

On the energy of Hořava-Lifshitz black holes

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Abstract

In this paper we calculate the energy distribution of the Mu-in Park, Kehagias-Sfetsos (KS) and Lü, Mei and Pope (LMP) black holes in the Hořava-Lifshitz theory of gravity. These black hole solutions correspond to the standard Einstein-Hilbert action in the infrared limit. For our calculations we use the Einstein and Møller prescriptions. Various limiting and particular cases are also discussed.

1 Introduction

Gravitational energy localization is a very interesting topic of general relativity theory, which has been tackled by many researchers over the years. Regardless of the type of approach and of the mathematical methods employed, there has not been yet developed a generally accepted expression of the gravitational energy density. This is the problem that needs to be solved and we use this opportunity to enumerate several definitions able to settle it, from superenergy tensors [1], quasi local quantities [2], energy-momentum complexes [3]-[9] and up to teleparallel theory of gravitation (TEGR) [10]. The prescriptions of Einstein [3], Landau and Lifshitz [4], Bergmann-Thomson [5], Qadir-Sharif [6], Weinberg [7], Papapetrou [8] and Møller [9] have been successfully used by the pseudotensorial theory for the evaluation of energy-momentum of various gravitational backgrounds. A common characteristic of the first six definitions is the fact that quasi-Cartesian-coordinates need

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to be used to calculate the energy-momentum. As for Møller's prescriptions, studies may be conducted in any system of coordinates.

As we have noted the use of pseudotensorial definitions to calculate energy-momentum has led to very good results for several space-times. One should also bear in mind the results obtained for the $3+1$, $2+1$ and $1+1$ dimensional space-times [11]. In recent years, the similar results provided by the pseudotensorial method and the teleparallel theory of gravitation have also been of the highest importance [12].

As concerns Møller's prescription, its use for the calculation of the energy-momentum of a given gravitational background is supported both by Cooperstock's important assumption [13] and by Lessner's opinion [14]. Einstein's and Møller's prescriptions have had very good results in gravitational energy localization, which have also been supported by the numerous interesting works of the last few years [15] and by the quasi-local theory recently defined by Chang, Nester and Chen [16], which establishes a direct connection between quasi-local quantities and pseudotensors.

The remainder of our paper is structured as follows: in Section 2 we present the solutions given by Mu-In Park [17], Kehagias-Sfetsos (KS) [18] and Lü, Mei and Pope (LMP) [19] in the Hořava-Lifshitz theory, and that correspond to the case of the Einstein-Hilbert action in the IR limit. We also present the Einstein and Møller energy-momentum complexes and calculate the energy-momentum for the Mu-in Park, (KS) and (LMP) black hole solutions. In Discussion we point out some limiting and particular cases. For the calculations we choose the signature $(1, -1, -1, -1)$, the geometrized units ($c = 1; G = 1$) and consider that Greek (Latin) indices take value from 0 to 3 and 1 to 3.

2 Energy-Momentum of the Mu-In Park, Kehagias-Sfetsos (KS) and Lü, Mei and Pope (LMP) Black Hole Solutions

We start this section by presenting the three black hole solutions that we use for our calculations and, after this, the pseudotensorial definitions that we employed for performing the evaluation of the energy-momentum.

Recently Hořava has proposed a renormalizable gravity theory [20] with higher spatial derivatives in four dimensions. This theory may be regarded as a UV complete candidate for general relativity. The theory comes back to Einstein gravity with a non-vanishing cosmological constant in IR, but it has improved UV behaviors. The Hořava-Lifshitz theory has been considered very interesting and after its formulation many researchers found out new black hole solutions [17, [18, [19] and [21] (and the references there in) and a lot of work has been done in connection with the Hořava-Lifshitz theory [22].

Consider a static and spherically symmetric solution given by

$$ds^2 = e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where the functions $\nu(r)$ and $\lambda(r)$ are the metric potentials.

Now by imposing $\lambda_g = 1$, which reduces to the Einstein-Hilbert action in the infrared limit, the following solution of the vacuum field equations in Hořava gravity [17] is obtained:

$$e^{\nu(r)} = e^{-\lambda(r)} = 1 + (w - \Lambda_W)r^2 - \sqrt{r[w(w - 2\Lambda_W)r^3 + \beta]}. \quad (2)$$

Here β is an integration constant and λ_g , Λ_W and w are constant parameters. Now the Kehagias-Sfetsos (KS) [18] black hole solution is obtained by considering $\beta = 4wM$ and $\Lambda_W = 0$

$$e^{\nu(r)} = 1 + wr^2 - wr^2\sqrt{1 + \frac{4M}{wr^3}}. \quad (3)$$

By considering $\beta = -\frac{\alpha^2}{\Lambda_W}$ and $w = 0$ the solution given by Eq. (2) reduces to the Lü, Mei and Pope (LMP) [19] solution, given by

$$e^{\nu(r)} = 1 - \Lambda_W r^2 - \frac{\alpha}{\sqrt{-\Lambda_W}}\sqrt{r}. \quad (4)$$

In the following, we present the Einstein and Møller energy-momentum complexes.

