

**Phenomenology of single spin asymmetries in  $p^\uparrow p \rightarrow \pi X$** M. Anselmino<sup>1</sup>, M. Boglione<sup>2</sup>, F. Murgia<sup>3</sup><sup>1</sup> Dipartimento di Fisica Teorica, Università di Torino and  
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C.P. 170, I-09042 Monserrato (CA), Italy**Abstract:**

A phenomenological description of single transverse spin effects in hadron-hadron inclusive processes is proposed, assuming a generalized factorization scheme and pQCD hard interactions. The transverse momentum,  $\mathbf{k}_\perp$ , of the quarks inside the hadrons and of the hadrons relatively to the fragmenting quark, is taken into account in distribution and fragmentation functions, and leads to possible non zero single spin asymmetries. The role of  $\mathbf{k}_\perp$  and spin dependent quark fragmentations – the so-called Collins effect – is investigated in details in  $p^\uparrow p \rightarrow \pi X$  processes: it is shown how the experimental data could be described, obtaining an explicit expression for the spin asymmetry of a polarized fragmenting quark, on which some comments are made. Predictions for other processes, possible further applications and experimental tests are discussed.

## 1. Introduction

Understanding the kinematics and the dynamics of the quarks inside the hadrons is a challenging and interesting problem. In simple quark-parton models the quarks inside a fast hadron are assumed to be collinear, *i.e.* their momentum is parallel to the momentum of the hadron. This assumption works well for unpolarized processes, like  $A+B \rightarrow C+X$  which are computed, at large energy and momentum transfer  $p_T$ , by the usual convolution of distribution and fragmentation functions with a hard perturbative elementary interaction, according to QCD factorization theorem [1]. However, in polarized reactions, the contribution of transverse intrinsic momenta often turns out to be crucial in understanding the experimental results.

In Ref. [2], we proved how the large single spin asymmetries

$$A_N(x_F, p_T) = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{2 d\sigma^{unp}}, \quad (1)$$

measured in  $p^\uparrow p \rightarrow \pi X$  by the E704 Group at Fermilab [3], can be accounted for by introducing  $\mathbf{k}_\perp$  and spin dependences in the distribution functions of quarks inside the initial polarized proton.  $d\sigma^{\uparrow,\downarrow}$  stands for the differential cross section  $E_\pi d^3\sigma^{p^{\uparrow,\downarrow}p \rightarrow \pi X} / d^3\mathbf{p}_\pi$ ;  $x_F = 2p_L / \sqrt{s}$ , where  $p_L$  is the pion longitudinal momentum in the  $pp$  c.m. frame, and  $\sqrt{s}$  is the total c.m. energy. The proton spin is “up” or “down” with respect to the scattering plane:  $\uparrow$  ( $\downarrow$ ) is the direction parallel (antiparallel) to  $\mathbf{p}_{p^\uparrow} \times \mathbf{p}_\pi$ . Other sizeable single spin asymmetries have been explained or predicted in Ref. [4].

The original suggestion that the intrinsic  $\mathbf{k}_\perp$  of the quarks in the distribution functions might give origin to single spin asymmetries was first made by Sivers [5]; such an effect is not forbidden by QCD time reversal invariance [6] provided one takes into account soft initial state interactions among the colliding hadrons [2]. A similar suggestion for possible origin of single spin asymmetries was later made by Collins [6], concerning transverse momentum effects in the fragmentation of a polarized quark. A first simple application of this idea was given in [7].

Qiu and Sterman [8, 9] have proven that a generalized factorization theorem holds in QCD with twist-3 distribution and/or fragmentation functions, which take into account initial or final state interactions. Recently [10] they have computed - using a simple model for a new twist-3 correlation function - single spin asymmetries for  $p^\uparrow p \rightarrow \pi X$  and  $\bar{p}^\uparrow p \rightarrow \pi X$  processes. Their approach establishes a sound theoretical ground for simple phenomenological applications: a new quantity is introduced for valence  $u$  and  $d$  quarks which represents a correlation between these quarks inside a polarized proton and an external gluonic field; a simple parametrization of the new function is fixed by fitting data and some predictions for other observables are given.

In this respect the computation of Ref. [10] is similar to that of Ref. [2]: also in the latter a new quantity was introduced for valence  $u$  and  $d$  quarks, a quantity which requires some initial state interactions between the colliding hadrons in order

not to be forbidden by time reversal invariance, and the assumption that a factorization scheme still holds; a simple parametrization was given and fixed by fitting the experimental data, and predictions were subsequently made for other processes [4].

The main difference is that in Sivers and our approach [2, 4, 5] – based on the QCD improved parton model – the new quantity, called  $\Delta^N f_{a/p^\uparrow}(x_a, \mathbf{k}_{\perp a})$  or  $2I_{+-}^{a/p}(x_a, \mathbf{k}_{\perp a})$ , has a partonic interpretation: it is the difference between the density numbers  $\hat{f}_{a/p^\uparrow}(x_a, \mathbf{k}_{\perp a})$  and  $\hat{f}_{a/p^\downarrow}(x_a, \mathbf{k}_{\perp a})$  of partons  $a$ , with all possible polarization, longitudinal momentum fraction  $x_a$  and intrinsic transverse momentum  $\mathbf{k}_{\perp a}$ , inside a transversely polarized proton with spin  $\uparrow$  or  $\downarrow$ :

$$\begin{aligned}\Delta^N f_{a/p^\uparrow}(x_a, \mathbf{k}_{\perp a}) &\equiv \hat{f}_{a/p^\uparrow}(x_a, \mathbf{k}_{\perp a}) - \hat{f}_{a/p^\downarrow}(x_a, \mathbf{k}_{\perp a}) \\ &= \hat{f}_{a/p^\uparrow}(x_a, \mathbf{k}_{\perp a}) - \hat{f}_{a/p^\uparrow}(x_a, -\mathbf{k}_{\perp a})\end{aligned}\quad (2)$$

where the second line follows from the first one by rotational invariance. Notice that  $\Delta^N f_{a/p^\uparrow}(x_a, \mathbf{k}_{\perp a})$  vanishes when  $\mathbf{k}_{\perp a} \rightarrow 0$ ; parity invariance also requires  $\Delta^N f$  to vanish when the proton transverse spin has no component perpendicular to  $\mathbf{k}_{\perp a}$ , so that

$$\Delta^N f_{a/p^\uparrow}(x_a, \mathbf{k}_{\perp a}) \sim k_{\perp a} \sin \alpha \quad (3)$$

where  $\alpha$  is the angle between  $\mathbf{k}_{\perp a}$  and the  $\uparrow$  direction.

$\Delta^N f$  by itself is then a leading twist distribution function, but its  $\mathbf{k}_{\perp}$  dependence, when convoluted with the elementary partonic cross-section, results in twist-3 contributions to single spin asymmetries. This same function (up to some factors) has also been introduced in Ref. [11] – where it is denoted by  $f_{1T}^\perp$  – as a leading twist  $T$ -odd distribution function. In Ref. [12] the relation between  $f_{1T}^\perp$  and twist-3 correlation functions is explained and an evaluation of single spin asymmetries in Drell-Yan processes, originating from a gluonic background and the so-called gluonic poles, is given. The exact relation between  $\Delta^N f$  and  $f_{1T}^\perp$  is discussed in Ref. [13].

