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Estimation of process capability indices in case of distribution unlike the normal one

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ABSTRACT

Purpose: The aim of this paper is to show how it could utilize the statistical methods for the process management.

Design/methodology/approach: The research methodology base on theoretical analysis and on empirical researches. It is presented a practical solution for estimation of process capability indices In case non-normal distribution.

Findings: The Clements' method brings sufficiently correct results for the analyzed case.

Research limitations/implications: The future research will contain analysis for other non-normal distributions.

Practical implications: Described methodology and results can be employed in industrial practice.

Originality/value: Comparative analysis of capability indices determination in case of distribution unlike the normal one.

Keywords: Quality management; Statistical methods; Process capability; Non-normal distribution

METHODOLOGY OF RESEARCH, ANALYSIS AND MODELLING

1. Introduction

Quality is a challenge that must be taken up by producers. A formal requirement of quality are among others ISO 9000 standards and specific branch requirements e.g. ISO/TS 16949 standard relating to the automotive industry. The verified and efficient method of achievement of demanded quality (within time shorter than competitors!) is the organizational culture Six Sigma.

There are plenty of different verified quality tools such as: team work methods (e.g. brainstorming, quality circles), problem solving methods (e.g. flow diagram, decisions diagram, cause-and-effect diagram) and finally statistical techniques [1-6].

In practical approach statistical methods in quality management are first of all: statistical process control (SPC), measurement system analysis (MSA),statistical acceptance plans and statistical methods in process improvement (ANOVA, DOE etc.) [7-11].

Among the statistical methods mentioned above nowadays the most significant place takes the statistical process control (SPC).

One of the fundamental tasks of SPC is the assessment of capability of a process/ machine relating to client's expectations. In this scope many different capability indices are applied in practice, e.g. Cp, Cpk, Pp, Ppk, Cm, Cmk, Tp, Tpk [12-15].

2. Capability process estimation – normal distribution

We are analyzing the performance of a process from the point of view of a determined technological parameter or the quality-determining parameter of a product/ half-finished product. The behavior of the considered parameter from the variability point of view (as a random variable) we describe by means of the following statistical parameters:

$\bar{x}$ - mean value; position measure,

s - standard deviation; dispersion measure.

In SPC, the 6s scope is defined as a natural tolerance of the process T_n [12, 14, 15]:

$$T_n = 6s \quad (1)$$

Formulating this the most generally, estimation of a process capability boils to a comparison of the process variability (it is so-called "process voice") with client's expectations defined through specification limits. The expression of this comparison is a series of mentioned capability indices, among others capability indices of the first generation Cp, Cpk (Tab. 1).

The calculation formulae presented in the Table 1 are right when the analyzed parameter is subject to a normal distribution or its distribution is close to the normal one. In such situations there is obligatory the rule of three standard deviations according to which within the range $\bar{x}$ +/- 3s (i.e. within the range determined by a natural tolerance T_n (1)) all possible realizations of the process should be contained (Fig. 1).

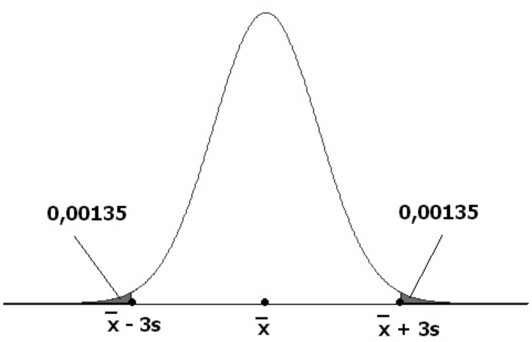

Fig. 1. Normal distribution; the rule of three standard deviations

3. Capability process estimation – distribution unlike the normal one researches

In case of distributions unlike the normal one the definition (1) is not valid; in this case a natural tolerance we define generally [16-18]:

$T_n = x_{0.99865} - x_{0.00135}$ (2)

where: $x_{0.00135}$ – value meeting the condition:

$P(x < x_{0.00135}) = 0.00135$

$x_{0.99865}$ – value meeting the condition

$P(x < x_{0.99865}) = 0.99865$

(i.e. $P(x <= x_{0.99865}) = 0.00135$), Fig. 2.

Let's give attention that the definition (2) guarantees exactly the same as the definition (1) in case of a normal distribution, i.e. within the range determined by a natural tolerance practically all possible realizations of the process should be contained, and participation of the realization outside the range determined by natural tolerance limits equals 0.0027 (i.e. 2700 ppm).

Additionally, let $x_{0.5}$ means the median ($P(x < x_{0.5}) = 0.5$), which, in case of an asymmetric distribution, will be a measure of midpoint of grouping of values (of course, in case of symmetric distributions e.g. a normal distribution there is the equality $x_{0.5}$=. $\bar{x}$).

