Phonon drag thermopower and weak localization

A. Miele, R. Fletcher and E. Zaremba

Physics Department, Queen's University, Kingston, Ontario, Canada, K7L 3N6.

Y. Feng

Microstructural Sciences, National Research Council, Ottawa, Ontario, Canada, K1A 0R6.

C. T. Foxon

Department of Physics, University of Nottingham, Nottingham NG7 2RD, UK.

J.J. Harris

Electronic and Electrical Engineering, University College, London WC1E 7JE, UK.
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Abstract

Previous experimental work on a two-dimensional (2D) electron gas in a Si-on-sapphire device led to the conclusion that both conductivity and phonon drag thermopower S^g are affected to the same relative extent by weak localization. The present paper presents further experimental and theoretical results on these transport coefficients for two very low mobility 2D electron gases in $\delta-$doped GaAs/Ga$_x$Al$_{1-x}$As quantum wells. The experiments were carried out in the temperature range 3-7K where phonon drag dominates the thermopower and, contrary to the previous work, the changes observed in the thermopower due to weak localization were found to be an order of magnitude less than those in the conductivity. A theoretical framework for phonon drag thermopower in 2D and 3D semiconductors is presented which accounts for this insensitivity of S^g to weak localization. It also provides transparent physical explanations of many previous experimental and theoretical results.

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I. INTRODUCTION

Weak localization (WL) refers to the quantum correction to the conductivity which arises because the phase of the electronic wavefunction is not randomized during elastic collisions. Through interference effects, this leads to a decrease in the longitudinal conductivity σ_{xx} which can be restored either by raising the temperature, thus increasing the frequency of phase-disrupting inelastic collisions, or by applying a low magnetic field which destroys the phase coherence of the electronic paths. WL has been thoroughly investigated[1] over the last two decades but the effect on other transport coefficients has received much less attention. It was predicted[2] that the electrical conductivity σ_{xx} and the electronic thermal conductivity λ_{xx}^e would be related by the Wiedemann-Franz relation in the usual way for elastic electronic scattering, and the available experimental data[3] on the longitudinal components in a quasi-2D system are in accord with this prediction. (We take the 2D electron gas (2DEG) to be in the xy plane and, unless otherwise stated, the magnetic field $\mathbf{B}$ along z).

The interpretation of thermopower data is complicated by the fact that there are two contributions, diffusion $\overset{\leftrightarrow}{S^d}$ and phonon drag $\overset{\leftrightarrow}{S^g}$. There is theoretical consensus that the longitudinal diffusion thermopower S_{xx}^d should be modified by WL,[4–7] the result being equivalent to the well-known Mott relation between S_{xx}^d and σ_{xx} for elastic electronic scattering. However, in 2DEGs, which are the focus of the present paper, S_{xx}^g is very large at low temperatures so that S_{xx}^d is difficult to observe. This situation is quite different from that encountered in 3D systems. Introducing impurities into a 3D lattice to reduce the elastic mean free path l_e of the electrons (so that WL is observable in σ_{xx}) inevitably leads to a strong reduction of phonon drag because of the scattering of phonons by the same impurities. In the 2D case, l_e is reduced by placing impurities into or near the 2DEG, but under normal circumstances these impurities have no effect on the phonons in the substrate and so drag is not thereby reduced.

As far as we aware, there is only one theoretical paper[8] concerned with WL and phonon drag. This dealt with S_{xx}^g but gave a result only for the range of magnetic field B where the localization term in σ_{xx} is quadratic in B, which is appropriate only at very low B, and the situation at arbitrary fields remained unclear.

Experimental data for a 2DEG in a Si-on-sapphire device were published by Syme et $al.$[9] which indicated that S_{xx}^g and σ_{xx} are affected equally by WL. The significance of this result is most conveniently discussed, as did the authors of that work, in terms of the phenomenological equation relating the electric current density $\mathbf{J}$ to the gradient of the electrochemical potential $\mathbf{E}' = \mathbf{E} + \nabla\mu/e$, where $\mathbf{E}$ is the electric field and μ the chemical potential, and the temperature gradient ∇T, i.e.

$$\mathbf{J} = \overset{\leftrightarrow}{\sigma}\,\mathbf{E}' - \overset{\leftrightarrow}{\epsilon}\,\nabla T \tag{1}$$

where $\overset{\leftrightarrow}{\epsilon}$ is a thermoelectric tensor. The thermopower $\overset{\leftrightarrow}{S}$ is the experimental quantity which gives access to $\overset{\leftrightarrow}{\epsilon}$ and is defined by $\mathbf{E} = \overset{\leftrightarrow}{S}\,\nabla T$ (the chemical part $\nabla\mu/e$ is not observed in experimental situations, e.g. see Gurevich et $al.$[10]) measured under the condition $\mathbf{J} = 0$. The coefficient $\overset{\leftrightarrow}{\epsilon}$ has two additive contributions, diffusion $\overset{\leftrightarrow}{\epsilon^d}$ and phonon drag $\overset{\leftrightarrow}{\epsilon^g}$, which are responsible for the two corresponding components $\overset{\leftrightarrow}{S^d}$ and $\overset{\leftrightarrow}{S^g}$.

S_{xx}^g varies rapidly with temperature and it is difficult to observe deviations that might be due to localization. It is therefore necessary to measure S_{xx} as a function of perpendicular magnetic field $\mathbf{B}$. From Eq. (1) with $\nabla T_y = 0$ we have $S_{xx} = \rho_{xx}\epsilon_{xx} + \rho_{yx}\epsilon_{xy}$, where $\overleftrightarrow{\rho} = \overleftrightarrow{\sigma}^{-1}$. The very low mobility of the previous (and present) samples means that it is an excellent approximation to write $S_{xx} = \epsilon_{xx}/\sigma_{xx}$. If we denote the relatively small changes due to B as $\triangle\sigma_{xx}$, etc., then we have

$$\frac{\triangle S_{xx}^g}{S_{xx}^g} = \frac{\triangle \epsilon_{xx}^g}{\epsilon_{xx}^g} - \frac{\triangle \sigma_{xx}}{\sigma_{xx}} \tag{2}$$

and the data were consistent with the first term on the right hand side being zero. Indeed in a further publication[11] it was assumed that this was precisely so and residual field dependent effects at the level of $< 1\%$ were analyzed in terms of WL corrections to S_{xx}^d.

For reasons which will be explained in detail in Section III, this experimental result is unexpected. Basically we will show that ϵ_{xx}^g is proportional to the electron-impurity scattering rate, $1/\tau_{ei}$, whereas S_{xx} is not. One can view WL to be a modification of τ_{ei} due to interference effects. Thus, we would have expected both ϵ_{xx}^g and σ_{xx} to be affected by WL, but not S_{xx}^g. This is just the opposite of that found by Syme et $al.$[9]

In view of these considerations, we have reexamined the question both experimentally and theoretically. The present paper gives new experimental data on two samples of 2DEGs in δ-doped GaAs/Ga$_{1-x}$Al$_x$As quantum wells. In contrast to the previous results, we find $\triangle S_{xx}^g \approx 0$ for $B \leq 1$ T, which implies $\triangle\sigma_{xx}/\sigma_{xx} = \triangle\epsilon_{xx}^g/\epsilon_{xx}^g$ (to an accuracy of $\sim 10\%$).

For several years we have been using various physical pictures of phonon drag thermopower to help in understanding experimental data on 2D and 3D systems in a qualitative way, e.g. Refs. 12 and 13. In Section III, we outline a theory of phonon drag thermopower which puts these ideas on a quantitative basis. In particular, it provides arguments to support the result that WL has no influence on phonon drag. However, we also show that the theory provides a useful framework to understand many previously published experimental and theoretical results in a transparent physical way. A brief, preliminary account of some of this work has been presented in conference form.[14]

II. EXPERIMENTAL TECHNIQUES AND RESULTS

Both our samples have been well characterized in previous work.[15] The electron densities are $n = 3.6 \times 10^{16}\,\mathrm{m}^{-2}$ for sample 1 (S1) and $n = 2.21 \times 10^{16}\,\mathrm{m}^{-2}$ for sample 2 (S2). Using the 4 K values of the conductivities, each has a mobility of $\mu \approx 0.12\,\mathrm{m}^2/\mathrm{Vs}$. Only a single subband is occupied in each sample.

The thermoelectric properties of S1 have been previously studied as a function of B over a wide temperature range.[16] At low temperatures, where S_{xx} for this device is dominated by phonon drag, no variation of S_{xx} with B up to about 1 T was seen to within the experimental accuracy of $\leq 2\%$ at low fields, even though changes $\approx 5 - 6\%$ of σ occurred due to localization effects.