A function analogous to  $\Delta^N f_{a/p^\uparrow}(x_a, \mathbf{k}_{\perp a})$  can be defined for the fragmentation process of a transversely polarized parton [6], giving the difference between the density numbers  $\hat{D}_{h/a^\uparrow}(z, \mathbf{k}_{\perp h})$  and  $\hat{D}_{h/a^\downarrow}(z, \mathbf{k}_{\perp h})$  of hadrons  $h$ , with longitudinal momentum fraction  $z$  and transverse momentum  $\mathbf{k}_{\perp h}$  inside a jet originated by the fragmentation of a transversely polarized parton with spin  $\uparrow$  or  $\downarrow$ :

$$\begin{aligned}\Delta^N D_{h/a^\uparrow}(z, \mathbf{k}_{\perp h}) &\equiv \hat{D}_{h/a^\uparrow}(z, \mathbf{k}_{\perp h}) - \hat{D}_{h/a^\downarrow}(z, \mathbf{k}_{\perp h}) \\ &= \hat{D}_{h/a^\uparrow}(z, \mathbf{k}_{\perp h}) - \hat{D}_{h/a^\uparrow}(z, -\mathbf{k}_{\perp h}).\end{aligned}\quad (4)$$

A closely related function is denoted by  $H_1^\perp$  in Refs. [11, 14] and its correspondence with  $\Delta^N D$  is discussed in Ref. [13]. Again we expect

$$\Delta^N D_{h/a^\uparrow}(z, \mathbf{k}_{\perp h}) \sim k_{\perp h} \sin \beta \quad (5)$$

where  $\beta$  is the angle between  $\mathbf{k}_{\perp h}$  and the  $\uparrow$  direction.

We first discuss here the role of the above functions, (2) and (4), in generating single spin asymmetries in the large  $p_T$  inclusive production of particles,  $A^\uparrow + B \rightarrow C + X$ , within a phenomenological QCD factorization model; we give explicit expressions for the single spin asymmetries taking into account the leading order contributions of parton transverse momentum (Section 2). In Section 3 we assume that only Collins effect is active – the analogous work for Sivers effect was performed in Refs. [2, 4] – and show that it can explain existing data on  $p^\uparrow p \rightarrow \pi X$ , with some possible problems at the largest  $x_F$  values; we obtain an explicit expression for the function given in Eq. (4). Such function is then used to predict other single spin asymmetries in Section 4, while further comments are made and future applications, with attention to planned experiments, are discussed in Section 5.

## 2. Single spin asymmetries in QCD parton model

We write the differential cross-section for the hard scattering of a polarized hadron  $A^\uparrow$  off an unpolarized hadron  $B$ , resulting in the inclusive production of a hadron  $C$  with energy  $E_C$  and three-momentum  $\mathbf{p}_C$ ,  $A^\uparrow + B \rightarrow C + X$ , in a factorized form, as

$$\begin{aligned}
d\sigma^\uparrow &\equiv \frac{E_C d\sigma^{A^\uparrow B \rightarrow CX}}{d^3\mathbf{p}_C} \\
&= \frac{1}{2} \sum_{a,b,c,d} \sum_{\lambda_a, \lambda'_a, \lambda_b, \lambda_c, \lambda'_c, \lambda_d, \lambda_C} \int \frac{dx_a dx_b}{\pi z 16\pi\hat{s}^2} d^2\mathbf{k}_{\perp a} d^2\mathbf{k}_{\perp b} d^2\mathbf{k}_{\perp C} \\
&\quad \rho_{\lambda_a, \lambda'_a}^{a/A^\uparrow} \hat{f}_{a/A^\uparrow}(x_a, \mathbf{k}_{\perp a}) \hat{f}_{b/B}(x_b, \mathbf{k}_{\perp b}) \hat{M}_{\lambda_c, \lambda_d; \lambda_a, \lambda_b} \hat{M}_{\lambda'_c, \lambda'_d; \lambda'_a, \lambda'_b}^* \hat{D}_{\lambda_C, \lambda_C}^{\lambda_c, \lambda'_c}(z, \mathbf{k}_{\perp C}).
\end{aligned} \tag{6}$$

If we choose the  $z$ -axis as the direction of motion of  $A^\uparrow$  and  $xz$  as the scattering plane, then the  $\uparrow$  direction is along the  $y$ -axis.

The above expression is proven for collinear configurations, according to the QCD factorization theorem: we assume here its validity also when taking into account the parton transverse motion. There is no demonstration of the factorization theorem in such a case, and Eq. (6) has to be considered as a plausible phenomenological model.

In our expression (6) we have explicitly shown the  $\mathbf{k}_\perp$  dependence in the parton distribution and fragmentation. The usual number density of partons  $a$  inside hadron  $A$  is given by

$$f_{a/A}(x_a) = \int d^2\mathbf{k}_{\perp a} \hat{f}_{a/A^\uparrow}(x_a, \mathbf{k}_{\perp a}).$$

Notice that while  $f_{a/A}(x_a)$  cannot depend on the parent hadron spin,  $\hat{f}_{a/A^\uparrow}(x_a, \mathbf{k}_{\perp a})$  can, provided some soft initial state interactions are taken into account [2, 5].

$\rho_{\lambda_a, \lambda'_a}^{a/A^\uparrow}(x_a)$  is the helicity density matrix of parton  $a$  inside the polarized hadron  $A$ . The  $\hat{M}_{\lambda_c, \lambda_d; \lambda_a, \lambda_b}$ 's are the helicity amplitudes for the elementary process  $ab \rightarrow cd$ ; if one wishes to consider higher order (in  $\alpha_s$ ) contributions also elementary

processes involving more partons should be included.  $\hat{D}_{\lambda_C, \lambda'_C}^{\lambda_c, \lambda'_c}(z, \mathbf{k}_\perp)$  is the product of *fragmentation amplitudes* for the  $c \rightarrow C + X$  process

$$\hat{D}_{\lambda_C, \lambda'_C}^{\lambda_c, \lambda'_c} = \rlap{-}\int_{X, \lambda_X} \hat{D}_{\lambda_X, \lambda_C; \lambda_c} \hat{D}_{\lambda_X, \lambda'_C; \lambda'_c}^*, \quad (7)$$

where the  $\rlap{-}\int_{X, \lambda_X}$  stands for a spin sum and phase space integration of the undetected particles, considered as a system  $X$ . The usual unpolarized fragmentation function  $D_{C/c}(z)$ , *i.e.* the density number of hadrons  $C$  resulting from the fragmentation of an unpolarized parton  $c$  and carrying a fraction  $z$  of its momentum, is given by

$$D_{C/c}(z) = \frac{1}{2} \sum_{\lambda_c, \lambda_C} \int d^2 \mathbf{k}_\perp \hat{D}_{\lambda_C, \lambda_c}^{\lambda_c, \lambda_c}(z, \mathbf{k}_\perp). \quad (8)$$

We shall neglect in Eq. (6), due to the limited  $Q^2$  range of its application, the (unknown)  $Q^2$  scale dependences in  $\hat{f}$  and  $\hat{D}$ ; the variable  $z$  is related to  $x_a$  and  $x_b$  by the usual imposition of energy momentum conservation in the elementary  $2 \rightarrow 2$  process, which reads, in the collinear case,  $z = -(x_a t + x_b u)/x_a x_b s$ , where  $s, t, u$  are the Mandelstam variables for the overall process  $AB \rightarrow CX$ , whereas  $\hat{s}, \hat{t}, \hat{u}$  are those for the elementary process  $ab \rightarrow cd$ . A similar expression holds when taking  $\mathbf{k}_\perp$  into account. The elementary process amplitudes are normalized so that

$$\frac{d\hat{\sigma}}{d\hat{t}} = \frac{1}{16\pi \hat{s}^2} \frac{1}{4} \sum_{\lambda_a, \lambda_b, \lambda_c, \lambda_d} |\hat{M}_{\lambda_c, \lambda_d; \lambda_a, \lambda_b}|^2. \quad (9)$$

The cross-section  $d\sigma^\downarrow$  is readily obtained from Eq. (6) by changing  $\uparrow$  into  $\downarrow$ ; in absence of  $\mathbf{k}_\perp$ , collinear configurations and helicity conservation in the pQCD elementary processes do not allow any single hadron spin dependence and it would result  $d\sigma^\uparrow = d\sigma^\downarrow$  [15]. With non zero  $\mathbf{k}_\perp$  instead, spin dependences might still remain in  $\hat{f}_{a/A^\uparrow}(x_a, \mathbf{k}_{\perp a})$  and/or in the fragmentation process of a polarized quark.

