Mon. Not. R. Astron. Soc. **000**, 000–000 (0000) Printed 2 February 2008 (MN LATEX style file v2.2)

Linearized perturbation on stationary inflow solutions in an inviscid and thin accretion disc

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2 February 2008

ABSTRACT
The influence of a linearized perturbation on stationary inflow solutions in an inviscid and thin accretion disc has been studied here, and it has been argued, that a perturbative technique would indicate that all possible classes of inflow solutions would be stable. The choice of the driving potential, Newtonian or pseudo-Newtonian, would not particularly affect the arguments which establish the stability of solutions. It has then been surmised that in the matter of the selection of a particular solution, adoption of a non-perturbative technique, based on a more physical criterion, as in the case of the selection of the transonic solution in spherically symmetric accretion, would give a more conclusive indication about the choice of a particular branch of the flow.

Key words: accretion, accretion discs – black hole physics – hydrodynamics – methods: analytical

1 INTRODUCTION

The object of this work has been to study the stability of the stationary inflow solutions in axially symmetric accretion, under the influence of a linearized perturbation. The chosen model has been that of an inviscid and thin accretion disc.

Use of this model, in which viscous effects have been ignored, has found a fairly widespread and consistent favour over the years (Abramowicz & Zurek 1981; Chakrabarti 1989; Molteni et al. 1996). As a consequence of such a physical prescription, there remains no mechanism for the outward transport of the angular momentum of the flow, and hence the specific angular momentum of the flow has been taken to be a constant of the motion. This naturally simplifies the analysis to a great extent, without significantly detracting from a true understanding of the underlying physics. The whole analysis has been based on the thin-disc configuration in accretion studies – a very well-known textbook model (Pringle 1981; Frank et al. 1992) that has been quite readily and regularly invoked by researchers in accretion astrophysics.

It has been argued here, that in an inviscid and thin accretion disc, the stationary inflow solutions of abiding interest, are all stable under the influence of a linearized time-dependent perturbation. The technique adopted has been similar to the one already existing for the case of spherically symmetric accretion (Petterson et al. 1980; Theuns & David 1992). Separate emphasis has been laid on discussing the stability of the so-called transonic inflow solution – a solution that passes through the critical point(s) of the flow, and whose velocity is subsonic very far away from the accretor, becoming supersonic as the accretor is approached. The subsonic inflows, a class of physically meaningful inflow solutions, the maximum of whose velocity always remains below the velocity of the inflow solution passing through the critical point(s), have been quite extensively considered as regards their stability, since, like the transonic inflow, the subsonic inflow solutions also obey the same outer boundary condition that the flow velocity should decrease very far away from the accretor. And it may also be pointed out that since the subsonic solutions do not pass through any singular point of the flow, the mathematical problem of studying their stability is much simpler compared with that of the transonic solution.

The classical Newtonian potential has been taken as the driving factor which effects the infall of matter, although it is easy to see that the use of the pseudo-Newtonian potential of Paczyński & Wiita (1980) – which satisfactorily represents general relativistic effects in a Newtonian framework and which is frequently in use to study the infall of matter under the gravitational influence of a black hole – alters nothing radically about the conclusions drawn from a linear stability analysis

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of the inflow solutions. What it changes, however, is the number of critical points of the flow. In the context of the use of the Paczyński-Wiita potential, mention may be made here of the fact that the number of critical points of the flow is also determined by various constants of the motion like accretion rate, specific angular momentum and the specific energy of the flow. It has been shown (Chakrabarti 1990) that for a given angular momentum of the flow, there exists a range of accretion rate, or equivalently a range of energy, such that the flow has three critical points. Of these, the two outer points are saddle points, flanking between themselves, a third centre-type point. The flow, however, has only one critical point for values of accretion rate and energy which do not lie within the given range that will develop three critical points.