In the present experiments a different cryostat was used with the sample mounting and associated instrumentation redesigned to increase the precision of the measurements by about an order of magnitude in the range 3-7K where phonon drag is dominant in S_{xx}, and also to allow the direction of the magnetic field to be set either perpendicular or parallel to

the 2DEG. The low temperature limit was chosen to avoid any problems at low magnetic fields caused by the superconducting/normal transition of In which is used to provide thermal contact between the sample and cold sink.

The experimental techniques are based on those published previously.[16] All data were taken by dc methods. Very slow field sweeps were used for ρ_{xx}, with a sample current for which self-heating was negligible. To measure the temperature and temperature gradient we used a matched pair of $22\,k\Omega$ Philips surface mount devices which have very high sensitivity at these temperatures and no observable magnetoresistance (even up to 8T). During a measurement of S_{xx}, B was held constant and the temperature gradient was repeatedly switched on and off by routing a fixed power either to the sample heater, which produced the temperature gradient, or to a matched heater mounted on the cold sink close to the sample. In this way the average power input to the sample stage remained constant and times to equilibrate were short. With temperature differences mostly in the range of $\sim$100-300mK we had a resolution for ∇T_x of $\leq 0.2\%$. It is important to note that ∇T_x was independently measured at each value of B. Thermoelectric voltages of 3.5-$22\,\mu V$ were measured with a resolution of a few nV by an EM 11 nanovoltmeter. The voltages were reproducible to $\leq 0.1\%$ when ∇T_x was switched on and off at fixed T.

During the measurement of S_{xx} the average sample T was found to increase slightly with B. The relative change was always largest at highest T; at 7.4K and 1.5T it reached a maximum of about 0.8% for S1, and 0.4% for S2. On lowering the field to 0.5T, the changes dropped to 0.2% for S1 and 0.1% for S2. These changes are similar for both perpendicular and parallel **B**. Each data point has been corrected to bring them all to the same temperature. Interestingly, when this is done, the results are essentially the same as if we had used the zero-field value for ∇T_x and the as-measured values for the thermoelectric voltages. This arises because the thermal conductivity of the substrate and S_{xx} have similar T dependences. The estimated relative error of S_{xx} as a function of B is typically $\sim 0.3\%$ and that of ρ_{xx} is $< 0.1\%$. There is a systematic overall uncertainty in S_{xx} of about 10% due to possible errors in thermometer and voltage contact spacing, but this should not depend on T or B.

The temperature dependence of S_{xx} for the two samples is shown in Fig. 1 along with S_{xx}^d which was measured for S1[16] but is estimated for S2. The increase in S_{xx} for S2 compared with S1 is due to the lower carrier density and somewhat higher substrate thermal conductivity. The data for S1 are in good agreement with those of previous measurements.[16]

The upper panels of Figs. 2 and 3 show data on the B dependence of $\triangle S_{xx}/S_{xx} = (S_{xx}(B) - S_{xx}(0))/S_{xx}(0)$ and $\triangle\sigma_{xx}/\sigma_{xx} = (\sigma_{xx}(B) - \sigma_{xx}(0))/\sigma_{xx}(0)$ (taking $\sigma_{xx} = 1/\rho_{xx}$) for the two samples. The plots are given in terms of $\ln B$ because if WL is dominant we expect $\sigma_{xx} \propto \ln B$ over a relatively wide field range. Although $\triangle\sigma_{xx}/\sigma_{xx}$ is about 6% and 10% respectively for the two samples, the variation of $\triangle S_{xx}/S_{xx}$ is roughly an order of magnitude smaller over the same field range, though it is not zero within experimental error. These data are contrary to those previously published[9] where $\triangle S_{xx}/S_{xx}$ showed essentially the same behaviour as $-\triangle\sigma_{xx}/\sigma_{xx}$.

Although there seems no doubt that WL is responsible for the behaviour of σ_{xx}, we briefly outline our analysis of the temperature and field dependence of these data to show that they do indeed behave as expected. We expect the Boltzmann value of σ_{xx} (which we write as $\pi\sigma_N k_F l_e$ where $\sigma_N = e^2/2\pi^2\hbar$, k_F is the magnitude of the Fermi wave vector and

l_e is the elastic mean free path) to be modified due to contributions from WL, $\Delta\sigma_{xx}^{WL}$, and electron-electron interaction, $\Delta\sigma_{xx}^{ee}$, of the form[1]

$$\sigma_{xx} = \pi\sigma_N k_F l_e + \Delta\sigma_{xx}^{WL} + \Delta\sigma_{xx}^{ee} = \sigma_N[\pi k_F l_e - (1+g)\ln(l_i/l_e)] \tag{3}$$

where $l_i = v_F\tau_i$ is the inelastic mean free path given in terms of the phase coherence time τ_i, and g is a measure of the strength of the electron-electron interaction. The temperature dependence arises from l_i. If one assumes $l_i \propto 1/T^p$, then $\sigma_{xx} = p(1+g)\sigma_N \ln T + \text{constant}$. Plots of σ_{xx} versus $\ln T$ over the range 2-9 K for both samples gave good straight lines with slopes of $1.93 \times 10^{-5}\,\Omega^{-1}$ for S1 and $1.85 \times 10^{-5}\,\Omega^{-1}$ for S2. From the analysis of the B dependence given below we find $p = 1.0$; these results imply $g = 0.57$ for S1 and 0.51 for S2, each $\sim \pm 10\%$.

Noting that $\Delta\sigma_{xx}^{ee}$ is B-independent for low fields, the B-dependence of σ_{xx} was fitted to the standard results of WL,[1] i.e.

$$\sigma_{xx} = \sigma_N\left[\pi k_F l_e - g\ln\left(\frac{l_i}{l_e}\right) + \psi\left(\frac{1}{2} + \frac{\hbar}{2eBl_i l_e}\right) - \psi\left(\frac{1}{2} + \frac{\hbar}{2eBl_e^2}\right)\right] \tag{4}$$

where ψ is the digamma function. Using an expression for the conductivity in this form puts a strong constraint on the value for l_e because it is required to fit simultaneously both the Boltzmann term and the correction terms. The usual practice has been to fit only the field dependent part in isolation. As shown in Figs. 2 and 3 this expression gives excellent fits to our experimental data in the range $-0.05\,\text{T} < B < 0.05\,\text{T}$, but at higher fields the fits become worse and are no longer acceptable for $|B| > 0.1\,\text{T}$. This agrees with expectations since the field dependence in Eq. (4) is accurate only when $2eBl_e^2/\hbar \ll 1$,[17] which corresponds to $|B| \ll 0.25\,\text{T}$ in our case. Within experimental error $l_i = a/T$ and l_e is precisely constant (using fits in the range $-0.05\,\text{T} < B < 0.05\,\text{T}$); a and l_e are given in Table 1, along with $k_F l_e$. If electron-electron effects are ignored in the above ($g = 0$), the fitted parameters are changed only weakly.

Our samples are in the dirty limit with $\hbar/\tau_e \gg k_B T$, where k_B is the Boltzmann constant and τ_e the impurity relaxation time. Under these conditions, Altshuler *et al.*[18] have predicted that l_i should be given by

$$\frac{1}{l_i} = \frac{k_B T}{2\epsilon_F l_e}\ln(\epsilon_F \tau_e \hbar) \tag{5}$$

where ϵ_F is the Fermi energy and τ_e is the electronic elastic relaxation time. This expression yields $l_i = 51/T\,\mu\text{m}$ for S1 and $31/T\,\mu\text{m}$ for S2, which are to be compared with the experimental values of $14/T\,\mu\text{m}$ and $21/T\,\mu\text{m}$. The agreement is certainly not as good as that obtained by Choi *et al.*[19] with GaAs heterojunctions, but the order of magnitude is correct, especially for S2. All these results confirm that our samples are dominated by WL, at least insofar as the B dependence of σ_{xx} is concerned.

In the previous work[9] data were also taken with the magnetic field parallel to the 2DEG, where no change is expected nor observed for σ_{xx}, and no change was seen for S_{xx}. We have also done this and typical data on S_{xx} (with $\mathbf{B} \parallel y$) are shown in the lower panels of Figs. 2 and 3 for both samples. Any changes are now at the level 0.2% which is within the range of the expected uncertainties.

