Sterile Neutrino and Accelerating Universe

P. Q. Hung*

Dept. of Physics, University of Virginia,

382 McCormick Road, P. O. Box 400714, Charlottesville, Virginia 22904-4714

(November 7, 2013)

Abstract

If all three neutrino oscillation data were to be confirmed in the near future, it is probable that one might need a sterile neutrino, in addition to the three active ones. This sterile neutrino, ν_S, would be very light with mass $m_{\nu_S} \leq 1 eV$ or even with $m_{\nu_S} \sim 10^{-3} eV$ according to some scenarios. Why would it be so light? On another front, recent cosmological observations and analyses appear to indicate that the present universe is flat and accelerating, and that the present energy density is dominated by a "dark variety", with $\rho_V \sim (10^{-3} eV)^4$. Is it a constant? Is there a link between these apparently unrelated phenomena?

98.80.Cq, 14.60.Pq, 14.60.St

For the past three years or so, a number of discoveries in particle physics and cosmology has begun to reveal startling results which, if verified, would have deep implications. Of particular relevance to this paper are the new results on neutrino oscillation which suggest that neutrinos have a mass, albeit a tiny one, and new evidence for an accelerating universe.

On the cosmology front, there is a most recent evidence for a flat universe, i.e. $\Omega = 1$, from the Boomerang collaboration [1]. In addition, discoveries of high red-shift (high Z) Supernovae IA and their use in determining the deceleration parameter q_0 have been most dramatic [2]. A *positive* q_0 implies that the universe is decelerating while a *negative* one- as implied by the high Z Supernovae data- means that it is *accelerating*. Furthermore, assuming that $\Omega_0 = \Omega_m + \Omega_\Lambda = 1$, where $\Omega_{m,\Lambda}$ refer to the Ω's coming from matter and a cosmological constant respectively, Supernovae (SNIA) results seem to indicate that $\Omega_m \sim 0.3$ and $\Omega_\Lambda \sim 0.7$, which although differ from one another by a factor of two, are of the same order of magnitude. The vacuum energy density coming from a cosmological constant would be approximately $\rho_V \sim (1.6 \times 10^{-3} eV)^4$. Why is it so small?

There are several appealing suggestions for such a "tiny" (although presently dominant) value for the vacuum energy (which come with their own difficulties), among which is the idea of a dynamical vacuum energy, or in other words "quintessence" [3]. If, on the other hand, this vacuum energy were to genuinely come from some phase transition, there can be interesting consequences, such as the links with current physical phenomena as presented in this paper. With $\rho_V = V(0) = \sigma^4$, one would then expect $\sigma \sim 1.6 \times 10^{-3} eV$. What might be the origin of of such a small σ? In the Standard Model, there are several symmetry breaking scales, e.g. the electroweak and the chiral symmetry breaking scales, each of which contributes a cosmological constant several orders of magnitude larger than the aforementioned constant. One generally agrees that a cancellation of something like $\rho_v \sim 4 \times 10^{45} eV^4$ (Electroweak) down to $\rho_v \sim 10^{-12} eV^4$ is highly unnatural. In the present *absence of a satisfactory solution* to the cosmological constant problem, one might then assume that there is a yet-unknown mechanism by which the various cosmological constants get cancelled out to zero, and the hint that one might have a non-zero vacuum energy at the present time indicates some new contribution, either in the form of quintessence or a genuinely ongoing new phase transition. It is the latter assumption that we shall exploit in this paper. In other words, this unknown mechanism would bring the total vacuum energy down to zero when *all* phase transitions are completed.

On another front, it is well accepted that if all three neutrino oscillation results (solar, atmostpherics, and LSND data) were to be proven correct, one would need, in addition to three standard neutrinos, ν_e, ν_μ, and ν_τ, an additional one which does not have normal electroweak interactions- the so-called sterile neutrino ν_S [4]. This neutrino would have a mass $m_{\nu_S} \leq 1eV$ or even $m_{\nu_S} \sim 10^{-3}eV$ according to some scenarios. In order to explain the data, this neutrino would mix with one or more active neutrinos. If the idea of a sterile neutrino proves to be correct in the future, one will be confronted with the following puzzling question: Why is ν_S so light and so close in mass to an active neutrino when it appears that they are of very different types of particles? In popular scenarios with the see-saw mechanism and Majorana mass, typically the sterile neutrino is *heavy* and the scale of new physics is rather large (a typical Grand Unified scale). Needless to say, the issues of neutrino mass are far from being resolved, including the important question of whether or not the mass is Dirac or Majorana. One should finally also notice that there are additional (astrophysical) arguments which claim the need for a sterile neutrino [5].