In solutions which admit of more than one saddle-type critical point, the existence of shocks is a possibility (Chakrabarti 1990). For accretion on to a black hole, a solution would then pass through the outermost critical point (which is a saddle point), undergo a shock and then pass through the innermost critical point (again a saddle point) to cross the horizon with supersonic velocities. For accretion on to a neutron star, a flow passing through the inner critical point must undergo another shock to meet the inner boundary condition. The behaviour of these shocks are understood by invoking the Rankine-Hugoniot conditions (Chakrabarti 1989). Once again it is to be emphasized that the outer boundary condition for all kinds of physically meaningful inflow solutions – shocked or continuous – is that the bulk velocity should decrease to zero very far away from the accretor.

2 THE EQUATIONS OF THE FLOW

For the radial drift velocity v and density ρ, the flow is governed by Euler's equation and the continuity equation. The radial component of Euler's equation is given by

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial R} + \frac{1}{\rho}\frac{\partial P}{\partial R} + \frac{\partial V}{\partial R} - \frac{L^2}{R^3} = 0 \tag{1}$$

where L, which is a constant, is the specific angular momentum, and the Newtonian gravity potential $V = -GM/R$. The choice of the pseudo-Newtonian potential of Paczyński & Wiita (1980) would give $V = -GM/(R - r_g)$, in which r_g is the Schwarzschild radius of a black hole.

Use has been made of a polytropic equation of state

$$P = k\rho^{\gamma} \tag{2}$$

where γ is the polytropic exponent, whose admissible range ($1 < \gamma < 5/3$) is restricted by the isothermal limit and the adiabatic limit respectively.

A surface density Σ has been defined by vertically integrating ρ over the disc thickness H, and in the thin-disc approximation (Frank et al. 1992) this gives $\Sigma \cong \rho H$. The continuity equation, in terms of the surface density Σ, in the thin-disc approximation, is thus given by

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{R}\frac{\partial}{\partial R}\left(\Sigma v R\right) = 0 \tag{3}$$

Again invoking the thin-disc approximation, the disc thickness H is obtained from the vertical component of Euler's equation as

$$H \cong \frac{c_s}{(GM)^{1/2}} R^{3/2} \tag{4}$$

where c_s, the speed of sound, obeys the relation

$${c_s}^2 = \gamma k \rho^{\gamma - 1}. \tag{5}$$

The set of equations (1)–(5), defines the whole problem completely. From (1) and (2) is obtained

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial R} + k\gamma\rho^{\gamma-2}\frac{\partial \rho}{\partial R} + \frac{\partial V}{\partial R} - \frac{L^2}{R^3} = 0 \tag{6}$$

while (3),(4) and (5) give

$$\frac{\partial}{\partial t}\left[\rho^{(\gamma+1)/2}\right] + R^{-5/2}\frac{\partial}{\partial R}\left[\rho^{(\gamma+1)/2} v R^{5/2}\right] = 0 \tag{7}$$

The two equations (6) and (7) above are those that govern the flow in the inviscid and thin accretion disc.

3 THE PERTURBATION EQUATION

The steady-state solutions of (6) and (7) are v_0 and ρ_0, on which are imposed small first-order perturbations v' and ρ' respectively. Following a similar prescription by Petterson et al. (1980) for spherically symmetric accretion, it is convenient to introduce a new variable, $f \equiv \rho^{(\gamma+1)/2} v R^{5/2}$, which in physical terms is the mass flux. Its steady-state value f_0, as can be seen from (7), is a constant that is identified as the mass accretion rate. A linearized perturbation f', about f_0 is given by,

$$f' = {\rho_0}^{(\gamma-1)/2} R^{5/2} \left(v' \rho_0 + \frac{\gamma+1}{2} v_0 \rho' \right) \tag{8}$$

Linearizing in the perturbation variables v' and ρ', gives from (7), the expression,

$$\frac{\partial \rho'}{\partial t} + \frac{2}{\gamma+1} {\rho_0}^{-(\gamma-1)/2} R^{-5/2} \frac{\partial f'}{\partial R} = 0 \tag{9}$$