Einstein's energy-momentum complex [3] in a four-dimensional space-time is given by:

$$\theta_\nu^\mu = \frac{1}{16\pi} h_{\nu,\lambda}^{\mu\lambda}. \quad (5)$$

The Einstein superpotential $h_\nu^{\mu\lambda}$ has the expression:

$$h_\nu^{\mu\lambda} = \frac{1}{\sqrt{-g}} g_{\nu\sigma} [-g(g^{\mu\sigma} g^{\lambda\kappa} - g^{\lambda\sigma} g^{\mu\kappa})]_{,\kappa} \quad (6)$$

and presents the antisymmetry property

$$h_\nu^{\mu\lambda} = -h_\nu^{\lambda\mu}. \quad (7)$$

θ_0^0 and θ_i^0 are the energy and momentum density components, respectively. The Einstein energy-momentum complex satisfies the local conservation law

$$\theta_{\nu,\mu}^\mu = 0. \quad (8)$$

The energy and momentum in Einstein's prescription are given by

$$P_\mu = \int \int \int \theta_\mu^0 dx^1 dx^2 dx^3 \quad (9)$$

and the energy of a physical system in a four-dimensional background is

$$E = \int \int \int \theta_0^0 dx^1 dx^2 dx^3. \quad (10)$$

In eq. (9) P_i , $i = 1, 2, 3$, are the momentum components. In the case of the Einstein definition the calculations are restricted to quasi-Cartesian coordinates. For performing the calculations we have to transform the metric given by (1) in Schwarzschild Cartesian coordinates, as given by

$$ds^2 = N dt^2 - (dx^2 + dy^2 + dz^2) - \frac{N^{-1} - 1}{r^2} (x dx + y dy + z dz)^2, \quad (11)$$

where N is employed by $e^{\nu(r)} = e^{-\lambda(r)}$ corresponding to the Mu-In Park, (KS) and (LMP) black hole solutions, respectively. We have $N = 1 + (w - \Lambda_W)r^2 - \sqrt{r[w(w - 2\Lambda_W)r^3 + \beta]}$ for the Mu-In Park black hole solution, $N = 1 + wr^2 - wr^2\sqrt{1 + \frac{4M}{wr^3}}$ for the Kehagias-Sfetsos (KS) gravitational background and $N = -\Lambda_W r^2 - \frac{\alpha}{\sqrt{-\Lambda_W}}\sqrt{r}$ for the Lü, Mei and Pope (LMP) black hole solution.

For the Mu-In Park gravitational background described by (2), with $N = 1 + (w - \Lambda_W)r^2 - \sqrt{r[w(w - 2\Lambda_W)r^3 + \beta]}$ and using (10) and (11) we obtain

$$E(r) = \frac{r}{2} \left[-(w - \Lambda_W)r^2 + \sqrt{r[w(w - 2\Lambda_W)r^3 + \beta]} \right]. \quad (12)$$

In the case of the (KS) black hole solution described by (3) and with $N = 1 + wr^2 - wr^2\sqrt{1 + \frac{4M}{wr^3}}$ we compute the energy distribution with the aid of (10) and (11) and we get

$$E(r) = \frac{r}{2} \left[wr^2 \left(-1 + \sqrt{1 + \frac{4M}{wr^3}} \right) \right]. \quad (13)$$

For the (LMP) black hole solution using $N = -\Lambda_W r^2 - \frac{\alpha}{\sqrt{-\Lambda_W}}\sqrt{r}$ (10) and (11) the energy distribution is given by

$$E(r) = \frac{r}{2} \left[\Lambda_W r^2 + \frac{\alpha}{\sqrt{-\Lambda_W}}\sqrt{r} \right]. \quad (14)$$

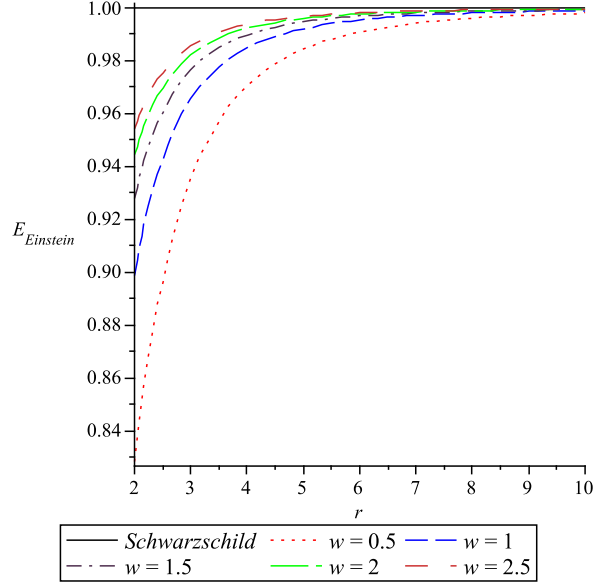


Figure 1: The plot for the Einstein energy E vs. r , in the case of the (KS) black hole solution, for various values of w and with $M = 1$.

