

Consistency of parity-violating pion-nucleon couplings extracted from measurements in ^{18}F and ^{133}Cs

W.S. Wilburn and J.D. Bowman

Los Alamos National Laboratory, Los Alamos, NM 87545

(February 8, 2008)

Abstract

The recent measurement of the nuclear anapole moment of ^{133}Cs has been interpreted to yield a value of the weak pion-nucleon coupling H_π^1 which contradicts the upper limit from the ^{18}F experiments. We argue that because of the sensitivity of the anapole moment to H_ρ^0 in the odd proton nucleus ^{133}Cs , there is a combination of weak meson-nucleon couplings which satisfies both experiments and which is (barely) in agreement with theory. In addition, the anapole moment measurement in ^{205}Tl gives a constraint which is inconsistent with the value from ^{133}Cs , calling into question the theory of nuclear anapole moments. We argue that measurements of directional asymmetry in $\bar{n} + p \rightarrow d + \gamma$ and in the photo-disintegration of the deuteron by circularly polarized photons, combined with results from pp scattering, would determine H_π^1 and several other weak meson-nucleon couplings in a model-independent way.

I. INTRODUCTION

The nucleon-nucleon (NN) weak interaction can be described as arising from meson exchange involving a parity-violating weak vertex and a parity-conserving strong vertex. This interaction leads to a meson-exchange potential involving seven weak meson-nucleon coupling constants¹, of which two, the $\Delta I = 1$ πN coupling H_π^1 and the $\Delta I = 0$ ρN coupling H_ρ^0 contribute the most to low-energy nuclear observables. The coupling H_π^1 is of interest because it is expected to be particularly sensitive to weak neutral currents. In the absence of neutral currents, H_π^1 is small. Some theories that include neutral currents predict large values of H_π^1 . Despite considerable experimental and theoretical effort, the size of the weak pion-nucleon coupling constant H_π^1 remains uncertain. Until recently, the conflict has centered around a discrepancy between theory, which predicts a relatively large value $H_\pi^1 = 1.1 \times 10^{-6}$ [1], and experiments in ^{18}F , which set an upper limit of $H_\pi^1 \leq 0.28 \times 10^{-6}$ [2], a disagreement of more than 3σ . In addition, Adelberger and Haxton [2] fit the available data to obtain

¹We use the notation of Adelberger and Haxton, where, for example $H_\pi^1 = F_\pi = g_\pi f_\pi / \sqrt{32}$ and $H_\rho^0 = F_0 = -g_\rho h_\rho^0 / 2$.

$H_\pi^1 = 0.5 \times 10^{-6}$, a value somewhat larger than the ^{18}F upper limit. The interpretation of the ^{18}F experiments is thought to be more reliable than for other nuclei [3]. Although experiments in other nuclei have been used to determine the weak meson-nucleon couplings (see [4,2] for reviews), their interpretation is considerably more model-dependent than ^{18}F since their interpretation involves *ab initio* calculations of transition matrix elements of the meson-exchange potential. For this reason, they will not be considered here.

Recent progress has deepened the controversy. First we consider progress in the theory. Kaplan and Savage [5] find that strangeness-changing currents, which were previously neglected, contribute substantially to H_π^1 . Their result, $H_\pi^1 = 1.2 \times 10^{-6}$, is in agreement with the Desplanques, Donoghue, and Holstein (DDH) “best” value prediction. In contrast, Henley, Hwang, and Kisslinger [6], using QCD sum rule techniques, find an almost complete cancellation between perturbative and non-perturbative contributions from weak neutral currents. This cancellation results in an extremely small $H_\pi^1 = 0.05 \times 10^{-6}$, a factor of 24 smaller than Kaplan and Savage and DDH.

The interpretation of new data on the nuclear anapole moment of ^{133}Cs has added a new dimension to the controversy. The non-zero measurement of the anapole moment of ^{133}Cs [7] has been analyzed by Flambaum and Murray [8] to extract a value for H_π^1 . Their result, $H_\pi^1 = 2.26 \pm 0.50(\text{expt}) \pm 0.83(\text{theor}) \times 10^{-6}$ is a factor of two larger than the DDH value and a factor of *seven* larger than the upper limit set by the ^{18}F experiments.