With the use of (9), successive differentiation of (8) with respect to time yields, first

$$\frac{\partial v'}{\partial t} = R^{-5/2} {\rho_0}^{-(\gamma+1)/2} \left(\frac{\partial f'}{\partial t} + v_0 \frac{\partial f'}{\partial R} \right) \tag{10}$$

and then

$$\frac{\partial^2 v'}{\partial t^2} = R^{-5/2} {\rho_0}^{-(\gamma+1)/2} \left[\frac{\partial^2 f'}{\partial t^2} + v_0 \frac{\partial}{\partial R} \left(\frac{\partial f'}{\partial t} \right) \right] \tag{11}$$

Now linearizing in terms of the perturbation variables, gives from (6) the equation,

$$\frac{\partial v'}{\partial t} + \frac{\partial}{\partial R} \left(v_0 v' + {c_{s0}}^2 \frac{\rho'}{\rho_0} \right) = 0 \tag{12}$$

where c_{s0} is the speed of sound in the steady state.

Differentiating (12) with respect to time, and using the conditions given by (9),(10) and (11) will finally deliver the perturbation equation as

$$\frac{\partial^2 f'}{\partial t^2} + 2 \frac{\partial}{\partial R} \left(v_0 \frac{\partial f'}{\partial t} \right) + \frac{1}{v_0} \frac{\partial}{\partial R} \left[v_0 \left({v_0}^2 - \frac{2}{\gamma+1} {c_{s0}}^2 \right) \frac{\partial f'}{\partial R} \right] = 0 \tag{13}$$

This expression bears a very close resemblance in form to a perturbation equation obtained by Theuns & David (1992), using similar methods, for the case of spherically symmetric accretion.

4 STABILITY ANALYSIS OF SOLUTIONS

Before taking up the stability analysis of solutions, as implied by (13), it would be worthwhile to have an understanding of the critical points of the flow. In the steady state, (6) and (7) can be combined to give the critical points – the required condition being that the numerator and the denominator of dv_0/dR vanish simultaneously at those points. This leads to the expressions

$$\begin{aligned} {v_0}^2 &= \frac{2}{\gamma+1} {c_{s0}}^2 \\ \frac{5}{\gamma+1} {c_{s0}}^2 &= \frac{GM}{R} - \frac{L^2}{R^2} \end{aligned} \tag{14}$$

The second equation above, being a quadratic in R, indicates that there would be two critical points for the flow. The choice of the pseudo-Newtonian potential of Paczyński & Wiita (1980), merely changes the number of critical points (Chakrabarti 1989), without altering the first of the two equations above.

It is seen that the transonic inflow which originates very far away from the accretor, with a very low subsonic velocity, will pass through the first point – which is a saddle point, and then tend to curl around the second point – which is a centre-type point. In case the accretor is a large and distended object like an ordinary star (which justifies the use of the conventional Newtonian potential in this analysis), it is quite likely then that while the flow solution tends to curl around the inner critical point, it would also meet the surface of the accretor. This, as will be discussed later, has important implications for the stability of the transonic inflow.

Returning now to (13), a solution of the form $f' = g(R) e^{-\mathrm{i}\Omega t}$ is chosen, in which $g(R)$ is the spatial part of the perturbation and Ω is real. This leads to the result

$$- g\Omega^2 - 2\mathrm{i}\Omega \frac{d}{dR}(v_0 g) + \frac{1}{v_0} \frac{d}{dR} \left[v_0 \left({v_0}^2 - \beta^2 {c_{s0}}^2 \right) \frac{dg}{dR} \right] = 0 \tag{15}$$

where $\beta^2 = 2/(\gamma+1)$.

