Dynamic scaling regimes of collective-decision making

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We investigate a social system of agents faced with a binary choice. We assume there is a correct, or beneficial, outcome of this choice. Furthermore, we assume agents are influenced by others in making their decision, and that the agents can obtain information that may guide them towards making a correct decision. The dynamic model we propose is of nonequilibrium type, converging to a final decision. We run it on random graphs and scale-free networks. On random graphs, we find two distinct regions in terms of the *finalizing time*—the time until all agents have finalized their decisions. On scale-free networks on the other hand, there does not seem to be any such distinct scaling regions.

I. INTRODUCTION

Designing simple, mechanistic models of social phenomena has been an off-the-mainstream theme in economics and sociology literature for several decades (1). Such models turn out to be very apt for the methods of statistical physics. So as the statistical physics community gets increasingly interdisciplinary it is not surprising that there is a growing interest in "sociophysics." One of the most common topics in this area is the spread of opinions (2; 3; 4; 5) or fads (6; 7). In models of these phenomena, the agents do not usually actively use information to optimize their behavior; the spread takes place more or less like an infection process. Another example of social systems studied by physicists is spatial, or networked, games (see Refs. (8; 9; 10) and references therein). In that case the agents actively try to maximize their individual gain. Between these two extremes is the problem of the present paper—collective decision making (11; 12). This is the situation when a population, through individual action and social interaction, has to make up their mind about a specific question. This problem differs from the opinion dynamics in that an individual can do better or worse. It also differs from evolutionary games in that there is no conflict built into the problem—the success of one agent is never the harm of others, or vice versa.

In this paper we model collective decisions as one at the same time social and individual process. The basic idea is that the process for an individual to make a decision contain elements of both social influence and individually obtained information. This is, since Janis's concept of "groupthink" (13), a well-established idea in psychology. Our model is a non-equilibrium, agent-based model for how N individuals each settle for one of two choices. This dynamic model is grounded on two precepts: First, the information that an agent bases its decision on is a sum of contributions from its social neighbors and direct information sources. Second, when the information in favor of one decision exceeds a certain threshold, the agent finalizes the decision. Furthermore, we assume the underlying network to be static. To study the effect of the network topology on the dynamics, we run it on different network models.

In the rest of the paper, we will first give a derivation and a detailed description of the dynamic model and the models to generate the underlying networks. Then we present our simulation results and finally relate them to psychosocial phenomena.

II. PRELIMINARIES

A. Definition of the model

We restrict ourselves to a binary decision process—a process where the outcome of the decision can be good or bad, right or wrong, correct or false. Without loss of generality, we let $+1$ represent the "correct" outcome. Consider a population of N agents connected via an underlying, static, social network. The social network is represented by a graph G consisting of N vertices, V; and M edges, E. The accumulated information used by an agent i at time t of the decision making process is represented by $S_t(i)$. The initial value $S_0(i)$, representing the direct information, is picked randomly. The probability of $S_0(i) = +1$ is p, otherwise $S_0(i)$ takes the value -1.

The social information-spreading process updates agents iteratively until all agents have reached fixed states. The current choice of an agent i is sgn $(S_t(i))$

where

$$\mathrm{sgn}\,(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}. \tag{1}$$

In our simulations $\mathrm{sgn}(S_t(i)) = 0$ represents the rare situation the agent i is completely undecided at time t. If the information in favor of a particular decision is strong enough, i.e. $|S_t(i)| > \theta$ for a threshold θ, the agent will finalize his/her decision, and $S_t(i)$ remain fix for the rest of the run. The threshold θ is for simplicity considered to be the same for every agent. A higher θ implies that the system needs more time to converge to its decision. The information exchange between two agents works as follows: Pick an active (not finalized) agent i at random, and a random neighbor j of i and let

$$S_{t+1}(i) = S_t(i) + \mathrm{sgn}\, S_t(j). \tag{2}$$

To summarize the algorithm:

1. Pick p and θ.
2. Assign each $S_0(1), \cdots, S_0(N)$; 1 with probability p, and -1 otherwise.
3. Pick a random active vertex i and a random of i's neighbors j.
   (a) Let $S_{t+1}(i) = S_t(i) + \mathrm{sgn}\, S_t(j)$.
   (b) If $|S_{t+1}(i)| > \theta$, let agent i be finalized.
4. If there are unfinalized vertices, go to step 3.