We argue that the because of the sensitivity of the anapole moment to H_ρ^0 , the two experimental results are not incompatible. However, the constraint on the weak couplings extracted from the ^{133}Cs result is inconsistent with that extracted from an earlier null measurement of the anapole moment of ^{205}Tl [9], calling into question the reliability of the theoretical interpretation of the anapole moment. The issues raised by the apparent inconsistencies between different determinations of H_π^1 point out the need for a model-independent determination of the weak meson-nucleon couplings.

II. EXTRACTION OF WEAK COUPLINGS FROM ^{18}F AND ANAPOLE MOMENT EXPERIMENTS

The determination of the weak-meson-nucleon couplings from experimental measurements in nuclei are discussed in the review by Adelberger and Haxton [2]. There are substantial uncertainties in interpreting most experiments in nuclei because one cannot make reliable *ab initio* calculations of amplitudes of the weak-meson-exchange potential operators. The experiment to measure the circular polarization P_γ of the 1081 keV transition in ^{18}F is an exception to this unfortunate situation because the matrix elements needed to extract a value for H_π^1 from experiments can be measured. The circular polarization of a $\Delta I = 1$ parity-forbidden gamma transition in ^{18}F has been measured in five different and internally consistent experiments (references given in [2]). To a good approximation the circular polarization is due to the parity-violating mixing between the $J = 0, I = 0$ parity-odd level ($|-\rangle$) in ^{18}F and the nearly degenerate $J = 0, I = 1$ level ($|+\rangle$). The circular polarization is given by:

$$P_\gamma = \frac{2}{\Delta E} \frac{\langle +|V_{pnc}|-\rangle \langle gs|M1|+\rangle}{\langle gs|E1|-\rangle}. \quad (1)$$

The magnitudes of the $M1$ and $E1$ transition amplitudes and the energy splitting $\Delta E = 39$ keV between the levels are known experimentally. Bennet, Lowry, and Krien [10] and Haxton [3] point out that the unknown amplitude, $\langle +|V_{pnc}|-\rangle$, could be related to the lifetime of the first forbidden beta decay between isobaric analog of $|+\rangle$ in ^{18}Ne and $|-\rangle$. For beta transitions between opposite parity $J = 0$ levels, spin and parity selection rules exclude all but two of the six possible transition amplitudes. One of these vanishes in the long wavelength limit leaving only the M_{00}^5 amplitude. Haxton use PCAC, current algebra, and the approximation that the neutron and proton densities are the same for the mass 18 system to argue that the two-body part of M_{00}^5 renormalizes the one body part and that the operator for the weak nucleon-nucleon potential due to pion exchange is to within a known constant an isospin rotation of the operator for M_{00}^5 . Haxton evaluates the contributions of ρ and ω to be a 5% correction to the pion term. Since the experimental value, $P_\gamma = 8 \pm 39 \times 10^{-5}$, of the asymmetry is consistent with zero, an upper limit for H_π^1 results. Adelberger and Haxton find $H_\pi^1 \leq 0.28 \times 10^{-6}$. This value is a fraction of the DDH “best” value and is an order of magnitude smaller than the “reasonable range”. It should be noted, however, that Kaplan and Savage have suggested that including two-pion contributions may reduce the sensitivity of P_γ to H_π^1 through interference with the one-pion term [5].

The anapole moment operator [11] is a parity-odd rank one tensor and is given by

$$\vec{a} = -\pi \int r^2 \vec{j}(r) d^3r, \quad (2)$$

where $\vec{j}(r)$ is the electromagnetic current density operator. The anapole moment has an expectation value of zero for states of definite parity. The size of the anapole moment is given by the dimensionless constant κ_a , whose value is thought to be theoretically stable for three reasons:

1. The anapole moment of a nuclear state is a diagonal matrix element, in contrast to a transition matrix element.
2. The nuclear wave function is constrained by the measured magnetic moment of the state. The single-particle estimate is $\mu_{sp} = 1.72$ for ^{133}Cs and $\mu_{sp} = 2.79$ for ^{205}Tl (see, for example, [12]) in relatively good agreement with the experimental values $\mu_{exp} = 2.58$ and $\mu_{exp} = 1.64$, respectively.
3. The contribution to anapole moment of the spin of the odd proton can be estimated analytically and gives results close to those from full many-body calculations² [13].