For subsonic flows, it is easy to understand that there are two values of R – one close to the accretor and one very far away – where the perturbation, which is in the form of a standing wave, can be constrained to die out. Multiplying (15) by $v_0 g$ and integrating over the range of R, which is bounded by the two points where the perturbation dies out, yields,

$$\Omega^2 \int v_0 g^2 \, dR = - \int v_0 \left({v_0}^2 - \beta^2 {c_{s0}}^2 \right) \left(\frac{dg}{dR} \right)^2 dR \tag{16}$$

The above result could be obtained because the boundary terms all vanish. With a real value of g, for the subsonic flows – all of them governed everywhere by the condition ${v_0}^2 < \beta^2 {c_{s0}}^2$ – Ω^2 will be positive, implying that the perturbation, which is in the nature of a standing wave, will be oscillatory and at least will not grow in time. This line of reasoning was analogously established for spherically symmetric subsonic flows by Petterson et al. (1980).

It is also possible to subject the subsonic flow solutions to an analysis where the perturbation may be treated as a travelling wave. The method for this, again follows a prescription used by Petterson et al. (1980) for the spherically symmetric flow. The perturbation would have to be confined to a short-wavelength regime, in which the wavelength of the propagating waves will be much less compared to a characteristic length-scale of the system, which, in this case, as Petterson et al. (1980) argued for the analogous spherically symmetric situation, could be the radius of the accretor, and that for an ordinary star is $R_\star$. Since such a length-scale would be significantly large, it is somewhat justifiable to restrict the study to a perturbation of comparatively short wavelengths. Consequently, Ω, the frequency of the waves, would be large.

It is convenient to recast (15) as

$$\left({v_0}^2 - \beta^2 {c_{s0}}^2 \right) \frac{d^2 g}{dR^2} + \left[3v_0 \frac{dv_0}{dR} - \beta^2 \frac{1}{v_0} \frac{d}{dR} \left(v_0 {c_{s0}}^2 \right) - 2\mathrm{i}\Omega v_0 \right] \frac{dg}{dR} - \left(2\mathrm{i}\Omega \frac{dv_0}{dR} + \Omega^2 \right) g = 0 \tag{17}$$

A solution to (17) may be obtained by expanding $g(R)$ as a power series (Petterson et al. 1980), in the form

$$g(R) = \exp \left[\sum_{n=-1}^{\infty} \frac{k_n(R)}{\Omega^n} \right] \tag{18}$$

and to solve for the first few terms.

For such a solution, terms in Ω^2, Ω and Ω^0 will respectively yield the conditions

$$\left({v_0}^2 - \beta^2 {c_{s0}}^2 \right) \left(\frac{dk_{-1}}{dR} \right)^2 - 2\mathrm{i}v_0 \frac{dk_{-1}}{dR} - 1 = 0 \tag{19}$$

$$\left({v_0}^2 - \beta^2 {c_{s0}}^2 \right) \left(\frac{d^2 k_{-1}}{dR^2} + 2\frac{dk_{-1}}{dR}\frac{dk_0}{dR} \right) + \left[3v_0 \frac{dv_0}{dR} - \frac{\beta^2}{v_0} \frac{d}{dR} \left(v_0 {c_{s0}}^2 \right) \right] \frac{dk_{-1}}{dR} - 2\mathrm{i}v_0 \frac{dk_0}{dR} - 2\mathrm{i}\frac{dv_0}{dR} = 0 \tag{20}$$

and

$$\left({v_0}^2 - \beta^2 {c_{s0}}^2 \right) \left[\frac{d^2 k_0}{dR^2} + 2\frac{dk_{-1}}{dR}\frac{dk_1}{dR} + \left(\frac{dk_0}{dR} \right)^2 \right] + \left[3v_0 \frac{dv_0}{dR} - \frac{\beta^2}{v_0} \frac{d}{dR} \left(v_0 {c_{s0}}^2 \right) \right] \frac{dk_0}{dR} - 2\mathrm{i}v_0 \frac{dk_1}{dR} = 0 \tag{21}$$

It is to be noted that as compared to the spherically symmetric case, the analogous results for the inviscid and thin accretion disc, given by (19), (20) and (21), vary by a scale factor of β for the speed of sound in the steady state. It is then quite straightforward to arrive at the expressions,

$$\frac{dk_{-1}}{dR} = \frac{\mathrm{i}}{v_0 \pm \beta c_{s0}} \tag{22}$$

and

$$k_0 = -\frac{1}{2} \ln \left(\beta v_0 c_{s0} \right) + \text{constant} \tag{23}$$