In the case of these black hole solutions all the momenta are found to be zero.

The Møller energy-momentum complex is defined by

$$\mathcal{J}_\nu^\mu = \frac{1}{8\pi} M_{\nu, \lambda}^{\mu\lambda}, \quad (15)$$

with the Møller superpotential $M_\nu^{\mu\lambda}$ given by

$$M_\nu^{\mu\lambda} = \sqrt{-g} \left(\frac{\partial g_{\nu\sigma}}{\partial x^\kappa} - \frac{\partial g_{\nu\kappa}}{\partial x^\sigma} \right) g^{\mu\kappa} g^{\lambda\sigma}, \quad (16)$$

which presents the antisymmetric property

$$M_\nu^{\mu\lambda} = -M_\nu^{\lambda\mu}. \quad (17)$$

Møller's energy-momentum complex satisfies the local conservation law

$$\frac{\partial \mathcal{J}_\nu^\mu}{\partial x^\mu} = 0, \quad (18)$$

where \mathcal{J}_0^0 is the energy density and \mathcal{J}_i^0 are the momentum density components.

In the Møller prescription the energy and momentum are given by

$$P_\mu = \int \int \int \mathcal{J}_\mu^0 dx^1 dx^2 dx^3. \quad (19)$$

The energy distribution is

$$E = \int \int \int \mathcal{J}_0^0 dx^1 dx^2 dx^3. \quad (20)$$

Using the Gauss theorem and evaluating the integral over the surface of a sphere of radius r the expression for energy is given by

$$E = \frac{1}{8\pi} \oint_r M_0^{01} \sin \theta d\theta d\varphi. \quad (21)$$

The Møller definition allows to make the calculations in any coordinate system, and for our purpose we use the metrics given by (2), (3) and (4). For the Mu-in Park black hole solution the non-zero components of the Møller energy-momentum complex are

$$M_2^{21} = -2r \sin \theta (-1 - r^2 w + r^2 \Lambda_W + \sqrt{r[w(w - 2\Lambda_W)r^3 + \beta]}), \quad (22)$$

$$M_3^{31} = -2r \sin \theta (-1 - r^2 w + r^2 \Lambda_W + \sqrt{r[w(w - 2\Lambda_W)r^3 + \beta]}), \quad (23)$$

$$M_3^{32} = 2 \cos \theta, \quad (24)$$

$$M_0^{01} = -\frac{1}{2} \frac{r^2 \sin \theta [4r \sqrt{r[w(w - 2\Lambda_W)r^3 + \beta]} (\Lambda_W - w) + 4w(w - 2\Lambda_W)r^3 + \beta]}{\sqrt{r[w(w - 2\Lambda_W)r^3 + \beta]}}. \quad (25)$$

Using (21) and (25) we obtain the expression for the energy distribution of the Mu-In Park black hole solution that is given by

$$E(r) = \frac{r^2}{2} \left[2(w - \Lambda_W)r - \frac{1}{2} \frac{4w(w - 2\Lambda_W)r^3 + \beta}{\sqrt{r[w(w - 2\Lambda_W)r^3 + \beta]}} \right]. \quad (26)$$

In the case of the (KS) gravitational background the non-zero Møller superpotentials are

$$M_2^{21} = -2r \sin \theta \left[-1 - wr^2 \left(1 - \sqrt{1 + \frac{4M}{wr^3}} \right) \right], \quad (27)$$

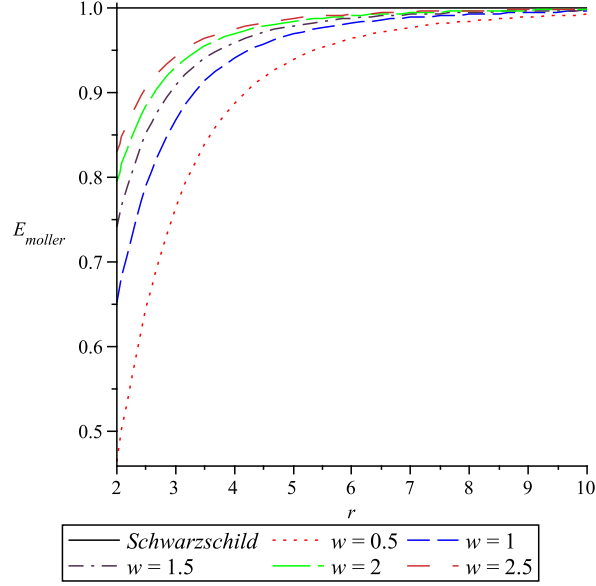


Figure 2: The plot for the Møller energy vs. r , in the case of the (KS) black hole solution, for various values of w and with $M = 1$.