5

III. THEORY

In keeping with the experimental situation, we shall restrict our attention to the case of a 2DEG in which the electrons occupy a single subband and have an isotropic energy dispersion. The generalization to multiple subbands is straightforward but will not be considered in this paper. Our treatment of the transport properties is based on the semiclassical Boltzmann equation and is similar to the approaches used by Butcher's group[20] and Lyo.[21] It is useful to provide a brief but complete summary of the theory in order to clearly reveal the underlying approximations that are made. In addition, our intention is to bring out various features of the theory, not immediately evident in the previous theoretical work, which give useful physical insights into the general behaviour of the phonon drag component of the thermopower in degenerate semiconductors. A result of particular interest is the definition of an effective electric field which accounts for the effect of a nonequilibrium flux of phonons on the electrons. Once these semiclassical results have been established, we indicate the way in which the theory can be extended to include the effects of weak localization.

In the presence of static electric and magnetic fields, the electron distribution function $f_{\mathbf{k}}(\mathbf{r})$ satisfies the steady-state equation

$$\mathbf{v_k} \cdot \nabla f_{\mathbf{k}} - \frac{e}{\hbar} \left(\mathbf{E} + \mathbf{v_k} \times \mathbf{B} \right) \cdot \nabla_{\mathbf{k}} f_{\mathbf{k}} = \left. \frac{\partial f_{\mathbf{k}}}{\partial t} \right|_{imp} + \left. \frac{\partial f_{\mathbf{k}}}{\partial t} \right|_{ph} \tag{6}$$

where the terms on the right hand side account for impurity and phonon scattering. The impurity scattering term is given by

$$\left. \frac{\partial f_{\mathbf{k}}}{\partial t} \right|_{imp} = \sum_{\mathbf{k}'} W_{\mathbf{k}\mathbf{k}'} \left[f_{\mathbf{k}'}(1 - f_{\mathbf{k}}) - f_{\mathbf{k}}(1 - f_{\mathbf{k}'}) \right] \tag{7}$$

where $W_{\mathbf{k}\mathbf{k}'}$ is an energy conserving impurity scattering rate. Similarly, the phonon scattering term is given by[20]

$$\left. \frac{\partial f_{\mathbf{k}}}{\partial t} \right|_{ph} = \sum_{\mathbf{k}'} \left[f_{\mathbf{k}'}(1 - f_{\mathbf{k}}) P_{\mathbf{k}'\mathbf{k}} - f_{\mathbf{k}}(1 - f_{\mathbf{k}'}) P_{\mathbf{k}\mathbf{k}'} \right] \tag{8}$$

with

$$P_{\mathbf{k}\mathbf{k}'} = \frac{2\pi}{\hbar} \sum_{Q} |M_{\mathbf{k}\mathbf{k}'}(Q)|^2 \left[N_Q \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} - \hbar\omega_Q) + (N_{-Q} + 1)\delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} + \hbar\omega_Q) \right]. \tag{9}$$

The variable Q represents both the three-dimensional phonon wavevector $\mathbf{Q}$ with components parallel ($\mathbf{Q}_{\parallel}$) and perpendicular ($Q_{\perp}$) to the 2DEG, and polarization index λ; ω_Q is the phonon frequency and N_Q is the nonequilibrium phonon occupation number. The term proportional to N_Q corresponds to phonon absorption whereas the $(N_{-Q} + 1)$ term corresponds to emission. The electron-phonon matrix element has the property $M_{\mathbf{k}\mathbf{k}'}(Q) = M^*_{\mathbf{k}'\mathbf{k}}(-Q)$ and is proportional to $\delta_{\mathbf{k}',\mathbf{Q}_{\parallel}+\mathbf{k}}$, due to momentum conservation in the plane of the 2DEG, and to the subband matrix element $I(Q_{\perp}) \equiv \int \exp(iQ_{\perp}z)|\phi(z)|^2 dz$. The detailed form of $M_{\mathbf{k}\mathbf{k}'}(Q)$ depends on the nature of the interaction; the acoustic phonon deformation potential and piezoelectric interactions are relevant to the GaAs/GaAlAs systems studied here.

Finally it should be noted that electronic screening of the electron-phonon interaction is included by dividing the bare electron-phonon matrix element by the dielectric function $\epsilon(\mathbf{Q}_\parallel)$ of the 2DEG.

The solution to Eq. (6) is obtained by linearizing about a local equilibrium (le) Fermi distribution, i.e.

$$f_\mathbf{k}(\mathbf{r}) = f_\mathbf{k}^{le}(T(\mathbf{r}), \mu(\mathbf{r})) - \frac{\partial f_\mathbf{k}^0}{\partial \varepsilon_\mathbf{k}} \Phi_\mathbf{k} \,, \tag{10}$$

where $T(\mathbf{r}) = T + \delta T(\mathbf{r})$ is the local temperature and $\mu(\mathbf{r}) = \mu + \delta\mu(\mathbf{r})$ is the local chemical potential. The thermodynamic equilibrium distribution at temperature T and chemical potential μ will be denoted by $f_\mathbf{k}^0 = (e^{\beta(\varepsilon_\mathbf{k} - \mu)} + 1)^{-1}$. The deviation from local equilibrium is written in the form shown for later convenience. Similarly, the phonon distribution is expanded as

$$N_Q = N_Q^{le} - \frac{\partial N_Q^0}{\partial(\hbar\omega_q)} G_Q \,, \tag{11}$$

where $N_Q^0 = (e^{\beta\hbar\omega_Q} - 1)^{-1}$ is the equilibrium Bose distribution at temperature T. We shall assume that the nonequilibrium phonon distribution G_Q is established by a temperature gradient and is insensitive to the interactions of the phonons with the electronic system. However, phonon-drag thermopower arises directly from the interaction of the electrons with the nonequilibrium distribution of phonons.

Substituting these forms of the electron and phonon distributions into Eq. (6), we obtain the linearized equation (details of the derivation of the phonon scattering terms can be found in the work of Cantrell and Butcher[20])

$$\mathbf{v}_\mathbf{k} \cdot \nabla T \frac{\partial f_\mathbf{k}^0}{\partial T} - e\mathbf{v}_\mathbf{k} \cdot \mathbf{E}' \frac{\partial f_\mathbf{k}^0}{\partial \varepsilon_\mathbf{k}} + \frac{e}{\hbar} \mathbf{v}_\mathbf{k} \times \mathbf{B} \cdot \nabla_\mathbf{k} \Phi_\mathbf{k} \frac{\partial f_\mathbf{k}^0}{\partial \varepsilon_\mathbf{k}} = \left.\frac{\partial f_\mathbf{k}}{\partial t}\right|_{imp} + \left.\frac{\partial f_\mathbf{k}}{\partial t}\right|_{ph} \tag{12}$$

where

$$\left.\frac{\partial f_\mathbf{k}}{\partial t}\right|_{imp} = -\sum_{\mathbf{k}'} W_{\mathbf{k}\mathbf{k}'} \left[\Phi_{\mathbf{k}'} - \Phi_\mathbf{k}\right] \frac{\partial f_\mathbf{k}^0}{\partial \varepsilon_\mathbf{k}} \tag{13}$$

and

$$\left.\frac{\partial f_\mathbf{k}}{\partial t}\right|_{ph} = \beta \sum_{\mathbf{k}',Q} \{\Phi_{\mathbf{k}'} - \Phi_\mathbf{k}\} \left[\Gamma_{\mathbf{k}\mathbf{k}'}(Q) + \Gamma_{\mathbf{k}'\mathbf{k}}(Q)\right] - \beta \sum_{\mathbf{k}',Q} G(Q) \left[\Gamma_{\mathbf{k}\mathbf{k}'}(Q) - \Gamma_{\mathbf{k}'\mathbf{k}}(Q)\right] \tag{14}$$

with

$$\Gamma_{\mathbf{k}\mathbf{k}'}(Q) = \frac{2\pi}{\hbar} |M_{\mathbf{k}\mathbf{k}'}(Q)|^2 f_\mathbf{k}^0 (1 - f_{\mathbf{k}'}^0) N_Q^0 \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_\mathbf{k} - \hbar\omega_Q) \,. \tag{15}$$

The effective electric field $\mathbf{E}'$ in Eq. (12) is now the gradient of the electrochemical potential, $\mathbf{E} + \nabla\mu/e$. The first sum on the right hand side of Eq. (14), which we call $\partial f_\mathbf{k}/\partial t|_{ph,1}$, represents the scattering of the nonequilibrium distribution of electrons from the thermal distribution of phonons which has the effect of equilibrating the electrons to the lattice. The

7

second sum, $\partial f_{\mathbf{k}}/\partial t|_{ph,2}$, accounts for the effect of the nonequilibrium phonons in driving the electrons out of equilibrium. When considered in isolation, the first sum is responsible for the resistivity due to phonon scattering. However, for the samples of interest here, impurity scattering is the dominant scattering mechanism at low temperatures and this term can be neglected in comparison to Eq. (13). The equilibrium-phonon scattering term is nevertheless of interest as will be discussed later.