The first two terms in the power series expansion of $g(R)$ are given by (22) and (23). As with the spherically symmetric case, here also self-consistency may be had by establishing

$$\Omega |k_{-1}(R)| \gg |k_0| \gg \Omega^{-1} |k_1(R)| \tag{24}$$

From the way it has been chosen, $\Omega \gg (v_0 \pm \beta c_{s0})/R_\star$, it can be seen that the condition given by (24) is satisfied, if the radial distance is not too small. For large R, the asymptotic behaviour of k_1, k_{-1} and k_0 obtained from (21), (22) and (23) respectively, is given by

$$k_1 \sim \frac{1}{R}, \quad k_{-1} \sim R, \quad k_0 \sim \ln R \tag{25}$$

The result given above upholds the self-consistency required by the condition in (24). The results in (25) follow from a very possible power-law dependence of the steady flow velocity on the radius, at great distances from the accretor.

The expression for the perturbation may then be written as

$$f' = \frac{e^{-\mathrm{i}\Omega t}}{\sqrt{\beta v_0 c_{s0}}} \left[A_+ \exp \left(\mathrm{i}\Omega \int \frac{dR}{v_0 + \beta c_{s0}} \right) + A_- \exp \left(\mathrm{i}\Omega \int \frac{dR}{v_0 - \beta c_{s0}} \right) \right] \tag{26}$$

In (26) there is a linear superposition of two solutions with arbitrary constants A_+ and A_-, both travelling with velocity βc_{s0}, relative to the fluid, which itself moves with a bulk velocity v_0. One solution moves along with the bulk flow, while the other moves against it.

Use of (9) and (26) gives the perturbation in density as

$$\rho' = \beta^2 R^{-5/2} {\rho_0}^{-(\gamma-1)/2} \frac{e^{-\mathrm{i}\Omega t}}{\sqrt{\beta v_0 c_{s0}}} \left[\frac{A_+}{v_0 + \beta c_{s0}} \exp\left(\mathrm{i}\Omega \int \frac{dR}{v_0 + \beta c_{s0}} \right) + \frac{A_-}{v_0 - \beta c_{s0}} \exp\left(\mathrm{i}\Omega \int \frac{dR}{v_0 - \beta c_{s0}} \right) \right] \tag{27}$$

Using (27), the perturbation in velocity can then be obtained from (8), and following some simple algebraic manipulation it can be rendered as

$$v' = \pm \frac{c_{s0}}{\beta} \frac{\rho'}{\rho_0} \tag{28}$$

in which the positive and negative signs indicate outgoing and incoming waves, respectively.

In a unit volume of the fluid, the kinetic energy content is

$$\mathcal{E}_{\mathrm{kin}} = \frac{1}{2} \left(\rho_0 + \rho' \right) \left(v_0 + v' \right)^2 \tag{29}$$

The potential energy per unit volume of the fluid is the sum of the gravitational energy, the rotational energy and the internal energy, and is given by

$$\mathcal{E}_{\mathrm{pot}} = \left(\rho_0 + \rho' \right) \frac{GM}{R} - \left(\rho_0 + \rho' \right) \frac{L^2}{2R^2} + \rho_0 \epsilon + \rho' \frac{\partial}{\partial \rho_0} \left(\rho_0 \epsilon \right) + \frac{1}{2} {\rho'}^2 \frac{\partial^2}{\partial {\rho_0}^2} \left(\rho_0 \epsilon \right) \tag{30}$$

where ϵ is the internal energy per unit mass (Landau & Lifshitz 1987).

