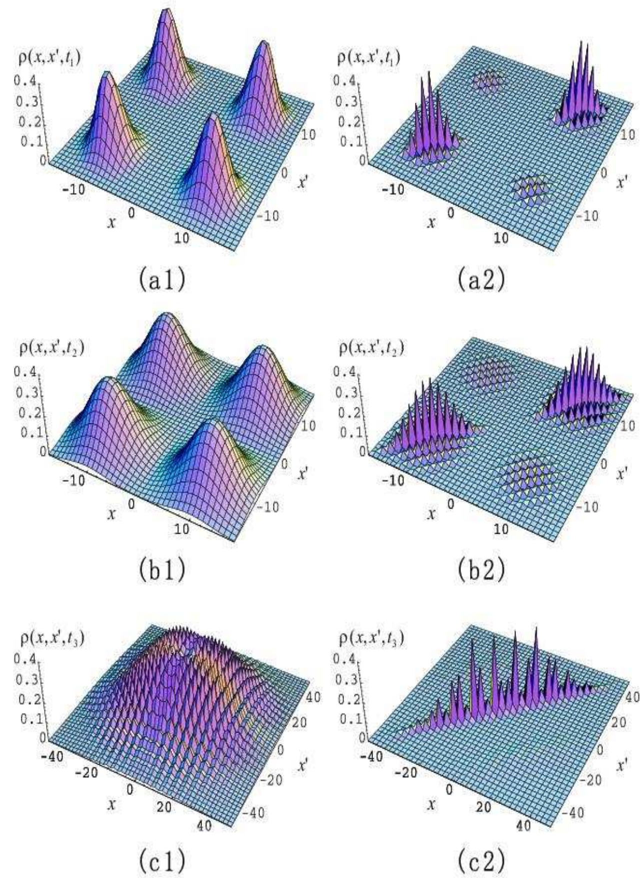

FIG. 3: Schematics of the evolution of the system state at three different times $t_1 = \frac{100}{\Gamma}, t_2 = \frac{200}{\Gamma}$ and $t_3 = \frac{1000}{\Gamma}$. The left three figures demonstrate the state evolution when there is no spontaneous emission while the right three figures represent the case when spontaneous emission exist. The third figure corresponds to the case when the two packets have overlapped. These Schematic illustrates that the quantum coherence of the system is lost in presente of spontaneous emission and thus the spreading wave packets are suppressed.

We take the initial state as a narrow Gaussian wave packet $G_+(x-a)$ for the relative position representation of the two-atom system. Obviously the narrowness of wave packet implies that the two atom system is initially in localization. If there would not exist the spontaneous emission, the relative position Gaussian wave packet would spread into the full space infinitely and the localization of wave packet is lost during the evolution of the system. Its breadth increases to infinity while its height decreases from its initial value to zero. In present of the spontaneous emission, we calculate the time evolution of

$$G_+(x-a,t \ = \ 0) = \int_{-\infty}^{\infty} C_p e^{i\frac{p}{\hbar}x} dp, \tag{23}$$

$$C_p \ = \ \frac{2d^2}{\pi\sqrt{2\pi}} e^{-\frac{d^2}{\hbar}p^2}. \tag{24}$$

According to Eq. (16) and Eq. (18), the reduced density matrix at time $t >> \frac{1}{\Gamma}$ is

$$\rho\left(x, x', t\right) = N' G_+(x-a,t) G_+^*(x'-a,t) F(x,x') \tag{25}$$

where

$$G_+(x-a,t) \ = \ \frac{1}{(2\pi)^{1/4}\sqrt{d + it\hbar/\left(2\mu d\right)}} \times \exp\left(-\frac{\left(1-\frac{i\hbar}{2\mu d^2}t\right)x^2}{4d^2+\frac{t^2\hbar^2}{\mu^2 d^2}}\right). \tag{26}$$

According to Eq. (25), we can conclude that the breadth of the spreading wave packet is suppressed as can also be seen in FIG. 4. We give the schematics of the evolution of the wave packet at three different times and in two cases: one is when there is no spontaneous emission and the other is when there exists spontaneous emission. From the Eq. (25) and FIG. 4, we can see the wave packet spreading is suppressed and the ultimate breadth is related to the wave length of the atomic radiation. Longer is the atomic radiative wave length, wider is the breadth of the ultimate wave packet.

