ROM2F/99-42

Discrete torsion in non-geometric orbifolds and their open-string descendants

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arXiv:hep-th/9910228v2 9 Dec 1999

Abstract

We discuss some $Z_N^L \times Z_N^R$ orbifold compactifications of the type IIB superstring to $D = 4, 6$ dimensions and their type I descendants. Although the $Z_N^L \times Z_N^R$ generators act asymmetrically on the chiral string modes, they result into left-right symmetric models that admit sensible unorientable reductions. We carefully work out the phases that appear in the modular transformations of the chiral amplitudes and identify the possibility of introducing discrete torsion. We propose a simplifying ansatz for the construction of the open-string descendants in which the transverse-channel Klein-bottle, annulus and Möbius-strip amplitudes are numerically identical in the proper parametrization of the world-sheet. A simple variant of the ansatz for the $Z_2^L \times Z_2^R$ orbifold gives rise to models with supersymmetry breaking in the open-string sector.

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1 Introduction

The microscopic description of Bogomolny-Prasad-Sommerfield (BPS) solitons carrying Ramond–Ramond (R-R) charge in terms of Dirichlet-branes (D-branes) and Orientifold-planes (O-planes) [1] has played a crucial rôle in the emerging non-perturbative picture of string theory. Although many interesting vacuum configurations of the type II superstring and their type I descendants can be easily accounted for in terms of D-branes and O-planes, these concepts tend to loose their clear meaning in non-trivial compactifications, such as asymmetric orbifolds [2], free fermionic models [3, 4] and Gepner models [5, 6]. Insisting on a 'geometric' target-space approach, that is expected to be valid in the large volume limit, is by far less useful than pursuing an 'algebraic' worldsheet approach based on the conformal field theory (CFT) description [7, 8, 9]. The construction of type I descendants of non-geometric type II vacuum configurations may shed some light on the necessary generalization of the above concepts.

It is the purpose of this paper to discuss some $Z_N^L \times Z_N^R$ models that, though left-right symmetric, do not admit a clear geometric description because of the chiral nature of the projections. Still, we are able to analyze their open-string descendants in terms of two-dimensional CFT's where the strict concepts of D-branes and O-planes are abandoned. Differently from what has been found for geometric orbifolds, we will find the presence of open strings belonging to twisted sectors of the orbifold group. Similar kinds of open strings have been recently found in a discussion of type I vacuum configurations in $D = 6$ that involve D-branes at angles [10]. We cannot exclude the possibility that some of the models discussed in the present paper may admit a geometric interpretation in terms of M-theory along the lines of [11].

We will identify the possibility of introducing additional phases (known as discrete torsion [12]) in the modular invariant combinations of characters that appear in the one-loop torus amplitude. Some brane configurations in the presence of discrete torsion have been considered in [13]. The introduction of discrete torsion allows one to relate models with different amount of supersymmetry. The resulting theories are symmetric under the interchange of the left and right movers for any choice of the discrete torsion and are thus good candidate parents of type I descendants. The very possibility of working in a rational context (only a highly restricted class of lattices admit chiral automorphisms! [14]) makes the construction of the type I descendants almost straightforward following the approach pioneered by Sagnotti [15] and then systematized in [4, 7, 16, 17].

In this paper we are able to go a step further and simplify the construction of at least one of the possible descendants by assuming that the transverse-channel Klein-bottle ($\widetilde{\mathcal{K}}$), Annulus ($\widetilde{\mathcal{A}}$) and Möbius-strip ($\widetilde{\mathcal{M}}$) amplitudes exactly coincide in the proper parametrization of the worldsheet. The numerical relation $\widetilde{\mathcal{K}} = \widetilde{\mathcal{A}} = \widetilde{\mathcal{M}}$ automatically enforces the tadpole conditions. With this simplifying ansatz, the only non-trivial issue consists in reconstructing the open-string spectrum in terms of Chan-Paton (CP) group assignments from the overall transverse-channel multiplicities.

In the transverse channel, the Klein-bottle amplitude $\widetilde{\mathcal{K}}$ represents a closed-string exchange between "crosscap-states" $|C\rangle$ whose target-space counterparts are the loci left invariant under the combined action of orientation reversal (Ω) and some target-space symmetry ($\mathcal{I}$). Although these objects are well defined in a CFT context, they do not necessarily admit a sensible large volume limit. They share with the standard O-planes the property of being charged with respect to the R-R fields. The cancellation of the R-R charge flowing through the compact space requires as usual the introduction of "boundary-states" $|B\rangle$ carrying opposite R-R charges. A "minimal choice" is provided by boundaries $|B\rangle$ with the same closed-string content as $|C\rangle$, where the difference in the R-R charges between the two objects is compensated by a correct assignment of CP multiplicities. We will consider this minimal ansatz, according to which, once the CP multiplicities have been plugged into the annulus amplitude $\widetilde{\mathcal{A}}$, the $\langle B|B\rangle$ exchange numerically coincides with the $\langle C|C\rangle$ exchange even for the whole tower of massive states. The Möbius-strip amplitude $\langle B|C\rangle + \langle C|B\rangle$, when expressed in terms of "hatted" quantities [4] as required by the reality of $\widetilde{\mathcal{M}}$, will again coincide with the transverse annulus and Klein-bottle amplitudes, up to alternating signs at the massive levels. The construction is not as restrictive as one could imagine. One may easily check that in the case of toroidal orbifolds it encompasses most of the Z_2, Z_3 orientifold models considered in the recent literature [18]. A general list of solutions, and a rather more precise conformal description of the models we consider, can be systematically found following the open-descendant techniques developed in [16, 4, 17, 7]. Indeed, it is explicitly shown in Appendix B that the $Z_2^L \times Z_2^R$ model provides an orientifold description of the open-string descendant associated to the A_{16} permutation invariant considered in [6]. The same is true for the $Z_3^L \times Z_3^R$ model in $D = 6$ indicated as A_{81} in [6], although we omit here the details of a similar correspondence.

Some circumstantial evidence for the validity of our simplifying ansatz can be illustrated in the simplest context of the so-called toroidal orientifold models [19]. These models are obtained by quotienting the type II superstrings by $\Omega\mathcal{I}$, where $\mathcal{I}$ is the inversion of the (internal) coordinates of a d-dimensional torus. They are T-dual to standard toroidal compactifications of the type I superstring in the presence of Wilson lines breaking $SO(32)$ to $SO(2^{5-d})^{2^d}$ [17]. In the absence of Wilson lines the relation $\widetilde{\mathcal{K}} = \widetilde{\mathcal{A}} = \widetilde{\mathcal{M}}$ does not hold because crosscap states only couple to even windings while boundary states couple to all windings. Precisely after introducing the proper Wilson lines [17, 1, 19] the relation $\widetilde{\mathcal{K}} = \widetilde{\mathcal{A}} = \widetilde{\mathcal{M}}$ is enforced. All the open-string KK momenta are shifted by one-half unit and the transverse-channel annulus only allows even windings to flow.

Slightly at variant with the minimal ansatz, in the last part of the paper we construct a non-supersymmetric R-R tadpole-free model. The prize to pay is an uncancelled Neveu-Schwarz–Neveu-Schwarz (NS-NS) tadpole.

It is amusing to observe that in the absence of discrete torsion untwisted and twisted open-string states combine with one another to reconstruct multiplets of the enhanced bulk supersymmetry. This could sound surprising since twisted open-strings are naturally interpreted as strings connecting "branes at angles" [10], each of them breaking one-half of the original 32 type IIB supercharges. However in the present context the two objects are properly identified by elements of the orbifold group and the distinction between them tends to loose meaning. In the $Z_2^L \times Z_2^R$ case this is in line with the identification of one of the two chiral projections as the T-duality group element that inverts all the radii.

The plan of the paper is as follows. In Section 2 we briefly review the construction of $Z_N^L \times Z_N^R$ asymmetric orbifolds of the type IIB superstring. Section 3 is devoted to the explicit construction of the $Z_2^L \times Z_2^R$ orbifold in $D = 6$ in the presence of discrete torsion and to the analysis of its type I descendants. We partly associate the rank reduction of the CP group to the presence of a quantized NS-NS antisymmetric tensor background [17, 20] and partly to the identification of would-be D5- and D9-branes imposed after quotienting by T-duality. In Sections 4 and 5 we perform similar analyses for $Z_3^L \times Z_3^R$ orbifolds in $D = 4, 6$ and discuss their type I descendants. In Section 6 we describe a variant of the $Z_2^L \times Z_2^R$ model in $D = 6$ that leads to brane supersymmetry breaking [22, 21, 23, 24]. Finally, Section 7 contains our conclusions and comments for future developments of the present approach to other non-geometric vacuum configurations that admit simple and handy algebraic descriptions [6, 9, 8]. In order to make the paper as self-contained as possible, we have added two appendices. In Appendix A we set up our conventions and define the conformal blocks that appear in the asymmetric orbifolds under consideration. In Appendix B we have included an expansion of the $Z_2^L \times Z_2^R$ models in terms of generalized characters.

2 Asymmetric orbifolds: a quick review

Thanks to the large degree of independence between left and right movers on the string worldsheet, one can conceive vacuum configurations in which one set of modes propagates in a target space and the other in a completely different one. For closed strings, modular invariance puts very tight constraints and the largest class of models of this kind that have been constructed are the asymmetric orbifolds of Narain, Sarmadi and Vafa [2]. Free fermionic [3] and covariant lattice [14] constructions have some overlap with asymmetric orbifolds.