Our model thus have two control parameters, p and θ.

B. The model: considerations and possible extensions

In a real situation the direct knowledge may be obtained any time during a group prediction process. We believe this does not change the qualitative behavior of the model much. Assuming the direct information is to the benefit of the agent, the sensible range of p is $[1/2, 1]$. If $p = 1/2$ the initial knowledge does not guide agents towards a correct decision at all; if $p = 1$ the knowledge or preference influences the final decision strongly.

One also has to consider the relative influence of the agents in the social network. As mentioned, in our model, one vertex i and a random neighbor of i's is selected for the information transfer. This means that, if the network has neutral degree-degree correlations, then the probability that information is obtained from a vertex with degree k is proportional to k. I.e., vertices with many connections are more likely to influence others than vertices with few connections. This we believe is a plausible situation: People with many social ties are more likely to function as opinion-makers.

C. Network models

The model we present can be applied to any underlying network. In this paper we will use three types of model networks: The first model is random graphs (14) constructed such that M edges are iteratively added between random pairs of vertices such that no multiple edge or loop (self-edge) occurs. Such graphs have a binomial degree distribution (which becomes Poissonian in the $N \to \infty$ limit) and no other network structure. Thus random graphs make a good starting point for investigating a dynamic model such as ours.

Since many real-world networks have skewed degree distributions we also use a model producing networks with a power-law degree distribution. These networks, that we will refer to as "scale-free networks," are constructed by first assigning desired degrees $\tilde{k}_i$ from a power-law distribution $\mathrm{Prob}(k) \sim k^{-\gamma}$. Then we assign edges to random pairs (i, j) of vertices as long as the degree of a vertex i is less than $\tilde{k}_i$. If (i, j) already is an edge, or $i = j$ we do not add an edge. This process is terminated when M edges are added.

III. NUMERICAL RESULTS

A. Success rate of the decision process

Let ϕ be the fraction of the population making the correct decision after the dynamics has converged:

$$\phi = \frac{1}{2} + \frac{1}{2N} \sum_{i=1}^{N} \mathrm{sgn}\, S_\infty(i). \tag{3}$$

We note that there are two types of qualitatively different behavior that can emerge. Either the system benefit from the social communication or it does not. To study this behavior we define the order parameter

$$\psi = \frac{\phi - p}{1 - p}, \tag{4}$$

i.e., the improvement of the decisions from the communication relative to the theoretically largest improvement. If no improvement of the decisions are made then $\psi = 0$. If all agents decisions are correct (i.e. the improvement is maximal) $\psi = 1$.

By construction $p = 0.5$ is the symmetry value for which $\psi = 0$. This means that as p is increased from $p = 0.5$, systems will on average benefit from the information diffusion.

In Fig. 1(a) we display ψ, for random networks. We see that the decisions improve with, not only p, but also θ. With a higher decision threshold the agents need more information, and thus more iterations to converge. As more information is integrated, the average agent can reach the correct decision with higher probability. However, there is also a possibility that a majority (or, indeed, any fraction) of the agents end up with the wrong decision.