The estimate of the one-body part of the anapole moment is similar to the calculation of the contribution of the spin of the unpaired nucleon to the magnetic moment of a nucleus. The analytical estimate [14] gives a value in rough agreement with detailed nuclear structure calculations by Haxton [15], Haxton, Henley, and Musolf [16], Flambaum and Murray [8],

²Dmitriev and Telitsin find that a naive harmonic oscillator model gives results which agree with their full calculation to within 10%.

and Dmitriev and Telitsin [13]. The latter is the most recent calculation and includes the effects of spin, spin-orbit, convection, and contact currents, many-body corrections, and RPA re-normalization of the weak interaction addition to the single-particle weak interaction. An additional attractive feature of anapole moment measurements is that they can in principle be made for many nuclei, thus providing an opportunity to test the consistency of the theory.

III. INTERPRETATION OF THE ^{133}Cs ANAPOLE MOMENT MEASUREMENT

Flambaum and Murray have argued that the value of H_π^1 determined from the ^{18}F measurement and the measured value of the ^{133}Cs anapole moment are inconsistent. As discussed in the introduction, they argue that the ^{133}Cs result requires a much larger value of H_π^1 than the ^{18}F result. The ^{133}Cs anapole moment is, however, almost as sensitive to H_ρ^1 as to H_π^1 , as we demonstrate below. Because of this sensitivity, and because of the lack of model-independent constraints on H_ρ^0 , it is possible to extract values of the two couplings which agree with both experiments.

We follow the method of Flambaum and Murray in relating the ^{133}Cs anapole moment to the weak couplings. Dmitriev and Telitsin [13] calculate for the case of ^{133}Cs

$$\kappa_a = 0.041g_p + 0.008g_n + 0.0052g_{pn} - 0.0006g_{np}. \quad (3)$$

Here, Flambaum and Murray keep only the g_p term. Since this approximation leads to a reduced sensitivity of κ_a to H_ρ^0 , we keep all of the terms in equation 3. Assuming only contributions from isovector π and isoscalar ρ exchange, the dimensionless constants g_{ab} can be related to the weak coupling constants [17]

$$g_{pp} = g_{nn} = 2(\mu_v + 2)W_\rho A_\rho H_\rho^0, \quad (4)$$

$$g_{pn} = 2(2\mu_v + 1)W_\rho A_\rho H_\rho^0 + \sqrt{32}W_\pi A_\pi H_\pi^1, \quad (5)$$

$$g_{np} = 2(2\mu_v + 1)W_\rho A_\rho H_\rho^0 - \sqrt{32}W_\pi A_\pi H_\pi^1, \quad (6)$$

$$g_p = \frac{Z}{A}g_{pp} + \frac{N}{A}g_{pn}, \quad (7)$$

$$g_n = \frac{Z}{A}g_{np} + \frac{N}{A}g_{nn}, \quad (8)$$

$$(9)$$

where

$$A_\rho = \frac{\sqrt{2}}{Gm_\rho^2} \approx 0.20 \times 10^6, \quad (10)$$

$$A_\pi = \frac{1}{Gm_\pi^2} \approx 4.4 \times 10^6, \quad (11)$$

with $W_\rho \approx 0.4$ and $W_\pi \approx 0.16$. This leads to the following expression for κ_a in terms of H_π^1 and H_ρ^0 :

$$\kappa_a \approx 1.05 \times 10^5 \left(H_\pi^1 + 0.69H_\rho^0 \right). \quad (12)$$

Equation 12 implies that it is possible to choose values of H_π^1 and H_ρ^0 which simultaneously satisfy both the ^{18}F and ^{133}Cs measurements. This solution, as can be seen in figure 1, consists of a small value for H_π^1 and a large value of H_ρ^0 , at the limit of the DDH “reasonable range”. In contrast, Flambaum and Murray assume the DDH “best” value of H_ρ^0 in their extraction of H_π^1 .

