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Solutions Of Optimization Problems Using Excel

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Abstract: Excel spreadsheet is used in solving optimization problems of one variable and several variables without constraints, and it is stated that the Solver of Tools menu can be used to solve the constrained optimization problems.

Keywords: Excel, Optimization and Constraints

Introduction

Microsoft Excel is used throughout the paper, and gives a simple approach in solving optimization problems with and without constraints. A spreadsheet is made up of cells as follows:

	A	B	C	D	E
1	A1	B1	C1	D1	E1
2	A2	B2	C2	D2	E2
3	A3	B3	C3	D3	E3
4	A4	B4	C4	D4	E4
5	A5	B5	C5	D5	E5

In these boxes we type the data into, the data that can be typed intó every cell, (A1, A2,.....,B1, B2,......, E1, E2, ...) can be numeric or algebraic.

Single Variable Optimization, Newton's Method:

Newton's method technique can be used to find the maximum and the minimum of a function *f(x)*, Radwan (2000) and Chapra (1998), by using the following formula:

$$x_{i+1} = x_i - \frac{f'(x_i)}{f''(x_i)},$$

where x_i is an initial guess of the extreme point, and x_{i+1} is a new estimation of the extreme point.

From calculus we have a maximum at $x = a$ if $f'(a) = 0$ and $f''(a) < 0$, and we have a minimum at $x = b$ if $f'(b) = 0$ and $f''(b) > 0$.

Example: Use Excel and Newton's method technique to find the maximum of the following function:

$f(x) = -2\sin x + \sin 2x - \frac{2\sin 3x}{3}$, using an initial guess of.

$x_0 = 3.5$

We can use Excel spreadsheet to solve this problem as follows:

	A	B	C	D	E	F
1	x_i	$f(x_i)$	$f'(x_i)$	$f''(x_i)$	X_{i+1}	$f(x_{i+1})$
2	3.5	=-2*SIN(A2)+SIN(2*A2)	=-2*COS(A2)+2*COS	=2*SIN(A2)4*SIN	=A2-	=-2*SIN(E2)+SIN(2*E2)-
3	=E2	=-2*SIN(A3)+SIN(2*A3)	=-2*COS(A3)+2*COS	=2*SIN(A3)4*SIN(2*A3)+6	=A3-	=-2*SIN(E3)+SIN(2*E3)-
4	=E3	------------------	-------------------	--------------------	-------	--------------------
5	=E4	------------------	-------------------	--------------------	-------	--------------------

The results are as follows:

	A	B	C	D	E	F
1	X_I	$f(x_i)$	$f'(x_i)$	$f''(x_i)$	X_{i+1}	$f(x_{i+1})$
2	3.5	1.9450169	4.3317917	-8.607687	4.0032469	2.8584528
3	4.0032469	2.8584528	-0.699558	-8.641379	3.9222924	2.8855113
4	3.9222924	2.8855113	0.0454955	-9.709395	3.9269782	2.8856181
5	3.9269782	2.8856181	0.0001223	-9.656998	3.9269908	2.8856181
6	3.9269908	2.8856181	9.07E-10	-9.656854	3.9269908	2.8856181
7	3.9269908	2.8856181	2.887E-15	-9.656854	3.9269908	2.8856181
8	3.9269908	2.8856181	-2E-15	-9.656854	3.9269908	2.8856181
9	3.9269908	2.8856181	-2E-15	-9.656854	3.9269908	2.8856181

	A	B	C	D	E	F
1	x_i	$f(x_i)$	$f'(x_i)$	$f''(x_i)$	x_{i+1}	$f(x_{i+1})$
2	2	-2.38912	-2.395334	3.1693118	2.75579	-2.0604027
3	2.75579	-2.060403	4.0900068	9.0363429	2.3031725	-2.8723427
4	2.3031725	-2.872343	-0.49478	8.9817684	2.3582596	-2.8855975
5	2.3582596	-2.885597	0.0199667	9.6800998	2.356197	-2.8856181
6	2.356197	-2.885618	2.39E-05	9.6568823	2.3561945	-2.8856181
7	2.3561945	-2.885618	3.466E-11	9.6568542	2.3561945	-2.8856181
8	2.3561945	-2.885618	0	9.6568542	2.3561945	-2.8856181
9	2.3561945	-2.885618	0	9.6568542	2.3561945	-2.8856181

This implies that the maximum value of the function $f(x)$ is 2.8856181 occurs at x = 3.9269908 as we have seen that $f''(3.9269908) < 0$.
Simply we can find the minimum of the same function just by changing the initial value in cell A2 from 3.5 to 2.0 and immediately we will have the results as above:
This implies that the minimum value of the function $f(x)$ is −2.8856181 occurs at x = 2.3561945 as we have seen that $f''(2.3561945) > 0$.