We specialize now Eq. (6) to the case of  $p^\uparrow p \rightarrow \pi X$  processes; we keep only the leading contributions in  $\mathbf{k}_\perp$ , *i.e.* we consider parton transverse motion only in those functions which would otherwise be zero [ $\Delta^N f_{a/p^\uparrow}(x_a, \mathbf{0}_{\perp a}) = \Delta^N D_{h/a^\uparrow}(z, \mathbf{0}_{\perp h}) = 0$ ] or integrate to zero, and neglect it elsewhere. From Eq. (6), after some straightforward but lengthy algebra, one then obtains, for  $p^\uparrow p \rightarrow \pi X$ :

$$\begin{aligned} d\sigma^\uparrow - d\sigma^\downarrow = & \sum_{a,b,c,d} \int \frac{dx_a dx_b}{\pi z} \times \\ & \left\{ \int d^2 \mathbf{k}_\perp \Delta^N f_{a/p^\uparrow}(x_a, \mathbf{k}_\perp) f_{b/p}(x_b) \frac{d\hat{\sigma}^{ab \rightarrow cd}}{d\hat{t}}(x_a, x_b, \mathbf{k}_\perp) D_{\pi/c}(z) \right. \\ & \left. + \int d^2 \mathbf{k}'_\perp P^{a/p^\uparrow} f_{a/p}(x_a) f_{b/p}(x_b) \Delta_{NN} \hat{\sigma}^{ab \rightarrow cd}(x_a, x_b, \mathbf{k}'_\perp) \Delta^N D_{\pi/c}(z, \mathbf{k}'_\perp) \right\}. \end{aligned} \quad (10)$$

Eq. (10) is worth some comments.

- The second line of Eq. (10) represents the Sivers effect: this expression of  $d\sigma^\uparrow - d\sigma^\downarrow$  is the one used in Refs. [2] and [4] to fit the data [3] on  $A_N$ , Eq. (1), and to obtain an explicit functional form for  $\Delta^N f_{u/p}$  and  $\Delta^N f_{d/p}$ , Eq. (2).
- The third line of Eq. (10) represents the Collins effect: this expression of  $d\sigma^\uparrow - d\sigma^\downarrow$  is the one we are going to use in the sequel to fit the data on  $A_N$  and to obtain an explicit expression for some  $\Delta^N D_{\pi/c}$ , defined in Eq. (4). Notice that

$$\Delta^N D_{\pi/c}(z, \mathbf{k}'_\perp) = 2 \text{Im} \hat{D}_{\pi/c}^{+-}, \quad (11)$$

where [see Eq. (7)]

$$\hat{D}_{\pi/c}^{+-} = \not{\mathcal{P}}_{X, \lambda_X} \hat{\mathcal{D}}_{\lambda_X; +} \hat{\mathcal{D}}_{\lambda_X; -}^* \quad (12)$$

is a purely imaginary quantity, by parity invariance.

- $P^{a/p^\uparrow} = 2i \rho_{+, -}^{a/p^\uparrow}$  is the polarization of parton  $a$  inside the transversely polarized proton  $p^\uparrow$ ; it is twice the average value of the  $\uparrow$  component of the parton spin. In this part only collinear configurations are taken into account.  $P^{a/p^\uparrow}$  might depend on  $x_a$ , but we will assume that, at least for large  $x_a$  valence quarks, it is constant. The product  $P^{a/p^\uparrow} f_{a/p}(x_a)$  is the transversity distribution, often denoted by  $\Delta_{Tq}(x_a)$  or  $h_1(x_a)$  [16].
- $\Delta_{NN} \hat{\sigma}(x_a, x_b, \mathbf{k}'_\perp)$  is a double spin asymmetry for the elementary process:

$$\begin{aligned} \Delta_{NN} \hat{\sigma}(x_a, x_b, \mathbf{k}'_\perp) &= \left[ \frac{d\hat{\sigma}^{a^\uparrow b \rightarrow c^\uparrow d}}{d\hat{t}} - \frac{d\hat{\sigma}^{a^\uparrow b \rightarrow c^\downarrow d}}{d\hat{t}} \right] = \\ &= \frac{1}{32 \pi \hat{s}^2} \sum_{\lambda_b, \lambda_d} \left[ \hat{M}_{+, \lambda_d; +, \lambda_b} \hat{M}_{-, \lambda_d; -, \lambda_b}^* - \hat{M}_{+, \lambda_d; -, \lambda_b} \hat{M}_{-, \lambda_d; +, \lambda_b}^* \right]. \quad (13) \end{aligned}$$

- The division of Eq. (10) into two separate pieces, corresponding respectively to spin and  $\mathbf{k}_\perp$  effects in distribution and fragmentation functions, is not due to keeping only leading terms in  $k_\perp$ ; the structure of Eq. (10) – within its range of validity – remains unchanged also at higher order in  $k_\perp$ , and one should only add the appropriate  $\mathbf{k}_\perp$  dependences in all terms.

### 3. Single spin asymmetries in $p^\uparrow p \rightarrow \pi X$ and Collins effect

In Refs. [2] and [4] we have considered  $\Delta^N f$  as the only possible source of single spin asymmetries in  $p^\uparrow p \rightarrow \pi X$  processes and have obtained an explicit expression for it, by fitting the experimental data. This expression of  $\Delta^N f$  has been used in Ref. [4] to compute the values of  $A_N$  for  $\bar{p}^\uparrow p \rightarrow \pi X$ , in agreement with data, and to predict  $A_N$  for  $p^\uparrow p \rightarrow KX$ , not yet measured.

We consider here  $\Delta^N D$  as the only possible source of single spin asymmetries and see if we can get an equally good description of the data, obtaining an explicit

expression for it, to be used in other processes. We will find that this is possible, with some difficulties, and shall comment on it at the end of this Section.

Of course, both effects – in distribution and fragmentation functions – might be at work at the same time and be in different proportions responsible for the observed asymmetries. By considering only one effect at a time we can obtain the maximum possible values for  $\Delta^N f$  and  $\Delta^N D$ ; such functions could then be used in processes where only one of them can be at work to produce single spin asymmetries, and see whether or not they give results in agreement with data. For example, in prompt photon production,  $A^\dagger B \rightarrow \gamma X$  [4] and Drell-Yan processes,  $A^\dagger B \rightarrow \mu^+ \mu^- X$ , only  $\Delta^N f$  can be active, whereas in semi-inclusive DIS,  $\ell p^\dagger \rightarrow \ell \pi X$ , only  $\Delta^N D$  should contribute, because initial state interactions in lepton-proton interactions are suppressed by powers of  $\alpha_{em}$  [17]. More data are expected in the future, from running or planned experiments at HERA, Jefferson Lab and RHIC; a richer and more precise amount of experimental information might also allow a simultaneous determination of both  $\Delta^N f$  and  $\Delta^N D$  by using the full equation (10).

There is another reason to explore here the effects of  $\Delta^N D$  alone. As we said, the function  $\Delta^N f$  is bound to be zero if the incoming hadrons are treated as free plane wave states [6, 12]; this conclusion may be avoided by invoking soft initial state interactions, different from those taken into account in the proof of the factorization theorem, together with the assumption that one can still use the factorized form (6) (see also Ref. [18]). The same does not apply to the fragmentation process, which requires anyway final state interactions; it might be – and this has to be determined experimentally – that the Collins effect is the main origin of single spin asymmetries.