An asymmetric orbifold is obtained by quotienting a string compactification, typically on a d-dimensional torus, by a discrete group, typically a cyclic group, that acts asymmetrically on the left and right movers. For the heterotic string, that is left-right asymmetric from the very beginning, this is rather natural. For the type II superstrings, in particular for the type IIB superstring that is left-right symmetric, this may sound slightly artificial, but undoubtedly represents an improvement and an implementation of the string potentialities beyond their field-theory limit [25]. Most of these asymmetric constructions are still waiting for some geometric interpretation in terms of M-theory or F-theory, if any.

We will concentrate most of our analysis on $Z_N^L \times Z_N^R$ asymmetric orbifolds of toroidal compactifications of the type IIB superstring[1] and their open-string descendants.

The basic building blocks in the construction of Z_N asymmetric orbifolds, are the chiral supertraces

$$\rho_{g,h} \equiv \mathrm{Tr}'_{\mathrm{NS,g}} \frac{1}{2}(1-(-)^F) h q^{L_0 - \frac{c}{24}} - \mathrm{Tr}'_{\mathrm{R,g}} \frac{1}{2}(1+(-)^F) h q^{L_0 - \frac{c}{24}} \quad , \tag{1}$$

where $g, h \in Z_N$ are elements of the chiral orbifold group and the trace runs over the g-twisted sector with a plus for NS states and minus for R states. We denote by a prime the omission in the trace (1) of the free bosonic contributions

$$X_D = (\sqrt{\tau_2}\eta\bar{\eta})^{2-D} \tag{2}$$

for non-compact bosons, and

$$\Lambda_\Gamma = \frac{1}{(\eta\bar{\eta})^d} \sum_{\mathbf{p}\in\Gamma_{d,d}} q^{\frac{1}{2}\mathbf{p}_L^2} \bar{q}^{\frac{1}{2}\mathbf{p}_R^2} \tag{3}$$

for compact bosons, with $D + d = 10$. In addition, string partition functions will be weighted by the volume factors

$$\mathcal{V}_D \equiv \tau_2^{\frac{D}{2}} \int \frac{d\mathbf{p}\, d\mathbf{x}}{(2\pi)^D} e^{-\pi\tau_2\alpha'\mathbf{p}^2} = \frac{V_D}{(4\pi^2\alpha')^{\frac{D}{2}}} \tag{4}$$

and by an integer $C_{g,h}$ counting the number of "fixed points" under a given g-action left invariant by the h-projection. It can be computed applying the formula [2]

$$C_{g,h} = \left| \frac{N_g}{(1-g)N_g^* + (1-h)N_g} \right| \quad , \tag{5}$$

where N_g represents the lattice orthogonal to the lattice left invariant by g and N_g^* its dual. Notice that a lattice admitting a chiral discrete automorphism must be very special. The choice is typically restricted to weight lattices of compact Lie algebras [14].

3 $T^4/Z_2^L \times Z_2^R$ orbifold and its open-string descendants

Let us start with the $T^4/Z_2^L \times Z_2^R$ orbifold of the type IIB superstring. As a choice for a T^4 admitting a chiral Z_2 isometry we take the torus associated to the weight lattice of $SO(8)$. For future reference, notice that this requires to turn on a quantized NS-NS antisymmetric tensor background of rank two. The construction is equivalent to a

[1] The $Z_3^L \times Z_3^R$ case in $D = 4$ has been previously considered in [26]. We will find results somewhat in disagreement with [26] for the oriented closed-string spectra.

T-duality orbifold of the standard geometric Z_2 orbifold. Orbifolding by T-duality has been considered in [26] and [30] as a way to freeze out some of the moduli of the theory. From the open-string perspective it has been considered in [31].

The torus partition function can be written as

$$\mathcal{T} = \mathcal{V}_6 \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} X_6 \sum_{g_L, g_R} \mathcal{T}_{g_L, g_R} \quad , \tag{6}$$

where $\mathcal{F}$ is the fundamental region of the one-loop moduli space and

$$\begin{aligned}
\mathcal{T}_{00} &= \frac{1}{4}\left[\rho_{00}\bar{\rho}_{00}\Lambda_{SO(8)} + \rho_{00}\bar{\rho}_{01}\Lambda_R + \rho_{01}\bar{\rho}_{00}\bar{\Lambda}_R + \rho_{01}\bar{\rho}_{01}\right] \\
\mathcal{T}_{01} &= \frac{2}{4}\left[\rho_{00}\bar{\rho}_{10}\Lambda_W^+ + \rho_{00}\bar{\rho}_{11}\Lambda_W^- + \epsilon\rho_{01}\bar{\rho}_{10} + \epsilon\rho_{01}\bar{\rho}_{11}\right] \\
\mathcal{T}_{10} &= \frac{2}{4}\left[\rho_{10}\bar{\rho}_{00}\Lambda_W^+ + \rho_{11}\bar{\rho}_{00}\Lambda_W^- + \epsilon\rho_{10}\bar{\rho}_{01} + \epsilon\rho_{11}\bar{\rho}_{01}\right] \\
\mathcal{T}_{11} &= \frac{16}{4}\left[\rho_{10}\bar{\rho}_{10} + \rho_{11}\bar{\rho}_{11} - \frac{\epsilon}{2}\rho_{10}\bar{\rho}_{11} - \frac{\epsilon}{2}\rho_{11}\bar{\rho}_{10}\right] \quad .
\end{aligned} \tag{7}$$

The explicit form of the chiral supertraces ρ_{gh} and lattice sums is given in Appendix A. The relative powers of two represent the number of "fixed points" under the asymmetric orbifold group actions. Most of the amplitudes in (7) are in the same modular orbit as the amplitudes in the untwisted sector and therefore the relevant number of fixed points (associated to $C_{g,1}$) is easily determined from modular transformations. This is not the case for the modular orbit (g_L, h_R) which is clearly disconnected from the untwisted sector. By inspection of (5) one can easily see that $C_{g_L,h_R} = C_{g_L,1}$ and therefore the multiplicity of this orbit is again determined through modular transformations up to a Z_2-phase, *i.e.* a sign. This phase, that we have denoted by ϵ in (7), represents the discrete torsion between the two Z_2 factors [12].

Depending on the choice of discrete torsion, $\epsilon = \pm 1$, the spectrum of massless oriented closed-string states corresponds to compactifications of the type IIB theory on spaces topologically equivalent to T^4 and $K3$, respectively. The resulting massless spectra of closed oriented strings are:

ϵ	Supersymmetry	Supermultiplets
$+$	$\mathcal{N} = (2,2)$	$\mathbf{G}_{(2,2)}$
$-$	$\mathcal{N} = (2,0)$	$\mathbf{G}_{(2,0)} + 21\, \mathbf{T}_{(2,0)}$

where **G** and **T** stand for gravity and tensor super-multiplets respectively.

The Klein-bottle amplitude is defined by the Ω-projection of (7). Clearly, left-right asymmetric sectors, such as $\mathcal{T}_{01}$ or $\mathcal{T}_{10}$, will not contribute to this trace since states in these sectors come always in Ω even-odd pairs. The result only involves the diagonal components $\delta_{g_1,g_2}\rho_{g_1,h_1-h_2}$ and reads

$$\mathcal{K} = \frac{\mathcal{V}_6}{2} \int_0^\infty \frac{d\tau_2}{\tau_2^4} \frac{1}{\eta^4} \left[\frac{1}{2}\rho_{00}\Lambda_W^+ + \frac{1}{2}\rho_{01} + 2^{-\frac{r}{2}}\, 8\, \rho_{10} - \epsilon\, 2^{-\frac{r}{2}}\, 4\, \rho_{11}\right](2i\tau_2) \quad , \tag{8}$$

where we have used the fact that the total number of $\pm$ Ω-eigenvalues in the twisted-sector ground-states is given by [31, 32]

$$n_\pm = \frac{n_F}{2}(1 \pm 2^{-\frac{r}{2}}) \quad , \tag{9}$$

with $r = 2$ the rank of the antisymmetric tensor background B_{ij} in the $SO(8)$ lattice and n_F the number of fixed points $C_{g,h}$ in the parent torus amplitude. Notice that this mechanism is automatic in terms of characters, *i.e.* $n_+ - n_-$ characters appear diagonally in the one-loop modular invariant (see Appendix B).

