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Exponentially Localized Solutions of Mel'nikov Equation

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Abstract

The Mel'nikov equation is a (2+1) dimensional nonlinear evolution equation admitting boomeron type solutions. In this paper, after showing that it satisfies the Painlevé property, we obtain exponentially localized dromion type solutions from the bilinearized version which have not been reported so far. We also obtain more general dromion type solutions with spatially varying amplitude as well as induced multi-dromion solutions.

1 Introduction

The identification of dromions which are exponentially localized solutions in (2+1) dimensional soliton equations [1-6] has been one of the most interesting developments in soliton theory in recent times, which has given a fillip to the understanding of integrable systems in (2+1) dimensions. Essentially, these localized solutions arise due to the presence of some additional nonlocal terms or effective local fields associated with "boundaries". Further, the advent of "explode decay dromions" which are again exponentially localized solutions with time varying amplitudes [7] and induced dromions [6,8] using arbitrary functions of space and time variables have set in motion the process of unearthing more and more novel localized entities in (2+1) dimensional nonlinear systems.

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1.1 Mel'nikov Equation

An interesting evolution equation in (2+1) dimensions which we consider here is the one proposed by Mel'nikov [9,10] that describes (under certain conditions) the interaction of two waves on the x-axis. This equation is of the form

$$3u_{tt} - [u_y + (3u^2 + u_{xx} + 8\kappa|\chi|^2)_x]_x = 0, \tag{1a}$$

$$i\chi_t = u\chi + \chi_{xx}, \tag{1b}$$

where u is the long wave amplitude (real), χ is the complex short wave envelope, and the parameter κ satisfies the condition $\kappa^2 = 1$. Eq. (1) may be considered either as a generalization of the K-P equation with the addition of a complex scalar field or as a generalization of the NLS equation with a real scalar field (after suitable interchange of coordinates y and t). Mel'nikov [10] has pointed out that Eq. (1) admits boomeron type solutions, which can be realized from an asymptotic analysis of the two soliton solution.

It is expected that the investigation of this equation may have wider ramifications in plasma physics, nonlinear optics and hydrodynamics. It is this diverse presence of this equation which prompts one to make a detailed investigation of their dynamics, particularly to identify whether localized solutions exist in this system. For this purpose, we first carry out a Painlevé singularity structure analysis and confirm that Eq. (1) does indeed satisfy the Painlevé property. Then bilinearizing the evolution equation and making use of certain arbitrary functions present in the solution, we obtain a large class of exponentially localized dromion solutions.

2 Singularity Structure Analysis of Mel'nikov Equation

We explore the singularity structure of Eq. (1), by rewriting χ=a and χ^*=b as

$$3u_{tt} - u_{xy} - 6u_x^2 - 6uu_{xx} - u_{xxxx} - 8\kappa(a_{xx}b + 2a_xb_x + ab_{xx}) = 0, \tag{2a}$$

$$ia_t = ua + a_{xx}, \tag{2b}$$

$$-ib_t = ub + b_{xx}. \tag{2c}$$

We now effect a local Laurent expansion in the neighbourhood of a noncharacteristic singular manifold $\phi(x, y, t) = 0$, $\phi_x \neq 0$, $\phi_y \neq 0$. Assuming the leading

orders of the solutions of Eq. (2) to have the form

$$u = u_0\phi^{\alpha}, a = a_0\phi^{\beta}, b = b_0\phi^{\gamma}, \tag{3}$$

where u_0, a_0 and b_0 are analytic functions of (x, y, t) and α, β, γ are integers to be determined, we now substitute (3) into (2) and balance the most dominant terms to get

$$\alpha = \beta = \gamma = -2, \tag{4}$$

with the condition

$$\kappa a_0 b_0 = -9\phi_x^4, u_0 = -6\phi_x^2. \tag{5}$$

Now, considering the generalized Laurent expansion of the solutions in the neighbourhood of the singular manifold