What we would like to propose in this article is a "simple" model which links the issue of the origin of the sterile neutrino mass to that of the dark energy. In a nutshell, it is simply this: the sterile neutrino obtains its mass through a Yukawa coupling with a "singlet" Higgs field whose effective potential is of a "slow-rolling" type. During this slow rolling, the vacuum energy would be given approximately by $V(0) = \sigma^4$. In our model, the *effective* mass of the sterile neutrino is proportional to the *present* value of the singlet Higgs field, $\phi_S(t_0)$ and can be as small as $10^{-3}eV$ provided $\phi_S(t_0)$ is itself sufficiently small. The Higgs field ϕ_S will eventually reach its global value v_S which can be *much greater* than $\phi_S(t_0)$. In this sense, a small sterile neutrino mass at the present time is, in our scenario, merely a reflection of the current value of ϕ_S. At the end of the phase transition, the sterile neutrino could, in principle, acquire a *much larger mass*. It is an intriguing possibility that, as ϕ_S evolves, the sterile neutrino mass will change and, as a result, the oscillation with the electron neutrino will end in some distant future.

In what follows, the active neutrinos will be assumed to obtain their masses by phase transitions which have *already* occured. These masses could either be Majorana or Dirac masses. (One example of a naturally light Dirac neutrino mass can be found in [6], along with numerous phenomenological consequences.) The singlet sterile neutrino, on the other hand, will be assumed to obtain primarily a mass through a Yukawa coupling with a singlet Higgs field as described above. In other words, this mass would be obtained during the "last" phase transition.

First, a phenomenological effective potential for a singlet Higgs field is presented along with a relationship between the vacuum expectation value, v_S, of the singlet Higgs field and its present classical value, $\phi_S(t_0)$. Second, we shall use this relationship *in conjunction* with the sterile neutrino mass to infer on a possible magnitude for v_S.

The scenario that we would like to discuss here is the following. 1) There are several phase transitions occuring during the course of history of the universe. 2) The "last" (perhaps) of those phase transitions is the one associated with ϕ_S. 3) We will assume that there exists a mechanism by which the vacuum energy vanishes after the associated phase transition is completed. As a result, the total energy density is given by $\rho_{tot} = \rho_m + \rho_V$, where ρ_m includes matter energy density of all types, and- this is the crucial assumption- ρ_V is the vacuum energy which will include, at any given time, the total cosmological constant arising from phase transitions which either have not started or have not been completed. In our scenario, before the "last"phase transition is completed, ρ_V will simply be given by $V(\phi_S = 0)$.

The *phenomenological* effective potential used to illustrate our scenario is as follows:

$$V(\phi_S) = \sigma^4(1 - ax^2 + bx^2 exp(-cx^2) + dx^3), \tag{1}$$

where $x \equiv \phi_S^2/v_S^2$ with v_S being the value at the global minimum, and where $V(\phi_S = 0) = \sigma^4$. The arbitrary coefficients a, b, c, d are chosen so as to make $V(\phi_S)$ *very flat*. Since this potential is purely *phenomenological*, issues such as quantum corrections, etc.., are already included in the choice of these parameters, i.e. $V(\phi_S)$ is presumably the "final" form arising from some unknown deeper theory. This peculiar form for the potential is inspired, in parts, by previous studies of inflationary models [7]. This potential obeys: $V(\phi_S = v_S) = 0$, $V'(\phi_S = v_S) = 0$. For a given set of values for a, c, d, these conditions restrict b to be $b = (d/2 - 1)exp(c)/c$. Let us furthermore assume that, whatever barrier that existed, the singlet field has already proceeded to *classically* "roll down" to its global minimum at v_S. The equation describing the evolution of $\phi_S(t)$ is a well-known one, namely $\ddot{\phi}+3H\dot{\phi}+V'(\phi_S) = 0$.

One remark about the potential (1) is in order here. The analysis presented below depends primarily on two points: 1) $V(0) = \sigma^4$; 2) With $V(\phi_S) = \sigma^4\tilde{V}(x)$, the present slow-rolling requirement would be satisfied if $2\tilde{V}(x)' + 4x\tilde{V}(x)'' \sim O(x)$. Although we use a potential of the form (1), any other potential which satisfies those two points might also work. One might assume that there is a class of models where the above conditions are satisfied.