As usual NS-NS (R-R) states flowing along the Klein-bottle (8) are (anti)symmetrized, while one half of the remaining ones survives the Ω-projection. The resulting spectra of massless unoriented closed-string states are given by:

ϵ	Supersymmetry	Supermultiplets
$+$	$\mathcal{N} = (1,1)$	$\mathbf{G}_{(1,1)} + 4\,\mathbf{V}^c_{(1,1)}$
$-$	$\mathcal{N} = (1,0)$	$\mathbf{G}_{(1,0)} + 14\,\mathbf{H}_{(1,0)} + 7\,\mathbf{T}_{(1,0)}$

In order to determine the unoriented open-string spectrum, that one has to couple to the above unoriented closed-string spectrum, we start by rewriting the Klein-bottle amplitude (8) in the transverse channel[2] ($\tau_2 \to 1/\tau_2$) as

$$\widetilde{\mathcal{K}} = 2^3 \frac{\mathcal{V}_6}{2} \int_0^1 \frac{dq}{2\pi q} \frac{1}{\eta^4} \left[\rho_{00}\Lambda_R + \rho_{01} + 2\rho_{10} + 2\epsilon\rho_{11}\right](q) \quad . \tag{10}$$

According to our simplifying ansatz, the transverse-channel annulus and Möbius-strip amplitudes read

$$\begin{aligned} \widetilde{\mathcal{A}} &= 2^{-3}\frac{\mathcal{V}_6}{2} \int_0^1 \frac{dq}{2\pi q} \frac{1}{4\eta^4} \Big[I_O^2(\rho_{00}O + \rho_{01} + 2\rho_{10} + 2\epsilon\rho_{11}) \\ &\quad + \rho_{00}(I_V^2\, V + I_S^2\, S + I_C^2\, C) + 2(\rho_{10} - \epsilon\rho_{11})(I_V^2 + I_S^2 + I_C^2)\Big](q) \\ \widetilde{\mathcal{M}} &= -2\frac{\mathcal{V}_6}{2} \int_0^1 \frac{dq}{2\pi q} \frac{I_O}{2\eta^4} \left[\rho_{00}O + \rho_{01} + 2\rho_{11} + 2\epsilon\rho_{10}\right](-q) \quad , \end{aligned} \tag{11}$$

with $I_O = 16$, $I_V = I_S = I_C = 0$. A simple CP group assignment of the boundary traces is given by

$$\begin{aligned} I_0 &= n_1 + n_2 + n_3 + n_4 \\ I_V &= n_1 + n_2 - n_3 - n_4 \\ I_S &= n_1 - n_2 + n_3 - n_4 \\ I_C &= n_1 - n_2 - n_3 + n_4 \quad , \end{aligned} \tag{12}$$

[2] Although tilded and untilded amplitudes coincide, we distinguish by a tilde the rewriting of the one-loop amplitudes in terms of the closed-string modular parameter of the common world-sheet double-cover.

with $n_1 = n_2 = n_3 = n_4 = 4$. Going to the direct-channel Annulus and Möbius-strip amplitudes through S and P modular transformations respectively yields

$$\begin{aligned}
\mathcal{A} &= \frac{\mathcal{V}_6}{2}\int_0^\infty \frac{d\tau_2}{\tau_2^4}\frac{1}{\eta^4}\left[\sum_i n_i^2[\frac{1}{2}\rho_{00}O + \frac{1}{2}\rho_{01} + \rho_{10} + \epsilon\rho_{11}] + \sum_{i<j} 2n_i n_j(\rho_{10} - \epsilon\rho_{11})\right. \\
&\quad \left. +(n_1n_2 + n_3n_4)\rho_{00}V + (n_1n_3 + n_2n_4)\rho_{00}S + (n_1n_4 + n_2n_3)\rho_{00}C\right](\frac{i\tau_2}{2}) \qquad (13) \\
\mathcal{M} &= (n_1 + n_2 + n_3 + n_4)\frac{\mathcal{V}_6}{2}\int_0^\infty \frac{d\tau_2}{\tau_2^4}\frac{1}{\eta^4}\left[\frac{1}{2}\rho_{00}O + \frac{1}{2}\rho_{01} + \rho_{11} + \epsilon\rho_{10}\right](\frac{i\tau_2}{2} + \frac{1}{2}) \quad .
\end{aligned}$$

Notice that for $\epsilon = +1$ terms proportional to I_V^2, I_S^2, I_C^2 do not contribute to tadpoles and therefore only the sum of the CP charges is fixed: $I_O = n_1+n_2+n_3+n_4 = 16$. This restricts only the total rank of the CP gauge group. At the massless level one thus finds:

ϵ	supersymmetry	Gauge group	Hypermultiplets
$+$	N= (1,1)	$\prod_{i=1}^4 Sp(n_i)$	$- - -$
$-$	N= (1,0)	$Sp(4)^4$	$(\mathbf{4},\mathbf{4},\mathbf{1},\mathbf{1}) + (\mathbf{4},\mathbf{1},\mathbf{4},\mathbf{1}) + (\mathbf{4},\mathbf{1},\mathbf{1},\mathbf{4}) +$
			$(\mathbf{1},\mathbf{4},\mathbf{4},\mathbf{1}) + (\mathbf{1},\mathbf{4},\mathbf{1},\mathbf{4}) + (\mathbf{1},\mathbf{1},\mathbf{4},\mathbf{4})$

The models with $\mathcal{N} = (1,1)$ correspond, at least topologically, to toroidal compactifications without vector structure, *i.e.* with a reduction of the rank of the CP group associated to a non-vanishing generalized second Stieffel-Whitney class and measured by the presence of a quantized NS-NS antisymmetric tensor [17, 20]. The model with $\mathcal{N} = (1,0)$ is chiral but anomaly-free thanks to the GSS mechanism [33, 34] that involves several antisymmetric tensors and is the field-theory counterpart of the R-R tadpole conditions [4]. Notice that because of the chiral Z_2 action, that implies a quotienting by the T-duality transformation $X_L^i \to -X_L^i$ with $X_R^i \to +X_R^i$ for $i = 1\ldots4$, would-be D9- and D5-branes are effectively identified and form some generalized brane bound-state. This accounts for a further reduction by half of the rank of the CP group. As mentioned in the Introduction, it is a difficult task to describe the above non-geometric vacuum configurations in terms of D-branes and O-planes. Their concepts become fuzzy in highly curved or non-geometric backgrounds [8, 21]. In the case under consideration, however, open-strings belonging to twisted sectors could be thought as strings with one end on a would-be D5-branes and the other end on the would-be D9-brane, here described by an unique CP charge n_i. It is interesting to observe that the $\mathcal{N} = (1,1)$ vector multiplets are recovered in the $\epsilon = +1$ case by mixing the would-be D9-D9 and D5-D5 states with the would-be D9-D5 states.

4 $T^4/Z_3^L \times Z_3^R$ orbifold and its open-string descendants

Let us now consider the $T^4/Z_3^L \times Z_3^R$ orbifold of the type IIB superstring. As a choice for a T^4 admitting a chiral Z_3 isometry we take the torus associated with the weight lattice of $SU(3)^2$. This requires to turn on a quantized NS-NS antisymmetric tensor background of rank four. Notice that, differently from what happens for geometric orbifolds, we will find the presence of open-string twisted sectors. Once again, no compelling D-brane interpretation is available for this kind of open strings in the present context.

The torus amplitude for this asymmetric orbifold is given by (6) with

$$
\begin{aligned}
\mathcal{T}_{00} &= \frac{1}{9}\left[\rho_{00}\bar\rho_{00}\Lambda_{SU(3)^2} + (\rho_{01}+\rho_{02})\bar\rho_{00}\bar\Lambda_R + \rho_{00}(\bar\rho_{01}+\bar\rho_{02})\Lambda_R + |\rho_{01}+\rho_{02}|^2\right] \\
\mathcal{T}_{01} &= \frac{1}{9}\left[\rho_{00}(\bar\rho_{10}\Lambda_W + \bar\rho_{11}\Lambda_W^{\omega} + \bar\rho_{12}\Lambda_W^{\bar\omega}) + (\epsilon\rho_{01}+\bar\epsilon\rho_{02})(\bar\rho_{10}+\bar\rho_{11}+\bar\rho_{12})\right] \\
\mathcal{T}_{02} &= \frac{1}{9}\left[\rho_{00}(\bar\rho_{20}\Lambda_W + \bar\rho_{22}\Lambda_W^{\omega} + \bar\rho_{21}\Lambda_W^{\bar\omega}) + (\epsilon\rho_{02}+\bar\epsilon\rho_{01})(\bar\rho_{20}+\bar\rho_{22}+\bar\rho_{21})\right] \\
\mathcal{T}_{11} &= \frac{1}{9}\left[9(\rho_{10}\bar\rho_{10}+\rho_{11}\bar\rho_{11}+\rho_{12}\bar\rho_{12}) - 3\epsilon(\rho_{10}\bar\rho_{12}+\rho_{11}\bar\rho_{10}+\rho_{12}\bar\rho_{11})\right. \\
&\quad \left. -3\bar\epsilon(\rho_{11}\bar\rho_{12}+\rho_{12}\bar\rho_{10}+\rho_{10}\bar\rho_{11})\right] \\
\mathcal{T}_{22} &= \frac{1}{9}\left[9(\rho_{20}\bar\rho_{20}+\rho_{22}\bar\rho_{22}+\rho_{21}\bar\rho_{21}) - 3\epsilon(\rho_{20}\bar\rho_{21}+\rho_{22}\bar\rho_{20}+\rho_{21}\bar\rho_{22})\right. \\
&\quad \left. -3\bar\epsilon(\rho_{22}\bar\rho_{21}+\rho_{21}\bar\rho_{20}+\rho_{20}\bar\rho_{22})\right] \\
\mathcal{T}_{12} &= \frac{1}{9}\left[9(\rho_{10}\bar\rho_{20}+\rho_{11}\bar\rho_{22}+\rho_{12}\bar\rho_{21}) - 3\epsilon(\rho_{10}\bar\rho_{22}+\rho_{11}\bar\rho_{21}+\rho_{12}\bar\rho_{20})\right. \\
&\quad \left. -3\bar\epsilon(\rho_{11}\bar\rho_{20}+\rho_{12}\bar\rho_{22}+\rho_{10}\bar\rho_{21})\right] \quad . \qquad (14)
\end{aligned}
$$

The $\mathcal{T}_{10}$, $\mathcal{T}_{20}$ and $\mathcal{T}_{21}$ torus amplitudes are given by the complex conjugate of $\mathcal{T}_{01}$, $\mathcal{T}_{02}$ and $\mathcal{T}_{12}$ respectively.