$$M_3^{31} = -2r \sin \theta \left[-1 - wr^2 \left(1 - \sqrt{1 + \frac{4M}{wr^3}} \right) \right], \quad (28)$$

$$M_3^{32} = 2 \cos \theta, \quad (29)$$

$$M_0^{01} = 2 \sin \theta \frac{\left[wr^3 \left(-1 + \sqrt{1 + \frac{4M}{wr^3}} \right) - M \right]}{\sqrt{1 + \frac{4M}{wr^3}}}. \quad (30)$$

For the (KS) black hole solution given by (3) we insert (30) into (21) and we get

$$E(r) = \frac{\left[wr^3 \left(-1 + \sqrt{1 + \frac{4M}{wr^3}} \right) - M \right]}{\sqrt{1 + \frac{4M}{wr^3}}}. \quad (31)$$

The non-zero components of the Møller energy-momentum complex in the case of the (LMP) black hole solution are

$$M_2^{21} = -2 \frac{r \sin \theta (-\sqrt{-\Lambda_W} + \Lambda_W r^2 \sqrt{-\Lambda_W} + \alpha \sqrt{r})}{\sqrt{-\Lambda_W}}, \quad (32)$$

$$M_3^{31} = -2 \frac{r \sin \theta (-\sqrt{-\Lambda_W} + \Lambda_W r^2 \sqrt{-\Lambda_W} + \alpha \sqrt{r})}{\sqrt{-\Lambda_W}}, \quad (33)$$

$$M_3^{32} = 2 \cos \theta, \quad (34)$$

$$M_0^{01} = -\frac{1}{2} \frac{r^2 \sin \theta (4\Lambda_W r^{3/2} \sqrt{-\Lambda_W} + \alpha)}{\sqrt{-\Lambda_W} \sqrt{r}}. \quad (35)$$

In the case of the (LMP) black hole solution described by (4) using (35) and (21) we obtain the energy distribution

$$E(r) = \frac{r^2}{2} \left[-2\Lambda_W r - \frac{1}{2} \frac{\alpha}{\sqrt{r} \sqrt{-\Lambda_W}} \right]. \quad (36)$$

For the Mu-In Park, Kehagias-Sfetsos (KS) and Lü, Mei and Pope (LMP) black hole solutions all the momenta are found to be zero.

3 Discussion

In this paper we calculate the energy distribution of Mu-In Park, Kehagias-Sfetsos (KS) and Lü, Mei and Pope (LMP) black holes in the Einstein and Møller prescriptions. For the gravitational background described by the Mu-in Park metric we found that the energy distribution depends on the w , Λ_W parameters and β both in the Einstein and Møller prescriptions. For the space-time given by the Kehagias-Sfetsos (KS) metric the energy depends on w and the mass M . In the case of the Lü, Mei and Pope (LMP) black hole solution the energy distribution presents a dependence in function of the Λ_W parameter and α . In both prescriptions for these gravitational backgrounds all the momenta are zero.

We present some particular and limiting cases.

For the Mu-In Park black hole solution a particular case is obtained for $r \gg [\beta/w(w - 2\Lambda_W)]^{1/3}$ when N has a new expression $N = 1 + \frac{\Lambda_W^2}{2w} r^2 - \frac{\beta}{2\sqrt{w(w-2\Lambda_W)}} \frac{1}{r} + O(r^{-4})$. This

condition combined with $\Lambda_W = 0$ and $\beta = 4wM$ independently of w leads to the usual behaviour of a Schwarzschild black hole solution. Using (11) with the new expression for N and (10) the energy distribution in the Einstein prescription is $E = M - O(r^{-3})$. The Møller definition gives for the energy distribution the expression $E = M - O(r^{-3})$. For large r both prescriptions yield the same result $E = M$, which is also equal to the ADM mass. These are expected results in the context of general relativity (at large distances the standard general relativity is recovered).

In the case of the (KS) gravitational background for $r \gg (M/\omega)^{1/3}$ the usual behavior of a Schwarzschild black hole is obtained. For large r the Einstein and Møller definitions yield for the energy distribution the expression $E = M$.

In the following, we study the case $r \rightarrow \infty$ for the Mu-In Park, Kehagias-Sfetsos (KS) and Lü, Mei and Pope (LMP) gravitational backgrounds. The results for the Einstein prescription are presented in Table 1.

The Einstein prescription	Energy	r
Mu-In Park	$signum(-\frac{1}{2}w + \frac{1}{2}\Lambda_W + \frac{1}{2}\sqrt{w^2 - 2w\Lambda_W}) \infty$	$r \rightarrow \infty$
Kehagias-Sfetsos (KS)	M	$r \rightarrow \infty$
Lü, Mei and Pope (LMP)	$signum(\Lambda_W) \infty$	$r \rightarrow \infty$

Table 1

The limiting cases for large r in the case of the Møller prescription are given in Table 2.

