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"quasi-particles" in bosonization theory of interacting fermion liquids at arbitrary dimensions

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Abstract

Within bosonization theory we introduce in this paper a new definition of "quasi-particles" for interacting fermions at arbitrary space dimenions. In dimensions higher than one we show that the constructed quasi-particles are consistent with quasi-particle descriptions in Landau Fermi liquid theory whereas in one-dimension the "quasi-particles" are non-perturbative objects (spinons and holons) obeying fractional statistics. The more general situation of Fermi liquids with singular Landau interaction is discussed.

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The concept of "quasi-particles", as excitations adiabatically connected to excitations in corresponding non-interacting Fermi gas has formed the basis for our understanding of Fermi liquids at dimensions higher than one [1]. Based on Bethe-ansatz solutions [2] of exactly solvable models, it is now understood that similar to fermions in higher dimensions, the low energy behaviours of interacting fermions in one dimension can also be described as gases of "free", or quasi-particles, except that these particles are not adiabatically connected to free fermions as in Landau fermi-liquid theory, but are non-perturbative objects called "spinon" and "holon" characterizing the spin- and charge- degrees of freedom of the system separately [3]. The spinons and holons are neither fermions or bosons, but are objects obeying fractional(exclusion) statistics [4].

The purpose of the present paper is to show that the two rather different pictures of "quasi-particles" in Landau Fermi liquid and in Bethe-ansatz solutions in one dimension, can be understood in bosonization theory by introducing a new definition of "quasi-particle" operators. Our approach provides a simple way of unifying the two "quasi-particle" pictures and provides a more general framework of understanding quasi-particles in interacting fermion systems. We shall also discuss the situation of generalized Fermi liquids with singular Landau interaction within our framework.

To warm up we first review briefly bosonization theory in one [5] and higher dimensions [6–8]. The main idea of bosonization theory is that the low energy excitation spectrum of Fermi and Luttinger liquids can both be described by effective harmonic theories describing elastic deformation of Fermi surfaces [7,8]. The deformation at position $\vec{k}_F$ on the Fermi surface is described by the coarse-grained Wigner operator [8]

$$\rho_{\vec{k}_F\sigma}(\vec{q}) = \frac{1}{V}\sum_{-\Lambda<k<\Lambda} c^{+}_{\vec{k}_F+k\hat{k}_F+\vec{q}/2\sigma}c_{\vec{k}_F+k\hat{k}_F-\vec{q}/2\sigma},$$

where V =volume, $\hat{k}_F$ is a unit vector along the direction of $\vec{k}_F$ and $\Lambda << k_F$ is a high momentum cutoff. $c(c^+)$ are the usual fermion annihilation(creation) operators. We shall set $\hbar = 1$ in this paper. Notice that in one dimension $\vec{k}_F \to \pm k_F$ and $\rho_{\vec{k}_F\sigma}(\vec{q}) \to \rho_{L(R)\sigma}(q)$, i.e. left and right Fermi points. $\rho_{\vec{k}_F\sigma}(\vec{q})$'s obey approximate commutation relations [7,8]

$$[\rho_{\vec{k}_F\sigma}(\vec{q}), \rho_{\vec{k}'_F\sigma'}(-\vec{q}')] = -\delta^D(\vec{q}-\vec{q}')\delta^{D-1}(\vec{k}_F-\vec{k}'_F)\delta_{\sigma\sigma'}\frac{\vec{q}.\vec{k}_F}{m}N_\Lambda(0), \tag{1}$$

where D=dimension and $N_\Lambda(0) = N(0)/S_d$, where $N(0)$ is the density of states of spin-σ fermions on Fermi surface, and $S_d = \int d\Omega$ = solid angle on the Fermi surface. For computation purpose it is convenient to introduce canonical boson operators $b^+(b)$ defined by $\rho_{\vec{k}_F\sigma}(\vec{q}) = \left(\frac{|\vec{k}_F.\vec{q}|}{m}N_\Lambda(0)\right)^{\frac{1}{2}}(\theta(\vec{k}_F.\vec{q})b^+_{\vec{k}_F\sigma}(\vec{q}) + \theta(-\vec{k}_F.\vec{q})b_{\vec{k}_F\sigma}(-\vec{q}))$. The $b(b^+)$ operators satisfy canonical boson commutation relations [8].