As above, depending on the choice of ϵ, the model enjoys $\mathcal{N}=(2,2)$ or $\mathcal{N}=(2,0)$ spacetime supersymmetry. The resulting massless oriented closed-string contents are:

ϵ	Supersymmetry	Supermultiplets
1	$\mathcal{N}=(2,2)$	$\mathbf{G}_{(2,2)}$
$e^{\pm\frac{2\pi i}{3}}$	$\mathcal{N}=(2,0)$	$\mathbf{G}_{(2,0)} + 21\,\mathbf{T}_{(2,0)}$

Notice that in this case the two choices $\epsilon = e^{+\frac{2\pi i}{3}}$ and $\epsilon = e^{-\frac{2\pi i}{3}}$ give equivalent theories. As before, the Klein-bottle amplitude is expressed only in terms of the chiral amplitudes $\delta_{g_L,g_R}\rho_{g_L,h_L-h_R}$, that appear diagonally in (14), and reads

$$
\begin{aligned}
\mathcal{K} &= \frac{\mathcal{V}_6}{2}\int_0^\infty \frac{d\tau_2}{\tau_2^4}\frac{1}{\eta^4}\left[\frac{1}{3}(\rho_{00}\Lambda_W + \rho_{01} + \rho_{02})\right. \\
&\quad \left. +3\rho_{10} - \epsilon\rho_{11} - \bar\epsilon\rho_{12} + 3\rho_{20} - \epsilon\rho_{22} - \bar\epsilon\rho_{21}\right](2i\tau_2) \quad . \qquad (15)
\end{aligned}
$$

The action of Ω on the fixed points is determined by the requirement that only Z_3 invariant states flow in the transverse $\langle C|C\rangle$ amplitude. Indeed, writing (15) in terms of the closed-string variables, one finds

$$
\begin{aligned}
\tilde{\mathcal{K}} &= 2^3\frac{\mathcal{V}_6}{2}\int_0^1 \frac{dq}{2\pi q}\frac{1}{\eta^4}\left[\rho_{00}\Lambda_R + \rho_{01} + \rho_{02}\right. \\
&\quad \left. +\rho_{10} + \bar\epsilon\rho_{11} + \epsilon\rho_{12} + \rho_{20} + \epsilon\rho_{21} + \bar\epsilon\rho_{22}\right](q) \quad . \qquad (16)
\end{aligned}
$$

Thus, only states with Z_3 eigenvalue equal to 1 in the untwisted sector and $\epsilon, \bar{\epsilon}$ in the twisted sectors flow in the $\langle C|C\rangle$ amplitude.

The Klein-bottle projection (anti)symmetrizes NS-NS (R-R) states in the left-right symmetric sectors, $\mathcal{T}_{00}, \mathcal{T}_{11}, \mathcal{T}_{22}$, and halves the ones in the remaining left-right asymmetric sectors. The unoriented massless closed-string states that survive the projections are given by:

ϵ	Supersymmetry	Supermultiplets
1	$\mathcal{N} = (1,1)$	$\mathbf{G}_{(1,1)} + 4\,\mathbf{V}^c_{(1,1)}$
$e^{\pm\frac{2\pi i}{3}}$	$\mathcal{N} = (1,0)$	$\mathbf{G}_{(1,0)} + 15\,\mathbf{H}_{(1,0)} + 6\,\mathbf{T}_{(1,0)}$

According to our simplifying ansatz, the open-string sectors are completely determined once (16) is given. The relevant transverse amplitudes read

$$\begin{aligned}
\widetilde{\mathcal{A}} &= 2^{-3}\frac{N^2}{2}\mathcal{V}_6\int_0^1 \frac{dq}{2\pi q}\frac{1}{\eta^4}\,[\rho_{00}\Lambda_R + \rho_{01} + \rho_{02} \\
&\quad +\rho_{10} + \bar{\epsilon}\rho_{11} + \epsilon\rho_{12} + \rho_{20} + \epsilon\rho_{21} + \bar{\epsilon}\rho_{22}]\,(q) \\
\widetilde{\mathcal{M}} &= -2\frac{N}{2}\mathcal{V}_6\int_0^1 \frac{dq}{2\pi q}\frac{1}{\eta^4}\,[\rho_{00}\Lambda_R + \rho_{01} + \rho_{02} \\
&\quad +\rho_{11} + \bar{\epsilon}\rho_{12} + \epsilon\rho_{10} + \rho_{22} + \epsilon\rho_{20} + \bar{\epsilon}\rho_{21}]\,(-q) \quad ,
\end{aligned} \tag{17}$$

with $N = 8$. In the direct channel we are finally left with

$$\begin{aligned}
\mathcal{A} &= \frac{N^2}{2}\mathcal{V}_6\int_0^\infty \frac{d\tau_2}{\tau_2^4}\frac{1}{\eta^4}\left[\frac{1}{3}(\rho_{00}\Lambda_W + \rho_{01} + \rho_{02})\right. \\
&\quad \left. +3\rho_{10} - \epsilon\rho_{11} - \bar{\epsilon}\rho_{12} + 3\rho_{20} - \bar{\epsilon}\rho_{21} - \epsilon\rho_{22}\right](\frac{i\tau_2}{2}) \\
\mathcal{M} &= -\frac{N}{2}\mathcal{V}_6\int_0^\infty \frac{d\tau_2}{\tau_2^4}\frac{1}{\eta^4}\left[\frac{1}{3}(\rho_{00}\Lambda_W + \rho_{01} + \rho_{02})\right. \\
&\quad \left. +3\rho_{11} - \epsilon\rho_{12} - \bar{\epsilon}\rho_{10} + 3\rho_{22} - \bar{\epsilon}\rho_{20} - \epsilon\rho_{21}\right](\frac{i\tau_2}{2} + \frac{1}{2}) \quad .
\end{aligned} \tag{18}$$

The additional phases are due to the fact that only "hatted" quantitities should enter $\mathcal{M}$ and $\widetilde{\mathcal{M}}$ as required by reality of the amplitudes.

The resulting massless open-string content is now given by:

ϵ	supersymmetry	Gauge group	Hypermultiplets
1	(1,1)	$SO(8)$	$- - -$
$e^{\pm\frac{2\pi i}{3}}$	(1,0)	$SO(8)$	$4\,(\mathbf{28})$

Once again, the model with $\mathcal{N} = (1,1)$ corresponds to a toroidal compactification without vector structure [17, 20], while the model with $\mathcal{N} = (1,0)$ is chiral but anomaly-free thanks to the GSS mechanism [33, 34]. Notice that because of the chiral Z_3 action the open-string spectrum involves states belonging to the twisted sectors. This should not sound too surprising given the form of the torus amplitude.

5 $T^6/Z_3^L \times Z_3^R$ Orbifold and its open-string descendants

Very similarly to the above model, we can now discuss the $Z_3^L \times Z_3^R$ orbifold of the type IIB superstring in $D = 4$ and its open-string descendants. As a choice for a T^6 admitting a chiral Z_3 isometry we take the torus associated with the weight lattice of $SU(3)^3$. This requires to turn on a quantized NS-NS antisymmetric tensor background of rank six.

The torus amplitude is given by

$$\mathcal{T} = \mathcal{V}_4 \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} X_4 \sum_{g_L, g_R} \mathcal{T}_{g_L, g_R} \quad , \tag{19}$$