$$u = u_0\phi^{\alpha} + ... + u_r\phi^{r+\alpha} + ..., \tag{6a}$$
$$a = a_0\phi^{\beta} + ... + a_r\phi^{r+\beta} + ..., \tag{6b}$$
$$b = b_0\phi^{\gamma} + ... + b_r\phi^{r+\gamma} + ..., \tag{6c}$$

the resonances (powers) at which arbitrary functions enter into (6) can be determined by substituting (6) into (2) and comparing the coefficients of $(\phi^{r-6},\phi^{r-4},\phi^{r-4})$ to give

$$\begin{pmatrix} \phi_x^2(r^2-5r-30)\hat{r} & 8\kappa b_0\hat{r} & 8\kappa a_0\hat{r} \\ a_0 & \phi_x^2(r^2-5r) & 0 \\ b_0 & 0 & \phi_x^2(r^2-5r) \end{pmatrix} \begin{pmatrix} u_r \\ a_r \\ b_r \end{pmatrix} = 0, \tag{7}$$

where $\hat{r} = (r-4)(r-5)$. Solving Eq. (7), one gets the resonance values as

$$r = -3, -1, 0, 4, 5, 5, 6, 8. \tag{8}$$

The resonance at r = -1 naturally represents the arbitrariness of the manifold $\phi(x, y, t) = 0$. In order to prove the existence of arbitrary functions at the other resonance values, we now substitute the full Laurent series

$$u = u_0\phi^{\alpha} + \sum_r u_r\phi^{r+\alpha}, \tag{9a}$$
$$a = a_0\phi^{\beta} + \sum_r a_r\phi^{r+\beta}, \tag{9b}$$
$$b = b_0\phi^{\gamma} + \sum_r b_r\phi^{r+\gamma} \tag{9c}$$

into Eq. (2). Now collecting the coefficients of $(\phi^{-6},\phi^{-4},\phi^{-4})$ and solving them, we obtain the relations (5), implying a resonance at $r = 0$.

Similarly collecting the following coefficients, we obtain the necessary information about the positive resonances:

(i) coefficients of (ϕ^{-5},ϕ^{-3},ϕ^{-3}):

$$u_1 = 0, a_1 = \frac{ia_0\phi_t}{2} - a_{0x}, b_1 = \frac{-ib_0\phi_t}{2} - b_{0x}. \tag{10}$$

(ii) coefficients of (ϕ^{-4},ϕ^{-2},ϕ^{-2}): u_2, a_2, and b_2 can be uniquely determined.

(iii) coefficients of (ϕ^{-3},ϕ^{-1},ϕ^{-1}): u_3, a_3, and b_3 can be uniquely determined.

(iv) coefficients of (ϕ^{-2},ϕ^{0},ϕ^{0}): Only two equations result for three unknowns u_4, a_4, b_4 and so one of them is arbitrary, corresponding to a resonance at r = 4.

(v) coefficients of (ϕ^{-1},ϕ^{1},ϕ^{1}): Only u_5 is determined, while a_5 and b_5 are arbitrary corresponding to double resonance at r = (5, 5).

(vi) coefficients of (ϕ^{0},ϕ^{2},ϕ^{2}): Only two equations result for three unknowns u_6, a_6, b_6 and so one of them is arbitrary, corresponding to r = 6.

(vii) coefficients of (ϕ^{1},ϕ^{3},ϕ^{3}): u_7, a_7, and b_7 can be determined in terms of earlier coefficients.

(viii) coefficients of (ϕ^{2},ϕ^{4},ϕ^{4}): Only two equations result for three unknowns u_8, a_8, b_8 and so one of them is arbitrary, corresponding to r = 8.

For the negative resonance r = -3, following the approach of Conte, Fordy and Pickering [11], we demand that both the solution of Mel'nikov Eq. (2) and the solution close to it represented by a peturbation series in a small parameter ϵ are free from movable critical manifolds. We identify that the first order peturbed series does admit an arbitrary function corresponding to a movable pole at the resonance r = -3. Consequently, for each of the eight resonances given by Eq. (8), one can associate an arbitrary function in the solution (and close to it) without the introduction of movable critical manifolds.