Multivariate Unconstrained Optimization, Steepest Ascent Method and Steepest Descent Method: The steepest ascent method and steepest descent method can be used to locate the maximum and the minimum respectively of a function of several variables, Radwan (2000) and Chapra (1998).
The steepest ascent method is the most straightforward of the gradient search techniques. In this method we start with an initial point (x_o, y_o), at this point; we determine the direction of the steepest ascent, that is, the gradient. We then search along the direction of the gradient, h_o, until we find a maximum. The process is then repeated. The same approach can be used for minimization, in which case the terminology steepest descent is used.
Starting at (x_o, y_o) the coordinates of any point in the gradient direction can be expressed as:

$$x = x_0 + \frac{\partial f}{\partial x} h \quad (1)$$

$$y = y_0 + \frac{\partial f}{\partial y} h \quad (2)$$

Where h is a distance along the h axis, and f is a function of x and y, and the gradient is $\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j$.

To find the maximum, we would search along the gradient direction, that is, along an h axis running along the direction of this vector. The function can be expressed along this axis as:

$f\left(x_0 + \frac{\partial f}{\partial x} h, y_0 + \frac{\partial f}{\partial y} h\right) = g(h)$, where $g(h)$ is a single variable function of h.

Now we have developed a function along the path of steepest ascent. How far along this path do we travel? One approach might be to move along this path until we find the maximum of this function. We will call the location of this maximum h^*. This is the value of the step that maximizes $g(h)$ (and hence, $f(x, y)$ in the gradient direction. This problem is equivalent to finding the maximum of a function of a single variable h.
This method is called steepest ascent when an arbitrary step size h is used. If a value of a single step $h^*$is found that brings us directly to the maximum along the gradient direction, the method is called the optimal steepest ascent.
Whether a maximum or a minimum occurs involves not only the first partial derivatives of $f(x, y)$ with respect to x and y but also the second partial derivatives. Assuming that these derivatives are continuous at and near the point being evaluated, determinant can be computed:

$|H| = \frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2$, Three cases can occur:

1. If $|H| > 0$ and $\frac{\partial^2 f}{\partial x^2} > 0$, then $f(x, y)$ has a local minimum.
2. If $|H| > 0$ and $\frac{\partial^2 f}{\partial x^2} < 0$, then $f(x, y)$ has a local maximum.
3. If $|H| < 0$, then $f(x, y)$ has a saddle point (a saddle point is a point where all the first partial derivatives of a function vanish but which is not a local maximum or minimum).

The quantity $|H|$ is equal to the determinant of a matrix made up of the second derivatives:

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix}$$

, Where this matrix is called the *Hessian* of $f(x, y)$.

Example: Use Steepest Ascent Method to maximize the following function $f(x\ y)=2xy+1.5y-1.25x^2-2y^2$, using initial guesses $x = 1$ and $y = 1$.

$\frac{\partial^2 f}{\partial x^2}=-2.5$, $\frac{\partial^2 f}{\partial x \partial y}=\frac{\partial^2 f}{\partial y \partial x}=2$, and $\frac{\partial^2 f}{\partial y^2}=-4$,

$\Rightarrow |H|=6>0$, then $f(x, y)$ has a local maximum.

We can use Excel spreadsheet to find the maximum as follows:

	A	B	C	D	E	F
1	x	y	f(x,y)	∂f/∂x	∂f/∂y	h
2	1	1	=2*A2*B2+1.5*B2-1.25*A2^2-2*B2^2	=2*B2-2.5*A2	=2*A2+1.5-4*B2	=L2
3	=A2+D2*F2	=B2+E2*F2	=2*A3*B3+1.5*B3-1.25*A3^2-2*B3^2	=2*B3-2.5*A3	=2*A3+1.5-4*B3	=L3
4	-------------	-----------	------------------------	----------------	-----------------	------------

	G	H	I	J	K	L
1	The constant in g(h)	Coefficient of h in g(h)	Coefficient of h^2 in g(h)	Coefficient of h* in g'(h)=f(h*)	The constant in f(h*)	The value of h*
2	=2*A2*B2+1.5*B2-1.25*A2^2-2*B2^2	=2*A2*E2+2*D2*B2+1.5*E2-1.25*2*A2*D2-2*2*B2*E2	=2*D2*E2-1.25*D2^2-2*E2^2	=2*I2	=H2	=-K2/J2
3	=2*A3*B3+1.5*B3-1.25*A3^2-2*B3^2	=2*A3*E3+2*D3*B3+1.5*E3-1.25*2*A3*D3-2*2*B3*E3	=2*D3*E3-1.25*D3^2-2*E3^2	=2*I3	=H3	=-K3/J3
4	------------------	-----------------	---------------	--------------------	------------	---------