We consider then the last line only of Eq. (10) to compute  $A_N$ , Eq. (1), for  $p^\dagger p \rightarrow \pi^{\pm,0} X$  processes:

$$d\sigma^\uparrow - d\sigma^\downarrow = \sum_{a,b,c,d} \int \frac{dx_a dx_b}{\pi z} d^2 \mathbf{k}_\perp \quad (14)$$

$$\times P^{a/p^\dagger} f_{a/p^\dagger}(x_a) f_{b/p}(x_b) \Delta_{NN} \hat{\sigma}^{ab \rightarrow cd}(x_a, x_b, \mathbf{k}_\perp) \Delta^N D_{\pi/c}(z, \mathbf{k}_\perp)$$

and

$$d\sigma^\uparrow + d\sigma^\downarrow = 2 d\sigma^{unp} = 2 \sum_{a,b,c,d} \int \frac{dx_a dx_b}{\pi z} \quad (15)$$

$$\times f_{a/p}(x_a) f_{b/p}(x_b) \frac{d\hat{\sigma}^{ab \rightarrow cd}}{d\hat{t}}(x_a, x_b) D_{\pi/c}(z).$$

Let us consider Eq. (14) and notice that  $\Delta^N D_{\pi/c}(z, \mathbf{k}_\perp)$ , Eq. (4), is an odd function of  $\mathbf{k}_\perp$ : this means that we cannot neglect the  $\mathbf{k}_\perp$  dependence of the other terms, and have to keep into account  $\mathbf{k}_\perp$  values in  $\Delta_{NN} \hat{\sigma}^{ab \rightarrow cd}$ . Eq. (14) can then be written as

$$d\sigma^\uparrow - d\sigma^\downarrow = \sum_{a,b,c,d} \int \frac{dx_a dx_b}{\pi z} d^2 \mathbf{k}_\perp P^{a/p^\dagger} f_{a/p^\dagger}(x_a) f_{b/p}(x_b) \Delta^N D_{\pi/c}(z, \mathbf{k}_\perp)$$

$$\times [\Delta_{NN} \hat{\sigma}(x_a, x_b, \mathbf{k}_\perp) - \Delta_{NN} \hat{\sigma}(x_a, x_b, -\mathbf{k}_\perp)] \quad (16)$$

where now the integration on  $\mathbf{k}_\perp$  runs only over one half-plane of its components. The only unknown functions in Eq. (16) are  $P^{a/p^\dagger}$  and  $\Delta^N D_{\pi/c}(z, \mathbf{k}_\perp)$ , which, at least in principle, are then measurable via the single spin asymmetry  $A_N$ .

In order to perform numerical calculations, we make some further assumptions, following Refs. [2] and [4]. Our first assumption is that the dominant effect is given by the valence quarks in the polarized protons. That is, we assume  $P^{a/p^\dagger}$  to be non-zero only for valence  $u$  and  $d$  quarks: it is natural to assume a smaller polarization for sea quarks which, moreover, do not contribute much to the production of large  $x_F$  pions. Secondly, we evaluate Eq. (16) by assuming that the main contribution comes from  $\mathbf{k}_\perp = \mathbf{k}_\perp^0$  where, as suggested by Eq. (5),  $\mathbf{k}_\perp^0$  lies in the overall scattering plane and its magnitude equals the average value of  $\langle \mathbf{k}_\perp^2 \rangle^{1/2} = k_\perp^0(z)$ . This average value does in general depend on  $z$ . The  $z$  dependence of the transverse momentum of charged pions inside jets was measured at LEP [19]. We have made a fit to their data points. The result is shown in Fig. 1 and corresponds to the curve

$$\frac{k_\perp^0(z)}{M} = 0.61 z^{0.27} (1-z)^{0.20}, \quad (17)$$

with  $M = 1 \text{ GeV}/c^2$ .

The residual  $z$  dependence in  $\Delta^N D_{\pi/c}$  not coming from  $k_\perp^0$  is taken as a simple power behaviour, so that

$$\Delta^N D(z, \mathbf{k}_\perp^0) = \frac{k_\perp^0(z)}{M} N_c z^{\alpha_c} (1-z)^{\beta_c}, \quad (18)$$

where  $N_c$ ,  $\alpha_c$  and  $\beta_c$  are free parameters.

The  $\mathbf{k}_\perp$  integration then produces the simple expression:

$$\begin{aligned} & \int d^2 \mathbf{k}_\perp \Delta^N D_{\pi/c}(z, \mathbf{k}_\perp) \left[ \Delta_{NN} \hat{\sigma}^{ab \rightarrow cd}(\mathbf{k}_\perp) - \Delta_{NN} \hat{\sigma}^{ab \rightarrow cd}(-\mathbf{k}_\perp) \right] \\ & \simeq \frac{k_\perp^0(z)}{M} N_c z^{\alpha_c} (1-z)^{\beta_c} \left[ \Delta_{NN} \hat{\sigma}^{ab \rightarrow cd}(\mathbf{k}_\perp^0) - \Delta_{NN} \hat{\sigma}^{ab \rightarrow cd}(-\mathbf{k}_\perp^0) \right], \end{aligned} \quad (19)$$

where  $k_\perp^0(z)$  is given by Eq. (17).

Exploiting isospin and charge conjugation invariance and assuming that a large  $z$  pion is mainly generated by the fragmentation of a quark which can be a valence quark for the pion, one can express all functions (18) – one for each flavour  $c$  – in terms of a single one,

$$\Delta^N D_{val}(z, \mathbf{k}_\perp^0) = N \frac{k_\perp^0(z)}{M} z^\alpha (1-z)^\beta. \quad (20)$$

In fact

$$\begin{aligned} \Delta^N D_{\pi^+/u}(z, k_\perp^0) &= \Delta^N D_{\pi^+/\bar{d}}(z, k_\perp^0) \\ &= \Delta^N D_{\pi^-/\bar{u}}(z, k_\perp^0) = \Delta^N D_{\pi^-/d}(z, k_\perp^0) = \Delta^N D_{val}(z, k_\perp^0) \end{aligned} \quad (21)$$



and

$$\begin{aligned} \Delta^N D_{\pi^0/u}(z, k_\perp^0) &= \Delta^N D_{\pi^0/d}(z, k_\perp^0) \\ &= \Delta^N D_{\pi^0/\bar{u}}(z, k_\perp^0) = \Delta^N D_{\pi^0/\bar{d}}(z, k_\perp^0) = \frac{1}{2} \Delta^N D_{val}(z, k_\perp^0). \end{aligned} \quad (22)$$

Notice that  $\Delta^N D_{\pi/c}$  can be different from zero only for polarized quarks, *i.e.* quarks resulting from processes  $ab \rightarrow cd$  for which  $\Delta_{NN}\hat{\sigma}$  is not zero. These are the processes:  $qq \rightarrow qq$ ,  $qq' \rightarrow qq'$ ,  $q\bar{q} \rightarrow q\bar{q}$ ,  $q\bar{q} \rightarrow \bar{q}q$  and  $qg \rightarrow qg$ , and the corresponding  $\Delta_{NN}\hat{\sigma}$  can be computed from Eq. (13).

As initial polarized partons, like we said before, we consider only  $u$  and  $d$  valence quarks. The value of  $P^{q/p^\dagger}$  is taken as a free parameter: given the overall normalization factor  $N$  contained in  $\Delta^N D_{val}$  only the relative value

$$P_{u/d} \equiv P^{u/p^\dagger} / P^{d/p^\dagger} \quad (23)$$

is a meaningful free parameter.  $P^{u/p^\dagger}$  is assumed to be  $2/3$ .