with

$$\begin{aligned}
\mathcal{T}_{00} &= \frac{1}{9}\Big[\rho_{00}\bar\rho_{00}\Lambda_{SU(3)^2} + (\rho_{01}+\rho_{02})\bar\rho_{00}\bar\Lambda_R + \rho_{00}(\bar\rho_{01}+\bar\rho_{02})\Lambda_R + |\rho_{01}+\rho_{02}|^2\Big] \\
\mathcal{T}_{01} &= \frac{1}{9}\left[\rho_{00}(\bar\rho_{10}\Lambda_W + \bar\rho_{11}\Lambda_W^{\omega} + \bar\rho_{12}\Lambda_W^{\bar\omega}) + (\epsilon\rho_{01} + \bar\epsilon\rho_{02})(\bar\rho_{10}+\bar\rho_{11}+\bar\rho_{12})\right] \\
\mathcal{T}_{02} &= \frac{1}{9}\left[\rho_{00}(\bar\rho_{20}\Lambda_W + \bar\rho_{22}\Lambda_W^{\omega} + \bar\rho_{21}\Lambda_W^{\bar\omega}) + (\epsilon\rho_{02} + \bar\epsilon\rho_{01})(\bar\rho_{20}+\bar\rho_{22}+\bar\rho_{21})\right] \\
\mathcal{T}_{11} &= \frac{1}{9}\Big[27(\rho_{10}\bar\rho_{10}+\rho_{11}\bar\rho_{11}+\rho_{12}\bar\rho_{12}) + 3\sqrt{3}i\epsilon(\rho_{10}\bar\rho_{12}+\rho_{11}\bar\rho_{10}+\rho_{12}\bar\rho_{11}) \\
&\quad -3\sqrt{3}i\bar\epsilon(\rho_{11}\bar\rho_{12}+\rho_{12}\bar\rho_{10}+\rho_{10}\bar\rho_{11})\Big] \\
\mathcal{T}_{22} &= \frac{1}{9}\Big[27(\rho_{20}\bar\rho_{20}+\rho_{22}\bar\rho_{22}+\rho_{21}\bar\rho_{21}) + 3\sqrt{3}i\epsilon(\rho_{20}\bar\rho_{21}+\rho_{22}\bar\rho_{20}+\rho_{21}\bar\rho_{22}) \\
&\quad -3\sqrt{3}i\bar\epsilon(\rho_{22}\bar\rho_{21}+\rho_{21}\bar\rho_{20}+\rho_{20}\bar\rho_{22})\Big] \\
\mathcal{T}_{12} &= \frac{1}{9}\Big[27(\rho_{10}\bar\rho_{20}+\rho_{11}\bar\rho_{22}+\rho_{12}\bar\rho_{21}) - 3\sqrt{3}i\epsilon(\rho_{10}\bar\rho_{22}+\rho_{11}\bar\rho_{21}+\rho_{12}\bar\rho_{20}) \\
&\quad +3\sqrt{3}i\bar\epsilon(\rho_{11}\bar\rho_{20}+\rho_{12}\bar\rho_{22}+\rho_{10}\bar\rho_{21})\Big] \quad .
\end{aligned} \tag{20}$$

The amplitudes $\mathcal{T}_{10}$, $\mathcal{T}_{20}$ and $\mathcal{T}_{21}$ are the complex conjugate of $\mathcal{T}_{01}$, $\mathcal{T}_{02}$ and $\mathcal{T}_{12}$, respectively.

Depending on the choice of discrete torsion ϵ, the massless oriented closed-string spectra are given by:

ϵ	Supersymmetry	Supermultiplets
1	$\mathcal{N}=4$	$\mathbf{G}_4 + 10\mathbf{V}_4$
$e^{+\frac{2\pi i}{3}}$	$\mathcal{N}=2$	$\mathbf{G}_2 + 19\,\mathbf{H}_2 + 6\,\mathbf{V}_2$
$e^{-\frac{2\pi i}{3}}$	$\mathcal{N}=2$	$\mathbf{G}_2 + 7\,\mathbf{H}_2 + 18\,\mathbf{V}_2$

and correspond, respectively, to compactifications on spaces topologically equivalent to $K3 \times T^2$ and to two mirror Calabi-Yau spaces with Hodge numbers $h_{1,1} = 18 = h'_{1,2}$, $h_{1,2} = 6 = h'_{1,1}$.

Notice that, differently from [26], we find for $\epsilon = 1$ an $\mathcal{N} = 4$ theory with only 10 rather than 28 vector multiplets. The number of vector multiplets is smaller than the maximum number, 22, found in superstring compactifications with $\mathcal{N} = 4$ in $D = 4$. From the type II perspective, they correspond to compactifications on manifolds that are locally, but not necessarily globally, of the form $K3 \times T^2$. From the type I or, equivalently, heterotic perspective, they correspond to toroidal compactifications possibly with non-commuting Wilson lines [17, 20, 27, 28]. In fact, some candidate dual pairs descend from the duality between type IIA superstring on $K3$ and heterotic string on T^4 [29]. In the case under consideration, because of the chiral nature of the projections and twistings, it is not easy to find a geometric interpretation for the above superstring vacuum configurations. Still, the left-right symmetry of the resulting oriented closed-string theory suggests that there is no reason to exceed the "experimental" bound of 22 on the number of vector multiplets. Moreover no "exotic" brane whose excitations could account for the extra vector multiplets has been proposed so far [26].

Let us therefore discuss the open-string descendants. The Klein-bottle projection is now given by

$$
\begin{aligned}
\mathcal{K} \; &= \; \frac{\mathcal{V}_4}{2} \int_0^\infty \frac{d\tau_2}{\tau_2^4} \frac{1}{\eta^4} \left[\frac{1}{3} (\rho_{00} \Lambda_W + \rho_{01} + \rho_{02}) \right. \\
&\left. +9\rho_{10} + i\sqrt{3}\epsilon\rho_{11} - i\sqrt{3}\bar{\epsilon}\rho_{12} + 9\rho_{20} + i\sqrt{3}\epsilon\rho_{22} - i\sqrt{3}\bar{\epsilon}\rho_{21} \right] (2i\tau_2) \quad , \qquad (21)
\end{aligned}
$$

and yields the following unoriented massless closed-string spectra:

ϵ	Supersymmetry	Supermultiplets
1	$\mathcal{N} = 2$	$\mathbf{G}_2 + 2\mathbf{V}_2 + 9\mathbf{H}_2$
$e^{+\frac{2\pi i}{3}}$	$\mathcal{N} = 1$	$\mathbf{G}_1 + 25\,\mathbf{C}_1$
$e^{-\frac{2\pi i}{3}}$	$\mathcal{N} = 1$	$\mathbf{G}_1 + 22\,\mathbf{C}_1 + 3\,\mathbf{V}_1$

with $\mathbf{C}_1$ denoting the chiral multiplet in $D = 4$.

The type I descendant is constructed by identifying the transverse-channel amplitudes

$$
\begin{aligned}
\widetilde{\mathcal{K}} \; &= \; 2^2 \frac{\sqrt{3}}{2} \mathcal{V}_4 \int_0^1 \frac{dq}{2\pi q} \frac{1}{\eta^2} \left[\rho_{00} \Lambda_R + \rho_{01} + \rho_{02} \right. \\
&\left. +\rho_{10} + \bar{\epsilon}\rho_{11} + \epsilon\rho_{12} + \rho_{20} + \epsilon\rho_{21} + \bar{\epsilon}\rho_{22} \right] (q) \\
\widetilde{\mathcal{A}} \; &= \; 2^{-2} \sqrt{3} \frac{N^2}{2} \mathcal{V}_4 \int_0^1 \frac{dq}{2\pi q} \frac{1}{\eta^2} \left[\rho_{00} \Lambda_R + \rho_{01} + \rho_{02} \right. \\
&\left. +\rho_{10} + \bar{\epsilon}\rho_{11} + \epsilon\rho_{12} + \rho_{20} + \epsilon\rho_{21} + \bar{\epsilon}\rho_{22} \right] (q) \\
\widetilde{\mathcal{M}} \; &= \; -2\sqrt{3} \frac{N}{2} \mathcal{V}_4 \int_0^1 \frac{dq}{2\pi q} \frac{1}{\eta^2} \left[\rho_{00} \Lambda_R + \rho_{01} + \rho_{02} \right. \\
&\left. +\rho_{11} + \bar{\epsilon}\rho_{12} + \epsilon\rho_{10} + \rho_{22} + \epsilon\rho_{20} + \bar{\epsilon}\rho_{21} \right] (-q) \quad , \qquad (22)
\end{aligned}
$$

with one another. This requires taking $N = 4$.

The direct-channel open-string amplitudes then read

$$
\mathcal{A} \; = \; \frac{N^2}{2} \mathcal{V}_4 \int_0^\infty \frac{d\tau_2}{\tau_2^4} \frac{1}{\eta^2} \left[\frac{1}{3} (\rho_{00} \Lambda_W + \rho_{01} + \rho_{02}) \right.
$$

$$+9\rho_{10}+i\sqrt{3}\epsilon\rho_{11}-i\sqrt{3}\bar{\epsilon}\rho_{12}+9\rho_{20}-i\sqrt{3}\bar{\epsilon}\rho_{21}+i\sqrt{3}\epsilon\rho_{22}\Big]\,(\frac{i\tau_2}{2})$$

$$\mathcal{M} \;=\; +\frac{N}{2}\mathcal{V}_4\int_0^\infty \frac{d\tau_2}{\tau_2^4}\frac{1}{\eta^2}\left[\frac{1}{3}(\rho_{00}\Lambda_W+\rho_{01}+\rho_{02})\right.$$

$$\left.+9\rho_{11}+i\sqrt{3}\epsilon\rho_{12}-i\sqrt{3}\bar{\epsilon}\rho_{10}+9\rho_{22}-i\sqrt{3}\bar{\epsilon}\rho_{20}+i\sqrt{3}\epsilon\rho_{21}\right](\frac{i\tau_2}{2}+\frac{1}{2}) \quad .(23)$$

The resulting CP group[3] is $Sp(4)$ and the massless open-string content is given by

ϵ	Supersymmetry	Gauge Group	Hypermultiplets
1	$\mathcal{N}=2$	$Sp(4)$	$4(\mathbf{10})$
$e^{+\frac{2\pi i}{3}}$	$\mathcal{N}=1$	$Sp(4)$	$6(\mathbf{10})$
$e^{-\frac{2\pi i}{3}}$	$\mathcal{N}=1$	$Sp(4)$	$12(\mathbf{10})$

The $\epsilon=1$ model is IR free. The CP group can be completely higgsed in the Higgs branch and the remaining massless hypermultiplets parametrize a non-trivial quaternionic manifold which becomes hyperkäler in the rigid limit. The models with $\epsilon=e^{\pm\frac{2\pi i}{3}}$ are non chiral and IR free. Although phenomenologically not very appealing, they are interesting in at least two respects. First, states belonging to the twisted sectors of the chiral orbifold group appear in the open-string spectrum. Second, at least for $\epsilon=e^{-\frac{2\pi i}{3}}$, additional vector multiplets appear in the unoriented closed-string spectrum thus decreasing the number of marginal deformations. The additional vectors belong to the R-R sector and as such are not minimally coupled to the perturbative string excitations. It would be interesting to identify the non-perturbative states that couple minimally to these R-R fields. They would represent the generalization of the concept of D-brane in these non-geometric compactifications.