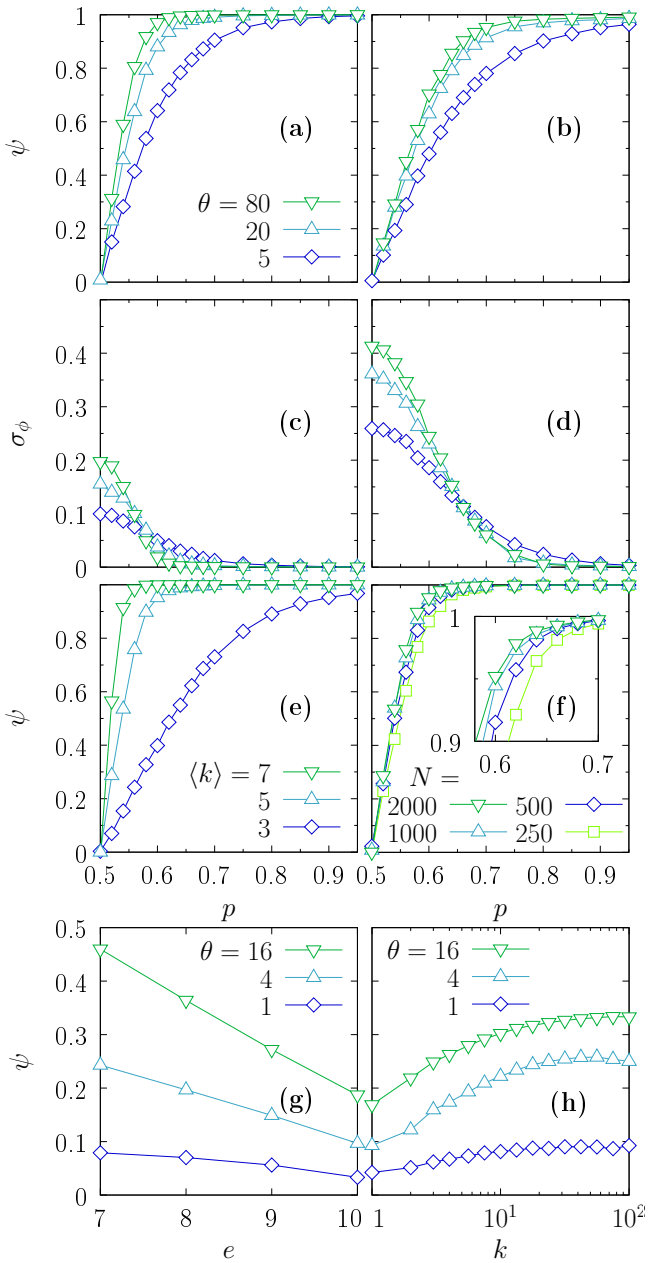

FIG. 1 Average relative increased performance ψ as a function of information quality p for (a) random graphs with different θ-values, using $N = 1000$ and $\langle k \rangle = 5$, and (b) scale-free networks with $N = 1000$, $\langle k \rangle = 5$ and $\gamma = 2.2$. The standard deviaton σ_ϕ of the success rate ϕ for (c) random graphs and (d) scale-free networks. The average relative increased performance ψ, (e) as function of average connectivity $\langle k \rangle$ for random graphs with $N = 1000$ and $\theta = 40$, and (f) as a function of system size N for random graphs with $\langle k \rangle = 5$ and $\theta = 40$. Panel (g) shows the average relative increased performance ψ as a function of eccentricity for random graphs with $N = 2000$, $\langle k \rangle = 5$ and $p = 0.53$. (h) displays ψ as a function of degree for scale-free networks with $N = 4000$, $\langle k \rangle = 5$, $\gamma = 2.2$ and $p = 0.53$. The curves are logarithmically binned. Lines are guides for the eyes. All standard errors are smaller than the symbols.

In Fig. 1(b) we display ψ, for scale-free networks. We see that, as with the random networks, decisions improve with both p and θ. Compared with random networks, the decisions for scale-free networks improve less, on average, from communication. Qualitatively however, the improvement from communication is similar to the case of random networks.

We note that the fluctuations of the success rate ϕ is larger for scale-free than random networks as can be seen from 1(c) and (d). These fluctuation, probably arising from the larger fluctuations in degree, is a possible cause for the comparatively low ψ-values of the scale-free networks (since, occasionally large hubs will initially be assigned $S = -1$ and influence its many neighbors towards an incorrect decision).

In Fig. 1(e) we display $\psi(p)$ for different average degrees $\langle k \rangle$ in random graphs. We see that ψ increases for with increasing k, that is, systems of agents with less restricted communication patterns will improve more from communications.

In Fig. 1(f) we display $\psi(p)$ for different system sizes. As we can see, not much of a difference appears with different system size. At most a smaller deviation is visible closer to the p values for which ψ approaches 1.