In the unpolarized cross-section we take into account all leading order QCD processes involving quarks and gluons. Expressions for the unpolarized partonic distributions and fragmentations can be found in the literature; we have chosen, as in Ref. [4], the MRSG parametrization [20] for the partonic distribution functions, and the parametrization (BKK1) of Ref. [21] for quark fragmentation functions into pions. These fragmentation functions are sufficiently well established and constrained by data; other sets available in the literature have similar large  $z$  behaviours and do not change significantly the quality of the fit and the conclusions which follow.

Eqs. (14) and (15) contain now known functions – the unpolarized distribution and fragmentation functions – computable elementary dynamics –  $d\hat{\sigma}/d\hat{t}$  and  $\Delta_{NN}\hat{\sigma}$  – and unknown functions –  $\Delta^N D_{\pi/c}$  and  $P^{a/p^\dagger}$  – parametrized in a simple way, Eqs. (17), (20)-(22) and (23), in terms of the four parameters  $N$ ,  $\alpha$ ,  $\beta$  and  $P_{u/d}$ .

We have computed  $A_N$ , Eq. (1), as a function of  $x_F$  in terms of these parameters and have compared with the existing data for  $p^\dagger p \rightarrow \pi^{\pm,0}$  processes [3]; in our computation the  $p_T$  of the produced pion in the  $pp$  c.m. frame has been fixed to  $p_T = 1.5$  GeV/ $c$ , the average transverse momentum of the data. We obtain that the  $\Delta^N D_{val}$  function which best fits the data is given by Eqs. (17) and (20) with:

$$\begin{aligned} N &= -0.22 \\ \alpha &= 2.33 \quad \text{for } z \leq 0.97742 \\ \beta &= 0.24 \end{aligned} \quad (24)$$

and

$$\Delta^N D_{val}(z, k_\perp^0) = -2D_{val}(z) = -2 \times 1.102 z^{-1}(1-z)^{1.2} \quad \text{for } z > 0.97742, \quad (25)$$

where  $z = 0.97742$  is the value at which  $|\Delta^N D_{val}(z, k_\perp^0)| = 2D_{val}(z)$ .  $D_{val}(z)$  is taken from Ref. [21]. The best fit value of  $P_{u/d}$  turns out to be

$$P_{u/d} = -0.76. \quad (26)$$

The corresponding fit is shown in Fig. 2. The fit is qualitatively good and satisfactory – one should not forget that we are considering an extreme situation, with only Collins effect taken into account and a simple  $\mathbf{k}_\perp$  dependence in  $\Delta^N D(z, \mathbf{k}_\perp)$ ; however, the last points at large  $x_F$  values seem difficult to fit. In our computation  $A_N \rightarrow 0$  when  $x_F \rightarrow 1$ , due to the fact that  $k_\perp^0(z) \rightarrow 0$  when  $z \rightarrow 1$ , Eq. (17). We can obtain the best results only by letting  $\Delta^N D(z, k_\perp^0)$  to be as large as possible when  $z \rightarrow 1$ , *i.e.* we have to saturate the necessary inequality:

$$|\Delta^N D(z, k_\perp^0)| \leq 2D(z). \quad (27)$$

In Fig. 3 we plot the ratio  $\Delta^N D_{val}(z, k_\perp^0)/2D_{val}$ ; it shows the elementary left-right asymmetry, in the fragmentation of a transversely polarized valence quark,

$$\hat{A}_{\pi/q}^N \equiv \frac{\hat{D}_{\pi/q^\uparrow}(z, k_\perp^0) - \hat{D}_{\pi/q^\downarrow}(z, k_\perp^0)}{\hat{D}_{\pi/q^\uparrow}(z, k_\perp^0) + \hat{D}_{\pi/q^\downarrow}(z, k_\perp^0)}, \quad (28)$$

which is needed to fit the data.

$|\hat{A}_{\pi/q}^N|$  has to reach the maximum value 1 at large  $z$ ; smaller values would give too small values of  $A_N$  at large  $x_F$ .

This was not the case when relying only on Siverts effect [2, 4]. In Fig. 4 we plot the value of the left-right asymmetry in the splitting of a transversely polarized proton into a valence quark,

$$\hat{A}_{q/p}^N \equiv \frac{\hat{f}_{q/p^\uparrow}(x, k_\perp^0) - \hat{f}_{q/p^\downarrow}(x, k_\perp^0)}{\hat{f}_{q/p^\uparrow}(x, k_\perp^0) + \hat{f}_{q/p^\downarrow}(x, k_\perp^0)} = \frac{\Delta^N f_{q/p}(x, k_\perp^0)}{2f_{q/p}}, \quad (29)$$

as obtained from Refs. [4] for  $u$  and  $d$  valence quarks<sup>1</sup>.

By comparing Figs. 3 and 4 we see that a small elementary left-right asymmetry  $\hat{A}_{u,d/p}^N$  in the proton distribution alone allows to fit the data on  $A_N$  [3, 4]; instead, one needs a much bigger (at large  $z$ ) left-right asymmetry  $\hat{A}_{\pi/q}^N$  in the quark fragmentation alone in order to fit the same data. Moreover, the fit obtained here with the large  $\hat{A}_{\pi/q}^N$  might show some difficulty at the largest  $x_F$  values. If such a discrepancy should be confirmed by further data it would be a significant indication towards the importance of spin and  $\mathbf{k}_\perp$  dependences in distribution functions.

We also notice that in the simple model proposed by Collins for  $\Delta^N D$  [6], one finds that  $\hat{A}_{\pi/q}^N \rightarrow 0$  when  $z \rightarrow 1$ ; such model could not explain the observed values of  $A_N$ .

The reason for the different results due solely to  $\Delta^N f$  [2, 4] or  $\Delta^N D$  (this paper) can be qualitatively understood by looking again at Eq. (10) and it turns out to be very interesting, as it is due to the different dynamical contributions in the two

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<sup>1</sup>Notice that, due to a mistake in the writing of the paper, the values of  $N_a$  given in Eq. (9) of Ref. [4] are wrong and have to be multiplied by a factor 4; the fit to the data, Fig. 1 of Ref. [4], remains the same.

cases: let us compare Eq. (19) with its analogue for Sivvers contribution, obtainable from the second line of Eq. (10) [4]:

$$\begin{aligned} & \int d^2\mathbf{k}_\perp \Delta^N f_{a/p^\uparrow}(x_a, \mathbf{k}_\perp) \left[ \frac{d\hat{\sigma}^{ab \rightarrow cd}}{d\hat{t}}(\mathbf{k}_\perp) - \frac{d\hat{\sigma}^{ab \rightarrow cd}}{d\hat{t}}(-\mathbf{k}_\perp) \right] \\ & \simeq \frac{k_\perp^0(x_a)}{M} N_a z^{\alpha_a} (1-z)^{\beta_a} \left[ \frac{d\hat{\sigma}^{ab \rightarrow cd}}{d\hat{t}}(\mathbf{k}_\perp^0) - \frac{d\hat{\sigma}^{ab \rightarrow cd}}{d\hat{t}}(-\mathbf{k}_\perp^0) \right]. \end{aligned} \quad (30)$$

In the expression for  $A_N$  Eqs. (19) or (30) have to be divided by the unpolarized cross section which contains  $f$ ,  $D$  and  $d\hat{\sigma}/d\hat{t}$ . Now, it turns out that not only  $\Delta_{NN}\hat{\sigma}$  is smaller than  $d\hat{\sigma}/d\hat{t}$ , but also the cancellation between the two terms in squared brackets is stronger for  $\Delta_{NN}\hat{\sigma}$ , Eq. (19), than for  $d\hat{\sigma}/d\hat{t}$ , Eq. (30). This explains why fitting the data requires a much bigger value of  $|\Delta^N D|/2D$  than  $|\Delta^N f|/2f$ .