We now concentrate on the nonequilibrium phonon scattering term, $\partial f_{\mathbf{k}}/\partial t|_{ph,2}$, which is responsible for phonon-drag thermopower. It depends on the nonequilibrium phonon distribution which is obtained from the phonon kinetic equation

$$\mathbf{v}_{ph}(Q) \cdot \nabla N_Q = \left.\frac{\partial N_Q}{\partial t}\right|_{coll} \equiv -\frac{1}{\tau_{ph}}\left(N_Q - N_Q^{le}\right) , \tag{16}$$

where the phonon relaxation time τ_{ph} is a parameter characterizing boundary and impurity scattering. In principle it is a function of Q but we will take it to be a simple Q-independent constant which can be obtained from the phonon thermal conductivity. With our earlier definition of G_Q, Eq. (16) implies that

$$G_Q = -\frac{\hbar\omega_Q}{T}\tau_{ph}\mathbf{v}_{ph}(Q) \cdot \nabla T . \tag{17}$$

Within an isotropic Debye model, the phonon velocity $\mathbf{v}_{ph}(Q) = s_\lambda \hat{\mathbf{Q}}$ has distinct values for the longitudinal and transverse modes.

The distribution function in Eq. (17) has the property $G(Q) = -G(-Q)$, which is the asymmetry one would expect to see in the presence of a temperature gradient. Using this property, the nonequilibrium phonon scattering term in Eq. (14) can be written in the form

$$\left.\frac{\partial f_{\mathbf{k}}}{\partial t}\right|_{ph,2} = -\beta \sum_{\mathbf{k}',Q} G(Q)\left[\Gamma_{\mathbf{k}\mathbf{k}'}(Q) + \Gamma_{\mathbf{k}'\mathbf{k}}(-Q)\right] , \tag{18}$$

and substituting Eq. (17) into Eq. (18), we find

$$\left.\frac{\partial f_{\mathbf{k}}}{\partial t}\right|_{ph,2} = \frac{1}{k_B T^2} \sum_{\mathbf{k}',Q} (\hbar\tau_{ph}s_\lambda^2)\nabla T \cdot \mathbf{Q}_\parallel \left[\Gamma_{\mathbf{k}\mathbf{k}'}(Q) + \Gamma_{\mathbf{k}'\mathbf{k}}(-Q)\right] . \tag{19}$$

The appearance of the in-plane projection $\mathbf{Q}_\parallel$ in the sum assumes that the temperature gradient is in the plane of the 2DEG.

To simplify Eq.(19) it is convenient to make use of the following identities

$$f^0(\varepsilon)[1 - f^0(\varepsilon + \hbar\omega)]N^0(\omega) = -\frac{1}{\beta}\frac{\partial f^0}{\partial\varepsilon}\left[N^0(\omega) + f^0(\varepsilon + \hbar\omega)\right]$$

$$f^0(\varepsilon - \hbar\omega)[1 - f^0(\varepsilon)]N^0(\omega) = -\frac{1}{\beta}\frac{\partial f^0}{\partial\varepsilon}\left[N^0(\omega) + 1 - f^0(\varepsilon - \hbar\omega)\right] , \tag{20}$$

for the thermal factors appearing in the scattering rates $\Gamma_{\mathbf{k}\mathbf{k}'}(Q)$ defined in Eq. (15). With these results, together with the momentum conservation condition $\mathbf{Q}_\parallel = \mathbf{k}' - \mathbf{k}$, Eq.(19) can be written in the form

8

$$\left.\frac{\partial f_{\mathbf{k}}}{\partial t}\right|_{ph,2} = \frac{\partial f_{\mathbf{k}}^0}{\partial \varepsilon_{\mathbf{k}}} \sum_\lambda \frac{m^* s_\lambda \Lambda_\lambda}{\tau_{ep}^\lambda(\varepsilon)} \mathbf{v_k} \cdot \frac{\nabla T}{T} , \tag{21}$$

where $\Lambda_\lambda = s_\lambda \tau_{ph}$ is the phonon mean free path and $\tau_{ep}^\lambda(\varepsilon)$ is an energy-dependent electron-phonon relaxation time defined as

$$\frac{1}{\tau_{ep}^\lambda(\varepsilon)} = \frac{2\pi}{\hbar} \sum_{\mathbf{k}',\mathbf{Q}} \left(1 - \frac{k'}{k}\cos\theta\right) |M_{\mathbf{k}\mathbf{k}'}(Q)|^2$$
$$\times \left\{\left[N_Q^0 + f^0(\varepsilon')\right]\delta(\varepsilon' - \varepsilon - \hbar\omega_Q) + \left[N_Q^0 + 1 - f^0(\varepsilon')\right]\delta(\varepsilon' - \varepsilon + \hbar\omega_Q)\right\} . \tag{22}$$

Here, θ is the scattering angle between the initial wavevector $\mathbf{k}$ and the final wavevector $\mathbf{k}'$ and we have introduced the short-hand notation $\varepsilon \equiv \varepsilon_{\mathbf{k}}$ and $\varepsilon' \equiv \varepsilon_{\mathbf{k}'}$. Provided that Eq.(17) is an accurate representation of the nonequilibrum phonon distribution, Eq.(21) is a rigorous result for the phonon collision integral.

If we now define the effective electric field

$$\mathbf{E}_{ph}(\varepsilon) = \sum_\lambda \frac{m^* s_\lambda \Lambda_\lambda}{e\tau_{ep}^\lambda(\varepsilon)} \frac{\nabla T}{T} , \tag{23}$$

Eq. (21) can be written in the suggestive form

$$\left.\frac{\partial f_{\mathbf{k}}}{\partial t}\right|_{ph,2} = e\mathbf{v_k} \cdot \mathbf{E}_{ph}(\varepsilon)\frac{\partial f_{\mathbf{k}}^0}{\partial \varepsilon_{\mathbf{k}}} . \tag{24}$$

In the context of the original Boltzmann equation in Eq. (12), the electron-phonon scattering term given by Eq. (24) (as stated earlier, we neglect the $\partial f_{\mathbf{k}}/\partial t|_{ph,1}$ term) can be grouped together with the term in Eq. (12) arising from the actual electric field $\mathbf{E}'$. In other words, the phonon-drag field, $\mathbf{E}_{ph}(\varepsilon)$, proportional to the temperature gradient, has exactly the same effect on the nonequilibrium electron distribution as the actual electric field. Its dependence on energy through the electron-phonon relaxation time is only an apparent complication. Since the dominant impurity scattering mechanism is elastic, the impurity Boltzmann equation can be solved at a given energy ε to define an energy-dependent impurity conductivity $\overleftrightarrow{\sigma}(\varepsilon; B)$. The contribution to the current density arising from the phonon-drag field can then be expressed in the form

$$\mathbf{J}_{ph} = \int d\varepsilon \left(-\frac{\partial f^0}{\partial \varepsilon}\right) \overleftrightarrow{\sigma}(\varepsilon; B) \, \mathbf{E}_{ph}(\varepsilon) . \tag{25}$$

It can be shown that this result for the current density is equivalent to that given by Zianni et al.[22] and therefore leads to the same results for the thermoelectric tensor and phonon-drag thermopower.

Our interest here is in the low temperature properties of a degenerate 2DEG. Since the impurity conductivity at low magnetic fields depends only weakly on energy, it can be taken out of the integral in Eq.(25) and we simply obtain the energy average of the effective field,

$$\mathbf{J}_{ph} = \overleftrightarrow{\sigma}(B) \langle \mathbf{E}_{ph}\rangle , \tag{26}$$

where $\overleftrightarrow{\sigma}(B)$ is the measurable conductivity appearing in Eq. (1). In view of Eqs. (23) and (26), the thermoelectric tensor is

$$\overleftrightarrow{\epsilon^g}(B) = -\sum_\lambda \frac{m^* s_\lambda \Lambda_\lambda}{eT} \left\langle \frac{1}{\tau_{ep}^\lambda} \right\rangle \overleftrightarrow{\sigma}(B), \tag{27}$$

where the average lifetime is given by

$$\left\langle \frac{1}{\tau_{ep}^\lambda} \right\rangle \equiv \int d\varepsilon \left(-\frac{\partial f^0}{\partial \varepsilon} \right) \frac{1}{\tau_{ep}^\lambda(\varepsilon)}. \tag{28}$$

The phonon drag thermopower tensor, defined as $\overleftrightarrow{S^g}(B) = \overleftrightarrow{\sigma}^{-1}(B) \overleftrightarrow{\epsilon^g}(B)$, is therefore given by $\overleftrightarrow{S^g}(B) = S^g \overleftrightarrow{1}$ with

$$S^g = -\sum_\lambda \frac{m^* s_\lambda \Lambda_\lambda}{eT} \left\langle \frac{1}{\tau_{ep}^\lambda} \right\rangle. \tag{29}$$

We see that the phonon drag thermopower is a diagonal tensor *independent* of the magnetic field. This result is a direct consequence of the thermoelectric tensor being a scalar multiple of the conductivity. Equations (23) through (29) are the main results of the theory.