6 $T^4/Z_2^L\times Z_2^R$ with non-supersymmetric open-string sectors

In recent times, there has been a lot of interest in constructing type I models with brane supersymmetry breaking either at the string scale [23] or at the compactification scale [22, 21, 24]. By this one means models in which supersymmetry is exact in the bulk but it is broken along (a subset of) the branes[4]. Clearly, because of the coupling between the brane excitations and the bulk modes, supersymmetry breaking is then transmitted to the whole system. So far, only geometric models have been considered. We would like to observe that the same mechanism of supersymmetry breaking can be exposed in the context of non-geometric models such as the ones discussed in the present paper. To keep the discussion as simple as possible we will restrict our attention to a non-supersymmetric version of the open-string descendant of the $T^4/Z_2^L\times Z_2^R$ orbifold

[3]This corrects an imprecise statement made in the last section of [35] concerning Z_3 orbifolds with quantized B_{ij}.

[4]Alternatively one may consider models in which the theory in the bulk is less supersymmetric than the theory on the branes [36].

discussed in Section 2. Supersymmetry breaking can be achieved by replacing the "*D*-branes", associated to the n_3, n_4 CP charges, with "anti-D-branes", and modifying the Ω action on the fixed-point space. The resulting unoriented descendant is encoded in the following amplitudes

$$\begin{aligned}
\mathcal{K}_{ns} &= \frac{\mathcal{V}_6}{2}\int_0^\infty \frac{d\tau_2}{\tau_2^4}\frac{1}{\eta^4}\left[\frac{1}{2}\rho_{00}(O+V-S-C)+\frac{1}{2}\rho_{01}+2\epsilon\rho_{11}\right](2i\tau_2)\\
\mathcal{A}_{ns} &= \frac{\mathcal{V}_6}{2}\int_0^\infty \frac{d\tau_2}{\tau_2^4}\frac{1}{\eta^4}\Big[(2n\bar{n}+2m\bar{m})(\frac{1}{2}\rho_{00}O+\frac{1}{2}\rho_{01}+\rho_{10}+\epsilon\rho_{11})\\
&\quad +(n^2+\bar{n}^2+m^2+\bar{m}^2)(\frac{1}{2}\rho_{00}V+\rho_{10}-\epsilon\rho_{11})\\
&\quad +(2n\bar{m}+2\bar{n}m)(\frac{1}{2}\rho'_{00}S+\rho'_{10}-\epsilon\rho'_{11})+(2nm+2\bar{n}\bar{m})(\frac{1}{2}\rho'_{00}C+\rho'_{10}-\epsilon\rho'_{11})\Big](\frac{i\tau_2}{2})\\
\mathcal{M}_{ns} &= \frac{\mathcal{V}_6}{2}\int_0^\infty \frac{d\tau_2}{\tau_2^4}\frac{1}{\eta^4}\Big[(n+\bar{n})(-\frac{1}{2}\rho_{00}V+\rho_{11}-\epsilon\rho_{10})\\
&\quad -(m+\bar{m})(\frac{1}{2}\rho''_{00}V+\rho''_{11}-\epsilon\rho''_{10})\Big](\frac{i\tau_2}{2}+\frac{1}{2}) \quad , \qquad (24)
\end{aligned}$$

where with a prime ($'$) and a double prime ($''$) we denote non-supersymmetric chiral traces obtained by modifying the sums over spin structures in $\rho_{g,h}$ in the way discussed in Appendix A. This correspond to changing the sign of the Ramond sector for the chiral supertraces with a double prime and to interchanging O, S with V, C in the space-time part for the amplitudes with a prime. The transverse amplitudes are then given by

$$\begin{aligned}
\widetilde{\mathcal{K}}_{ns} &= 2^3\frac{\mathcal{V}_6}{2}\int_0^1 \frac{dq}{2\pi q}\frac{1}{\eta^4}\left[\rho_{00}V+2\rho_{10}-2\epsilon\rho_{11}\right](q)\\
\widetilde{\mathcal{A}}_{ns} &= 2^{-3}\frac{\mathcal{V}_6}{2}\int_0^1 \frac{dq}{2\pi q}\frac{1}{\eta^4}\Big[\frac{1}{8}(I_O^2+I_V^2)(\rho_{00}(O+V)+\rho_{01}+4\rho_{10})\\
&\quad +\frac{1}{8}(I_O^2-I_V^2)(\rho''_{00}(O-V)+\rho''_{01}+4\epsilon\rho''_{11})\\
&\quad -\frac{1}{8}(I_S^2+I_C^2)(\rho_{00}(S+C)+4\rho_{10}-4\epsilon\rho_{11})-\frac{1}{8}(I_S^2-I_C^2)\rho''_{00}(S-C)\Big](q)\\
\widetilde{\mathcal{M}}_{ns} &= -2\frac{\mathcal{V}_6}{2}\int_0^1 \frac{dq}{2\pi q}\frac{1}{\eta^4}\Big[\frac{(I_V+I_O)}{4}(-\rho_{00}V+2\rho_{11}-2\epsilon\rho_{10})\\
&\quad +\frac{(I_V-I_O)}{4}(\rho''_{00}V+2\rho''_{11}-2\epsilon\rho''_{10})\Big](-q) \quad , \qquad (25)
\end{aligned}$$

where it is convenient to parametrize the CP charge assignments as

$$\begin{aligned}
I_0 &= n+\bar{n}+m+\bar{m}\\
I_V &= n+\bar{n}-m-\bar{m}\\
I_S &= n-\bar{n}+m-\bar{m}\\
I_C &= n+\bar{n}-m+\bar{m} \quad , \qquad (26)
\end{aligned}$$

with $n=\bar{n}=m=\bar{m}=4$ fixed by R-R tadpole cancellation. Comparing the above amplitudes in the transverse channel with the ones found in the supersymmetric case one can see that the replacements correspond precisely to flipping the signs of the mixed (nm) RR-sectors, so that $\rho_{gh} \rightarrow \rho''_{gh}$. As already discussed, this is interpreted as a replacement of a (sub)set of would-be D-branes by the corresponding would-be anti-D-branes. Moreover, as mentioned in the introduction, we have relaxed our simplifying ansatz $\widetilde{\mathcal{K}} = \widetilde{\mathcal{A}} = \widetilde{\mathcal{M}}$. Imposing this condition would have resulted into the supersymmetric model of Section 2 and would not have reached our aim of implementing the mechanism of brane supersymmetry breaking.

Depending on the choice of discrete torsion, the new Klein-bottle projection leads to the following massless unoriented closed-string spectra:

ϵ	Supersymmetry	Supermultiplets
$+$	$\mathcal{N} = (1,1)$	$\mathbf{G}_{(1,1)} + 4\,\mathbf{V}^c_{(1,1)}$
$-$	$\mathcal{N} = (1,0)$	$\mathbf{G}_{(1,0)} + 10\,\mathbf{H}_{(1,0)} + 11\,\mathbf{T}_{(1,0)}$

Once again, let us stress that supersymmetry is exact in the bulk while, as expected, it is broken in the open-string sector. At the massless level one finds

$\epsilon = 1$

matter	$U(4)^2$-representations
(V+4O-2S-2C)	$(\mathbf{16}, \mathbf{1}) + (\mathbf{1}, \mathbf{16})$
2 (4O)	$(\mathbf{4}, \mathbf{4}) + (\bar{\mathbf{4}}, \bar{\mathbf{4}}) + (\mathbf{4}, \bar{\mathbf{4}}) + (\bar{\mathbf{4}}, \mathbf{4})$

$\epsilon = -1$

matter	$U(4)^2$-representations
(V-2S)	$(\mathbf{16}, \mathbf{1}) + (\mathbf{1}, \mathbf{16})$
(4O-2C)	$(\mathbf{10}, \mathbf{1}) + (\bar{\mathbf{10}}, \mathbf{1})$
(4O-2S)	$(\mathbf{4}, \mathbf{4}) + (\bar{\mathbf{4}}, \bar{\mathbf{4}}) + (\mathbf{4}, \bar{\mathbf{4}}) + (\bar{\mathbf{4}}, \mathbf{4})$
(4O)	$(\mathbf{1}, \mathbf{6}) + (\mathbf{1}, \bar{\mathbf{6}})$
(-2C)	$(\mathbf{1}, \mathbf{10}) + (\mathbf{1}, \bar{\mathbf{10}})$

Notice, in particular, that supersymmetry is broken only by open strings charged under the anti D-brane gauge group (the second $U(4)$ factor above). It is easy to check that both gauge and gravitational anomalies are absent, thanks to the vanishing of the R-R tadpoles. Indeed, for the potential gauge anomaly of each $U(4)$ CP group one finds $2n - 2(n+8) + 4m = 0$, while for the potential gravitational anomaly, one finds $(4 \times 10 - 4 \times 16 - 2 \times 16)_o + (29 \times 11 + 10 - 273)_c = 0$. Once the minimal ansatz is left aside, it is interesting to exploit some extra freedom available in the $\epsilon = 1$ case. Since, as one can immediately see, no massless tadpoles are present in the transverse-channel Klein-bottle amplitude, there is no need of adding open strings and no brane supersymmetry breaking is induced in this case. The resulting six dimensional model is one of the most economic string realizations of $N = (1,1)$ supergravity coupled to 4 vector supermultiplets.