It must be mentioned that the above system (2) admits another leading order behaviour with $\alpha = -2$, $\beta = \gamma = -1$, $u_0 = -2\phi_x^2$, a_0 and b_0 are arbitrary. The Laurent series with the above leading order leads to resonances $0, 0, -1, 3, 3, 4, 5, 6$, corresponding to a normal branch and the existence of sufficient number of arbitrary functions can be established at these resonance values. This can also be verified from the corresponding bilinear form as studied by Grammaticos, Ramani and Hietarinta [12]. Consequently one can be assured that the Mel'nikov equation (1) or (2) indeed satisfies the Painlevé

property.

3 Bilinearization of Mel'nikov Equation and Localized Solutions

We next Hitora bilinearize the Mel'nikov Eq. (1) to bring out the existence of exponentially localized solutions of Mel'nikov equation. Making the transformation

$$u = 2\frac{\partial^2}{\partial x^2}\ln F, \tag{11a}$$

$$\chi = \frac{G}{F}, \tag{11b}$$

identifiable from the Painlevé analysis, Eq. (1) gets converted into the following Hirota bilinear form,

$$(3D_t^2 - D_xD_y - D_x^4)F.F = 8\kappa|G|^2, \tag{12a}$$

$$iD_tG.F = D_x^2G.F, \tag{12b}$$

where the D's are the usual bilinear operators. However, this bilinearization have been already done by Y. Hase etal [13] where they have given soliton solutions whereas we have brought out localised solutions here. For a completion, we proceed as follows. Introducing the series expansion,

$$G = \epsilon g^{(1)} + \epsilon^3 g^{(3)} + ..., \tag{13a}$$

$$F = 1 + \epsilon^2 f^{(2)} + \epsilon^4 f^{(4)} + ... \tag{13b}$$

into the above bilinear form and gathering terms with various powers of the small parameter ϵ, we obtain the following set of equations,

$$O(\epsilon) : ig_t^{(1)} = g_{xx}^{(1)}, \tag{14a}$$

$$O(\epsilon^2) : 3f_{tt}^{(2)} - f_{xy}^{(2)} - f_{xxxx}^{(2)} = 4\kappa g^{(1)}g^{(1)*}, \tag{14b}$$

etc. Solving Eq. (14a), we can immediately write down the following solution,

$$g^{(1)} = \sum_{j=1}^{N} exp(\psi_j), \psi_j = l_jx + m_jy + \omega_jt + \psi_j^{(0)}, i\omega_j = l_j^2, \tag{15}$$

where the spectral parameters l_j, m_j, ω_j and $\psi_j^{(0)}$ are all complex. Confining to $N = 1$ in Eq. (15) and substituting (15) into (14b), we obtain

$$f^{(2)} = \exp(\psi_1 + \psi_1^* + 2A),\ \exp(2A) = \frac{\kappa}{(3\omega_{1R}^2 - l_{1R}m_{1R} - 4l_{1R}^4)}. \tag{16}$$

Here $l_1 = l_{1R} + il_{1I}$, $m_1 = m_{1R} + im_{1I}$ and $\omega_1 = \omega_{1R} + i\omega_{1I}$ and also $i\omega_1 = l_1^2$. Choosing $g^{(2j+1)} = 0, f^{(2j)} = 0$, for $j > 1$, in Eq. (13) and using Eqs. (15) and (16) alongwith the transformation (11), the physical field u and the potential χ can be easily seen to be driven by the envelope soliton and pulse soliton respectively as

$$\chi = \left(\frac{\sqrt{l_{1R}}(12l_{1R}l_{1I}^2 - m_{1R} - 4l_{1R}^3)^{\frac{1}{2}}}{2\sqrt{\kappa}} \right) \mathrm{sech}(\psi_{1R} + A)e^{i\psi_{1I}}, \tag{17a}$$

$$u = 2l_{1R}^2 \mathrm{sech}^2(\psi_{1R} + A). \tag{17b}$$

where $\psi_{1R} = l_{1R}x + m_{1R}y + 2l_{1R}l_{1I}t + \psi_{1R}^{(0)}$ and $\psi_{1I} = l_{1I}x + m_{1I}y - (l_{1R}^2 - l_{1I}^2)t + \psi_{1I}^{(0)}$. One can proceed further in the standard way to obtain higher order soliton solutions also.