An equation similar to Eq. (29) was first given by Herring[23] for 3D non-degenerate semiconductors, and is explained in terms of the following physical picture. The non-equilibrium phonons preferentially flow down the temperature gradient ∇T carrying a crystal momentum current $\propto \Lambda \nabla T$. Scattering of electrons by phonons leads to momentum being transferred to the electrons at a rate proportional to $1/\tau_{ep}$, where τ_{ep} is the e-p momentum relaxation time. This acceleration of the electrons proceeds for a mean time determined by the e-i momentum relaxation time τ_{ei}, (we are assuming, as above, that $\tau_{ep} \ll \tau_{ei}$) at which point impurity scattering randomizes the momentum. The mean drift velocity of the electrons established in this way gives rise to the electric current contribution $\mathbf{J}_{ph}$. Thus $\mathbf{J}_{ph} = -\epsilon^g \nabla T \propto (\tau_{ei}/\tau_{ep}) \Lambda \nabla T$. Since the thermopower is measured with $\mathbf{J} = 0$, a compensating drift current, $\mathbf{J}_\sigma = \sigma \mathbf{E}$, is established. With $\mathbf{J}_{ph} + \mathbf{J}_\sigma = 0$ and $\sigma \propto \tau_{ei}$, the induced electric field is $\mathbf{E} \propto \Lambda \nabla T/\tau_{ep}$, and S^g has the form given in Eq. (29).

Before proceeding, we would like to indicate the usefulness of the results we have obtained so far. In the Appendix we show that the electron-phonon relaxation time appearing in Eq. (29) is closely related to the electron-phonon transport lifetime that arises in the context of the phonon-limited mobility. The latter is defined by $\mu_{ep} \equiv e\langle 1/\tau_{tr} \rangle^{-1}/m^*$ where the transport lifetime is given in Eq. (A4) of the Appendix. At low temperatures, $\langle 1/\tau_{tr}^\lambda \rangle$ and $\langle 1/\tau_{ep}^\lambda \rangle$ are essentially the same, in which case the phonon-drag thermopower can be expressed as $S^g \simeq -\sum_\lambda (s_\lambda \Lambda_\lambda / \mu_{ep}^\lambda T)$ with $\mu_{ep}^\lambda = e\langle 1/\tau_{tr}^\lambda \rangle^{-1}/m^*$. This result has been applied to degenerate 2DEGs (e.g. see Ref. 24), but it has been viewed as a semi-quantitative result, which indeed is the case for non-degenerate semiconductors. The fact that it is quantitatively accurate for the degenerate case (at least in the low temperature limit) was discovered empirically in recent experimental work[13] concerned with the thermopower of 2DEGs and composite fermions, and was used there to evaluate μ_{ep} for these systems. The importance of this connection is that the contribution to the resistivity from phonon scattering can be very small and difficult to measure at low temperatures, even in high mobility systems.

Through Eq. (29), the thermopower provides an alternate way to measure μ_{ep} which can be much more accurate.

Equation (29) shows that the longitudinal phonon drag thermopower S_{xx}^g should be independent of magnetic field, and that the phonon drag contribution to the Nernst-Ettingshausen coefficient S_{yx}^g should be zero. These results are contained in the theoretical work of Zianni et $al.$[22] but were presented in a form which is not as physically transparent as that given here. The field-independence of S_{xx}^g has been demonstrated experimentally in a number of 2DEG systems, e.g. Refs. 16 and 25, though it is also evident in much of the earlier work. However in contrast to the theoretical prediction, it was found that S_{yx}^g is not zero for the same systems. Butcher and Tsaousidou[26] have suggested anisotropy of the electrons and phonons as a possible reason for this discrepancy, but this explanation would also imply some field variation of S_{xx}^g.

Although the above results were derived with 2DEGs in mind, it should be emphasized that they are equally valid in three dimensions when the same physical conditions prevail. In Eq. (19) for example, the summation over $\mathbf{k}'$ is simply interpreted as a three-dimensional sum, all electron-phonon matrix elements are evaluated using three-dimensional plane wave states and $\mathbf{Q}_\parallel$ is replaced by the total phonon wave vector $\mathbf{Q}$. The subsequent analysis leading to the thermopower in Eq. (29) remains unchanged. It was recently demonstrated experimentally that $S_{yx}^g = 0$ to high accuracy in a degenerate 3D semiconductor, even when S_{xx}^g is completely dominant in the longitudinal case,[12] and that S_{xx}^g is independent of magnetic field. In this respect, the 3D situation conforms even more closely to the theoretical predictions.

We now turn to the aspect of immediate concern in this paper, that of WL. The weak field dependence shown by our experimental results for the thermopower is consistent with the conclusions reached above on the basis of the semiclassical Boltzmann equation, even though the behaviour of the conductivity in our samples is manifestly nonclassical. We argue that these observations have a natural explanation in terms of a generalized Boltzmann equation developed by Hershfield and Ambegaokar.[27] These authors showed that the effects of coherent backscattering can be included with the addition of an extra term in the semiclassical Boltzmann equation. Although their final results are derived in the absence of a magnetic field, they claim that their approach can be extended to include a magnetic field and that all the standard WL results for the magnetoresistance can be reproduced. The significance of this is that WL effects are in principle accessible within an otherwise semiclassical approach.

We accept the argument that a suitable modification of the impurity scattering term in Eq. (6) can be made which, in the absence of phonon scattering, leads to a conductivity with WL effects included, and now consider the additional effects of phonon scattering. This is one inelastic scattering mechanism contributing to the phase-coherence lifetime τ_i appearing in the WL correction, though electron-electron scattering is usually the dominant mechanism at low temperatures. Of interest here are the additional effects of phonon collisions which impart momentum to the electrons and induce a nonequilibrium electron distribution.

Syme et $al.$[11,28] argued that, since phonon scattering events are phase disrupting, and since phonon drag originates in these events, ϵ_{xx}^g will not exhibit WL effects. We believe that this is incorrect. Within a semiclassical description of the electron dynamics, we have shown that the effect of phonon collisions is equivalent to an effective electric field in so far

as the subsequent current response is concerned. Whether the impulse is provided by an electric field or a phonon collision, the induced current is limited by impurity scattering and is proportional to the impurity conductivity in either case. The same conclusion should also apply in the WL regime when $\tau_{ep} \gg \tau_{ei}$. Once an impulse has been applied, an electron propagates in the presence of the impurities and the full effects of quantum interference are operative up to a time determined by τ_i. It is important to emphasize that the frequency of electron-phonon scattering events is unchanged when the phonon distribution is displaced from equilibrium by the temperature gradient. This means that the frequency of phase disrupting events is unchanged and the relevant conductivity determining the current response is the one including the WL effects. Thus with the use of the generalized Boltzmann equation to incorporate WL effects, our final results for the phonon drag thermoelectric tensor given in Eq. (27) should still be valid with the simple replacement of the conductivity by the one including weak-localization corrections.

As a final comment, we note that the above argument used for phonon drag can also be made in the case of the diffusion thermopower. The first term on the left hand side of Eq. (12) gives rise to the diffusion current

$$\mathbf{J}_d = \frac{1}{e} \int d\varepsilon \left(-\frac{\partial f^0}{\partial \varepsilon} \right) (\varepsilon - \mu) \overleftrightarrow{\sigma}(\varepsilon; B) \frac{\nabla T}{T} . \tag{30}$$

Using a conductivity with WL corrections in this expression leads to the accepted form of the diffusion thermopower which, in contrast to the phonon-drag thermopower, does exhibit WL corrections.[4−7]

To summarize, we have argued that the role of phonon drag in establishing an electric current is equivalent to an applied electric field in the impurity-dominated regime, with the consequence that the phonon drag thermoelectric tensor is proportional to the impurity conductivity. The WL corrections within the conductivity also appear in the thermoelectric tensor, resulting in a cancellation of WL effects in phonon drag thermopower. These results are essentially in accord with the present experimental data.