The Møller prescription	Energy	r
Mu-In Park	$-signum(\frac{-\sqrt{w^2 - 2w\Lambda_W}(w - \Lambda_W) - 2w\Lambda_W + w^2}{\sqrt{w^2 - 2w\Lambda_W}}) \infty$	$r \rightarrow \infty$
Kehagias-Sfetsos (KS)	M	$r \rightarrow \infty$
Lü, Mei and Pope (LMP)	$-signum(\Lambda_W) \infty$	$r \rightarrow \infty$

Table 2

In Fig. 3 and Fig. 4 are plotted the energy distributions in the Einstein and Møller's prescription for the (KS) black hole solution in two cases. In Fig. 3 we have the plot of

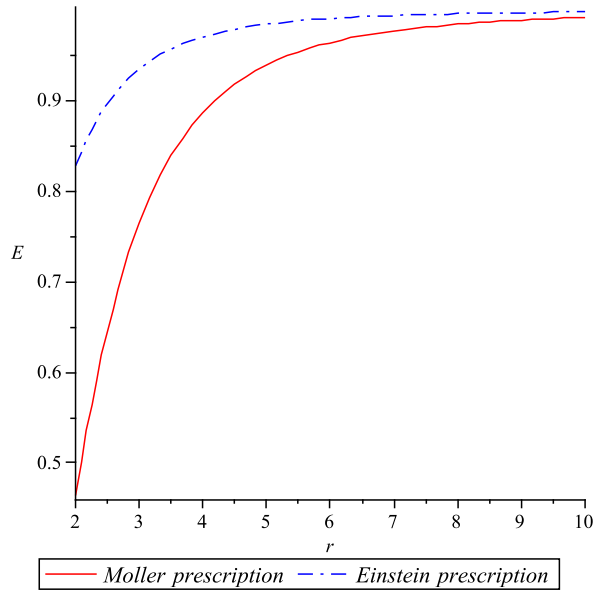


Figure 3: Energy distributions in the Einstein and Møller's prescription vs. r for small values of w like $w = 0.5$ and with $M = 1$.

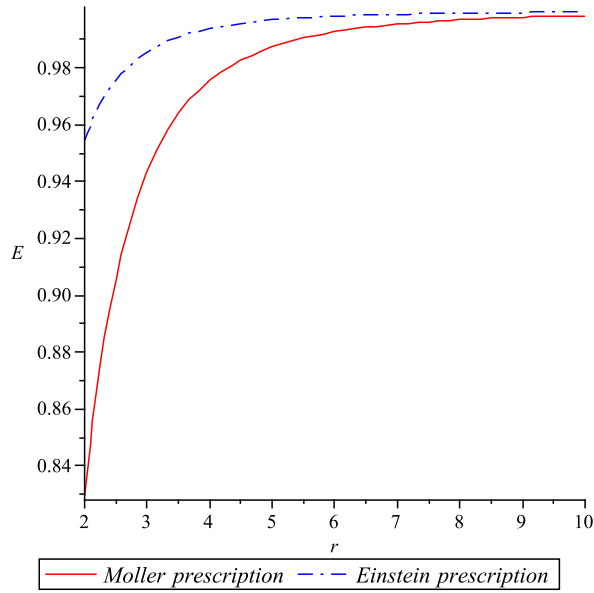


Figure 4: Energy distributions in the Einstein and Møller's prescription vs. r for large values of w like $w = 2.5$ and with $M = 1$.

the energy distributions vs. r for small values of w like $w = .5$ and with $M = 1$. In Fig. 4 we present the plot of the energy distributions vs. r for large value of w like $w = 2.5$ and with $M = 1$.

An interesting conclusion is that for small values of r , the amount of energy can be much smaller for a Kehagias-Sfetsos (KS) black hole obtained by using both Einstein and Møller's prescriptions than for a Schwarzschild black hole. However, for large r both the Einstein and Møller prescriptions give the same expression for energy distribution $E = M$. In the case of general relativity this represents the ADM mass. This result also confirms the fact that at large distances the Hořava-Lifshitz theory leads to the Einstein gravity, in the context of a particular coupling $\lambda_g = 1$ and both the Einstein and Møller prescriptions yield the same expression for energy, as is expected in this case. One can also note that in the case of small values of r the energy of Kehagias-Sfetsos (KS) black hole solution obtained by using Einstein's prescription is greater than the energy distribution obtained by using Møller's prescription. This difference is decreasing with the increasing of the values of w .

One can note that for $r \rightarrow 0$ the energy $E \rightarrow 0$. But in realistic situation, one should consider the value of $r > r_h$ (horizon).