The first-order terms vanish on time averaging. In that case, the contribution to the total energy in the perturbation comes from the second-order terms, which is given by

$$\mathcal{E}_{\mathrm{pert}} = \frac{1}{2} \rho_0 {v'}^2 + v_0 \rho' v' + \frac{1}{2} {\rho'}^2 \frac{\partial^2}{\partial {\rho_0}^2} \left(\rho_0 \epsilon \right) \tag{31}$$

For an adiabatic perturbation, such as the one being prescribed here, the condition $ds = 0$ can be imposed on the thermodynamic relation, $d\epsilon = T ds + \left(P/\rho^2 \right) d\rho$, which will then give

$$\left. \frac{\partial^2}{\partial {\rho_0}^2} \left(\rho_0 \epsilon \right) \right|_s = \frac{{c_{s0}}^2}{\rho_0} \tag{32}$$

Combining the results from (28),(31) and (32) will give

$$\mathcal{E}_{\mathrm{pert}} = \frac{c_{s0}}{\beta^2} \frac{{\rho'}^2}{\rho_0} \left[\frac{c_{s0}}{2} \left(\beta^2 + 1 \right) \pm \beta v_0 \right] \tag{33}$$

The expression for ρ' is to be obtained from (27), and upon time averaging, the result will be

$$\mathcal{E}_{\mathrm{pert}} = \frac{1}{2} \frac{\beta A^2}{f_0} \frac{R^{-5/2} {\rho_0}^{-(\gamma-1)/2}}{\left(v_0 \pm \beta c_{s0} \right)^2} \left[\frac{c_{s0}}{2} \left(\beta^2 + 1 \right) \pm \beta v_0 \right] \tag{34}$$

where $f_0 \equiv {\rho_0}^{(\gamma+1)/2} v_0 R^{5/2}$, is a constant that in physical terms is the steady-state value of the accretion rate. The factor 1/2 in (34) derives from the averaging of ${\rho'}^2$.

The total energy flux in the perturbation is obtained by multiplying $\mathcal{E}_{\mathrm{pert}}$ by the propagation velocity $(v_0 \pm \beta c_{s0})$ and then by integrating over the area of the cylindrical face of the accretion disc, which is $2\pi RH$. Here H is to be substituted from (4) and (5), and together with the fact that $H \ll R$ in the thin-disc approximation (Frank et al. 1992), an expression for the energy flux will be delivered as

$$\mathcal{F} = \frac{\pi A^2}{f_0} \left(\frac{\gamma k}{GM} \right)^{1/2} \beta^2 \left[\pm 1 + \frac{1 - \beta^2}{2\beta (\mathcal{M} \pm \beta)} \right] \tag{35}$$

where $\mathcal{M} = v_0 / c_{s0}$, is the Mach number.

It is to be seen that for purely subsonic flows, $\mathcal{M}$ always remains less than β, which is a condition that is easily understood from the first of the two critical point conditions, given by (14). This would then imply that the total energy in the wave remains finite as the wave propagates. Indeed, for very great radial distances, when $\mathcal{M} \longrightarrow 0$, the flux is less compared to what it is near the outer critical point of the flow. The perturbation does not manifest itself as a runaway increase in the energy of the system. The system would then remain stable under the influence of such a perturbation.

For the case of the transonic flow, a different line of reasoning would have to be adopted, since the condition ${v_0}^2 < \beta^2 {c_{s0}}^2$ would not hold everywhere, with the flow passing through a singular point. The physically meaningful flow in this case will be subsonic beyond the saddle-type outer critical point, pass through it to attain a supersonic velocity and then tend to curl around the inner centre-type critical point, but not actually flow through it. Theuns & David (1992) have established the stability of the transonic solution in spherical symmetry by arguing that the stability of the subsonic region of the flow, with the critical point as one boundary, will depend on conditions at the far outer end of the flow. In so far as the thin accretion disc is dependent only on R (the disc being inviscid and hence, the related equations are independent of any angle variable), the analogous argument of Theuns & David (1992) is expected to hold good to ensure the stability of the subsonic region of the transonic solution.

For the supersonic region, which is finite, the reasoning of Garlick (1979) that a disturbance in this region will be carried away in a finite time, will also ensure the stability of the flow in this region.

