Quantum Interferometry: Some Basic Features Revisited

Markus Simonius

Inst. f. Teilchenphysik, Eidg. Techn. Hochschule, CH-8093 Zürich, Switzerland

The reduction paradigm of quantum interferometry is reanalyzed. In contrast to widespread opinion it is shown to be amenable to straightforward mathematical treatment within "every-users" simple-minded single particle quantum mechanics (without reduction postulate or the like), exploiting only its probabilistic content.

Consider a typical interferometer arrangement as sketched in Fig. 1. Its properties are well known: even if only one quantum (photon or neutron etc.) is inside the arrangement at a given time, the configuration of Fig. 1a can reveal interference between the states passing the two arms I and II provided none of them is blocked. Correspondingly, a pure single-particle state within the interferometer is represented by a normed wave function of the form

$$\varphi = c_1\varphi_1 + c_2\varphi_2, \quad \|\varphi\| = \|\varphi_1\| = \|\varphi_2\| = 1, \tag{1}$$

where, φ_1 and φ_2 represent states passing completely along one of the two paths I or II, respectively, in the interferometer and have zero component in the other (and thus are mutually orthogonal). For ideal 50:50 beam-splitting $|c_1|^2 = |c_2|^2 = \frac{1}{2}$, of course.

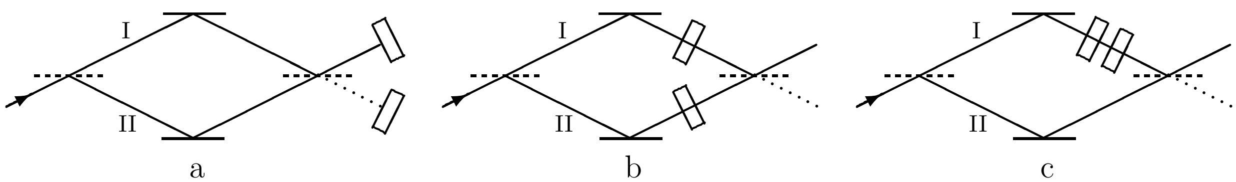

Figure 1: Sketch of typical interferometer arrangements with differently placed detectors.

Now consider instead coincidences between two detectors inserted into the two paths of the interferometer as shown in Fig. 1b. From the appearance of the wave function (1) one might suspect that such coincidences should occur. On the other hand, it is the fundamental proposition of quantum physics that quanta (particles or photons) are *indivisible entities* and thus a *single quantum* is not able to trigger both detectors in the arrangement of Fig. 1b (nor in Fig. 1a, of course). Though all this is much discussed textbook wisdom (tested also experimentally [3]), one looks in vain in the literature for a satisfying *derivation* of this fact which uses only the basic structure of single particle quantum theory and does not amount to just stating the fact as postulate in one way or another. But clearly it should be possible to deduce such basic properties mathematically once the general premises of the theory are laid down [1]. It is the object of this note to show that this is indeed so. Astonishingly, though the analysis to be presented is simple and straightforward, no such treatment is found in the literature, let alone in textbooks where it would belong.

For simplicity only pure states will be considered here explicitly, but the same results are obtained also directly from the general definition of superpositions in Ref. [2] which applies also to non-pure states (density operators).

Only the following minimal probabilistic set of postulates of quantum mechanics is used. No state reduction and no axiom of measurement etc.!

I Pure states are represented by normed elements $\varphi \in \mathcal{H}$ of a Hilbert space $\mathcal{H}$.

II The probability for a given kind of event for a system in a state represented by φ is given by an expectation value $\langle\varphi|A|\varphi\rangle$ where A is a hermitian operator on $\mathcal{H}$ which obviously must obey $0 \leq \langle\varphi|A|\varphi\rangle \leq 1$ for all $\varphi \in \mathcal{H},\ \|\varphi\| = 1$.

The positivity postulate in II immediately leads to the following "silly" but far reaching **Theorem:** *Let A be a positive operator on $\mathcal{H}$ such that $\langle\psi_1|A|\psi_1\rangle = \langle\psi_2|A|\psi_2\rangle = 0$ for given $\psi_1, \psi_2 \in \mathcal{H}$. Then $\langle\psi|A|\psi\rangle = 0$ for all superpositions $\psi = c_1\psi_1 + c_2\psi_2$ between them.*

In order to apply this theorem to Fig. 1b with the two detectors in coincidence one only has to remark that one of the fundamental requirements for a coincidence between the two detectors is that the probability for a coincidence event be zero for any state which has zero component in one of the two arms of the interferometer, i.e if either $c_1 = 0$ or c_2=0 in eq. (1). (This is actually what careful experimenters check in order to verify that their arrangement does not produce spurious coincidence events!) Thus $\langle\varphi_1|A|\varphi_1\rangle = \langle\varphi_2|A|\varphi_2\rangle = 0$ where A is the operator describing the probability of coincidence events. It then follows from the above theorem that $\langle\varphi|A|\varphi\rangle = 0$ also for arbitrary $\varphi = c_1\varphi_1 + c_2\varphi_2$ and thus that *also for arbitrary superpositions between the states in the two arms of the interferometer the two detectors in Fig. 1b have zero probability to produce a coincidence event.*

Thus "what must be" is graciously born out by the mathematical analysis in spite of appearance of the wave function in eq. (1), "with nothing left to the discretion of the theoretical physicist" [1] (except to formulate the problems properly).

A corresponding "reduction theorem" is obtained similarly (using anticoincidence, this time) for an arrangement of the kind shown in Fig. 1c where now both detectors are in the same path and it is assumed that at least the first detector transmits (does not absorb) the quanta: If the two detectors have unit efficiency either both of them fire or none.

It is emphasized that postulate II is an integral part of the mathematical structure of quantum theory and not just a supplementary interpretation. In fact this postulate distinguishes quantum theory from classical wave theory.

The structure of the "silly" theorem on which this is based and of the corresponding physical statements is emphasized: Conclusions for arbitrary superpositions between two states φ_i are obtained from conditions imposed only for the two states φ_i themselves. There is no need to *postulate* what happens in the case of superpositions. The results presented depend exclusively on properties of the Hilbert space used to describe an elementary system and the operators acting on it. Indeed, in the case analyzed explicitly here, it suffices to take into account the two-dimensional space spanned by φ_1 and φ_2 in eq. (1) which supports only four linearly independent operators.

Two-slit or Stern-Gerlach arrangements can be analyzed correspondingly.

Of course, the features proved here must be revealed also if the detection devices or whatever are taken into account explicitly. However, against widespread opinion [4], it is definitely not *necessary* to do so let alone to invoke corresponding macroscopic properties.

References

[1] J. S. Bell, Physics World **3** No. 8, August 1990, p. 33.
[2] M. Simonius, Helv. Phys. Acta **66**, 721 (1993); **65**, 884 (1992).
[3] P. Grangier, G. Roger, and A. Aspect, Europhysics Letters **1**, 173 (1986).
[4] K. Gottfried, Physics World **4** No. 10, October 1991, p.34.