Although there is no claim of phenomenological appeal, we would like to stress that the potential applications of the above construction to non-geometric models are undoubtly far reaching and deserve an extensive study. It would be interesting to study the connection of the above kind of models with stable non-BPS configurations of branes studied in the recent literature (see *e.g.* [37] and references therein).

7 Conclusions and discussion

We have discussed some asymmetric orbifolds of the type IIB superstring and their open and unoriented string descendants.

A nice feature of the models is that the introduction of discrete torsion allows to break half of the supersymmetries and relate "topologically" different compactifications. We have also shown that prior to the introduction of a non-trivial discrete torsion no exotic phenomena [26] appear in the oriented closed-string theories. This has been obtained after carefully taking into account the subtle phases that appear in the modular transformations of the chiral amplitudes.

From the algebraic point of view, the introduction of discrete torsion allows one to relate modular combinations of characters with extended symmetry and permutation modular invariants. This gives a rationale for the various unoriented descendants found by varying the choice of the modular invariant one-loop amplitudes in the type II parent theories. In particular, one is lead to speculate that some of the permutation modular invariants can be found by chiral projections much in the same way as in the simple asymmetric orbifold instances that have been considered above. In these cases the perturbatively different unoriented descendants may turn out to be non-perturbatively equivalent when a proper action of the projection is implemented on the solitonic spectra and interactions.

The consistency of the construction does not require a D-brane interpretation, that would be fuzzy in non-geometric environments such as the above ones and to some extent useless. At the string level, *i.e.* when high curvatures are present or when the vacuum configuration has no direct geometric interpretation, algebraic constructions such as boundary and crosscap states that can be extracted via techniques of unoriented descendants are much more rewarding. It is worth stressing that, although the geometrical meaning of boundaries and crosscaps in the present context is obscure, string techniques [38] can still allow one to determine the WZ anomalous couplings of the properly generalized solitonic objects to the bulk fields. The question of identifying the non-perturbative (non)BPS solitons in the above non-geometric backgrounds is still open.

8 Acknowledgements

We would like to thank C. Angelantonj, A. Dabholkar, A. Hammou, K.S. Narain, K. Ray, A. Sagnotti, H. Sarmadi, C.A. Scrucca, M. Serone and Ya.S. Stanev for fruitful discussions. We would like to thank the organizers of the "Extended Workshop in String Theory" for the kind hospitality at Abdus Salam ICTP, Trieste, where this work has been initiated. One of us (M.B.) would like to thank the organizers of the workshop on "String Theory: Duality, Gravity & Field Theory" for the stimulating atmosphere at the Aspen Center for Physics, where part of this investigation was carried on.

9 Appendix A: Conformal blocks in $D = 4, 6$

In a Z_N-orbifold, the chiral traces of an element h over states in a given g-twisted sector read

$$\begin{aligned}
\rho_{00} &\equiv \frac{1}{2} \sum_{\alpha,\beta=0,1/2} (-)^{2\alpha+2\beta+4\alpha\beta} \frac{\vartheta\left[{\alpha \atop \beta}\right]^4}{\eta^4} \\
\rho_{0h} &\equiv \frac{1}{2} \sum_{\alpha,\beta=0,1/2} (-)^{2\alpha+2\beta+4\alpha\beta} \left(\frac{\vartheta\left[{\alpha \atop \beta}\right]}{\eta}\right)^{4-d/2} \prod_{i=1}^{d/2} (2 sin\pi h_i) \frac{\vartheta\left[{\alpha \atop \beta+h_i}\right]}{\vartheta\left[{\frac{1}{2} \atop \frac{1}{2}+h_i}\right]} \qquad h \neq 0 \\
\rho_{gh} &\equiv -(i)^{\frac{d}{2}} \frac{1}{2} \sum_{\alpha,\beta=0,1/2} (-)^{2\alpha+2\beta+4\alpha\beta} \left(\frac{\vartheta\left[{\alpha \atop \beta}\right]}{\eta}\right)^{4-d/2} \prod_{i=1}^{d/2} \frac{\vartheta\left[{\alpha+g_i \atop \beta+h_i}\right]}{\vartheta\left[{\frac{1}{2}+g_i \atop \frac{1}{2}+h_i}\right]} \qquad g, h \neq 0 \quad , \qquad (27)
\end{aligned}$$

where $\vartheta\left[{\alpha \atop \beta}\right]$ are the standard Jacobi theta functions with characteristics and $\sum_i^{d/2} g_i = \sum_i^{d/2} h_i = 0(mod1)$.

The behaviour under S-modular transformations ($\tau \to -1/\tau$) is as follows

$$\begin{aligned}
\rho_{00} &\to \rho_{00} \\
\rho_{0h} &\to (2 sin\pi h)^{\frac{d}{2}} \rho_{h0} \quad h \neq 0 \\
\rho_{ho} &\to (2 sin\pi h)^{-\frac{d}{2}} \rho_{0,-h} \quad h \neq 0 \\
\rho_{gg} &\to (i)^{\frac{d}{2}} \rho_{g,-g} \quad g \neq 0 \\
\rho_{g,-g} &\to (-i)^{\frac{d}{2}} \rho_{-g,-g} \quad g \neq 0 \quad . \qquad (28)
\end{aligned}$$

The behaviour under T-modular transformations ($\tau \to \tau + 1$) is as follows

$$\eta^{-\frac{D-2}{2}} \rho_{gh} \to \eta^{-\frac{D-2}{2}} \rho_{g,g+h} \quad . \qquad (29)$$

The modular transformation $P = ST^2ST$ then relates chiral traces in the transverse and direct Möbius-strip amplitudes and corresponds to $\widehat{P} = T^{1/2}ST^2ST^{1/2}$ on "hatted" real characters [4].

The chiral traces entering the non-supersymmetric models (Section 6) are defined by

$$
\begin{aligned}
\rho'_{gh} &\equiv \frac{1}{2}\sum_{\alpha,\beta}(-)^{2\alpha+4\alpha\beta}\frac{\vartheta\left[{\alpha \atop \beta}\right]^2}{\eta^2}\prod_{i=1}^{2}\frac{\vartheta\left[{\alpha+g_i \atop \beta+h_i}\right]}{\vartheta\left[{\frac{1}{2}+g_i \atop \frac{1}{2}+h_i}\right]} \quad g,h\neq 0 \\
\rho''_{gh} &\equiv \frac{1}{2}\sum_{\alpha=,\beta}(-)^{2\beta+4\alpha\beta}\frac{\vartheta\left[{\alpha \atop \beta}\right]^2}{\eta^2}\prod_{i=1}^{2}\frac{\vartheta\left[{\alpha+g_i \atop \beta+h_i}\right]}{\vartheta\left[{\frac{1}{2}+g_i \atop \frac{1}{2}+h_i}\right]} \quad g,h\neq 0 \quad ,
\end{aligned}
\tag{30}
$$

with similar replacements for the remaining traces with g and/or h equal to zero.

Some relevant lattice sums for compact $SO(8)$ bosons are

$$
\begin{aligned}
\Lambda_{SO(8)} &= |O_8|^2+|V_8|^2+|S_8|^2+|C_8|^2 \\
\Lambda_W^{\pm} &= O_8 \pm V_8 \pm S_8 \pm C_8 \quad ,
\end{aligned}
\tag{31}
$$

where O_n, V_n, S_n, C_n are $SO(n)$ characters at level one. Some relevant lattice sums for compact $SU(3)^\ell$ bosons are

$$
\begin{aligned}
\Lambda_{SU(3)^\ell} &= (|\chi_1|^2+|\chi_3|^2+|\chi_{\bar 3}|^2)^\ell \\
\Lambda_R &= \chi_1^\ell \\
\Lambda_W^{\omega} &= (\chi_1+\omega\chi_3+\omega\chi_{\bar 3})^\ell \quad ,
\end{aligned}
\tag{32}
$$

where $\chi_{\mathbf{1}}, \chi_{\mathbf{3}}, \chi_{\bar{\mathbf{3}}}$ are $SU(3)$ characters at level one.