We finally comment on the value found for  $P_{u/d}$  which might be surprising: in  $SU(6)$ , for example, one has  $P^{u/p^\uparrow} = 2/3$  and  $P^{d/p^\uparrow} = -1/3$ , so that  $P_{u/d} = -2$ . We find here that, indeed, the  $u$  and  $d$  valence quark polarizations inside a polarized proton have opposite signs, but the  $d$  polarization is, in magnitude, slightly bigger than the  $u$  polarization, Eq. (26).

#### 4. Application to other processes, $\bar{p}^\uparrow p \rightarrow \pi X$ and $p^\uparrow p \rightarrow KX$

We now apply the  $\Delta^N D_{val}(z, \mathbf{k}_\perp^0)$  we have obtained by fitting the E704 experimental data on pion production, to predict single spin asymmetries in other processes.

Recently the E704 Collaboration at Fermilab has presented results on single spin asymmetries for inclusive production of pions also in the collision of transversely polarized antiprotons off a proton target [22]. The kinematical conditions are the same as in the case of polarized protons [3]. Within our model the connection between single spin asymmetries with polarized protons or antiprotons is very simple: since only valence quark contributions are taken into account, we just have to exploit the charge conjugation relations (21) and (22) for the  $\Delta^N D$  and similar ones for the  $P^{\bar{q}/\bar{p}^\uparrow}$ . In particular, this means that we should expect

$$A_N(\bar{p}^\uparrow p \rightarrow \pi^\pm) \simeq A_N(p^\uparrow p \rightarrow \pi^\mp) \quad A_N(\bar{p}^\uparrow p \rightarrow \pi^0) \simeq A_N(p^\uparrow p \rightarrow \pi^0). \quad (31)$$

Our results are shown in Fig. 5, and indeed they agree with Eq. (31). The experimental data are in very good agreement with our predictions for  $\pi^+$  and somewhat smaller than our predictions for  $\pi^0$  and  $\pi^-$ , although the overall agreement is good. The expectations (31) are essentially based on charge conjugation invariance and the dominance of certain elementary dynamical contributions and it is difficult to see how they could not be true; we expect that further data both on  $p^\uparrow p \rightarrow \pi X$  and  $\bar{p}^\uparrow p \rightarrow \pi X$  should lead to a better agreement with Eq. (31).

We consider now the case where charged and neutral kaons are detected in the final state,  $p^\uparrow p \rightarrow KX$ ; the kinematical conditions are the same as for the other

processes. Lacking a determination of  $\Delta^N D(z, \mathbf{k}_\perp^0)$  for the fragmentation of quarks into kaons we assume that the  $\Delta^N D_{val}$  are the same for  $K$  and  $\pi$  production, up to a different normalization factor which we fix by imposing, for valence quarks,

$$\frac{\Delta^N D_{K/q}(z, \mathbf{k}_\perp^0)}{\Delta^N D_{\pi/q}(z, \mathbf{k}_\perp^0)} = \frac{\langle N_K \rangle}{\langle N_\pi \rangle}, \quad (32)$$

where  $\langle N_h \rangle = \int_0^1 dz D_{h/q}(z)$ . We have used unpolarized kaon fragmentation function sets from Refs. [21, 23, 24, 25] and have computed the appropriate normalization factor  $\langle N_K \rangle / \langle N_\pi \rangle$  for each set.

We should keep in mind that here only large  $x$  *valence* quarks from the initial polarized proton are taken into account: this means that contribution from strange quarks from the polarized proton sea are completely neglected and the production of kaons is through non-strange quarks only. The eventual role of strange polarized quarks (and antiquarks) could only be studied within a more refined model for the transversely polarized distribution functions,  $h_1(x) = P^{q/p^\uparrow} f_{q/p}(x)$ , where contributions from  $q = s$  and  $\bar{s}$  are included. However, we have checked that, even assuming a large value of  $P^{s/p^\uparrow}$  and taking  $\Delta^N D_{K/s} = \Delta^N D_{val}$ , the contribution of strange quarks to  $A_N$  is negligible at large  $x_F$ , as  $f_{s/p}(x_a)$  is small at large  $x_a$  values.

Fig. 6 shows the results we obtain for  $A_N$  in the production of either charged or neutral kaons,  $(K^+ + K^-)/2$  and  $(K^0 + \bar{K}^0)/2$  respectively. The positive values of  $A_N$  refer to charged kaons, while the negative ones to neutral ones: the values obtained are large for all sets of unpolarized fragmentation functions, whose features are further discussed in Ref. [4]. Notice that the different sets of fragmentation functions give qualitatively similar results, contrary to the findings of Refs. [4], where, for neutral kaons, the results show a strong dependence on the choice of the fragmentation functions.

Let us stress once more that these results for  $K$  production are rather qualitative and based on the simple assumption about the form of  $\Delta^N D_{K/q}(z, \mathbf{k}_\perp^0)$ , Eq. (32); nevertheless, they clearly show how  $A_N$  might be large and measurable also for  $p^\uparrow p \rightarrow KX$  processes and its measurement should give more information about the Collins effects in the fragmentation of a polarized quark into a kaon.

## 5 Comments and conclusions

Single spin asymmetries in inclusive hadron production can be explained within QCD; they are twist-3 effects which can be large and measurable in the  $p_T$  range of few GeV/ $c$ . Several possible origins – leading to the introduction of new basic functions – have been discussed in the literature, but a clear and comprehensive understanding has not been reached yet: more theoretical and experimental work is still necessary.

In this paper and previously in Refs. [2] and [4] we attempt a consistent phenomenological description and computation of single spin asymmetries based on QCD parton model and the introduction of fundamental spin asymmetries at the

quark level, both in the splitting of polarized protons into quarks, Eq. (2), and in the fragmentation of polarized quarks into hadrons, Eq. (4). The link between this approach and other approaches based on the introduction of parton correlation functions [10, 26] and/or gluonic poles [12] has to be clarified. As suggested by Ratcliffe [26] the data might also be analysed using a general parametrization of the partonic correlators.

In Ref. [2] we showed how the so-called Sivers effect alone could account for data on  $A_N$  and we derived an explicit expression for the function  $\Delta^N f$ ; here we have shown that also the Collins effect – with some more difficulties – can account for the same data, and we have obtained an explicit expression for the function  $\Delta^N D$ . We have also introduced a general formalism which includes both effects, Eq. (10).

Their relative importance has to be established by further experimental data; single spin asymmetries could be measured in the near future at Jefferson Lab, HERA [27] and RHIC. Of particular interest are those processes in which only one of the two effects is expected to be active: in semi-inclusive DIS,  $\ell p^\uparrow \rightarrow \ell hX$ , and in  $\gamma^* p^\uparrow \rightarrow hX$  initial state interactions are suppressed by higher powers of  $\alpha_{em}$  and single spin asymmetries should originate only from the fragmentation of a final polarized quark [28]. It would be very interesting to have data on such processes.

On the other hand processes like  $p^\uparrow N \rightarrow \gamma X$  and  $p^\uparrow N \rightarrow \mu^+ \mu^- X$  could exhibit single spin asymmetries only due to spin and  $\mathbf{k}_\perp$  dependent distribution functions [4] or quark-gluon correlations [8, 12, 29]; also data on these processes would be of great help.

A comprehensive phenomenological description and a physical understanding of subtle single spin effects, within perturbative QCD and a simple factorization model, seems indeed possible; more data should allow the determination of new basic distribution and fragmentation functions, allowing genuine predictions for new processes and a deeper knowledge of the nucleon structure and the hadronization mechanism. More theoretical work, towards a true generalization of the factorization theorem to include transverse momenta, is also needed and might be motivated by our phenomenological study.

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## Figure Captions

Figure 1. The pion intrinsic average transverse momentum  $k_{\perp}^0$  in a jet as a function of  $z$ . The diamonds are the data from Abreu *et al.* [19], the continuous line is our fit, as given by Eq. (17).