3.1 Dromions

Looking at the above solutions (17), we realize the fact that as the parameter $l_{1R} \to 0$, both u and χ vanish. But, when $m_{1R} \to 12l_{1R}l_{1I}^2 - 4l_{1R}^3$, the potential χ vanishes, whereas the physical field u survives and is driven by a ghost soliton of the form,

$$u = 2l_{1R}^2 \mathrm{sech}^2(\psi_{1R} + c), \tag{18}$$

where c is a new constant. This predicts the existence of exponentially localized solution for the complex field variable χ in Eq. (1).

3.1.1 (1,1) Dromion

To generate a (1,1) dromion, we now make the ansatz

$$F = 1 + e^{\tilde{\psi}_1 + \tilde{\psi}_1^*} + e^{\tilde{\psi}_2 + \tilde{\psi}_2^*} + Ke^{\tilde{\psi}_1 + \tilde{\psi}_1^* + \tilde{\psi}_2 + \tilde{\psi}_2^*}, \tag{19}$$

where

$$\tilde{\psi}_1 = px + \omega t, \tag{20a}$$

$$\tilde{\psi}_2 = qy. \tag{20b}$$

Here K is a real constant and p, ω and q are complex constants. Substituting (19) in (12a), we obtain

$$G = \rho e^{\tilde{\psi}_1 + \tilde{\psi}_2}, \tag{21a}$$

$$4\kappa|\rho|^2 = (p + p^*)(q + q^*)(1 - K) \tag{21b}$$

for the parametric choice $3\omega_R^2 = 4p_R^4$. Substituting (19) and (21) in (12b), we find that $\omega = -ip^2$, where $p_R = \pm\sqrt{3}p_I$, p_R and p_I are the real and imaginary parts, respectively of p. Hence, the exponentially localized solution with one bound state for the potential field χ for the above choice of ω and p takes the form

$$\chi = \frac{\rho e^{\tilde{\psi}_1 + \tilde{\psi}_2}}{1 + e^{\tilde{\psi}_1 + \tilde{\psi}_1^*} + e^{\tilde{\psi}_2 + \tilde{\psi}_2^*} + K e^{\tilde{\psi}_1 + \tilde{\psi}_1^* + \tilde{\psi}_2 + \tilde{\psi}_2^*}}, \tag{22}$$

while the scalar field has the form

$$u = \frac{4p_R^2(1 + e^{\tilde{\psi}_2 + \tilde{\psi}_2^*})(e^{\tilde{\psi}_1 + \tilde{\psi}_1^*} + K e^{\tilde{\psi}_1 + {\psi_1}^* + \tilde{\psi}_2 + \tilde{\psi}_2^*})}{(1 + e^{\tilde{\psi}_1 + \tilde{\psi}_1^*} + e^{\tilde{\psi}_2 + \tilde{\psi}_2^*} + K e^{\tilde{\psi}_1 + \tilde{\psi}_1^* + \tilde{\psi}_2 + \tilde{\psi}_2^*})^2}, \tag{23}$$

which is always bounded, but localized everywhere except in the neighbourhood of the line $\tilde{\psi}_1 + \tilde{\psi}_1^* = 0$ in the (x, y) plane. A snapshot of the (1,1)