The results are as follows:

	A	B	C	D	E	F
1	X	y	f(x, y)	∂f/∂x	∂f/∂y	h
2	1	1	0.25	-0.5	-0.5	0.8
3	0.6	0.6	0.45	-0.3	0.3	0.19047619
4	0.54285714	0.65714286	0.46714286	-0.0428571	-0.0428571	0.8
5	0.50857143	0.62285714	0.46861224	-0.0257143	0.02571429	0.19047619
6	0.50367347	0.6277551	0.46873819	-0.0036735	-0.0036735	0.8
7	0.50073469	0.62481633	0.46874899	-0.0022041	0.00220408	0.19047619
8	0.50031487	0.62523615	0.46874991	-0.0003149	-0.0003149	0.8
9	0.50006297	0.62498426	0.46874999	-0.0001889	0.00018892	0.19047619
10	0.50002699	0.62502024	0.46875	-2.699E-05	-2.699E-05	0.8
11	0.5000054	0.62499865	0.46875	-1.619E-05	1.6193E-05	0.19047619
12	0.50000231	0.62500173	0.46875	-2.313E-06	-2.313E-06	0.8
13	0.50000046	0.62499988	0.46875	-1.388E-06	1.388E-06	0.19047619
14	0.5000002	0.62500015	0.46875	-1.983E-07	-1.983E-07	0.8
15	0.50000004	0.62499999	0.46875	-1.19E-07	1.1897E-07	0.19047619
16	0.50000002	0.62500001	0.46875	-1.7E-08	-1.7E-08	0.8
17	0.5	0.625	0.46875	-1.02E-08	1.0197E-08	0.19047619
18	0.5	0.625	0.46875	-1.457E-09	-1.457E-09	0.80000014
19	0.5	0.625	0.46875	-8.741E-10	8.7407E-10	0.19047609
20	0.5	0.625	0.46875	-1.249E-10	-1.249E-10	0.79999877
21	0.5	0.625	0.46875	-7.492E-11	7.492E-11	0.19047647
22	0.5	0.625	0.46875	-1.07E-11	-1.07E-11	0.80000517
23	0.5	0.625	0.46875	-6.422E-12	6.422E-12	0.1904788
24	0.5	0.625	0.46875	-9.173E-13	-9.175E-13	0.79982473
25	0.5	0.625	0.46875	-5.509E-13	5.5067E-13	0.19046585
26	0.5	0.625	0.46875	-7.86E-14	-7.905E-14	0.79681658
27	0.5	0.625	0.46875	-4.796E-14	4.7518E-14	0.19118666
28	0.5	0.625	0.46875	-6.883E-15	-7.105E-15	0.78403478
29	0.5	0.625	0.46875	-4.219E-15	3.9968E-15	0.1973924
30	0.5	0.625	0.46875	0	0	