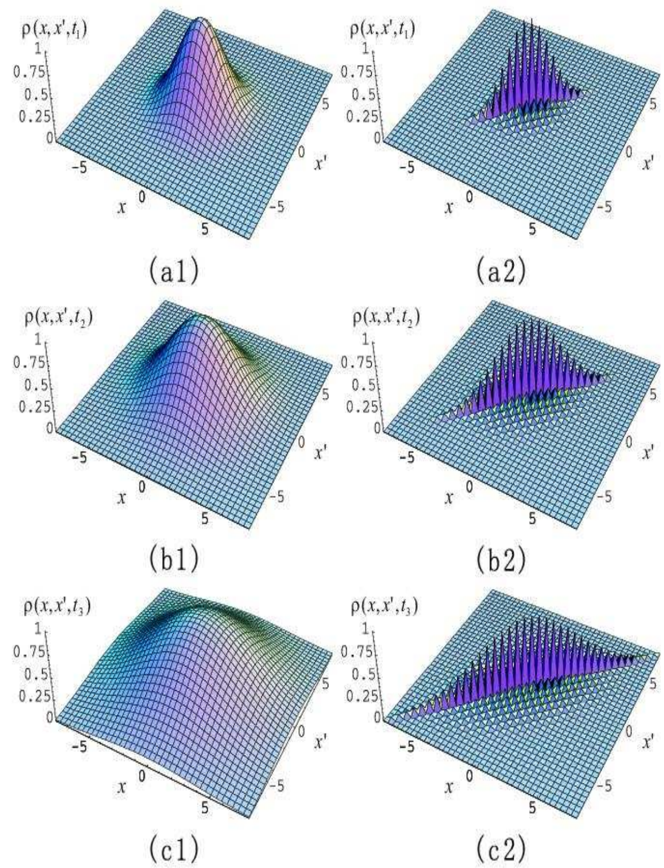

FIG. 4: Schematics of the suppression of a wave packet spreading. The left three figures correspond to the evolution of the single wave packet at three different instance when there is no spontaneous emission and the right three figures represent the cases when the spontaneous emissions exist. Here $t_1 = \frac{2}{\Gamma}, t_2 = \frac{3}{\Gamma}$ and $t_3 = \frac{5}{\Gamma}$. They demonstrate how the atomic spontaneous emission suppresses the spreading of wave packet.

V. CONCLUSIONS

In conclusion, we have investigated the atomic spontaneous emission induced quantum decoherence phenomenon in association with the localization of the relative position of a two atom system. The spontaneous emission or the interaction with vacuum electromagnetic field may be a fundamental process destroying the quantum effects in macroscopic objects. By analyzing two simple examples, we demonstrate how the spontaneous emission suppresses the spreading wave packet and thus localizes a macroscopic object.

This work is supported by NSFC with Numbers 90203018, 10474104 and 10447133. It is also funded by the National Fundamental Research Program of China with No. 2001CB309310. Z.L thanks for the helpful discussions with Zhang Peng.

APPENDIX: SOLUTIONS OF THE SCHRÖDINGER EQUATION

In this appendix we give the calculations on solving the system of equations consisting of Eq. (5) to Eq. (7). We take the Laplace transformation to Eq. (5) to Eq. (7) and obtain

$$\begin{aligned}&\left(L + i\left(\omega_A - \omega_0\right)\right) A\left(p, L\right) \\ &= \ -2i\sum_{\mathbf{k}} B_{\mathbf{k},p}\left(L\right) g(\mathbf{k}) + C_p,\end{aligned} \tag{A.1}$$

$$\begin{aligned}&\left(L + i\left(\omega_B\left(\mathbf{k}\right) - \omega_0\right)\right) B_{\mathbf{k},p}\left(L\right) \\ &= \ -iA\left(p, L\right) g(\mathbf{k}) - i\sum_{\mathbf{k}} D_{\mathbf{k},\mathbf{k}',p}\left(L\right) g(\mathbf{k}'),\end{aligned} \tag{A.2}$$

$$\begin{aligned}&\left[L + i\left(\omega_D\left(\mathbf{k},\mathbf{k}'\right) - \omega_0\right)\right] D_{\mathbf{k},\mathbf{k}',p}\left(L\right) \\ &= \ -iB_{\mathbf{k},p}\left(L\right) g(\mathbf{k}') - iB_{\mathbf{k},p}\left(L\right) g(\mathbf{k}).\end{aligned} \tag{A.3}$$