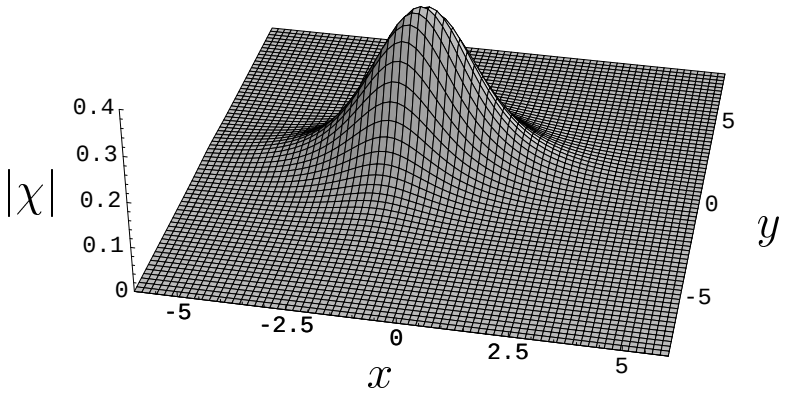

Fig. 1. Snapshot of the (1,1) dromion solution of the Mel'nikov equation (see Eq. (22))

dromion solution for the magnitude of the potential field χ is shown in Fig. 1. One can proceed to find multi-dromion solutions also, generalizing the above (1,1) dromions.

3.1.2 Dromions with Spatially Varying Amplitude

It can be seen from Eq. (14a) that the differential equation for g involves only the variables t and x. Hence, an arbitrary function of y can also enter into its solution so that the most general form of it can be given as

$$g^{(1)} = \sum_{j=1}^{N} exp(\tilde{\psi_j}), \tilde{\psi_j} = l_j x + f_j(y) + w_j t + \psi_j^{(0)}, iw_j = l_j^2, \tag{24}$$

where $f_j(y)$s are arbitrary functions of y. This fact can be harnessed in a suitable way to construct a more general class of localized solutions. Following the above procedure to derive the (1,1) dromion solution (22), one can easily obtain the generalized dromion solution involving arbitrary function of y in the same form as Eq. (22) except that $\tilde{\psi_2}$ is now given by

$$\tilde{\psi_2} = f(y), \tag{25}$$

instead of (20b), where the arbitrary function $f(y)$ is in general complex. The amplitude of the localized solution with arbitrary function of y is now defined by the equation

$$4\kappa|\rho|^2 = 2(p + p^*)(1 - K)f_R'(y). \tag{26}$$

where $f_R'(y)$ is the derivative of the real part of f with respect to y. Thus, the amplitude of the above localized solution varies with the spatial coordinate y by virtue of Eq. (26). This situation is reminiscent of the explode-decay dromion of the variable coefficient DSI equation [7] where the amplitude varies with time. It should be mentioned that, to our knowledge this is the first time the amplitude of a localized solution of a (2+1) dimensional nonlinear partial differential equation has been found to vary as a function of the spatial coordinate y. This can be easily generalized to multi-dromions with spatially varying amplitude.

3.1.3 Induced dromions

We also wish to point out that the existence of an arbitrary function in the solution of $g^{(1)}$ in Eq. (14a) can be further utilized to obtain new induced dromion solutions [6] for χ. For example, Eqs. (14a) and (14b) can also be solved in terms of arbitrary functions as

$$g^{(1)} = a(y)e^{\hat{\psi_1}}, \hat{\psi_1} = l_1 x + \omega_1 t, \tag{27}$$

where $a(y)$ is an arbitrary complex function of y and l_1 and ω_1 are complex constants constrained by the condition $i\omega_1 = l_1^2$. Substituting the form (27) into (14b) and solving the resultant equation, we obtain

$$f^{(2)} = b(y)e^{\hat{\psi_1}+\hat{\psi_1}^*}, \tag{28}$$

where $b(y)$ is a real function of y and is given by the condition

$$6\omega_{1R}^2 b(y) - l_{1R} b_y - 8l_{1R}^4 b(y) = 2\kappa |a(y)|^2. \tag{29}$$