The low energy physics of the systems is described by effective Hamiltonian $H = H_0+H_1$, where H_0 is the kinetic energy and H_1 is the interaction. In terms of $b(b^+)$ operators, H_0 takes the form [8]

$$H_0 = \frac{1}{V}\sum_{\vec{k}_F,\vec{q},\sigma}\frac{|\vec{k}_F.\vec{q}|}{m}b^+_{\vec{k}_F\sigma}(\vec{q})b_{\vec{k}_F\sigma}(\vec{q}), \tag{2}$$

where we have linearized the fermion dispersion around the Fermi surface to obtain (2). H_1 can be written as

$$H_1 = \frac{1}{2V} \sum_{\vec{k}_F,\sigma,\vec{k}'_F,\sigma',|\vec{q}|<\Lambda} f_{\vec{k}_F\sigma\vec{k}'_F\sigma'}(\vec{q})\rho_{\vec{k}_F\sigma}(\vec{q})\rho_{\vec{k}'_F\sigma'}(-\vec{q}), \qquad (3)$$

where $f_{\vec{k}_F\sigma\vec{k}'_F\sigma'}(\vec{q})$'s are effective parameters characterizing the residual (marginal) fermion-fermion interactions in the low energy and small wavevector limit [8,9]. With Eqs. (1) to (3), the Heisenberg equation of motion for $\rho_{\vec{k}_F\sigma}(\vec{q})$ is [8]

$$\left(i\frac{\partial}{\partial t} + \frac{\vec{k}_F.\vec{q}}{m}\right)\rho_{\vec{k}_F\sigma}(\vec{q}) = -\frac{\vec{k}_F.\vec{q}}{m}N_\Lambda(0)\sum_{\vec{k}'_F\sigma'} f_{\vec{k}_F\sigma\vec{k}'_F\sigma'}(\vec{q})\rho_{\vec{k}'_F\sigma'}(\vec{q}), \qquad (4)$$

and is the same as Landau transport equation for quasi-particles in Fermi liquid theory in the $\vec{q}\to 0$ limit where $f_{\vec{k}_F\sigma\vec{k}'_F\sigma'}(\vec{q}\to 0)$ can be identified as Landau parameters [8,9]. The same form of equation of motion for $\rho_{L(R)\sigma}(q)$ is also obtained at one dimension.

Although the equations of motion have the same form at one and higher dimensions, the resulting eigen-state spectrums are very different. At higher dimensions there are two kinds of eigen-state solutions to Eq. (4), the particle-hole continuum and collective modes [10]. The particle-hole continuum are eigenstates adiabatically connected to the particle-hole spectrum of non-interacting fermions, whereas collective modes are non-perturbative eigenstates that disappear in the absence of interaction. The validity of Fermi liquid theory is warranted by the existence of adiabatic particle-hole spectrum [10]. In one dimension *only* non-perturbative collective density and spin wave excitations exist [5]. The collective excitations are not adiabatically connected to excitation spectrum of non-interacting fermions showing that the systems are not Landau Fermi liquids.

Next we consider the construction of quasi-particle operators. Our goal is to search for a bosonization representation of the quasi-particle operator that is valid in the low energy, long wave length limit. To gain insight we first consider non-interacting fermions. In the spirit of Landau Fermi liquid theory we consider a (spin-σ) quasi-particle wavepacket with momentum $\vec{k}_F$ at position $\vec{r}$. Notice that quantum mechanics requires that there exist uncertainties $\delta r\sim\Lambda^{-1}$ and $\delta k\sim\Lambda$ in the position and momentum of the particle. For non-interacting fermions the quasi-particle wavepacket can be identified as a free spin-σ fermion wavepacket described by

$$\psi_{\vec{k}_F\sigma}(\vec{r}) \sim \frac{1}{2\pi}\int_{-\Lambda}^{\Lambda} k^{D-1}dk e^{-ik\hat{k}_F.\vec{r}}c_{\vec{k}_F+k\hat{k}_F\sigma},$$

where we have assumed that there is no uncertainty in the *direction* $\hat{k}_F$. In the limit $\Lambda << k_F$ it is easy to see that the wave-packet operator satisfies the equation of motion

$$i\frac{\partial}{\partial t}\psi_{\vec{k}_F\sigma}(\vec{r}) = [\psi_{\vec{k}_F\sigma}(\vec{r}), H_o-\mu N] = \frac{i\vec{k}_F.\nabla}{m}\psi_{\vec{k}_F\sigma}(\vec{r}) + O(\Lambda/k_F). \qquad (5)$$

where N = number of particles and μ is the chemical potential.