From Eq. (A.3), we have

$$D_{\mathbf{k},\mathbf{k}',p}\left(L\right)=\frac{-iB_{\mathbf{k},p}\left(L\right)g(\mathbf{k}')-iB_{\mathbf{k},p}\left(L\right)g(\mathbf{k})}{L+i\left(\omega_{D}\left(\mathbf{k},\mathbf{k}'\right)-\omega_{0}\right)}. \quad (A.4)$$

Submitting Eq. (A.4) into Eq. (A.2), we can rewrite Eq. (A.2) as

$$\begin{aligned} & \left(L+i\left(\omega_{B}\left(\mathbf{k}\right)-\omega_{0}\right)\right)B_{\mathbf{k},p}\left(L\right) \\ = & -iA\left(p,L\right)g(\mathbf{k}) \\ & -B_{\mathbf{k},p}\left(L\right)\sum_{\mathbf{k}'}\frac{g^{2}(\mathbf{k}')}{L+i\left(\omega_{D}\left(\mathbf{k},\mathbf{k}'\right)-\omega_{0}\right)} \\ & -\sum_{\mathbf{k}'}\frac{B_{\mathbf{k}',p}\left(L\right)g(\mathbf{k}')g(\mathbf{k})}{L+i\left(\omega_{D}\left(\mathbf{k},\mathbf{k}'\right)-\omega_{0}\right)}. \quad (A.5)\end{aligned}$$

Since we have ignored the higher order multi-photon processes, we can omit the last term in the right of Eq. (A.5). Then we obtain

$$B_{\mathbf{k},p}\left(L\right)=\frac{-iA\left(p,L\right)g(\mathbf{k})}{L+i\left(\omega_{B}\left(\mathbf{k}\right)-\omega_{0}\right)+\sum_{\mathbf{k}'}\frac{g^{2}(\mathbf{k}')}{L+i(\omega_{D}(\mathbf{k},\mathbf{k}')-\omega_{0})}}. \quad (A.6)$$

According to the Weisskopf-Wigner approximation[20], we can obtain

$$\frac{\Gamma}{2}+i\triangle\omega=\sum_{\mathbf{k}'}\frac{g^{2}(\mathbf{k}')}{L+i\left(\omega_{D}\left(\mathbf{k},\mathbf{k}'\right)-\omega_{0}\right)}$$

where $\Gamma=\omega_{0}^{2}\left|\mathbf{d}\right|^{2}/\left(4\varepsilon_{0}\hbar c^{2}\right)$ is the decay rate of an atom from state $|e\rangle$ to state $|g\rangle$ and $\triangle\omega$ is the Lamb shift which is omitted in our following calculations since it can be merged into the transition frequency ω_{0}. Eq. (A.6) can be simplified as

$$B_{\mathbf{k},p}\left(L\right)=\frac{-iA\left(p,L\right)g(\mathbf{k})}{L+i\left(\omega_{B}\left(\mathbf{k}\right)-\omega_{0}\right)+\frac{\Gamma}{2}}. \quad (A.7)$$

Submitting Eq. (A.7) into Eq. (A.1), we have

$$A\left(p,L\right)=\frac{C_{p}}{L+i\left(\omega_{A}-\omega_{0}\right)+2\sum_{\mathbf{k}}\frac{g^{2}(\mathbf{k})}{L+i(\omega_{B}(\mathbf{k})-\omega_{0})+\frac{\Gamma}{2}}}. \quad (A.8)$$

In the Weisskopf-Wigner approximation, we can also obtain

$$\frac{\Gamma}{2}+i\triangle\omega=\sum_{\mathbf{k}}\frac{g^{2}(\mathbf{k})}{L+i\left(\omega_{B}\left(\mathbf{k}\right)-\omega_{0}\right)+\frac{\Gamma}{2}}.$$

So Eq. (A.8) can be written as

$$A\left(p,L\right)=\frac{C_{p}}{L+i\left(\omega_{A}-\omega_{0}\right)+\Gamma}. \quad (A.9)$$