The evolution of the scale factor $R(t)$, under the assumption of zero curvature ($k = 0$) as is presumably the case experimentally, is given by

$$H(t)^2 = (8\pi/3m_{Pl}^2)(\rho_m + \rho_V), \tag{2}$$

where $H(t) \equiv \dot{R}/R$ and where m_{Pl} is the Planck mass. One can also rewite the above equation using the definition: $\Omega(t) \equiv (8\pi/3m_{Pl}^2)(\rho/H(t)^2)$, namely $\Omega_m(t) + \Omega_\Lambda(t) = 1$. From Eq. (2), one observes that, because ρ_m *decreases* with time, the Hubble parameter $H(t)$ also *decreases* and tend towards $H_V^2 \sim (8\pi/3m_{Pl}^2)\rho_V$, although the present value for $H(t_0) \equiv H_0$ does not differ by much from H_V. In fact, with $\Omega_m \sim 0.3$ and $\Omega_\Lambda \sim 0.7$, it is easy to see that $H_0^2 = H_V^2(1 + \rho_m(t_0)/\rho_V) \sim 1.43H_V^2$. Furthermore, since ρ_V remains constant, $\Omega_\Lambda(t)$ *increases* with time, implying that $\Omega_m(t)$ decreases with time. The universe will become more and more dominated by the vacuum energy.

From Eq. (2), we can set a constraint on the parameter σ appearing in $V(\phi_S)$. First, in our scenario, $\rho_V = V(\phi_S = 0) = \sigma^4$. Denoting the present Hubble rate $H(t_0)$ by H_0, we can write down the following inequality:

$$\sigma^4 \leq (3m_{Pl}^2/8\pi)H_0^2. \tag{3}$$

Putting in the present value for H_0, one obtains $\sigma \leq 10^{-3}eV$.

What range of values might one expect for v_S? To find this out, we assume that the present universe is in a stage where ϕ_S is "slowly rolling"- and this is what the above form for $V(\phi_S)$ is supposed to do. As stated, it is irrelevant at this stage to try to determine the exact value for v_S. Consequently, we will approximate H to be a constant, namely $H \sim H_V$, noting that $H_0 \sim 1.2H_V$ as shown above. One obtains the usual constraint: $|V''(\phi_S)| \lesssim 9H_V^2$. If we write $V(\phi_S) = \sigma^4\tilde{V}(x)$ with $x \equiv (\phi_S/v_S)^2$, the previous constraint translates into:

$$|2\tilde{V}' + 4x\tilde{V}''| \lesssim 24\pi(\frac{v_S}{m_{Pl}})^2, \tag{4}$$

where the primes denote derivatives with respect to x. If the universe is in the stage where $x \ll 1$, the constraint (4) can be translated into a constraint on v_S as a function of what one might think the present value of ϕ_S would be, namely

$$v_S \gtrsim (\frac{a-b}{2\pi})^{1/4}\sqrt{\phi_{S,0}m_{Pl}}, \tag{5}$$

where $\phi_{S,0}$ refers to the present value of ϕ_S. (Notice that this bound is independent of the value of σ.) To be able to make use of the bound (5), one should specify what the parameters a, b are as well as the value for $\phi_{S,0}$. The question we would like to ask is the following: Can the lower bound on v_S be so small as to allow v_S to be as little as O(eV)? The arguments presented below suggest that this might not be the case.

To estimate what $\phi_{S,0}$ might be, we now return to the issue of the sterile neutrino. Let us assume that there is a Yukawa coupling between the sterile neutrino, denoted by ν_S, and ϕ_S, of the form:

$$\mathcal{L}_S = g_S\bar{\nu}_{SL}\phi_S\nu_{RS} + h.c.. \tag{6}$$

When the phase transition has been completed, the sterile neutrino (Dirac) mass would be $m_{\nu_S} = g_S v_S$. But while ϕ_S is "coasting" towards its global minimum, the effective Dirac mass of the sterile neutrino would be given by

$$m_{\nu_S,eff} = g_S \phi_{S,0}, \tag{7}$$

where $\phi_{S,0}$ is the present value of the *classical* field. As alluded to in the Introduction, there are several reasons to think that there might be a sterile neutrino. However, its mass will depend on a particular scenario. For definiteness, we shall assume that $m_{\nu_S,eff} \sim 10^{-3} eV$, keeping in mind that other values which are less than 1 eV are possible. Also, for the sake of argument, let us assume that $g_S \sim O(1)$ and thus $\phi_{S,0} \sim O(10^{-3} eV)$. This would then imply that