We shall now show that we can use Eq. (5), which specifies the dynamics of a free-particle wavepacket, as the *definition* of quasi-particle operators for free fermions in bosonization theory. To see this we write $\psi_{\vec{k}_F\sigma}(\vec{r})=\hat{f}_{\hat{k}_F}e^{J_{\vec{k}_F\sigma}(\vec{r})}$, where $J_{\vec{k}_F\sigma}(\vec{r})\sim\sum_{\vec{q}}\alpha_{\vec{k}_F\sigma}(\vec{q},\vec{r})\rho_{\vec{k}_F\sigma}(\vec{q})$ is a linear combination of $\rho_{\vec{k}_F\sigma}(\vec{q})$ operators, i.e. $\psi_{\vec{k}_F\sigma}(\vec{r})$ represents a coherent state of bosonic waves in bosonization theory. $\hat{f}_{\hat{k}_F}$ is an operator (Klein factor) introduced to ensure the anti-communication relation between $\psi_{\vec{k}_F\sigma}(\vec{r})$ fields in different directions $\vec{k}_F$'s [6]. The linear coefficients α's characterizing $J_{\vec{k}_F\sigma}(\vec{r})$ are determined by requiring that $\psi_{\vec{k}_F\sigma}(\vec{r})$ satisfies the equation of motion (5) with the bosonized H_0 (Eq. (2)). After some straightforward algebra, we obtain

$$J_{\vec{k}_F\sigma}(\vec{r})\sim-(e^*)\frac{1}{V}\sum_{\vec{q}}\frac{me^{-i\vec{q}.\vec{r}}}{N_\Lambda(0)\vec{k}_F.\vec{q}}\rho_{\vec{k}_F\sigma}(\vec{q}), \tag{6}$$

where e^* is an arbitrary number that is not fixed by the equation of motion. The meaning of e^* can be seen by examining the communication relation between ψ operator and the total charge operator $\rho(\vec{q})=\sum_{\vec{k}_F,\sigma}\rho_{\vec{k}_F\sigma}(\vec{q})$. We obtain

$$[\psi_{\vec{k}_F\sigma}(\vec{r}),\rho(-\vec{q})]=(e^*)e^{-i\vec{q}.\vec{r}}\psi_{\vec{k}_F\sigma}(\vec{r}),$$

showing that e^* represents the charge carried by the quasi-particle and is equal to one for free fermions. The reason why we cannot determine e^* by the equation of motion is that the solutions of Eq. (5) represent coherent superposition of boson waves travelling with constant velocity $\vec{v}_F=\vec{k}_F/m$, and superposition of different solutions is again a solution to the equation. We note that in usual bosonization theory $\psi_\sigma(\vec{r})=\sum_{\vec{k}_F}\psi_{\vec{k}_F\sigma}(\vec{r})$ is identified as the bosonization representation of fermion operator and is usually derived by requiring that $\psi_\sigma(\vec{r})$ satisfies the correct commutation relation with the density operator [6,8]. We define $\psi_\sigma(\vec{r})$ as the quasi-particle operator through the equation of motion in our approach. For free fermions the two definitions give identical result.

In the presence of interaction we define quasi-particle operators as operators representing coherent states of bosonic waves satisfying equations of motion of form

$$i\frac{\partial}{\partial t}\psi^{(Q)}_{\vec{k}_F\gamma}(\vec{r})=[\psi^{(Q)}_{\vec{k}_F\gamma}(\vec{r}),H-\mu N]\sim i\vec{v}_\gamma.\nabla\psi^{(Q)}_{\vec{k}_F\gamma}(\vec{r})+O(\Lambda/k_F). \tag{7}$$

where H is the full bosonized Hamiltonian and γ is a branch index. $\vec{v}_\gamma\sim\vec{k}_F/m_\gamma$ is the velocity of branch γ quasi-particles at position $\vec{k}_F$ on the Fermi surface. The nature of the different branches and corresponding $\vec{v}_\gamma$ are determined self-consistently by requiring that $\psi^{(Q)}_{\vec{k}_F\gamma}(\vec{r})$ satisfies the equation of motion (7). Writing $\psi^{(Q)}_{\vec{k}_F\gamma}(\vec{r})=\hat{f}_{\hat{k}_F}e^{J^{(Q)}_{\vec{k}_F}(\gamma,\vec{r})}$ where $J_{\vec{k}_F\sigma}(\vec{r})\sim\sum_{\vec{q}}\alpha_{\vec{k}_F\sigma}(\vec{q},\vec{r})\rho_{\vec{k}_F\sigma}(\vec{q})$ as before, we obtain from (7) a linear eigenvalue equation for the coefficents determining α. The nature of the quasi-particle branches and $\vec{v}_\gamma$ are determined by the eigenvectors and eigenvalues of the eigenvalue equation.