How does the position of a vertex in the network affect its ability to make correct decisions? To investigate this, we plot the average success of vertices as a function of their eccentricity (the maximal distance to any other vertex in the network) in Fig. 1(g) (for the random network model). A central (low eccentricity) vertex has a higher probability of making a correct decision than a more eccentric vertex. Since centrality measures are usually strongly correlated (15) this suggests that being central in the information flow, in general, is beneficial for the individual vertex. Furthermore, in Fig. 1(h) we plot ψ versus degree in a scale-free network. As degree can be regarded as a local centrality measure it is no surprise that these curves reveal a positive correlation.

B. Dynamic properties of the decision making

Being a non equilibrium model, a natural quantity to study is the average time for systems to reach the fixed state (where all have finalized their decisions). We call it the *finalizing time* τ. To obtain an intensive time scale (i.e. independent of the system size N), we let $t = t'/N$, where t' is the simulation time for the system. Since an agent has to update S at least θ times to finalize its decision, we see that τ cannot scale slower than linearly with θ, that is if $\tau \sim \theta^z$, we in all situations (all p) have $z \geq 1$. In Fig. 2(a) we display $\theta^{-1}\tau$ for random graphs as underlying topology. We observe a crossing point at p close to $p^* = 0.56$, below which $\theta^{-1}\tau$ is increasing with θ and for larger p, $\theta^{-1}\tau$ is decreasing with θ. Since $z \geq 1$ for all p we should, in the limit of large θ, have $z = 1$ for $p > p^*$. From Fig. 2(b) we observe for $p < p^*$, and for large θ, that $\theta^{-1}\tau$ seems to scale linearly with θ. In

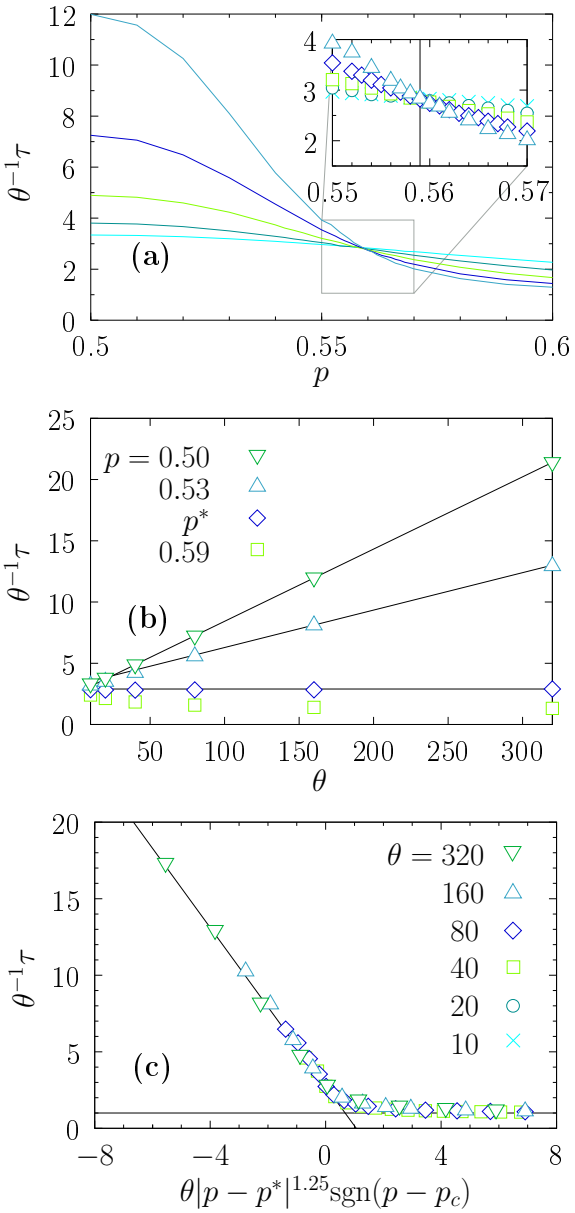

FIG. 2 Scaling analysis of τ in random graphs, where (a) is a crossing plot used to determine $p^* = 0.559 \pm 0.001$. The θ-values in (a) are, from top to bottom in the left side of the figure (or bottom to top in the right side): 160, 80, 40, 20, and 10 (as indicated by the legend in panel (c)). (b) shows τ's θ-scaling. Straight lines are fitted to the data points. (c) displays a scaling-collapse plot verifying $F(x)$'s asymptotic behavior using $\theta = 320$, 160, 80 and 40. For all networks $N = 4000$ and $\langle k \rangle = 5$. Standard errors are smaller than symbol sizes.