IV. DISCUSSION

The origin of the discrepancies between the present and previous work with regard to S_{xx}^g in a perpendicular magnetic field is not clear. The fact that the previous data were obtained on a MOSFET and the present data are for GaAs/Ga$_{1-x}$Al$_x$As quantum wells should be irrelevant if WL is the cause of the field dependence of S_{xx}^g. This in itself is significant because if the differences are real, it implies that something other than weak localization is involved in at least one of these experiments. One possibility might be that the previous samples had $k_F l_e \sim 2 - 4$, though WL theory is expected to be valid only for $k_F l_e \gg 1$. The present samples have $k_F l_e > 10$ (Table 1). Much less is known, both experimentally and theoretically, about the situation at small $k_F l_e$ where the system is approaching strong localization. Ashe et al.[29] have probed σ_{xx} in this limit but were unable to find satisfactory agreement with WL, even with extensions to the theory. How phonon drag might behave in this limit is unknown.

There are also some features of the experimental technique used in the previous work that might be cause for concern. Syme et al.[9,11,28] used an ac method for which the temperature

difference could be measured only when $B = 0$ and it was assumed to remain unchanged at finite B. Also, although constantan potential leads were used to minimize heat losses when thermal conductivity was measured, these were replaced by Cu leads for S_{xx}. The authors argued that this gives isothermal conditions in the direction transverse to the applied temperature gradient. However, the use of Cu leads implies a significant heat leak to the surroundings and unknown thermal boundary conditions on the 2DEG; this might be particularly important with an AC measurement where the heat flow is continuously changing.

Although the change we see in S_{xx}^g is much smaller than that in σ_{xx}, nevertheless it is outside experimental error, especially for S2, and we must ask how it arises. An obvious possibility is that it is due to diffusion. The effect of WL on S_{xx}^d, say ΔS_{xx}^d, is believed to be[4–7]

$$\frac{\Delta S_{xx}^d}{S_{xx}^d} \approx \frac{\ln(l_i/l_e)}{\pi k_F l_e}. \tag{31}$$

For our samples at $3\,\mathrm{K}$ ($7\,\mathrm{K}$), $\ln(l_i/l_e)/\pi k_F l_e = 0.08$ (0.06) for S1, and 0.12 (0.01) for S2. From Fig. 1 we see that for S1 (S2) the measurements show that S_{xx}^g/S_{xx}^d varies between about 25-70 (40-65) for the temperature range 3-7K. This suggests that changes due to this cause would be $\sim 0.3\%$ at 3K and $\sim 0.1\%$ at 7K. These estimates agree with the observations for S1, where $\Delta S_{xx}/S_{xx}$ is very small, but seem rather too low to account for the variations for S2. We would expect the effect to decrease as temperature increases thereby decreasing the relative importance of S_{xx}^d, but the experimental results are not unambiguous on this point. We also note that these effects might be much larger, and basically unknown, for the samples of Syme *et al.*[9,11] with $k_F l_e \sim 2 - 4$.

Another possibility is that we may be observing the effects of anisotropy in the electron and phonon system. As we noted in Section III, Butcher and Tsaousidou[26] have suggested this as the origin of the finite S_{yx}^g observed in 2DEGs. If this is correct, it will be accompanied by a field dependence of S_{xx}^g, and this should be essentially independent of temperature, not inconsistent with the observations.

V. CONCLUSIONS

Our measurements show that the longitudinal phonon drag thermopower S_{xx}^g is essentially independent of magnetic field, which implies that it is independent of weak localization, a result contrary to earlier work. We have given a theory of phonon drag based on the semiclassical Boltzmann approach which shows that S_{xx}^g is independent of magnetic field. Using the results of Hershfield and Ambegaokar[27], we have argued that this result remains valid in the presence of weak localization. Our theory also gives results which are useful in understanding the origin and behaviour of phonon drag thermopower in 2D and 3D degenerate systems.

ACKNOWLEDGMENTS

APPENDIX A:

It is apparent that Eq. (19) bears a strong similarity to the first phonon scattering term in Eq. (14) which is dependent on the nonequilibrium electron distribution. This similarity can be exploited to provide a relation between the phonon drag driving term given by Eq. (19) and the electron-phonon momentum relaxation time which determines the phonon-limited mobility. This latter quantity is obtained by solving the following transport equation

$$e\mathbf{v_k} \cdot \mathbf{E}\frac{\partial f_\mathbf{k}^0}{\partial \varepsilon_\mathbf{k}} = -\beta \sum_{\mathbf{k}',Q} \{\Phi_{\mathbf{k}'} - \Phi_\mathbf{k}\} \left[\Gamma_{\mathbf{k}\mathbf{k}'}(Q) + \Gamma_{\mathbf{k}'\mathbf{k}}(-Q)\right] , \tag{A1}$$

which describes the transport of electrons scattering exclusively from phonons, that is, in the absence of impurities. It should also be noted that we have not included a magnetic field or thermal gradient in this equation. This of course is not the physical situation of interest in Eq. (12), but rather represents an auxiliary problem which identifies a useful transport property that is relevant to the phonon-drag calculation. In addition, a discussion of Eq. (A1) will allow us to make contact with earlier work[30−32] on the calculation of electron-phonon mobilities.

The right hand side of Eq. (A1) follows from Eq. (14) with the replacement $Q \to -Q$ in the $\Gamma_{\mathbf{k}'\mathbf{k}}(Q)$ term. This puts it in the same form as the corresponding term in Eq. (19). To proceed with the solution of (A1), we make use of the identities in Eq.(20). The appearance of the Fermi function derivatives allows the transport equation to be simplified as

$$e\mathbf{v_k} \cdot \mathbf{E} = \frac{2\pi}{\hbar} \sum_{\mathbf{k}',Q} \{\Phi_{\mathbf{k}'} - \Phi_\mathbf{k}\} |M_{\mathbf{k}\mathbf{k}'}(Q)|^2 \left\{ \left[N_Q^0 + f^0(\varepsilon')\right] \delta(\varepsilon' - \varepsilon - \hbar\omega_Q) \right.$$
$$\left. + \left[N_Q^0 + 1 - f^0(\varepsilon')\right] \delta(\varepsilon' - \varepsilon + \hbar\omega_Q) \right\}. \tag{A2}$$

We now look for a solution to this equation which has the form $\Phi_\mathbf{k} = -e\mathbf{v_k} \cdot \mathbf{E}\tau_{tr}(\varepsilon)$. We shall refer to $\tau_{tr}(\varepsilon)$ as the electron-phonon transport lifetime to distinguish it from the electron-phonon relaxation time defined in Eq. (22). With this ansatz, Eq. (A2) becomes

$$1 = \frac{2\pi}{\hbar} \sum_{\mathbf{k}',Q} \left\{\tau_{tr}(\varepsilon) - \frac{k'\cos\theta}{k}\tau_{tr}(\varepsilon')\right\}|M_{\mathbf{k}\mathbf{k}'}(Q)|^2 \left\{ \left[N_Q^0 + f^0(\varepsilon')\right] \delta(\varepsilon' - \varepsilon - \hbar\omega_Q) \right.$$
$$\left. + \left[N_Q^0 + 1 - f^0(\varepsilon')\right] \delta(\varepsilon' - \varepsilon + \hbar\omega_Q) \right\}. \tag{A3}$$

This is an integral equation for $\tau_{tr}(\varepsilon)$ and is equivalent to Eq. (22) in Ref. [32]. The form of Eq. (A3) is obviously very similar to Eq. (22). It simplifies if we now make the quasielastic approximation[32] which assumes that the energies of interest are near the Fermi energy ε_F and $\hbar\omega_Q \ll \varepsilon_F$. We can then replace ε' by ε in the τ-dependent factor in Eq. (A3), and we obtain the following explicit expression for the relaxation time:

$$\frac{1}{\tau_{tr}(\varepsilon)} = \frac{2\pi}{\hbar} \sum_{\mathbf{k}',Q} (1 - \cos\theta)|M_{\mathbf{k}\mathbf{k}'}(Q)|^2$$
$$\times \left\{ \left[N_Q^0 + f^0(\varepsilon')\right] \delta(\varepsilon' - \varepsilon - \hbar\omega_Q) + \left[N_Q^0 + 1 - f^0(\varepsilon')\right] \delta(\varepsilon' - \varepsilon + \hbar\omega_Q) \right\} . \tag{A4}$$

It can be shown that this expression for $\tau_{tr}(\varepsilon)$ is identical to Eq. (39) in Ref. [32] where the quasielastic approximation is also invoked. The summation over Q includes the polarization

index λ so that $1/\tau_{tr}(\varepsilon) = \sum_\lambda 1/\tau_{tr}^\lambda(\varepsilon)$. It is clear that if the quasielastic approximation is also made in Eq. (22), $1/\tau_{ep}^\lambda(\varepsilon)$ and $1/\tau_{tr}^\lambda(\varepsilon)$ would in fact be identical. We expect any quantitative differences to be minimal at low tempertures where the two relaxation times can be used interchangeably.