To illustrate we consider interacting fermions at dimensions $D>1$. A easy way to obtain the quasi-particle operators is to notice that at dimenions higher than one, a continuous spectrum of particle-hole pair solution $\bar{\rho}_{\vec{p}_Fs}(\vec{q})=\rho_{\vec{p}_Fs}(\vec{q})+\sum_{\vec{k}_F\sigma}\xi_{\vec{p}_Fs\vec{k}_F\sigma}(\vec{q})\rho_{\vec{k}_F\sigma}(\vec{q})$ to Eq. (4) exists, with eigen-energy $\epsilon=\vec{p}_F.\vec{q}/m$ and

$$\xi_{\vec{p}_Fs\vec{k}_F\sigma}(\vec{q})=\frac{1}{V}\left(\frac{\frac{\vec{p}_F.\vec{q}}{m}}{\frac{\vec{k}_F.\vec{q}}{m}-\frac{\vec{p}_F.\vec{q}}{m}}\right)N_\Lambda(0)A_{\vec{k}_F\sigma\vec{p}_Fs}(\vec{q}),$$

where $A_{\vec{k}_F\sigma\vec{p}_Fs}(\vec{q})$ is the quasi-particle scattering matrix in Landau Fermi Liquid theory [8,10]. A quasi-particle operator with momentum $\vec{p}_F$, $\psi^{(Q)}_{\vec{p}_Fs}(\vec{r}) = \hat{f}_{\hat{p}_F}e^{J^{(Q)}_{\vec{p}_F}(s,\vec{r})}$ satisfying equation of motion (7) can be obtained by choosing

$$J^{(Q)}_{\vec{p}_F}(s,\vec{r}) = -\frac{1}{V}\sum_{\vec{q}}\frac{me^{-i\vec{q}.\vec{r}}}{N_\Lambda(0)\vec{p}_F.\vec{q}}\bar{\rho}_{\vec{p}_F\sigma}(\vec{q}). \tag{8}$$

The branch index is given by $\gamma = s =$ spin index and $m_\gamma = m$ as for non-interacting fermions. The quasi-particle operators constructed this way represent bare-fermions dressed by particle-hole pair excitations (described by the coefficients ξ) and are adiabatically connected to the non-interacting fermions. Notice that we have set $e^* = 1$ to ensure adiabaticity. We expect that the quasi-particle operator constructed this way corresponds to the eigen-quasi-particle state in Fermi liquid theory [10]. To confirm we compute the charge carried by the quasi-particle operator we constructed. We obtain

$$[\psi^Q_{\vec{p}_Fs}(\vec{r}),\rho(-\vec{q})] = \left(1-\sum_{\vec{k}_F\sigma}\xi_{\vec{k}_F\sigma\vec{p}_Fs}(\vec{q})\right)e^{-i\vec{q}.\vec{r}}\psi^Q_{\vec{p}_Fs}(\vec{r}), \tag{9}$$

suggesting that the charge $<\rho(\vec{q})>$ carried by the quasi-particle excitation is $<\rho(\vec{q})>= 1-\sum_{\vec{k}_F\sigma}\xi_{\vec{k}_F\sigma\vec{p}_Fs}(\vec{q})$, in exact agreement with Fermi liquid theory where the ξ factors describe screening effect [10]. The same agreement with Fermi liquid theory is also obtained for the current and spin operators.

Next we consider fermions in one dimension. We consider bosonized Hamiltonian H with $\vec{k}_F \to \pm k_F = R, L$ and $f_{\vec{k}_F\sigma\vec{k}'_F\sigma'}(\vec{q}) = f_s(q) + f_a(q)\vec{\sigma}.\vec{\sigma}'$, i.e., we neglect current-current interactions. The resulting equation of motion (4) for bosonic excitations has two branches of solution with dispersions $\epsilon_{s(a)}(q) = (\sqrt{v_F^2 + 2f_{s(a)}(q)v_F/\pi})q = v_{s(a)}q$, where $v_F = k_F/m$ and s, a represents density and spin wave fluctuations, respectively [5]. Notice that $v_s \neq v_a$ as long as $f_s(q) \neq f_a(q)$, reflecting general spin-charge separation of interacting fermions at one dimension.