The black hole in Hořava-Lifshitz gravity in Mu-In Park gravitational background is more general than KS and LPM solutions. The solution contains a number of parameters and for specific choices of the parameters, one could come back to KS and LPM solutions. Also, one can see that for large w , the KS case comes back to the Schwarzschild case (Einstein gravity). Also, we have shown a comparison between the two gravitational theories, based mostly on the plots.

The results that we obtained for the energy in the case of the Hořava-Lifshitz black hole space-times demonstrate that the pseudotensorial prescriptions are useful concepts for the evaluation of the energy and momentum.

References

- [1] L. Bel, C. R. Acad. Sci. Paris **246**, 3105 (1958); L. Bel, C. R. Acad. Sci., Paris, **247**, 1094 (1958); L. Bel, C. R. Acad. Sci. Paris **248**, 1297 (1959); I. Robinson, unpublished lectures; I. Robinson, Kings College, London (1958); I. Robinson, Class. Quant Grav. **14**, 4331 (1997); R. Debever, C. R. Acad. Sci., Paris **249**, 1324 (1959); R. K. Sachs, Zeitschrift fur Physik, **157**, 462 (1960); C. D. Collinson, Proc. Camb. Philo. Soc. **58**, 346 (1962); S. Deser, J. S. Franklin and D. Seminara, Class. Quant Grav. **16**, 2815 (1999); S. Deser and D. Seminara, Phys. Rev. **D62**, 084010 (2000); M. ´ A. G. Bonilla and J. M. M. Senovilla, Gen. Rel. Grav. **29**, 91 (1997); J. M. M. Senovilla, 1999 "Remarks on superenergy tensors" Gravitation and Relativity in General, Proc. of the Spanish Relativity Meeting in Honour of the 65th Birthday

- of L. Bel, ERE98 (Salamanca) ed A. Molina et al (Singapore: World Scientific) pp. 175–182; J. M. M. Senovilla, *Class. Quantum Grav.* **17**, 2799 (2000); J. M. Pozo and J. M. Parra, *Class. Quantum Grav.* **19**, 967 (2002); X. Jaen, A. Balfagon, *Class. Quant. Grav.* **17**, 2491 (2000).
- [2] J. D. Brown and J. W. York, *Quasilocal energy in general relativity*, Mathematical aspects of classical field theory (Seattle, WA, 1991), pp. 129-142, *Contemp. Math.*, 132, Amer. Math. Soc., Providence, RI, 1992; J. D. Brown and J. W. York, *Phys. Rev.* **D47**, 1407 (1993); S. W. Hawking and G. T. Horowitz, *Class. Quantum Grav.* **13**, 1487 (1996); C-C.M. Liu and S. T. Yau, *Phys. Rev. Lett.* **90**, 231102 (2003); Sean A. Hayward, *Phys. Rev.* **D49**, 831 (1994); G. Bergqvist, *Class. Quantum Grav.* **9**, 1753 (1992); R. Penrose, *Proc. R. Soc. London*, **A381**, 53 (1982); K. P. Tod, *Proc. R. Soc. London*, **A388**, 457 (1983); A. Komar, *Phys. Rev.* **113**, 934 (1959); Chiang-Mei Chen, James M. Nester, Roh Suan Tung, *Phys. Lett.* **A203**, 5 (1995); C. M. Chen, J. M. Nester, *Class. Quant. Grav.* **16**, 1279 (1999).
- [3] A. Einstein, *Preuss. Akad. Wiss. Berlin* **47**, 778 (1915); Addendum-*ibid.* **47**, 799 (1915); A. Trautman, in *Gravitation: an Introduction to Current Research*, ed. L. Witten (Wiley, New York, 1962, p. 169).
- [4] L. D. Landau and E.M. Lifshitz, *The Classical Theory of Fields* (Pergamon Press, 1987, p. 280).
- [5] P. G. Bergmann and R. Thompson, *Phys. Rev.* **89**, 400 (1953).
- [6] A. Qadir and M. Sharif, *Phys. Lett.* **A 167**, 331 (1992).
- [7] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of General Theory of Relativity* (John Wiley and Sons, Inc., New York, 1972, p. 165).
- [8] A. Papapetrou, *Proc. R. Irish. Acad.* **A 52**, 11 (1948).
- [9] C. Møller, *Ann. Phys. (NY)* **4**, 347 (1958).
- [10] T. Shirafuji, G. G. L. Nashed and K. Hayashi, *Prog. Theor. Phys.* **95**, 665 (1996); T. Shirafuji and Gamal G. L. Nashed, *Prog. Theor. Phys.* **98** (1997); Gamal G. L. Nashed, *Phys. Rev.* **D66**, 064015 (2002); Gamal G. L. Nashed, *Mod. Phys. Lett.* **A22**, 1047 (2007); Gamal G. L. Nashed and T. Shirafuji, *Int. J. Mod. Phys.* **D16**, 65 (2007); Gamal G. L. Nashed, *Eur. Phys. J.* **C49**, 851 (2007); Gamal G. L. Nashed, *Chin. Phys. Lett.* **25**, 1202 (2008); J. W. Maluf, *J. Math. Phys.* **36**, 4242 (1995); J. W. Maluf and A. Kneip, *J. Math. Phys.* **38**, 458 (1997); J. W. Maluf, E. F. Martins and A. Kneip; *J. Math. Phys.* **37**, 6302 (1996); J. W. Maluf, *J. Math. Phys.* **37**, 6293 (1996); J. W. Maluf, J. F. da Rocha-Neto, *J. Math. Phys.* **40**, 1490 (1999); J. W. Maluf, F. F. Faria and S. C. Ulhoa, *Class. Quant. Grav.* **24**, 2743 (2007); J. W. Maluf, M. V. O. Veiga and J. F. da Rocha-Neto, *Gen. Rel. Grav.* **39**, 227 (2007); A. A. Sousa, R. B. Pereira, A. C. Silva, *Grav. Cosmol.*, **16**, 25 (2010); M. Sharif, Sumaira Taj, *Astrophys. Space Sci.*, **325**, 75 (2010); J. M. Nester, Lau Loi So and T. Vargas, *Phys. Rev.* **D78**, 044035 (2008).