The above solutions can then be used to generate curved line soliton for the field variable and the potential as

$$\chi = \frac{a(y)}{2\sqrt{b(y)}} e^{i\hat{\psi_{1I}}} \text{sech}[\hat{\psi_{1R}} + \frac{1}{2}\text{log}b(y)], \tag{30a}$$

$$u = 2l_{1R}^2 \text{sech}^2[\hat{\psi_{1R}} + \frac{1}{2}\text{log}b(y)]. \tag{30b}$$

where $\hat{\psi_{1R}}$ and $\hat{\psi_{1I}}$ are the real and imaginary parts, repectively of $\hat{\psi_1}$. Thus, by choosing the arbitrary functions $a(y)$ and $b(y)$ suitably which are constrained by the Eq. (29), one can induce localized solutions for the field variable χ as in the case of Zakharov-Strachan equation [6]. Eventhough there exists two functions $a(y)$ and $b(y)$, only one of them is found to be arbitrary, which is evident from the Eq. (29) above. For example, by choosing

$$\frac{a(y)}{\sqrt{b(y)}} = 2\text{sech}(m_1 y), \tag{31}$$

where m_1 is a real constant, we can find from Eq. (29) that

$$b(y) = \exp\left(-\frac{1}{l_{1R}}(8\kappa \text{tanh} m_1 y + (8l_{1R}^4 - 6\omega_{1R}^2)y)\right). \tag{32}$$

Then we obtain the induced localized solution

$$\chi = e^{i\hat{\psi_{1I}}} \text{sech}(m_1 y) \text{sech}\left[l_{1R}(x + 2l_{1I}t) - \frac{1}{2l_{1R}}\left(8\kappa \text{tanh} m_1 y + (8l_{1R}^4 - 6\omega_{1R}^2)y\right)\right]. \tag{33}$$

Thus, by choosing $a(y)$ and $b(y)$ suitably, one can induce a wide class of localized solutions for the Mel'nikov Eq. (1). For example, choosing an algebraic form

$$\frac{a(y)}{\sqrt{b(y)}} = \frac{2}{(y + y_0)^2 + 1}, \tag{34}$$

we obtain an algebraically decaying localized solution

$$\chi = \frac{1}{(y + y_0)^2 + 1} e^{i\hat{\psi_{1I}}} \text{sech}\left[l_{1R}(x + 2l_{1I}t) - \frac{1}{2l_{1R}}\left((8l_{1R}^4 - 6\omega_{1R}^2)y + 8\kappa \int \frac{dy}{((y + y_0)^2 + 1)^2}\right)\right]. \tag{35}$$

One can as well generalize this procedure to construct even wider class of localized solutions. In fact, multi-induced dromions take the simple form

$$\chi_N = \left[\frac{\sum_{j=1}^{N} a_j(y)}{2\sqrt{b(y)}}\right] e^{i\hat{\psi_{1I}}}\text{sech}[\hat{\psi_{1R}} + \frac{1}{2}\text{log}b(y)], \tag{36}$$

where a_j, $j = 1, 2...N$ are arbitrary functions of y and they are related to $b(y)$ by the relation

$$6\omega_{1R}^2 b(y) - l_{1R}b_y - 8l_{1R}^4 b(y) = 2\kappa \sum_{p=1}^{N}\sum_{q=1}^{N} a_p(y)a_q^*(y). \tag{37}$$

Note that the sum of the arbitrary functions of y on the right hand side of eq.(36) becomes possible due to the structure of eqs. (1). In practice one can choose $\frac{\sum_{j=1}^{N} a_j(y)}{2\sqrt{b(y)}}$ conveniently, for example, as a combination of algebraic and hyperbolic functions. With N=2, choosing the functions as

$$\frac{a_1(y)}{\sqrt{b(y)}} = 2\text{sech}(m_1 y + \delta_1), \frac{a_2(y)}{\sqrt{b(y)}} = 2\text{sech}(m_2 y + \delta_2), \tag{38}$$

where m_1, m_2, δ_1 and δ_2 are parameters, solving eq.(29), one can obtain

$$\begin{aligned} b(y) = \exp\Bigg(&- \frac{1}{l_{1R}}((8l_{1R}^4 - 6\omega_{1R}^2)y + 8\kappa(\tanh(m_1 y + \delta_1) \\ &+\tanh(m_2 y + \delta_2) + 2\int \text{sech}(m_1 y + \delta_1)\text{sech}(m_2 y + \delta_2)dy))\Bigg). \end{aligned} \tag{39}$$