	G	H	I	J	K	L
1	The constant in *g(h)*	Coefficient of *h* in *g(h)*	Coefficient of h^2 in *g(h)*	Coefficient of *h** in *g'(h)=f(h*)*	The constant in *f(h*)*	The value of *h**
2	0.25	0.5	-0.3125	-0.625	0.5	0.8
3	0.45	0.18	-0.4725	-0.945	0.18	0.1904762
4	0.46714286	0.00367347	-0.0022959	-0.0045918	0.00367347	0.8
5	0.46861224	0.00132245	-0.0034714	-0.0069429	0.00132245	0.1904762
6	0.46873819	2.6989E-05	-1.687E-05	-3.374E-05	2.6989E-05	0.8
7	0.46874899	9.716E-06	-2.55E-05	-5.101E-05	9.716E-06	0.1904762
8	0.46874991	1.9828E-07	-1.239E-07	-2.479E-07	1.9828E-07	0.8
9	0.46874999	7.1383E-08	-1.874E-07	-3.748E-07	7.1383E-08	0.1904762
10	0.46875	1.4568E-09	-9.105E-10	-1.821E-09	1.4568E-09	0.8
11	0.46875	5.2444E-10	-1.377E-09	-2.753E-09	5.2444E-10	0.1904762
12	0.46875	1.0703E-11	-6.689E-12	-1.338E-11	1.0703E-11	0.8
13	0.46875	3.853E-12	-1.011E-11	-2.023E-11	3.853E-12	0.1904762
14	0.46875	7.8634E-14	-4.915E-14	-9.829E-14	7.8634E-14	0.8
15	0.46875	2.8308E-14	-7.431E-14	-1.486E-13	2.8308E-14	0.1904762
16	0.46875	5.7772E-16	-3.611E-16	-7.221E-16	5.7772E-16	0.8
17	0.46875	2.0798E-16	-5.459E-16	-1.092E-15	2.0798E-16	0.1904762
18	0.46875	4.2445E-18	-2.653E-18	-5.306E-18	4.2445E-18	0.8000001
19	0.46875	1.528E-18	-4.011E-18	-8.022E-18	1.528E-18	0.1904761
20	0.46875	3.1184E-20	-1.949E-20	-3.898E-20	3.1184E-20	0.7999988
21	0.46875	1.1226E-20	-2.947E-20	-5.894E-20	1.1226E-20	0.1904765
22	0.46875	2.2911E-22	-1.432E-22	-2.864E-22	2.2911E-22	0.8000052
23	0.46875	8.2482E-23	-2.165E-22	-4.33E-22	8.2482E-23	0.1904788
24	0.46875	1.683E-24	-1.052E-24	-2.104E-24	1.683E-24	0.7998247
25	0.46875	6.0665E-25	-1.593E-24	-3.185E-24	6.0665E-25	0.1904659
26	0.46875	1.242E-26	-7.793E-27	-1.559E-26	1.242E-26	0.7968166
27	0.46875	4.5691E-27	-1.195E-26	-2.39E-26	4.5691E-27	0.1911867
28	0.46875	9.7819E-29	-6.238E-29	-1.248E-28	9.7819E-29	0.7840348
29	0.46875	3.471E-29	-8.792E-29	-1.758E-28	3.471E-29	0.1973924
30	0.46875	0	0	0	0	

So the maximum value of the function is 0.46875 occurs at $x = 0.5$ and $y = 0.625$.

Example: Use steepest descent method to minimize the following function *f(x,y)*= $5x^2$ $5xy+2.5y^2-x-1.5y$, using initial guesses x=1 and y=2.

$$\frac{\partial^2 f}{\partial x^2}=10,\quad \frac{\partial^2 f}{\partial x\partial y}=\frac{\partial^2 f}{\partial y\partial x}=-5,\quad \text{and}\quad \frac{\partial^2 f}{\partial y^2}=5,$$

$\Rightarrow |H| = 25 > 0,$ then *f(x, y)* has a local minimum.

We can use Excel spreadsheet to find the minimum as follows:

	A	B	C	D	E	F
1	*x*	*y*	*f(x, y)*	$\partial f/\partial x$	$\partial f/\partial y$	*h*
2	1	2	=5*A2^2-5*A2*B2+2.5*B2^2-A2-1.5*B2	=10*A2-5*B2-1	=-2*A2+5*B2-1.5	=L2
3	=A2+D2*F2	=B2+E2*F2	=5*A2^2-5*A3*B3+2.5*B3^2-A3-1.5*B3	=10*A3-5*B3-1	=-2*A3+5*B3-1.5	=L3
4	-------------	------------	-------------------------	-----------------	---------------------	------------

	G	H	I	J	K	L
1	The constant in *g(h)*	Coefficient of *h* in *g(h)*	Coefficient of h^2in *g(h)*	Coefficient of *h**in *g'(h)=f(h*)*	The constant in *f(h*)*	The value of *h**
2	=5*A2-5*A2*B2+2.5*B2-A2-1.5*B2	=5*2*A2*D2-5*A2*E2-5*B2*D2+2.5*2*B2*E2-D2-1.5*E2	=5*D2^2-5*D2*E2+2.5*E2^2	=2*I2	=H2	=-K2/J2
3	=5*A3-5*A3*B3+2.5*B3-A3-1.5*B3	=5*2*A3*D3-5*A3*E3-5*B3*D3+2.5*2*B3*E3-D3-1.5*E3	=5*D3^2-5*D3*E3+2.5*E3^2	=2*I3	=H3	=-K3/J3
4	----------------	--------------------	---------------	-------------------	------------	---------

The results are as follows:

	A	B	C	D	E	F
1	*x*	*y*	*f(x, y)*	∂f/∂x	∂f/∂y	*h*
2	1	2	1	-1	6.5	-0.0829694
3	1.08296943	1.46069869	0.01473799	2.52620087	3.63755459	-0.20469
4	0.5658813	0.71612754	-0.7830837	1.07817534	0.94887508	-0.076659
5	0.48322947	0.64338773	-0.8004076	0.61535603	0.75047971	0.07360594
6	0.52852333	0.6986275	-0.8057837	0.79209581	0.93609083	-0.0059999
7	0.52377081	0.69301102	-0.8058421	0.77265301	0.91751347	0.000931
8	0.52449015	0.69386522	-0.8058434	0.77557538	0.92034581	-0.0001339
9	0.52438632	0.69374202	-0.8058434	0.77515316	0.91993743	1.9468E-05
10	0.52440141	0.69375993	-0.8058434	0.77521452	0.9199968	-2.827E-06
11	0.52439922	0.69375733	-0.8058434	0.77520561	0.91998818	4.1047E-07
12	0.52439954	0.6937577	-0.8058434	0.7752069	0.91998943	-5.961E-08
13	0.5243995	0.69375765	-0.8058434	0.77520671	0.91998925	8.6562E-09
14	0.5243995	0.69375766	-0.8058434	0.77520674	0.91998928	-1.257E-09
15	0.5243995	0.69375766	-0.8058434	0.77520674	0.91998927	1.8255E-10
16	0.5243995	0.69375766	-0.8058434	0.77520674	0.91998927	-2.651E-11
17	0.5243995	0.69375766	-0.8058434	0.77520674	0.91998927	3.8493E-12
18	0.5243995	0.69375766	-0.8058434	0.77520674	0.91998927	-5.587E-13
19	0.5243995	0.69375766	-0.8058434	0.77520674	0.91998927	8.0976E-14
20	0.5243995	0.69375766	-0.8058434	0.77520674	0.91998927	-1.15E-14
21	0.5243995	0.69375766	-0.8058434	0.77520674	0.91998927	1.7138E-15
22	0.5243995	0.69375766	-0.8058434	0.77520674	0.91998927	0
23	0.5243995	0.69375766	-0.8058434	0.77520674	0.91998927	0

	G	H	I	J	K	L
1	The constant in *g(h)*	Coefficient of *h* in *g(h)*	Coefficient of h^2 in *g(h)*	Coefficient of *h**in *g'(h)*= *f(h*)*	The constant in *f(h*)*	The value of *h**
2	-4	23.75	0	286.25	23.75	-0.0829694
3	-2.1168837	7.79541294	19.0419948	38.0839896	7.79541294	-0.20469
4	0.95343683	0.45197398	2.94795156	5.89590312	0.45197398	-0.076659
5	1.02178605	-0.1460789	0.99230364	1.98460728	-0.1460789	0.0736059
6	0.96651616	0.01944429	1.62037584	3.24075168	0.01944429	-0.0059999
7	0.97319955	-0.0028767	1.54494307	3.08988613	-0.0028767	0.000931
8	0.97219845	0.00041665	1.55618911	3.11237823	0.00041665	-0.0001339
9	0.97234318	-6.053E-05	1.55456226	3.10912452	-6.053E-05	1.947E-05
10	0.97232215	8.7893E-06	1.55479864	3.10959728	8.7893E-06	-2.827E-06
11	0.97232521	-1.276E-06	1.55476432	3.10952864	-1.276E-06	4.105E-07
12	0.97232476	1.8535E-07	1.5547693	3.1095386	1.8535E-07	-5.961E-08
13	0.97232483	-2.692E-08	1.55476858	3.10953716	-2.692E-08	8.656E-09
14	0.97232482	3.9088E-09	1.55476868	3.10953737	3.9088E-09	-1.257E-09
15	0.97232482	-5.676E-10	1.55476867	3.10953734	-5.676E-10	1.825E-10
16	0.97232482	8.2432E-11	1.55476867	3.10953734	8.2432E-11	-2.651E-11
17	0.97232482	-1.197E-11	1.55476867	3.10953734	-1.197E-11	3.849E-12
18	0.97232482	1.7373E-12	1.55476867	3.10953734	1.7373E-12	-5.587E-13
19	0.97232482	-2.518E-13	1.55476867	3.10953734	-2.518E-13	8.098E-14
20	0.97232482	3.5749E-14	1.55476867	3.10953734	3.5749E-14	-1.15E-14
21	0.97232482	-5.329E-15	1.55476867	3.10953734	-5.329E-15	1.714E-15
22	0.97232482	0	1.55476867	3.10953734	0	0
23	0.97232482	0	1.55476867	3.10953734	0	0

So the minimum value of the function is −0.8058434 occurs at x= 0.52058434 and y = 0.69375766.

Conclusion

As we have seen that we can use Excel spreadsheet to solve optimization problems of functions of one variable or of several variables without constraints, also we can use any initial guesses and we will obtain the results immediately. For constraint optimization we can use the Solver in the Tools menu in Excel spreadsheet, Radwan (2000) and Chapra (1998).

References

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