- [11] K. S. Virbhadra, Phys. Rev. **D41**, 1086 (1990); K. S. Virbhadra, Phys. Rev. **D42**, 2919 (1990); N. Rosen and K.S. Virbhadra, Gen. Rel. Grav. **25**, 429 (1993); K. S. Virbhadra and J. C. Parikh, Phys. Lett. **B331**, 302 (1994); J. M. Aguirregabiria, A. Chamorro and K. S. Virbhadra, Gen. Rel. Grav. **28**, 1393 (1996); S. S. Xulu, Int. J. Theor. Phys. **37**, 1773 (1998); S. S. Xulu, Int. J. Mod. Phys. **D7**, 773 (1998); I. Radinschi, Acta Physica Slovaca, **49(5)**, 789 (1999); I. Radinschi, Mod. Phys. Lett. **A15**, Nos. 11&12, 803 (2000); I-Ching Yang and I. Radinschi, Chin. J. Phys. **41**, 326 (2003); I. Radinschi and Th. Grammenos, Int. J. Mod. Phys. **A21**, 2853 (2006); Ragab M. Gad, Gen. Rel. Grav. **38**, 417 (2006); Th. Grammenos and I. Radinschi, Int. J. Theor. Phys. **46(4)**, 1055 (2007); T. Bringley, Mod. Phys. Lett. **A17**, 157 (2002); M. Sukenik and J. Sima, arXiv:grqc/0101026; M. Sharif, Nuovo Cim. **B19**, 463 (2004); P. Halpern, Astrophys. Space. Sci. **306**, 279 (2006); E. C. Vagenas, Int. J. Mod. Phys. **D14**, 573 (2005); E. C. Vagenas, Int. J. Mod. Phys. **A18**, 5781 (2003); Th. Grammenos, Mod. Phys. Lett., **A20**, 1741 (2005); T. Multamaki, A. Putaja, E. C. Vagenas and I. Vilja, Class. Quant. Grav. **25**, 075017 (2008).
- [12] Gamal G. L. Nashed, Nuovo Cim. **117B**, 521 (2002); Gamal G. L. Nashed, Phys. Rev. **D66**, 064015 (2002); Gamal G. L. Nashed, Int. J. Mod. Phys. Lett. **A21**, 3181 (2006); Gamal G. L. Nashed, Nuovo Cim. **B119**, 967 (2004); Gamal G. L. Nashed, Mod. Phys. Lett. **A22**, 1047 (2007); Gamal G. L. Nashed, Chin. Phys. Lett. **25**, 1202 (2008); Gamal G. L. Nashed, Int. J. Mod. Phys. **A23**, 1903 (2008).
- [13] F.I. Cooperstock, Mod. Phys. Lett. **A14**, 1531 (1999).
- [14] G. Lessner, Gen. Relativ. Gravit. **28**, 527 (1996).
- [15] S. S. Xulu, Mod. Phys. Lett. **A15**, 1511 (2000); S. S. Xulu, PhD Thesis, arXiv:hep-th/0308070; S. S. Xulu, Astrophys. Space. Sci. **283**, 23 (2003); I-Ching Yang, Ching-Tzung Yeh, Rue-Ron Hsu and Chin-Rong Lee, Int. J. Mod. Phys. **D6**, 349 (1997); I-Ching Yang, Wei-Fui Lin and Rue-Ron Hsu, Chin. J. Phys. **37**, 118 (1999); E. C. Vagenas, Int. J. Mod. Phys. **A18**, 5949 (2003); E. C. Vagenas, Int. J. Mod. Phys. **A18**, 5781 (2003); E. C. Vagenas, Mod. Phys. Lett. **A19**, 213 (2004); E. C. Vagenas, Mod. Phys. Lett. **A21**, 1947 (2006); Th. Grammenos, Mod. Phys. Lett. **A20**, 1741 (2005); I. Radinschi, Mod. Phys. Lett. **A15(35)**, 2171 (2000); I. Radinschi, Fizika **B9(4)**, 43 (2000); I. Radinschi, Chin. J. Phys. **39(3)**, 231 (2001); I-Ching Yang and I. Radinschi, Chin. J. Phys. **41**, 326 (2003); I-Ching Yang, Chi-Long Lin and I. Radinschi, Int. J. Theor. Phys. **48(1)**, 248 (2009); I-Ching Yang, Chi-Long Lin and I. Radinschi, Int. J. Theor. Phys. **48(8)**, 2454 (2009); I-Ching Yang, Chin. J. Phys. **45(5)**, 497 (2007); M. Sharif and Tasnim Fatima, Int. J. Mod. Phys. **A20**, 4309 (2005); M. Sharif, Nuovo Cim. **B119**, 463 (2004); M. Sharif, Int. J. Mod. Phys. **D13**, 1019 (2004); M. Sharif and Tasnim Fatima, Nuovo Cim. **B120**, 533 (2005); M. Sharif and Tasnim Fatima, Astrophys. Space Sci. **302**, 217 (2006); M. Sharif, M. Azam, Int. J. Mod. Phys. **A22**, 1935 (2007); Ragab M. Gad, Mod. Phys. Lett. **A19**, 1847 (2004); Ragab M. Gad, Astrophys. Space Sci. **295**, 451 (2005); Ragab M. Gad, Astrophys. Space Sci. **293**, 453 (2004); Ragab M. Gad, Astrophys. Space Sci. **295**, 459 (2005); Ragab M. Gad, Astrophys. Space Sci. **302**, 141 (2006);