There is another important argument to uphold the stability of the transonic flow in a thin accretion disc. In the case of the spherically symmetric transonic flow, the velocity near the accretor has an unbridled growth, such that any perturbation on it cannot be constrained to die out by a conceivable physical mechanism (Petterson et al. 1980). In the inviscid and thin accretion disc, the transonic solution has no such runaway growth in the inner region of the flow, tending to curl around the inner critical point as it does. And while doing so, it may reach the surface of the accretor, if the accretor is as distended as an ordinary star. This can ensure that a perturbation on the solution in this region will not behave in an unconstrained manner.

A note of caution has to be sounded here. The discussion presented above, pertains only to flows which are continuous everywhere. However, for spherically symmetric flows with discontinuities like shocks, Theuns & David (1992) have argued that the expectation would also be one in favour of the stability of those flows. Under the present assumptions governing an inviscid and thin accretion disc, the mathematical problem becomes very similar to the case of spherically symmetric accretion. In that case the argument of Theuns & David (1992) for spherical symmetry could very likely be extended to the axially symmetric system being considered here. In any case, regardless of the geometry of the problem, it is a matter of general understanding that a disturbance in the supersonic region in a shocked solution, is transmitted completely through the shock and that in the subsonic region a disturbance is reflected at the shock, with a diminished reflected amplitude for the disturbance as compared with the incident amplitude (Theuns & David 1992). More to the point, by a local stability analysis, it has been demonstrated (Chakrabarti 1989) that Rankine-Hugoniot shocks in an inviscid and thin accretion disc are stable under the influence of a short-wavelength perturbation in real time.

It can then be safely established that a physically well-behaved and meaningful inflow solution is indeed stable under the influence of a linearized perturbation. This conclusion holds good for both the transonic inflow solution, as well as the entire class of subsonic inflow solutions.

5 CONCLUDING REMARKS

So far, it has been demonstrated that linear stability analysis, which is essentially a perturbative technique, offers no clue about the exclusive choice of a particular solution – indeed, to the extent that a perturbative technique can be relied upon, every solution (transonic or subsonic) seems just as much realizable as any other. It is then to be conjectured that the criterion for the selection of a particular solution would have to be non-perturbative in character and would have to be based on fundamentally physical arguments, like the maximization of the accretion rate or the minimization of the total energy associated with a solution, as in the case of the transonic inflow solution in spherically symmetric accretion (Bondi 1952; Garlick 1979).

A recent work (Ray & Bhattacharjee 2002) on spherically symmetric accretion has indicated that such an approach, however, would necessitate the addressing of the whole issue of the selection of a particular solution, from the viewpoint of the temporal evolution of the flow, instead of considering it solely on the basis of the stationary picture. The study has shown that it is the transonic solution which is decisively preferred to all the others. Such an expectation would also be valid for the case of the inviscid and thin accretion disc being discussed here.

Support for such a contention can also be had from a very different quarter. Study carried out over the last two decades has established that there is a very close one-to-one correspondence between certain features of black hole physics and the physics of supersonic acoustic flows. More specifically, for an inviscid, barotropic and irrotational fluid flow (such as the conventional spherically symmetric accretion) the equation describing an acoustic disturbance can be rendered in the form of a metric that relates very closely to the Schwarzschild metric, represented in the Painlevér-Gullstrand form (Visser 1997). Close to the sonic point of the flow, the similarity is particularly evident. Such a close correspondence between the unidirectionality of trajectories near a black hole and of flows passing through the sonic point, is a reason strong enough to suggest that transonic trajectories are the favoured ones. Extending that argument, a comparison may also be drawn for rotational, inviscid fluid flows, to have an understanding of the favoured status of the trajectories in this case.

ACKNOWLEDGMENTS

This research has made use of NASA's Astrophysics Data System. The financial assistance provided by the Council of Scientific and Industrial Research, Government of India, is being gratefully acknowledged. Gratitude is also to be expressed to Prof. J. K. Bhattacharjee for his very helpful advice, given unreservedly throughout this work.

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