Then, the induced two dromion solution is given by

$$\begin{aligned} \chi_2 = e^{i\hat{\psi_{1I}}}&(\text{sech}(m_1 y + \delta_1) + \text{sech}(m_2 y + \delta_2))\text{sech}\Bigg[l_{1R}(x + 2l_{1I}t) \\ &-\frac{1}{2l_{1R}}\Bigg((8l_{1R}^4 - 6\omega_{1R}^2)y + 8\kappa(\tanh(m_1 y + \delta_1) \\ &+\tanh(m_2 y + \delta_2) + 2\int \text{sech}(m_1 y + \delta_1)\text{sech}(m_2 y + \delta_2)dy)\Bigg)\Bigg] \end{aligned} \tag{40}$$

and is shown in Fig. 2. One can identify the mutual influence of one dromion over the other from the additional terms occurring in the square bracket of eq. (40).

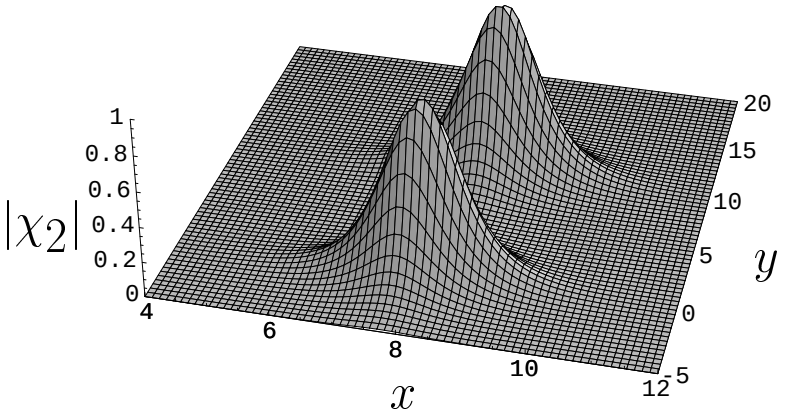

Fig. 2. Snapshot of the induced two dromion solution of the Mel'nikov equation (see Eq. (40))

If one chooses both the functions a_j, $j = 1, 2$, algebraically,

$$\frac{a_1(y)}{\sqrt{b(y)}} = \frac{2}{(m_1 y + y_{10})^2 + 1}, \frac{a_2(y)}{\sqrt{b(y)}} = \frac{2}{(m_2 y + y_{20})^2 + 1} \tag{41}$$

or one of the functions to be algebraic and the other one to be hypebolic

$$\frac{a_1(y)}{\sqrt{b(y)}} = 2\mathrm{sech}(m_1 y + \delta_1), \frac{a_2(y)}{\sqrt{b(y)}} = \frac{2}{(m_2 y + y_{20})^2 + 1} \tag{42}$$

one can generate different kinds of induced lump-lump or dromion-lump solutions, respectively.

We have also tried to generate more general two soliton solutions by choosing

$$g^{(1)} = a_1(y)e^{\hat{\psi}_1} + a_2(y)e^{\hat{\psi}_2}, \hat{\psi}_1 = l_1 x + \omega_1 t, \hat{\psi}_2 = l_2 x + \omega_2 t, \tag{43}$$

which lead to a condition $l_1 = l_2$ thereby reducing to our original form (36) for $N = 2$. This is also true for $N > 2$. Thus we believe that the solution (36) constitutes the most general localized solution we could construct for the Mel'nikov equation through our procedure.

4 Conclusion

In this paper, we have pointed out the interesting fact that the (2+1) dimensional Mel'nikov equation admits exponentially localized solutions of different classes. We have also checked its integrability through Painlevé analysis. We

have in particular constructed localized dromion solutions and obtained new classes of localized solutions such as dromions with spatially varying amplitude and induced dromions.

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