IV. INTERPRETATION OF THE ^{205}Tl ANAPOLE MOMENT MEASUREMENT

The ^{133}Cs experiment is as yet the only non-zero measure of an anapole moment. There is, however, an earlier null measurement in ^{205}Tl [9] which is of sufficient accuracy to be relevant.³ It is interesting to compare these two cases. Dmitriev and Telitsin treat both nuclei in a consistent way, so that we can use the two measurements as a check on the stability of the theory. We analyze the ^{205}Tl result, $\kappa_a = -0.22 \pm 0.30$, by the same method we used for ^{133}Cs and find

$$\kappa_a = 4.73 \times 10^5 (H_\pi^1 + 0.55H_\rho^0). \quad (13)$$

Because they are both odd-proton nuclei, the anapole moments of the two show a similar dependence on the weak coupling constants. The experimental values $\kappa_a = -0.22 \pm 0.30$ for ^{205}Tl [9] and $\kappa_a = 0.364 \pm 0.062$ for ^{133}Cs [8], however, lead to inconsistent constraints on H_π^1 and H_ρ^0 , as shown in figure 1. The inconsistency is statistically significant, 2.5σ . This result suggests that nuclear structure effects which are not included in the theory may be important in interpreting the measurements, assuming both measurements are correct.

V. DISCUSSION

The relationship between the values of H_π^1 and H_ρ^0 extracted from the ^{18}F and ^{133}Cs experiments are shown in figure 1. Also indicated on the plot are the values we extract from the ^{205}Tl experiment and the DDH “best” values and “reasonable range”. If one assumes, as do Flambaum and Murray, the DDH “best” value of H_ρ^0 value in interpreting the ^{133}Cs result, there is indeed a statistically significant disagreement between the values of H_π^1 from the ^{133}Cs anapole moment and from the ^{18}F measurements. No model-independent determination of H_ρ^0 , analogous to the method used for H_π^1 in ^{18}F , is possible from nuclear processes because the one-body part of the corresponding potential is $\Delta I = 0$. When the sensitivity of the anapole moment measurement to H_ρ^0 is considered, there is a solution consistent with both experiments, albeit an extreme one. This solution, taken at face value, is at the upper limit of the DDH “reasonable range” and implies that weak hadronic interaction phenomena in nuclei are determined primarily by one constant, H_ρ^0 . The apparent inconsistency between the ^{133}Cs and ^{205}Tl results, suggests that the nuclear theory of anapole moments is not yet reliable.

³There are also null results for the anapole moment of ^{207}Pb [18,19], however the statistical accuracy of the measurements is not sufficient to usefully constrain the weak couplings.

The present controversy concerning the interpretation of measured nuclear anapole moments highlights the need to determine the weak couplings from experiments in few-nucleon systems whose interpretation is free from uncertainties in nuclear structure. If the weak meson-nucleon couplings were known, the present controversy could be more definitively addressed. One would then calculate the values of the anapole moments using the measured values of the couplings from few-nucleon experiments; if the measured values of the anapole moments disagreed with these predictions, the problem would lie with the measurements, the nuclear theory, or the applicability of the meson-exchange model to nuclei.

We now outline a technically feasible program of measurements to determine the most important weak meson-nucleon couplings; H_π^1 , H_ρ^0 , H_ρ^2 , and the combination $H_\omega^0 + H_\omega^1$. Experiments which measured the longitudinal asymmetry A_z in pp elastic scattering at 15 and 45 MeV have considerably increased our knowledge of the weak meson-nucleon couplings [20,21]. These experiments determine a linear combination of the $\Delta I = 0, 1$, and 2 ρ and the $\Delta I = 0$ and 1 ω couplings. For example, at 45 MeV, A_z is given by [2]

$$A_z = -0.053 \left(H_\rho^0 + H_\rho^1 + H_\rho^2/\sqrt{6} \right) - 0.016 \left(H_\omega^0 + H_\omega^1 \right). \quad (14)$$

An experiment in progress at TRIUMF [22] to measure A_z at 221 MeV is sensitive to only the ρ couplings:

$$A_z = 0.028 \left(H_\rho^0 + H_\rho^1 + H_\rho^2/\sqrt{6} \right). \quad (15)$$

Further experiments are necessary to separate the relative contributions of the individual couplings, H_ρ^0 , H_ρ^1 , and H_ρ^2 . Since the $\Delta I = 1$ ρ coupling is believed to be small [1], only one other independent measurement is necessary. Measurements in the np system can provide the missing information.