Combining Eq. (A.4), Eq. (A.7) and Eq. (A.9), we can obtain

$$\begin{aligned} B_{\mathbf{k},p}\left(L\right) = & \frac{-iC_{p}g(\mathbf{k})}{L+i\left(\omega_{B}\left(\mathbf{k}\right)-\omega_{0}\right)+\frac{\Gamma}{2}} \\ & \times\frac{1}{L+i\left(\omega_{A}-\omega_{0}\right)+\Gamma}, \quad (A.10)\end{aligned}$$

$$\begin{aligned} D_{\mathbf{k},\mathbf{k}',p}\left(L\right) = & \frac{-g(\mathbf{k}')g(\mathbf{k})C_{p}}{L+i\left(\omega_{D}\left(\mathbf{k},\mathbf{k}'\right)-\omega_{0}\right)} \\ & \times\frac{1}{L+i\left(\omega_{A}-\omega_{0}\right)+\Gamma} \\ & \times\left(\frac{1}{\left(L+i\left(\omega_{B}\left(\mathbf{k}\right)-\omega_{0}\right)+\frac{\Gamma}{2}\right)}\right. \\ & \left.+\frac{1}{\left(L+i\left(\omega_{B}\left(\mathbf{k}'\right)-\omega_{0}\right)+\frac{\Gamma}{2}\right)}\right). \quad (A.11)\end{aligned}$$

Taking the inverse Laplace transformation to the above three equations, we obtain the solutions to equations from Eq. (5) to Eq. (7):

$$A\left(p,t\right)=C_{p}e^{-\Gamma t}e^{-i(\omega_{A}-\omega_{0})t}, \quad (A.12)$$

$$B_{\mathbf{k},p}\left(t\right)=-ig(\mathbf{k})C_{p}\frac{e^{(i\omega_{0}-\frac{\Gamma}{2}t)}\left(e^{-i\omega_{B}t}-e^{-i\omega_{A}t}\right)}{i\left(\omega_{A}-\omega_{B}\left(\mathbf{k}\right)\right)+\frac{\Gamma}{2}}, \quad (A.13)$$

$$\begin{aligned} & D_{\mathbf{k},\mathbf{k}',p}\left(t\right) \\ = & \frac{g(\mathbf{k}')g(\mathbf{k})C_{p}}{i\left(\omega_{A}-\omega_{D}\left(\mathbf{k},\mathbf{k}'\right)\right)+\Gamma} \\ & \times[\frac{e^{-i\left(\omega_{D}\left(\mathbf{k},\mathbf{k}'\right)-\omega_{0}\right)t}}{i\left(\omega_{B}\left(\mathbf{k}\right)-\omega_{D}\left(\mathbf{k},\mathbf{k}'\right)\right)+\frac{\Gamma}{2}}-\frac{e^{-i(\omega_{A}-\omega_{0})t-\Gamma t}}{i\left(\omega_{B}\left(\mathbf{k}\right)-\omega_{A}\right)-\frac{\Gamma}{2}} \\ & +\frac{e^{-i(\omega_{B}(\mathbf{k})-\omega_{0})t-\frac{\Gamma}{2}t}}{\left[i\left(\omega_{D}\left(\mathbf{k},\mathbf{k}'\right)-\omega_{B}\left(\mathbf{k}\right)\right)-\frac{\Gamma}{2}\right]\left[i\left(\omega_{A}-\omega_{B}\left(\mathbf{k}\right)\right)+\frac{\Gamma}{2}\right]} \\ & +\frac{e^{-i\left(\omega_{D}\left(\mathbf{k},\mathbf{k}'\right)-\omega_{0}\right)t}}{i\left(\omega_{B}\left(\mathbf{k}'\right)-\omega_{D}\left(\mathbf{k},\mathbf{k}'\right)\right)+\frac{\Gamma}{2}}-\frac{e^{-i(\omega_{A}-\omega_{0})t-\Gamma t}}{i\left(\omega_{B}\left(\mathbf{k}'\right)-\omega_{A}\right)-\frac{\Gamma}{2}} \\ & +\frac{e^{-i(\omega_{B}(\mathbf{k})-\omega_{0})t-\frac{\Gamma}{2}t}}{\left[i\left(\omega_{D}\left(\mathbf{k},\mathbf{k}'\right)-\omega_{B}\left(\mathbf{k}'\right)\right)-\frac{\Gamma}{2}\right]\left[i\left(\omega_{A}-\omega_{B}\left(\mathbf{k}'\right)\right)+\frac{\Gamma}{2}\right]}]. \quad (A.14)\end{aligned}$$

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