10 Appendix B: Open-string descendant of $T^4/Z_2^L \times Z_2^R$

The $T^4/Z_2^L \times Z_2^R$ model with $SO(8)$ lattice is a T-duality orbifold of the geometric Z_2 orbifold. Denoting by Q_O, Q_V, Q_S, Q_C the supersymmetric characters

$$
\begin{aligned}
Q_O &= V_4O_4 - C_4C_4 \\
Q_V &= O_4V_4 - S_4S_4 \\
Q_S &= O_4C_4 - S_4O_4 \\
Q_C &= V_4S_4 - C_4V_4 \quad ,
\end{aligned}
\tag{33}
$$

one is lead to introduce the following 16 characters [4]

$$
\begin{aligned}
\chi_1 &= Q_OO_4O_4 + Q_VV_4V_4 & \tilde\chi_1 &= Q_SS_4O_4 + Q_CC_4V_4 \\
\chi_2 &= Q_OO_4V_4 + Q_VV_4O_4 & \tilde\chi_2 &= Q_SS_4V_4 + Q_CC_4O_4 \\
\chi_3 &= Q_OC_4C_4 + Q_VS_4S_4 & \tilde\chi_3 &= Q_SV_4C_4 + Q_CO_4S_4 \\
\chi_4 &= Q_OC_4S_4 + Q_VS_4C_4 & \tilde\chi_4 &= Q_SV_4S_4 + Q_CO_4C_4 \\
\chi_5 &= Q_OV_4V_4 + Q_VO_4O_4 & \tilde\chi_5 &= Q_SC_4V_4 + Q_CS_4O_4 \\
\chi_6 &= Q_OV_4O_4 + Q_VO_4V_4 & \tilde\chi_6 &= Q_SC_4O_4 + Q_CS_4V_4 \\
\chi_7 &= Q_OS_4S_4 + Q_VC_4C_4 & \tilde\chi_7 &= Q_SO_4S_4 + Q_CV_4C_4 \\
\chi_8 &= Q_OS_4C_4 + Q_VC_4S_4 & \tilde\chi_8 &= Q_SO_4C_4 + Q_CV_4S_4 \quad .
\end{aligned}
\tag{34}
$$

The chiral Z_2 generators act by $Q_V \to -Q_V$, $Q_C \to -Q_C$, on the spacetime characters and by $V_4 \to -V_4$, $C_4 \to -C_4$ on the second $SO(4)$ factor in the decomposition of the internal $SO(8)$ into $SO(4)^2$. In the character basis the Z_2 generators then act diagonally with plus eigenvalues for $\chi_i, \tilde{\chi}_i$, with $i = 1, 4, 6, 7$ and minus eigenvalues for the remaining ones. The two modular invariant combinations, corresponding to the presence or absence of discrete torsion between the two chiral Z_2's, can be defined by projecting onto states with $Z_2^L = \epsilon Z_2^R = 1$ ($Z_2^L = \epsilon Z_2^R = -1$) in the (un)twisted sector. For $\epsilon = +1$, one finds

$$\begin{aligned} \mathcal{T} &= \mathcal{V}_6 \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} X_6 \, [\, |\chi_1|^2 + |\chi_4|^2 + |\chi_6|^2 + |\chi_7|^2 + |\tilde{\chi}_2|^2 + |\tilde{\chi}_3|^2 + |\tilde{\chi}_5|^2 + |\tilde{\chi}_8|^2 \\ &\quad + \chi_1 \bar{\tilde{\chi}}_8 + \chi_4 \bar{\tilde{\chi}}_5 + \chi_6 \bar{\tilde{\chi}}_3 + \chi_7 \bar{\tilde{\chi}}_2 + + \tilde{\chi}_8 \bar{\chi}_1 + \tilde{\chi}_5 \bar{\chi}_4 + \tilde{\chi}_3 \bar{\chi}_6 + \tilde{\chi}_2 \bar{\chi}_7 \,] \quad , \end{aligned} \tag{35}$$

while, for $\epsilon = -1$, one finds

$$\begin{aligned} \mathcal{T} &= \mathcal{V}_6 \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} X_6 \, [\, |\chi_1|^2 + |\chi_4|^2 + |\chi_6|^2 + |\chi_7|^2 + |\tilde{\chi}_1|^2 + |\tilde{\chi}_4|^2 + |\tilde{\chi}_6|^2 + |\tilde{\chi}_7|^2 \\ &\quad + \chi_2 \bar{\tilde{\chi}}_3 + \chi_3 \bar{\tilde{\chi}}_2 + \chi_5 \bar{\tilde{\chi}}_8 + \chi_8 \bar{\tilde{\chi}}_5 + + \tilde{\chi}_2 \bar{\chi}_3 + \tilde{\chi}_3 \bar{\chi}_2 + \tilde{\chi}_5 \bar{\chi}_8 + \tilde{\chi}_8 \bar{\chi}_5 \,] \quad . \end{aligned} \tag{36}$$

One can check that the $\epsilon = -1$ combination corresponds to the permutation modular invariant denoted by A_{16} in [6].

Notice that for $\epsilon = 1$ the torus amplitude can be rewritten as

$$\mathcal{T} = \mathcal{V}_6 \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} X_6 \, [\, |\xi_O|^2 + |\xi_V|^2 + |\xi_S|^2 + |\xi_C|^2 \,] \tag{37}$$

in terms of the extended characters

$$\begin{aligned} \xi_O &= \chi_1 + \tilde{\chi}_8 = QO_8 \\ \xi_V &= \chi_6 + \tilde{\chi}_3 = QV_8 \\ \xi_S &= \chi_7 + \tilde{\chi}_2 = QS_8 \\ \xi_C &= \chi_4 + \tilde{\chi}_5 = QC_8 \quad , \end{aligned} \tag{38}$$

where $Q = V_8 - S_8$, reflecting the fact that the orbifold correspond to a toroidal compactification of the type IIB superstring.

For $\epsilon = 1$, the corresponding open-string descendant (for the simplest CP group assignments) is then given by

$$\begin{aligned} \mathcal{K} &= \frac{\mathcal{V}_6}{2} \int_0^\infty \frac{d\tau_2}{\tau_2^4} \frac{1}{\eta^4} [\xi_O + \xi_V + \xi_S + \xi_C] \, (2i\tau_2) \\ \mathcal{A} &= \frac{\mathcal{V}_6}{2} \int_0^\infty \frac{d\tau_2}{\tau_2^4} \frac{1}{\eta^4} \Big[(n_1^2 + n_2^2 + n_3^2 + n_4^2)\xi_O + (2n_1 n_2 + 2n_3 n_4)\xi_V + \\ &\quad (2n_1 n_3 + 2n_2 n_4)\xi_S + (2n_1 n_4 + 2n_2 n_3)\xi_C \Big] \, (\frac{i\tau_2}{2}) \\ \mathcal{M} &= \frac{\mathcal{V}_6}{2} (n_1 + n_2 + n_3 + n_4) \int_0^\infty \frac{d\tau_2}{\tau_2^4} \frac{1}{\eta^4} \hat{\xi}_O (\frac{i\tau_2}{2} + \frac{1}{2}) \quad . \end{aligned} \tag{39}$$

The expression for $\epsilon = -1$ is similar, it only amounts to substituting the characters $\xi_0, \xi_V, \xi_S, \xi_C$ with the characters

$$\begin{aligned}
\tilde{\xi}_O &= \chi_1 + \tilde{\chi}_4 \\
\tilde{\xi}_V &= \chi_6 + \tilde{\chi}_7 \\
\tilde{\xi}_S &= \chi_7 + \tilde{\chi}_6 \\
\tilde{\xi}_C &= \chi_4 + \tilde{\chi}_1 \quad ,
\end{aligned} \tag{40}$$

respectively. Notice that, unlike in the $\epsilon = 1$ case, $\tilde{\xi}_V, \tilde{\xi}_S$ and $\tilde{\xi}_C$ are massless characters providing with additional massless matter in the bifundamentals.

For $\epsilon = 1$, the non-supersymmetric choice corresponds to the following amplitudes

$$\begin{aligned}
\mathcal{K}_{ns} &= \frac{\mathcal{V}_6}{2} \int_0^\infty \frac{d\tau_2}{\tau_2^4} \frac{1}{\eta^4} \left[\xi_O + \xi_V - \xi_S - \xi_C\right] (2i\tau_2) \\
\mathcal{A}_{ns} &= \frac{\mathcal{V}_6}{2} \int_0^\infty \frac{d\tau_2}{\tau_2^4} \frac{1}{\eta^4} \Big[(2n\bar{n} + 2m\bar{m})\xi_O + (n^2 + \bar{n}^2 + m^2 + \bar{m}^2)\xi_V + \\
&\quad (2n\bar{m} + 2\bar{n}m)\xi'_S + (2nm + 2\bar{n}\bar{m})\xi'_C\Big] \left(\frac{i\tau_2}{2}\right) \\
\mathcal{M}_{ns} &= \frac{\mathcal{V}_6}{2} \int_0^\infty \frac{d\tau_2}{\tau_2^4} \frac{1}{\eta^4} \Big[(n + \bar{n})\hat{\xi}_V - (m + \bar{m})\hat{\xi}''_V\Big] \left(\frac{i\tau_2}{2} + \frac{1}{2}\right) \quad ,
\end{aligned} \tag{41}$$

where with a prime ($'$) and a double prime ($''$) we denote characters corresponding to non-supersymmetric chiral traces much in the same way as discussed in Appendix A. Similar expressions, with $\xi \to \tilde{\xi}$, correspond to the non-supersymmetric choice with $\epsilon = -1$. It should be noticed that in this case only ξ_V ($\tilde{\xi_V}$) enters the transverse Klein-bottle amplitude. As already shown, ξ_V is massive, and a model without open-string sector is perfectly consistent with both worldsheet and target-space requirements.

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