The problem of separately determining the weak ρ couplings could be resolved by measuring the circular polarization P_γ of the gammas emitted in the $\vec{n} + p \rightarrow d + \gamma$ reaction. This observable is primarily sensitive to the $\Delta I = 0$ and 2 ρ couplings [2]:

$$P_\gamma = 0.022H_\rho^0 + 0.043H_\rho^2/\sqrt{6} - 0.002H_\omega^0. \quad (16)$$

The combination of this measurement and the TRIUMF pp measurement would then independently determine H_ρ^0 and H_ρ^2 . In practice, it is experimentally easier to measure the inverse reaction, the directional asymmetry in the photo-disintegration of the deuteron by circularly polarized photons, due to the low efficiency of γ -ray polarization analyzers. As in the case of A_γ , existing measurements of P_γ (or of the inverse reaction) [23–25] are not of sufficient precision to accomplish this task. With the availability of very intense gamma beams from Compton back-scattering using free electron lasers [26], a more precise measurement of the deuteron photo-disintegration asymmetry is possible and should be pursued.

The problem of determining H_π^1 can be addressed by measuring the gamma ray asymmetry with respect to neutron spin direction A_γ in the $\vec{n} + p \rightarrow d + \gamma$ reaction. According to Danilov's theorem [27], at thermal energies and below A_γ is due entirely to weak processes that have $\Delta I = 1$. Detailed calculations [28,2] show that contributions from heavier mesons are suppressed and the process is dominated by π exchange. A_γ is therefore almost completely determined by H_π^1 , with other couplings contributing at the few-percent level [2]:

$$A_\gamma = -0.045 \left(H_\pi^1 - 0.02H_\rho^1 + 0.02H_\omega^1 + 0.04H_\rho'^1 \right). \quad (17)$$

Unfortunately, existing measurements of A_γ [29,30] are not of sufficient statistical precision necessary to determine H_π^1 . A precision measurement of A_γ would be a nuclear structure independent determination of H_π^1 , with the other couplings contributing only at the few-percent level. A precision measurement of A_γ should be pursued.

Finally, we point out that additional measurements in few-nucleon systems, for example neutron spin rotation in H_2 or ${}^4\text{He}$ [31], would then test the applicability of the meson-exchange model to the weak interaction in nuclei, provided that the five-body calculations can be reliably performed.

VI. SUMMARY

The recent measurement of the anapole moment of ${}^{133}\text{Cs}$ appears to yield a value of H_π^1 which contradicts the upper limit from the earlier ${}^{18}\text{F}$ experiments. We argue that because of the sensitivity to H_ρ^0 in the ${}^{133}\text{Cs}$ measurement, there is a combination of weak couplings which satisfies both experiments and which is (barely) in agreement with theory. In addition, the interpretation of the ${}^{133}\text{Cs}$ result is inconsistent with an earlier null measurement in ${}^{205}\text{Tl}$, calling into question the reliability of the theory of nuclear anapole moments. All three experiments require assumptions about nuclear structure. We argue that measurements of directional asymmetry in $\vec{n} + p \rightarrow d + \gamma$ and in the photo-disintegration of the deuteron by circularly polarized photons, combined with results from pp scattering, would determine the weak meson-nucleon couplings in a model-independent way.

We would like to thank J.L. Friar, S.K. Lamoreaux, and S.I. Penttilä for useful discussions. This research was supported by the U.S. Department of Energy under contract W-7405-ENG-36.