To construct quasi-particle operators we proceed as before and define quasi-particle operators using Eq. (7), with $\psi^{(Q)}_{\vec{p}_F\gamma}(\vec{r}) \to \psi^{(Q)}_{L(R)\gamma}(x)$, and $(\vec{k}_F.\nabla)/m \to \pm v_F\partial/\partial x$. Writing $\psi^{(Q)}_{L(R)\gamma}(x) = \hat{f}_{L(R)}e^{J^{(Q)}_{L(R)}(\gamma,x)}$ where $J^{(Q)}$ is linear in the $\rho_{L(R)\sigma}(\vec{q})$ operators, we obtain from solving Eq. (7),

$$J^{(Q)}_{R(L)}(\gamma,x) = \frac{e^*_\gamma}{2V}\sum_q\frac{e^{-iqx}}{q}\left((\frac{+(-)1}{\eta_\gamma})(\rho_{L\gamma}(q)+\rho_{R\gamma}(q)) + (\rho_{R\gamma}(q)-\rho_{L\gamma}(q))\right), \tag{10}$$

where $\gamma = s, a$ and $\rho_{L(R)[s(a)]}(q) = \sum_\sigma[1(\sigma)]\,\rho_{L(R)\sigma}(q)$ is the coarse-grained density(spin) Wigner operator. Notice that $\psi^{(Q)}_{L(R)s(a)}(x)$'s constructed this way represent "quasi-particles" corresponding to coherent states formed by density (s) and spin (a) bosonic waves separately (spin-charge separation) and are not adiabatically connected to the original fermions. The self-determined quasi-particle velocities $v_{s(a)}$ are equal to the corresponding boson density(spin) wave velocities and $\eta_{s(a)} = v_F/v_{s(a)}$. The magnitude of charge(spin) carried by the quasi-particle $e^*_{s(a)}$ can be determined if we identify the quasi-particle excitations we constructed as holons ($\gamma = s$) and spinons ($\gamma = a$) in Bethe Ansatz solutions. The charge(spin) carried by the holon(spinon) equals 1(1/2) and $e^*_s = e^*_a = 1$ correspondingly.

The quasi-particles we constructed carry fractional (excluson) statistics. The statistics of the quasi-particles can be determined directly by examining the commutation rules between quasi-particles. We find that the spin and charge quasi-particles satisfy non-trivial commutation relation among themselves. We obtain

$$\psi^{(Q)}_{L(R)\gamma}(x)\psi^{(Q)}_{L(R)\gamma}(x')=e^{(i\pi/\eta_\gamma)sgn(x-x')}\psi^{(Q)}_{L(R)\gamma}(x')\psi^{(Q)}_{L(R)\gamma}(x),$$

indicating that the charge and spin quasi-particles are exclusons with statistical parameter $1/\eta_s$ and $1/\eta_a$, respectively, in agreement with previous results [4,11].

A natural question that arise is what is the most general criteria that quasi-particle operators can be constructed in a fermionic system at dimensions > 1, given that the low energy dynamics of the system is described by equation of motion of form (4)? This question is important because it has been observed in recent years that non-Fermi liquid behavior appears in certain systems at two dimension, for example, fermions in half-filled Landau level or in strongly fluctuating gauge fields [12,13]. The low energy physics of these systems are believed to be described by transport equation similar to Eq. (4), except that the effective Landau interactions and corresponding effective masses are frequency dependent and are singular in the $\omega, q \rightarrow 0$ limit [12,13].

For frequency-independent Landau interactions it can be shown that quasi-particle operators defined by Eq. (7) can be constructed as long as there exists a continuum eigenspectrum of the equation of motion (4) with eigenenergy $\epsilon_{\vec{p}_F}(\vec{q})=\vec{v}_\gamma.\vec{q}$ at small q, at every point $\vec{p}_F$ on the Fermi surface. The solutions may or may not be adiabatically connected to the non-interacting fermions. The physical reason for this requirement is clear: if the low energy excitations of the system is characterized by quasi-particle occupation numbers with quasi-particle dispersion that is continuous across the Fermi surface, then particle-hole spectrum of the form $\epsilon_{\vec{p}_F}(\vec{q})=\vec{v}_\gamma.\vec{q}$ must exist for all $\vec{p}_F$ at small q. What is surprising is that this is also a *sufficient* condition for existence of quasi-particles in bosonization theory with our definition.

For transport equations with frequency-dependent Landau parameters the above conclusion does not hold because the eigenstates of the transport equations are not orthogonal to each other, although we expect that the conclusion may be qualitatively similar. This seems to be the case for fermions in half-filled Landau level or in fluctuations transverse gauge fields [12]. Notice that in the case when the particle-hole continuum solution is non-perturbative, we expect that some Ward identities derived for Landau Fermi liquids may be violated, and non-Fermi liquid behaviours may appear in the physical response functions.

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