$$\sqrt{\phi_{S,0} m_{Pl}} \sim 3\, TeV. \tag{8}$$

Next, one might want to see if there is any reason for the lower bound on v_S to be as low as O(eV). For this to happen and taking into account (8, 5), one would need $a - b \sim 10^{-48}$, which means that they are either degenerate to 48 decimal places, or that they are as small as 10^{-48}. This is unlikely and undesirable for the following reasons. First, although the potential is purely phenomenological, there is no reason to expect a and b to be of that nature. In fact, as we have discussed above, for a given set of a, c, d, the conditions $V(\phi_S = v_S) = 0$, $V'(\phi_S = v_S) = 0$ constrain b to be $b = (d/2 - 1) exp(c)/c$. Hence, there is no reason to expect b and a to be degenerate to 48 decimal places. It is also highly unnatural to expect both a and b to be of O(10^{-48}). We shall henceforth assume that both a and b are of O(1), with b obeying the minimum constraint discussed above. With $a = 3.37, b = 3.3$ ($c = 4.5, d = 2.33333$) chosen for the sole purpose of illustration, one obtains

$$v_S \gtrsim 1\, TeV. \tag{9}$$

This bound opens up a whole host of interesting possibilities.

First, one could not help but notice an interesting point which might possibly have a deeper meaning. For the sake of argument, let us simply assume that $v_S \sim 3 TeV$, for example. Then, according to the above analysis, this means that we are in a midst of an era where the scalar field ϕ_S is "slowly rolling" toward its global minimum value. The present universe is dominated by the vacuum energy $V(0) = \sigma^4 \sim (10^{-3} eV)^4$, implying $\sigma \ll v_S$.

It then appears that σ and v_S are completely unrelated (which might still be the case in a deeper theory). However, one notices that

$$v_S^2/m_{Pl} \sim 10^{-3} eV. \tag{10}$$

if $v_S \sim 3TeV$. Does this numerical exercise imply that $\sigma \equiv v_S^2/m_{Pl}$? After all, in our scenario, there are three scales: $\sim 10^{-3}eV$, $\sim 1TeV$, and m_{Pl}. It might not be too surprising that one of the scales (e.g. $\sim 10^{-3}eV$) is related to the other two. This intriguing possibility prompts us to rewrite the phenomenological potential $V(\phi_S)$ as

$$V(\phi_S) = (v_S^2/m_{Pl})^4(1 - ax^2 + bx^2 exp(-cx^2) + dx^3), \tag{11}$$

where $b = (d/2 - 1)exp(c)/c$ and where v_S is of order of a few TeV's. The value of v_S which is determined from the constraint of "slow rolling" and the presumed mass of a sterile neutrino, independently of the value of the vacuum energy, can in turns be used to *fix* the vacuum energy itself if the potential has the form (11). What might be the origin of such a potential and of such a scale? Is that scale related to the scale of extra dimensions or of SUSY breaking? These questions are beyond the scope of this paper.

To complete the discussion, let us make a rough estimate of the time it takes to get from the present era to the point $\phi_{S,e}$ where ϕ_S starts to evolve rapidly. To be specific,let us take $\phi_{S,0} \sim O(10^{-3}eV)$ and $v_S \sim 3\,TeV$. From (9), $\phi_{S,e}$ will be approximately $3\,10^{-3}eV$. This means that we will be looking at the evolution from $x_0 \sim 10^{-31}$ to $x_e \sim 10^{-30}$, where $x \equiv (\phi_S/v_S)^2$. Let us also make the approximation that H is constant ($\approx H_0$) during that period (it does not vary much as we have shown above). One can then estimate the additional time $\Delta t \equiv t_e - t_0$ (t_0 is the present time) where the slow rolling begins to be invalid. It is straightforwardly given by :

$$\Delta t \approx \frac{2\pi}{H_0}\frac{v_S^2}{m_{Pl}^2}\int_{x_0}^{x_e}\frac{\tilde{V}}{-x\tilde{V}'}dx, \tag{12}$$

where $\tilde{V}$ has been defined as $V = \sigma^4\tilde{V}$, and where the second on the right-hand side of Eq. (12) is a rewriting of the well-known formula used to compute the number of e-folds in an inflationary scenario. A numerical integration of Eq. (12) gives $\Delta t \equiv t_e - t_0 \approx 36/H_0$. With $1/H_0 \sim 15 \times 10^9$ yr, one obtains roughly $\Delta t \sim 540 \times 10^9$ yr. The universe will still be accelerating for a long, long time! (It turns out that Δt gets even bigger as v_S gets larger.) In fact, the universe will undergo an inflationary period until the phase transition is completed. The small latent heat will be converted into the production of very massive (TeV) ϕ_S, which could decay into very massive ν_S if the masses allow for it to be so.