REFERENCES

- [1] B. Desplanques, J.F. Donoghue, and B.R. Holstein, *Ann. Phys.* **124**, 449 (1980).
- [2] E.G. Adelberger and W.C. Haxton, *Ann. Rev. Nucl. Part. Sci.* **35**, 501 (1985).
- [3] W.C. Haxton, *Phys. Rev. Lett.* **46**, 698 (1981).
- [4] W. Haeberli and B.R. Holstein, in *Symmetries and Fundamental Interactions in Nuclei*, edited by W. Haxton and E. Henley (World Scientific, Singapore, 1995), p. 17.
- [5] D.B. Kaplan and M.J. Savage, *Nucl. Phys.* **A556**, 653 (1993).
- [6] E.M. Henley, W.-Y.P. Hwang, and L.S. Kisslinger, *Phys. Lett. B* **367**, 21 (1996).
- [7] C.S. Wood, S.C. Bennett, D. Cho, B.P. Masterson, J.L. Roberts, C.E. Tanner, and C.E. Wieman, *Science* **275**, 1759 (1997).
- [8] V.V. Flambaum and D.W. Murray, *Phys. Rev. C* **56**, 1641 (1997).
- [9] P.A. Vetter, D.M. Meekhof, P.K. Majumder, S.K. Lamoreaux, and E.N. Fortson, *Phys. Rev. Lett.* **74**, 2658 (1995).
- [10] C. Bennet, M.M. Lowry, and K. Krien, proposal (unpublished).
- [11] Ya.B. Zel'dovich, *Zh. Eksp. Teor. Fiz.* **33**, 1531 (1957) [*Sov. Phys. JETP* **6**, 1184 (1958)].
- [12] A. Bohr and B.R. Mottelson, *Nuclear Structure* (W.A. Benjamin, New York, 1969), Vol. I.
- [13] V.F. Dmitriev and V.B. Telitsin, *Nucl. Phys.* **A613**, 237 (1997).
- [14] V.V. Flambaum and I.B. Kriplovich, *Zh. Eksp. Teor. Fiz.* **79**, 1656 (1980).
- [15] W.C. Haxton, *Science* **275**, 1753 (1997).
- [16] W.C. Haxton, E.M. Henley, and M.J. Musolf, *Phys. Rev. Lett.* **63**, 949 (1989).
- [17] O.P. Sushkov and V.B. Telitsin, *Phys. Rev. C* **48**, 1069 (1993).
- [18] D.M. Meekhof, P.A. Vetter, P.K. Majumder, S.K. Lamoreaux, and E.N. Fortson, *Phys. Rev. A* **52**, 1895 (1995).
- [19] S.J. Phipp, N.H. Edwards, P.E.G. Baird, and S. Nakayama, *J. Phys. B* **29**, 1861 (1996).
- [20] P.D. Eversheim *et al.*, *Phys. Lett. B* **256**, 11 (1991).
- [21] J.M. Potter, J.D. Bowman, C.F. Wang, J.L. McKibben, R.E. Mischke, D.E. Nagle, P.G. Debrunner, H. Fraunfelder, and L.B. Sorensen, *Phys. Rev. Lett.* **33**, 1307 (1974).
- [22] A.R. Berdoz *et al.*, in *Proceedings of the Sixth Conference on Intersections between Particle and Nuclear Physics*, edited by T. Donnelly (AIP Press, Woodbury, New York, 1997), Vol. 412.
- [23] V.M. Lobashov, D.M. Kaminker, G.I. Kharkevich, V.A. Kniazkov, N.A. Lozovoy, V.A. Nazarenko, L.F. Sayenko, L.M. Smotritsky, and A.I. Yegorov, *Nucl. Phys.* **A197**, 241 (1972).
- [24] V.A. Knyaz'kov, E.M. Kolomenskii, V.M. Lobashov, V.A. Nazarenko, A.N. Pirozhov, A.I. Shablii, E.V. Shalgina, Y.V. Sobolev, and A.I. Yegorov, *Nucl. Phys.* **A417**, 209 (1984).
- [25] E.D. Earle, A.B. McDonald, S.H. Kidner, E.T.H. Clifford, J.J. Hill, G.H. Keech, T.E. Chupp, and M.B. Schneider, *Can. J. Phys.* **66**, 534 (1988).
- [26] V.N. Litvinenko *et al.*, *Phys. Rev. Lett.* **78**, 4569 (1997).
- [27] G.S. Danilov, *Phys. Lett.* **18**, 40 (1965).
- [28] B. Desplanques, to be published (unpublished).
- [29] J.F. Caviagnac, B. Vignon, and R. Wilson, *Phys. Lett.* **B67**, 148 (1977).
- [30] J. Alberi *et al.*, *Can. J. Phys.* **66**, 542 (1988).

[31] B. Heckel, Nucl. Instrum. Methods A **284**, 66 (1989).

FIGURES

FIG. 1. Weak coupling constants H_{π}^1 and H_{ρ}^0 extracted from the ^{18}F (light band), ^{205}Tl (medium band), and ^{133}Cs (dark band) experiments. Also shown are the DDH “best” values (square) and “reasonable range” (box).