- Ragab M. Gad, *Int. J. Theor.Phys.* **46**, 3263 (2007); Ragab M. Gad, *Astrophys. Space Sci.* **310**, 135 (2007); M. Abdel-Megied, Ragab M. Gad, *Adv. High Energy Phys.* **2010**, 379473 (2010); I. Radinschi and Th. Grammenos, *Int. J. Mod. Phys.* **A21**, 2853 (2006); Th. Grammenos and I. Radinschi, *Int. J. Theor. Phys.* **46**, 1055 (2007); Saeed Mirshekari, Amir M. Abbassi, *Int. J. Mod. Phys.* **A24**, 789 (2009); Saeed Mirshekari, Amir M. Abbassi, *Mod. Phys. Lett.* **A24**, 747 (2009); Amir M. Abbassi, Saeed Mirshekari, Amir H. Abbassi, *Phys. Rev.* **D78**, 064053 (2008); I. Radinschi and Th. Grammenos, *Int. J. Theor. Phys.* **47**, 1363 (2008); I. Radinschi, F. Rahaman and A. Ghosh, *Int. J. Theor. Phys.* **49**, 943 (2010), arXiv:1003.2926 [gr-qc] ; J. Matyjasek, *Mod. Phys. Lett.* **A23(8)**, 591 (2008); K. S. Virbhadra, *Phys. Rev.* **D60**, 104041 (1999).
- [16] Chia-Chen Chang, J. M. Nester and Chiang-Mei Chen, *Phys. Rev. Lett.* **83**, 1897 (1999).
- [17] Mu-In Park, *JHEP* **0909**, 123 (2009).
- [18] A. Kehagias and K. Sfetsos, *Phys. Lett.* **B678**, 123 (2009).
- [19] H. Lu, J. Mei and C. N. Pope, *Phys. Rev. Lett.*, **103**, 091301 (2009).
- [20] P. Hořava, *Phys. Lett.* **B694**, 172 (2010); P. Hořava, *JHEP* **0903**, 020 (2009); P. Hořava, *Phys. Rev.* **D79**, 084008 (2009); P. Hořava, *Phys. Rev. Lett.* **102**, 161301 (2009).
- [21] T. Harko, Z. Kovács, Francisco S. N. Lobo, arXiv:0908.2874
- [22] Yun Soo Myung, *Phys. Lett.* **B685**, 318 (2010); Hyung Won Lee, Yong-Wan Kim, Yun Soo Myung, arXiv:1008.2243; R. A. Konoplya, *Phys. Lett.* **B679**, 499 (2009); G. Koutsoumbas, E. Papantonopoulos, P. Pasipoularides, M. Tsoukalas, arXiv:1004.2289; G. Koutsoumbas, P. Pasipoularides, arXiv:1006.3199; G. Leon, E. N. Saridakis, *JCAP*, 0911, **006** (2009); M. Jamil, E. N. Saridakis, M. R. Setare, *JCAP* **11**, 032 (2010); M. Jamil, E. N. Saridakis, *JCAP* **1007**, 028 (2010); A. Ghodsi, E. Hatefi, arXiv:0906.1237