A couple of other remarks are in order here. Since $\sqrt{V''(\phi_{S,0})}$ is the present effective mass of the Higgs field ϕ_S, the slow rolling condition would indicate that this effective mass would be at most $10^{-33} eV$. Would such a low mass cause any problem with long range forces? There are strong constraints on the couplings of such a low mass scalar to ordinary matter [3]. In our case, this singlet scalar field only couples to another singlet neutrino, ν_S, which in turns has a tiny mixing with ν_e. The effects of this low mass scalar on ordinary matter appear to be negligible. A rough estimate of the coupling of ϕ_S to an electron via a W-mediated loop diagram gives an efective coupling $\sim G_F m^2_{\nu_S} |V_{e\nu_S}|^2$. With $|V_{e\nu_S}|^2 \sim 10^{-3}$ and $m^2_{\nu_S} \sim 10^{-6} eV^2$, one expects this coupling to be less than 10^{-30}. When it is squared to provide the coupling for a "long range" potential, one expects it to be $\sim 10^{-60}$, and this is considerably less than $G m_e^2/m_{Pl}^2 \sim 10^{-44}$ for the gravitational potential. For all practical purposes, the effective coupling to matter (electron) is so weak that one can safely ignore it. This subject will be dealt with in future work. Another topic of interest for a future work is the effect of such light scalars in supernovae process (such as the r-process for example). A preliminary investigation indicates that, because of the extreme weakness of the interaction, there is practically no effect. The second remark has to do with the question of why $\Omega_m \sim 0.3$ and $\Omega_\Lambda \sim 0.7$ differ only by a factor of 2 or so at the present time. Tracker models of quintessence [3]are supposed to address this issue, although it is still controversial about the magnitude of ρ_{vacuum} (fine-tuning problem). At this point, the statement that one can make about the scenario presented here is the following: Some yet-unknown physics gives rise to a potential of the form (11) with a scale v_S of a few TeV's, so that if $\Omega_\Lambda \sim 0.7$ then $\Omega_m(t_0) + \Omega_\Lambda(t_0) = 1$ implies $\Omega_m \sim 0.3$. These issues are certainly beyond the scope of this paper.

In conclusion, we have presented a scenario in which the physics of the *accelerating universe* is intrincately linked to that of *a sterile neutrino*. It is amusing to note that, although the idea of inflation appears to be strengthened by the new astrophysical results, it must have happened at the dawn of the universe. The fact that the present universe appears to be accelerating prompts us to think that we are starting to *experience a late-time inflation*, of a different nature from the one at the birth of the present universe. In addition, the dark energy density of our picture is truly a constant, $V(\phi_S = 0)$, in contrast with scenarios based on quintessence in which it is time-dependent. This is something which could be tested within (hopefully) the next ten years.

I would like to thank Marc Sher for bringing my attention to an earlier paper [8] whose motivation was similar to the one presented here: the link of the physics of neutrinos to the present dark energy, and for useful comments. That model is however completely different from ours. To the best of our knowledge, the model presented here is the first attempt to link the dark energy to the physics which gives rise to the mass of the *sterile neutrino*. My thanks also go to Paul Frampton and Manfred Lindner for useful comments.

This work is supported in parts by the US Department of Energy under grant No. DE-A505-89ER40518.

REFERENCES

* e-mail: pqh@virginia.edu

[1] P. de Bernadis *et al.*, Nature **404**, 955 (2000).

[2] A. G. Riess *et al.*, Astron. J. **116**, 1009 (1998); S. Perlmutter *et al.* Astrophys. J. **517**, 565 (1999).

[3] For an extensive review and list of authors, see Sean M. Carroll, *The Cosmological Constant*, submitted to *Living Reviews in Relativity*, astro-ph/00004075.

[4] For an update on the current status of a sterile neutrino and for a complete list of experimental results, see V. Barger, B. Kayser, J. Learned, T. Weiler, and K. Whisnant, hep-ph/0008019.

[5] See e.g. David O. Caldwell, hep-ph/0003250.

[6] P. Q. Hung, Phys. Rev. **D59**, 113008 (1999); *ibid* Phys. Rev. **D62**, 053015 (2000); hep-ph/0006355, in Particle Physics and Cosmology: 2nd Tropical Workshop, edited by Jose F. Nieves, AIP Conference Proceedings, American Institute of Physics 2000.

[7] See Edward W. Kolb and Michael S. Turner, *The Early Universe*, (Redwood City, California: Addison-Wesley, 1990) for a pedagogical review.

[8] A. Singh, Phys. Rev. **D52**, 6700 (1995).