Figure 2. Single spin asymmetry for pion production,  $p^{\uparrow} p \rightarrow \pi X$ . The data points are the E704 experimental single spin asymmetries [3] for  $\pi^+$  (diamonds),  $\pi^0$  (squares) and  $\pi^-$  (triangles). The solid line is our best fit for  $\pi^+$ , the dashed line for  $\pi^0$  and the dotted line for  $\pi^-$ , obtained under the assumption of Collins effect only.

Figure 3. The ratio between  $|\Delta^N D_{val}|$  and twice the valence unpolarized fragmentation function  $D$ , Eq. (28), as a function of  $z$ . Notice that  $|\Delta^N D|/2D = 1$  for  $z \geq 0.97742$ .

Figure 4. The ratio between  $|\Delta^N f_{val}|$  and twice the valence unpolarized distribution function  $f$ , Eq. (29), as a function of  $x$ . The solid line refers to  $u$  quarks and the dashed line to  $d$  quarks. Notice that in both cases  $|\Delta^N f|/2f \rightarrow 0$  for  $x \rightarrow 1$ .

Figure 5. Single spin asymmetry for pion production in  $\bar{p}^{\uparrow} p \rightarrow \pi X$ . The data points are the E704 experimental single spin asymmetries [22] for  $\pi^+$  (diamonds),  $\pi^0$  (squares) and  $\pi^-$  (triangles). The solid line is the result of our model (with Collins effect only) for  $\pi^+$ , the dashed line for  $\pi^0$  and the dotted line for  $\pi^-$ .

Figure 6. Single spin asymmetry, as predicted by our model (with Collins effects only), for the production of charged (upper curves) and neutral (lower curves) kaons,  $(K^+ + K^-)/2$  and  $(K^0 + \bar{K}^0)/2$  respectively. The solid lines correspond to the BKK1 set of unpolarized fragmentation functions [21], the dashed lines correspond to the BKK2 set [23], the dotted lines to the GR set [24], and the dashed-dotted lines to the IMR set [25]; see Ref. [4] and text for further details.

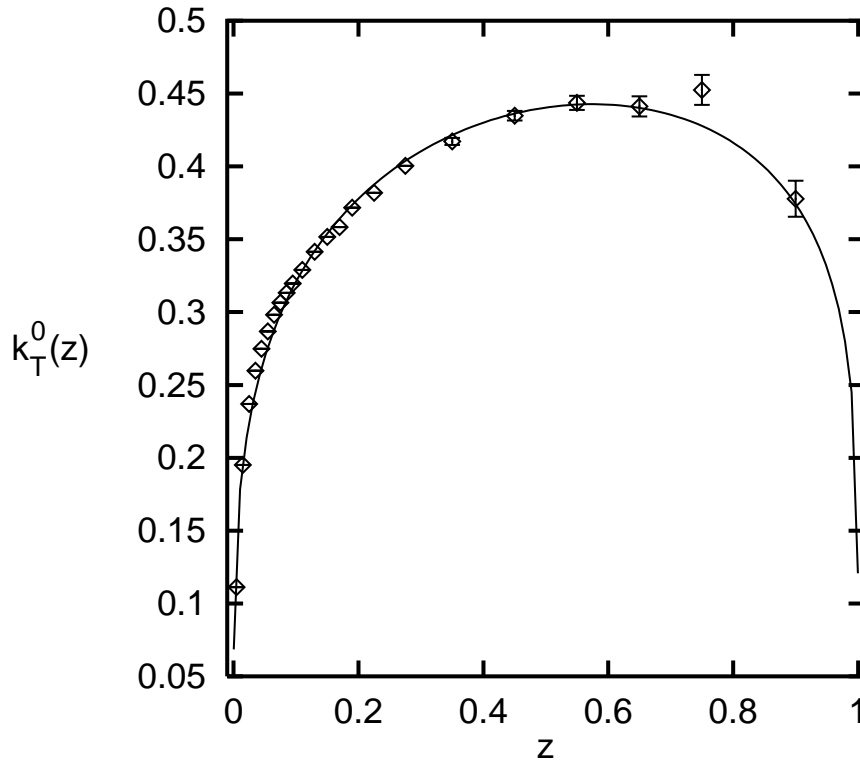


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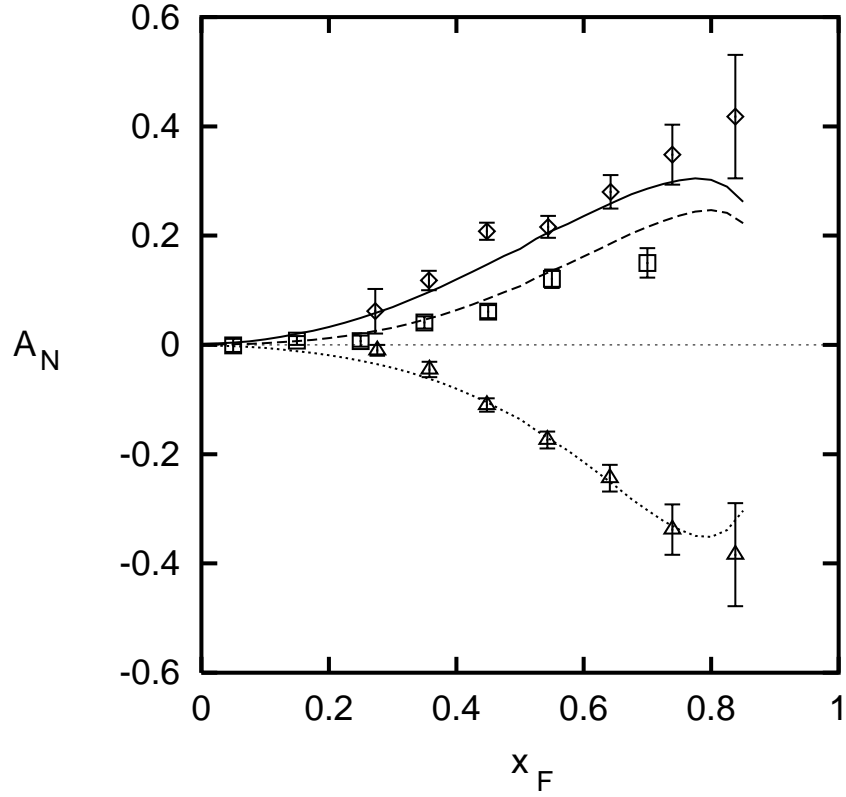


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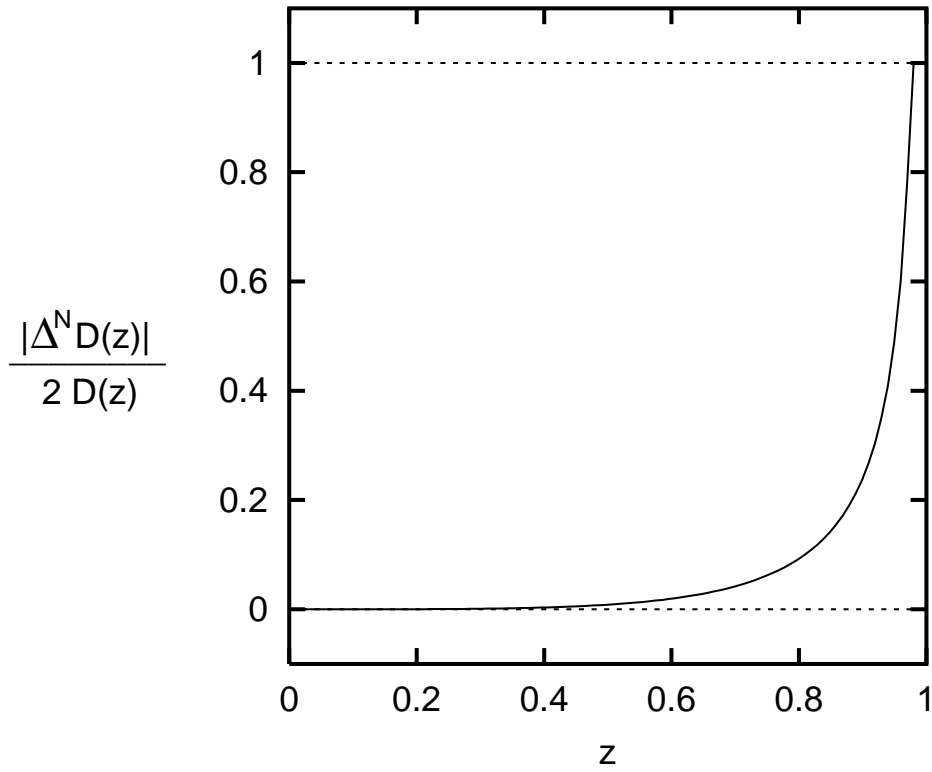