Finally, we can make contact with the average electron-phonon scattering rate which was derived by Stormer et al.[31] on the basis of the work of Price[30]. If Eq. (A4) is averaged over energy with respect to the weight function $(-\partial f^0/\partial \varepsilon)$, we obtain the result

$$
\left\langle \frac{1}{\tau_{tr}} \right\rangle = \frac{2\pi}{\hbar} \sum_{\mathbf{k}',Q} (1 - \cos\theta) |M_{\mathbf{k}\mathbf{k}'}(Q)|^2 \beta \{ f^0(\varepsilon')[1 - f^0(\varepsilon' + \hbar\omega_Q)] N_Q^0
$$
$$
+ f^0(\varepsilon')[1 - f^0(\varepsilon' - \hbar\omega_Q)](N_Q^0 + 1) \} . \tag{A5}
$$

In arriving at this expression, we have again used the identities in Eq. (20) as well as the identity $f^0(\varepsilon - \hbar\omega)[1 - f^0(\varepsilon)]N^0(\omega) = f^0(\varepsilon)[1 - f^0(\varepsilon - \hbar\omega)](N^0(\omega) + 1)$. The result actually given by Stormer et al.[31] follows from Eq.(A5) by performing the $\mathbf{Q}_\parallel$ sum with the understanding that $k = k'$, so that $Q_\parallel = 2k'\sin(\theta/2)$. The phonon-limited mobility defined as $\mu_{ep} = e\langle 1/\tau_{tr}\rangle^{-1}/m^*$ was used by Stormer et al.[31] to interpret their observation of a transition into the Bloch-Grüneisen regime of the mobility of a 2DEG. Eq. (A5) is applicable at arbitrary temperatures, but a simpler expression can be obtained in the limit of low temperatures. Since the derivative of the Fermi function is then sharply peaked at the chemical potential μ, the average in Eq. (A5) may be replaced by the value at the chemical potential if $\tau_{tr}(\varepsilon)$ is a slowly varying function of energy. In this case we have (see Eq. (A4))

$$
\frac{1}{\tau_{tr}(\mu)} = \frac{2\pi}{\hbar} \sum_Q (1 - \cos\theta) \frac{|M(Q)|^2}{\sinh(\beta\hbar\omega_Q)} \left\{ \delta(\varepsilon_{\mathbf{k}+\mathbf{Q}_\parallel} - \mu - \hbar\omega_Q) + \delta(\varepsilon_{\mathbf{k}+\mathbf{Q}_\parallel} - \mu + \hbar\omega_Q) \right\} . \tag{A6}
$$

In obtaining this result, we have made use of the momentum conservation condition $|M_{\mathbf{k}\mathbf{k}'}(Q)|^2 = |M(Q)|^2 \delta_{\mathbf{k}',\mathbf{k}+\mathbf{Q}_\parallel}$ to perform the sum over $\mathbf{k}'$. The magnitude of $\mathbf{k}$ in Eq. (A6) is now restricted to the Fermi wavevector. A similar approximation in Eq. (28) would allow the average lifetime defined there to be replaced by Eq. (A6).

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TABLES

TABLE I. The various parameters for the samples. l_e and $l_i = a/T$ were obtained from the fits of the data to the field dependence of σ_{xx}.

	$n\,(10^{16}\mathrm{m}^{-2})$	$\mu\,(\mathrm{m}^2/\mathrm{Vs})$	$l_e\,(\mathrm{nm})$	$a\,(\mu\mathrm{m\,K})$	$k_F l_e$
S1	3.6	0.134	35	14	20
S2	2.21	0.144	42	21	13

FIGURES

FIG. 1. The open circles give the measured thermopower S_{xx} of the two samples. The lines are the diffusion components S_{xx}^d. In the case of S1, $S_{xx}^d = -0.41\,\mu\text{V/K}$ is a measured quantity taken from Ref. 13. For S2, $S_{xx}^d = -0.67\,\mu\text{V/K}$ is estimated from that of S1 assuming $S_{xx}^d \propto 1/n$.

FIG. 2. A selection of the experimental data on the field dependence of σ_{xx} ($\circ$) and S_{xx} ($\bullet$) at various temperatures for S1. The results are averages of $\pm B$ data, though any dependence on the direction of B is weak. The upper and lower panels give data with $\mathbf{B}$ perpendicular and parallel to the 2DEG respectively. For clarity the data have been offset vertically by multiples of 3%. The solid line through the data for σ_{xx} in the upper panel is a theoretical fit according to Eq. (4).