other words, for large θ and $p < p^*$, τ is proportional to θ^2. To connect the different behaviors above and below p^*, we assume a scaling function of the form

$$\tau = \theta\, F\Big(\theta|p - p^*|^{\nu}\mathrm{sgn}(p - p^*))\Big). \tag{5}$$

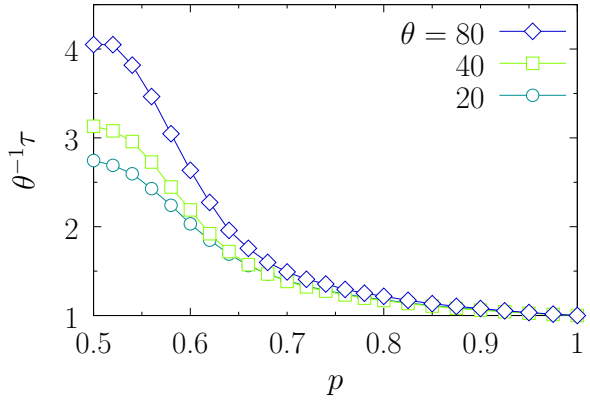

FIG. 3 The scaling of the finalization time for scale-free networks. In this figure, $N = 4000$, $\langle k \rangle = 5$ and $\gamma = 2.2$.

More specifically, to validate our scaling assumption $F(X)$ must be constant in the limit $X \to \infty$ (to verify $\tau \sim \theta$ above p^*), and $F(X) \sim X$ in the limit $X \to -\infty$ (to verify $\tau \sim \theta^2$ below p^*). In Fig. 2(c) we display a collapse to verify the scaling relation. The collapse is in agreement with our anticipated scaling behavior and thus demonstrates the asymptotic behavior of $F(X)$. To summarize, we have two distinct dynamical regions $\tau \sim \theta^z$ with the critical indices $z = 1$ and $z = 2$ that gives the following asymptotic scaling properties of the finalizing time for the system:

$$\tau \sim \begin{cases} \theta & \text{if } p \geq p^* \\ \theta^2 & \text{if } p < p^* \end{cases} . \tag{6}$$

In Fig. 3 we investigate the θ-dependence of τ for scale-free networks. Besides the trivial point at $p = 1$, we do not observe any crossing point of τ. This suggests that we do not have two distinct scaling regions for scale-free networks, rather z seems to be continuously varying, $z = z(p)$, with $1 = z(1) < z(p < 1) \leq p(1/2) \approx 1.4$. In comparison with random graphs the scaling of τ for scale-free networks is faster for $p < p^*$ but, in the large-θ limit, slower for $p > p^*$.

To get a closer view of the time evolution, we plot (in Fig. 4(a)) the average number of finalized agents, η_f, as function of time. We choose three p-values (0.53, 0.56 and 0.59)—over, at, and below the transition between the scaling regimes for random graphs. We plot the corresponding curves for scale-free networks in Fig. 4(b). The curves for $p = 0.53$, 0.56 and $p = 0.59$ are similar, as can be expected from Fig. 3. The initial increase of η_f is as fast for the curve of the scale-free networks as for the $p = 0.59$-curve of random networks, but the approach to $\eta_f = 1$ at later times slows down considerably. Thus, even if the time a whole population reaches finalization faster for random networks, the time an intermediate fraction of the population have finalized their decisions may be faster for the SF-networks.