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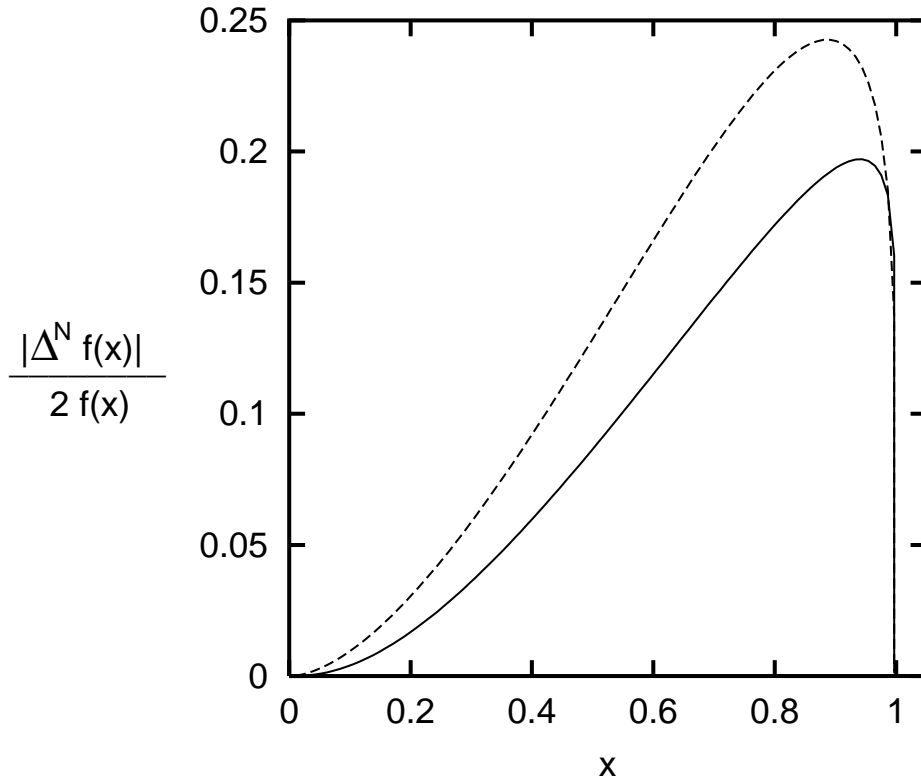


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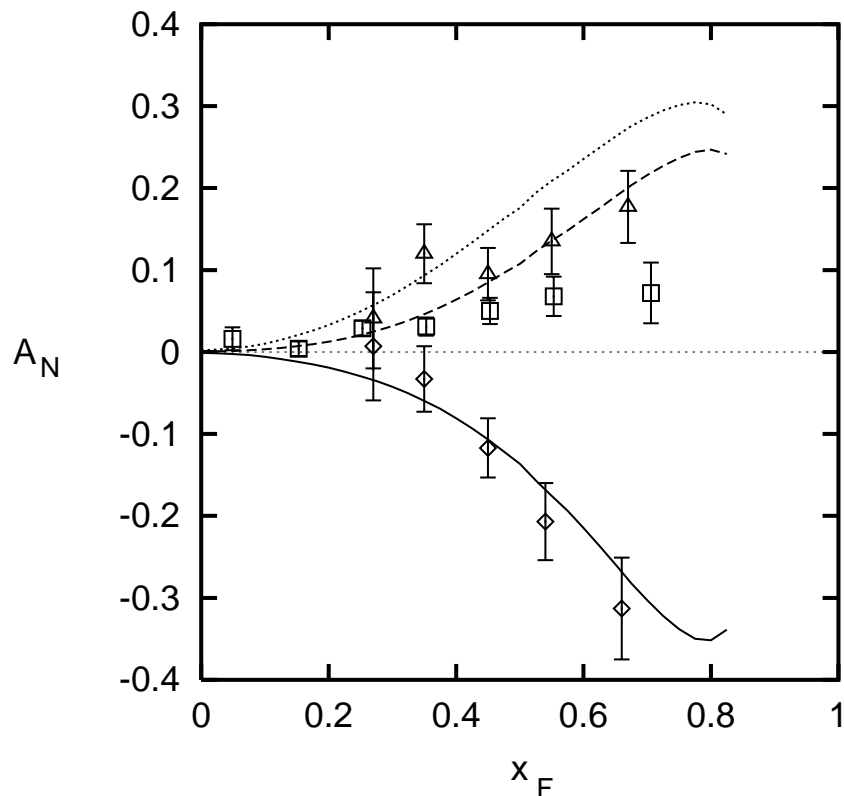


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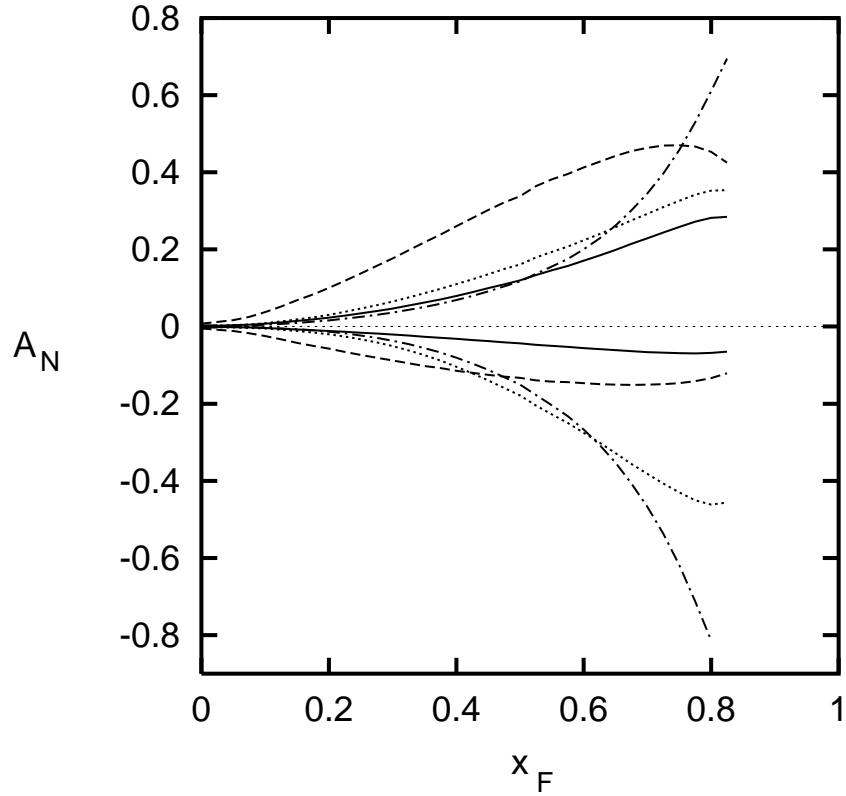


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