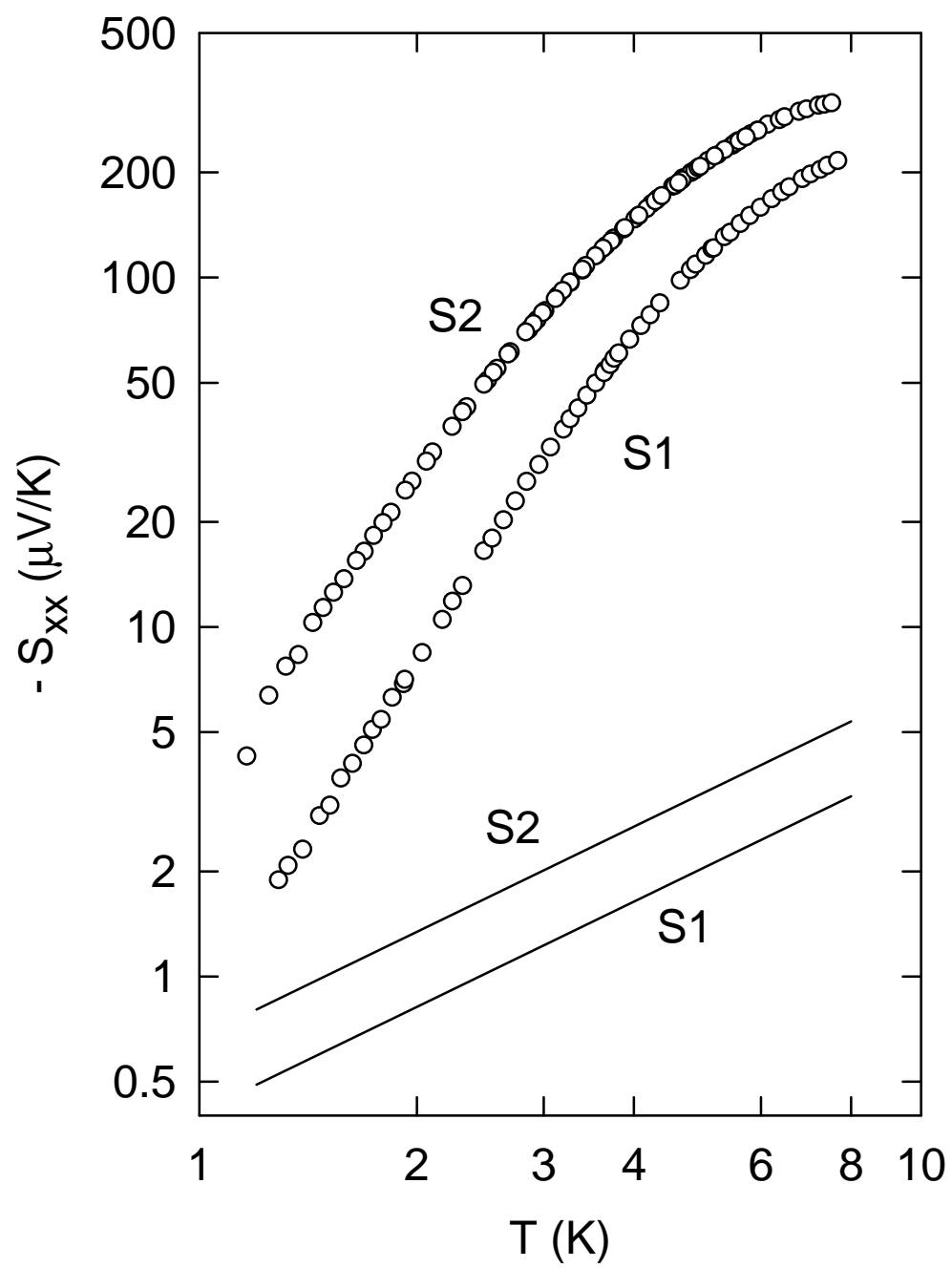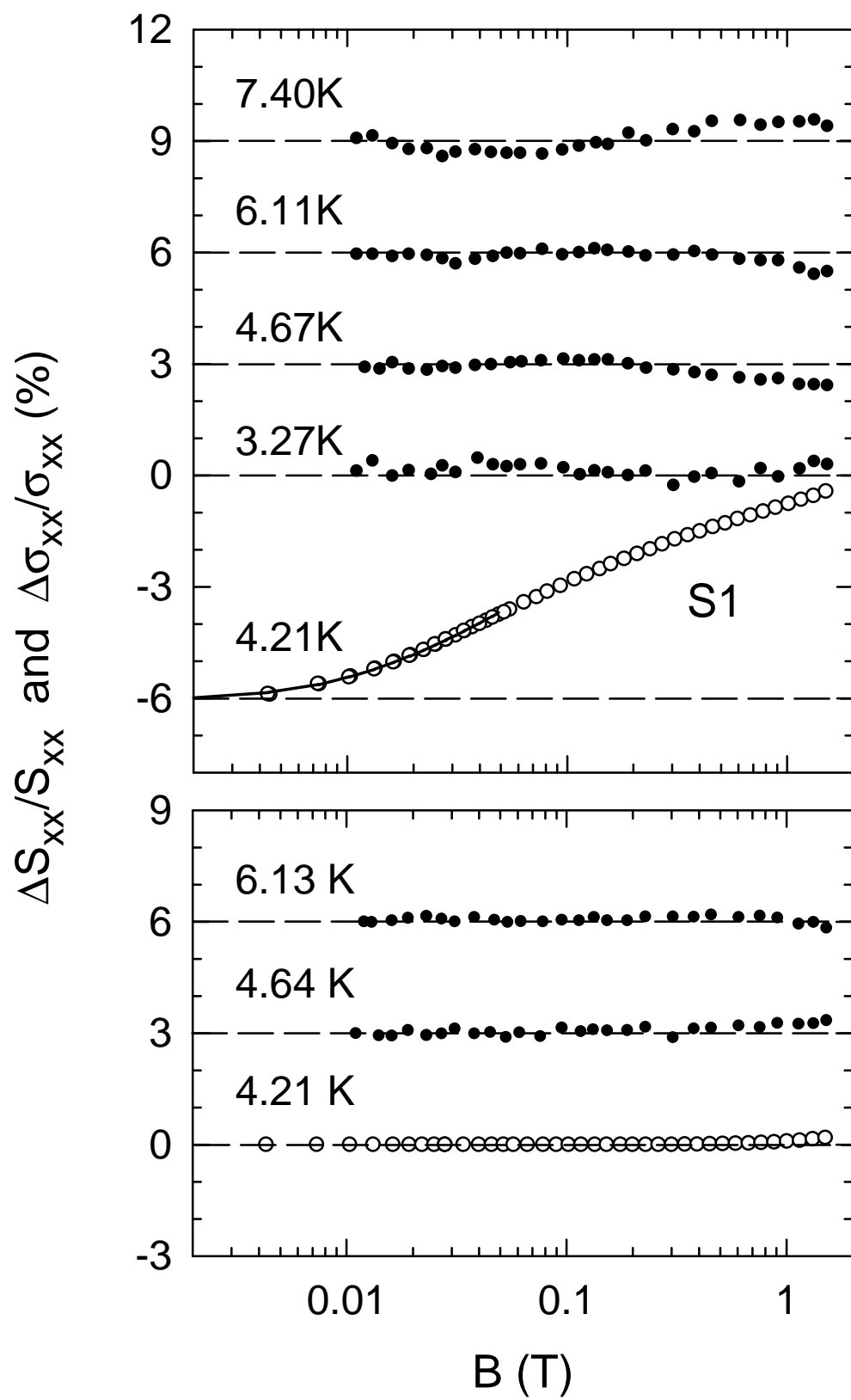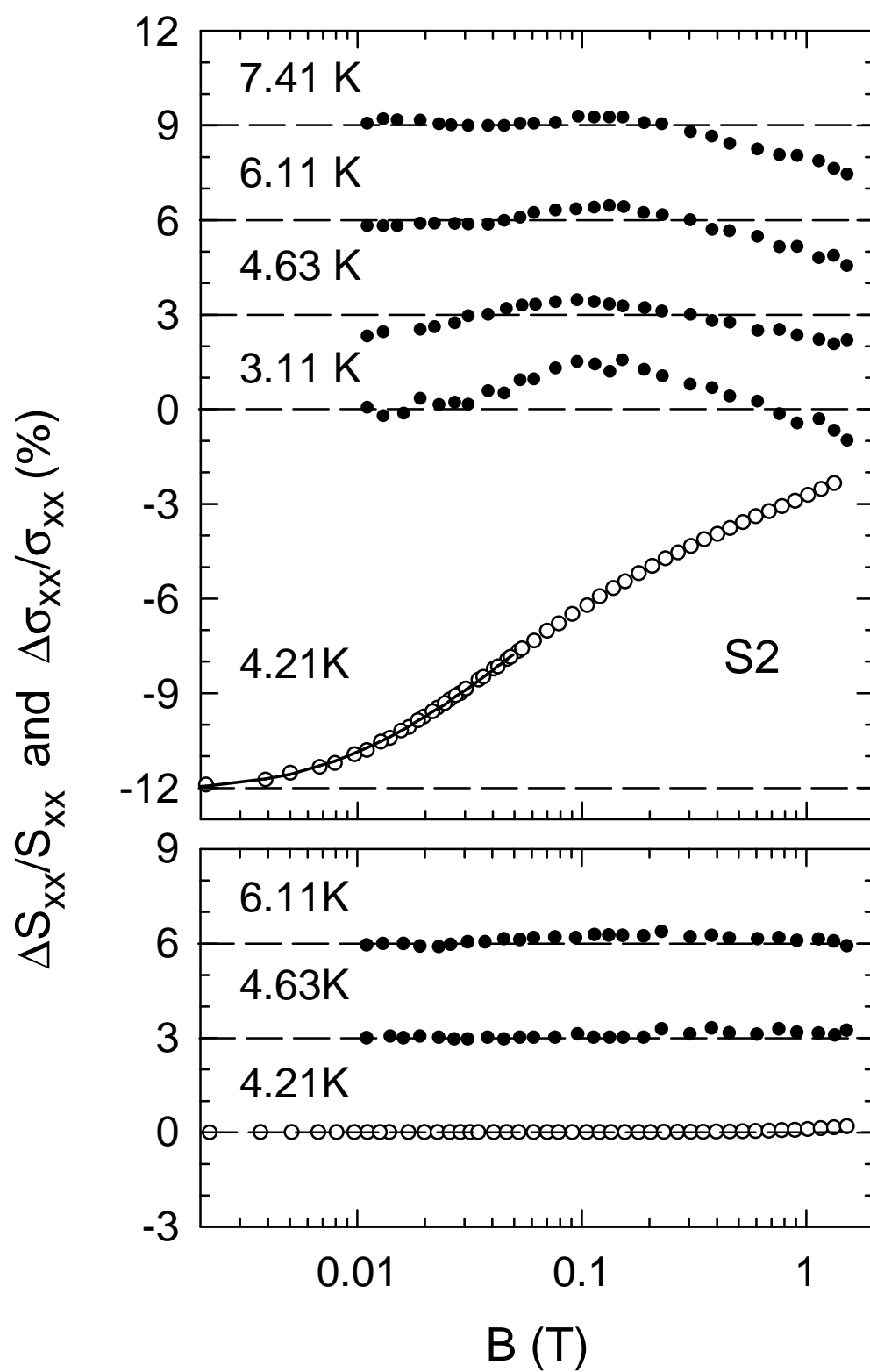FIG. 3. The same as Fig. 2 except these data are for S2.

Fig. 1

Fig. 2

$\Delta S_{xx}/S_{xx}$ and $\Delta\sigma_{xx}/\sigma_{xx}$ (%)

7.41 K
6.11 K
4.63 K
3.11 K
4.21K
S2

B (T)

6.11K
4.63K
4.21K

Fig. 3