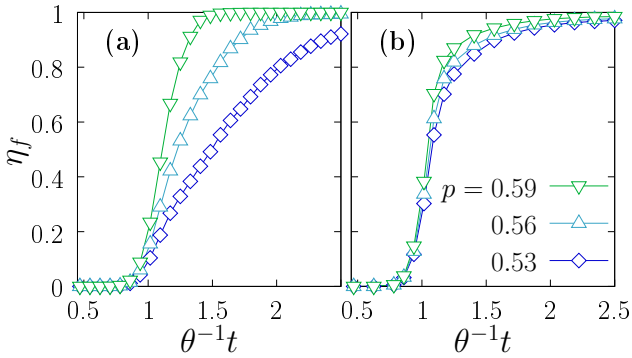

FIG. 4 The fraction of finalized vertices η_f over time in (a) random graphs, $N = 8000$, and (b) scale-free networks with $N = 4000$ and $\gamma = 2.2$. In all runs we have $\langle k \rangle = 5$ and $\theta = 80$. Lines are guides for the eyes. Standard errors are smaller than symbol sizes.

IV. SUMMARY AND DISCUSSION

We have proposed a dynamic model for collective decision processes. A decision is a choice of one of two values—the "correct" value +1 and the "incorrect" −1. We assume an individual's decision is affected by both directly obtained information (with some degree of helpful content) and information from social neighbors over social networks. We find that the community will benefit from social interaction whenever directly obtained information is the least helpful. We also observe that systems of agents with less restricted communication patterns will improve more from sharing information. Different global network topologies do not affect the decision ability quantitatively very much. In other studies, dynamical network systems—like spatial games (9; 10; 17), disease spreading (18; 19), congestion sensitive transport (20), and so on—are highly sensitive to the structure of the underlying networks. The local network structure makes a quantitative difference between the agents. An agent that is central in the information flow has a significantly higher chance of making the correct decision, than more peripheral agents.

Generally speaking, the population reach, on average, a decision that is more likely to be correct than a random guess. There might however be clusters of incorrect predictions, occasionally spanning a large fraction of the population. In psychology this result, that collective predictions may be irrational, has traditionally been explained by the "groupthink" concept of Irving Janis (13). In Janis's work the driving force is that the members of a tight-knit group try to conform to, what they believe, is the group's consensus, this creates an unstable situation that may lead to an illogical conclusion. (For modern works see Refs. (11; 21; 22) and further references therein.) In our model, in contrast, the agents do not actively try to avoid behaving aberrant; furthermore, the underlying network need not be a fully connected network; yet sometimes the majority of the population may predict falsely. In networks with broader degree distributions this tendency is stronger—the large fluctuation, gives large fluctuations in influence of the vertices and obscures the system's ability to integrate the information of a large fraction of the agents. To epitomize, the dynamics of social information diffusion may, in rare cases, be enough to misguide a population and this is another process that may lead to the same result as groupthink.

When we run our model on random graphs we find two distinct dynamic regions of the convergence of the finalizing time, $\tau \sim \theta^z$, with the critical indices $z = 1$ ($p > p^*$) and $z = 2$ ($p < p^*$). In contrast, for scale-free networks we find no distinct dynamic regions and the scaling is slower than $\tau \sim \theta$ for all values $p < 1$. In other words, if the average initial knowledge is high enough ($p > p^*$), the decision process on random graphs reach collective decisions faster than on scale-free graphs. If, on the other hand, the average knowledge is low ($p < p^*$) the decision making is faster on scale-free networks. We have investigated the limit $\theta \to \infty$ for parameter values such that the average distances d in the network is significantly less than θ. This means that all the vertices of the network, in principle, can influence all other vertices. A potentially different case, which we leave for future studies, is if $\theta \ll d$ as N grows. Since, in almost any random network, $d(N)$ is a very slowly growing function (logarithmic or, in the case of SF networks, slower still), it seems reasonable to assume that natural systems belong to the case we study.

Finally we note that there is a potential application of collective decision models to computer science. In the theory of distributed computing the *Byzantine agreement* problem is to coordinate concurrent processes where a number of the processes are faulty (23; 24). In our context that would be to make all vertices converge to the +1-prediction. We will not go into details about algorithms to solve the Byzantine agreement problem, but note that there is an area in common for mechanistic models of social processes and algorithmic computer science. Also algorithms for inference problems, such as belief propagation (25) or models of associative memory (26) have partly common ground with decision making models such as ours.

Acknowledgments

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