Based on the values defined above, i.e. $x_{0.00135}$, $x_{0.5}$, $x_{0.99865}$ process capability indices Cp, Cpk we define as below (Tab. 2).

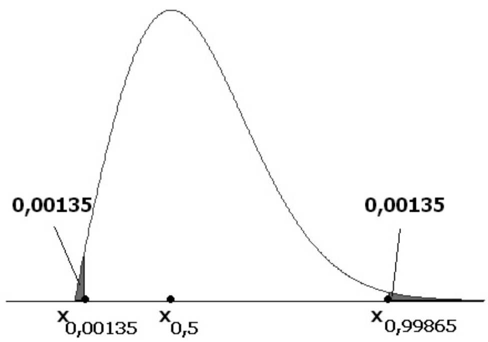

Fig. 2. Non-normal distribution; definition of natural tolerance

Let's give attention once again that in case of a normal distribution the expressions presented in Tables 1 and 2 lead to the same result because: $x_{0.00135} = \bar{x} - 3s$ and $x_{0.99865} = \bar{x} + 3s$. Therefore the expressions given in Table 2 we can treat as a generalized rule of determination of Cp, Cpk indices, apart from a distribution shape.

In practice, estimation of Cp, Cpk indices in case of distributions unlike the normal one boils to determination of three values $x_{0.00135}$, $x_{0.5}$, $x_{0.99865}$ and making use of adequate expressions placed in Table 2.

There are two methods of determination of the values $x_{0.00135}$, $x_{0.5}$, $x_{0.99865}$ [16-18]:

- Clements' method; it is an approximate method basing on values of shape parameters i.e. kurtosis and skewness; in this method there is no need to know the form of distribution of the analyzed parameter.
- Exact method requiring knowledge of a density function f(x) determining a distribution of the analyzed parameter; in this case the values $x_{0.00135}$, $x_{0.5}$, $x_{0.99865}$ we determine from the relationships:

$$\int_{-\infty}^{x_{0.00135}} f(x)dx = 0.00135 \quad (3)$$

$$\int_{-\infty}^{x_{0.5}} f(x)dx = 0.5 \quad (4)$$

$$\int_{-\infty}^{x_{0.99865}} f(x)dx = 0.99865 \quad (5)$$

4. Experimental procedure

The aim of investigations has been the estimation of Cp, Cpk capability indices. The analyzed parameter has been an absolute value of deviation from a nominal Δ for a thickness of the hot-rolled steel (DD12) strip (Δ = |g-T|, g – thickness in a given place, T – nominal value; T = 3 mm). Measurements have been performed with the aid of a X-ray thickness gauge. In case of the analyzed parameter the upper specification limit has been defined USL; USL= 0,018 mm. Data for tests came from the laboratory of tests of mechanical and geometrical properties.

Table 1.
Capability indices of the first generation – calculation formulae

	potential capability Cp		real capability Cpk	
double-sides specification limits	$\frac{USL - LSL}{6 \cdot s}$	(6)	$\min\left(\frac{\bar{x} - LSL}{3 \cdot s}; \frac{USL - \bar{x}}{3 \cdot s}\right)$	(7)
lower specification limit only	N / A		$\frac{\bar{x} - LSL}{3 \cdot s}$	(8)
upper specification limit only	N / A		$\frac{USL - \bar{x}}{3 \cdot s}$	(9)

Symbols: USL – upper specification limit LSL – lower specification limit N / A – not available

Table 2.
Capability indices for non-normal distribution – calculation formulae

	potential capability Cp		real capability Cpk	
double-sides specification limits	$\frac{USL - LSL}{x_{0.99865} - x_{0.00135}}$	(10)	$\min\left(\frac{x_{0.5} - LSL}{x_{0.5} - x_{0.00135}}; \frac{USL - x_{0.5}}{x_{0.99865} - x_{0.5}}\right)$	(11)
lower specification limit only	N / A		$\frac{x_{0.5} - LSL}{x_{0.5} - x_{0.00135}}$	(12)
upper specification limit only	N / A		$\frac{USL - x_{0.5}}{x_{0.99865} - x_{0.5}}$	(13)

Symbols: USL – upper specification limit LSL – lower specification limit N / A – not available

Table 3.
Statistical characteristic of data

Parameters	n	$\bar{x}$	s	x_{min}, x_{max}	Sk	Ku
Δ	180	0.0326	0.0258	0.001; 0.148	1.40	3.12

(n – simple size, $\bar{x}$ – mean value, s – standard deviation, x_{min}, x_{max} - min, max value respectively, Sk – skewness, Ku – kurtosis)

The results of the preliminary analysis (the values of shape parameters i.e. kurtosis and skewness (Tab. 3), the empirical distribution (Fig. 3) and especially the graphical test of normality (Fig. 4) indicate that the analyzed parameter is not subject to a normal distribution.

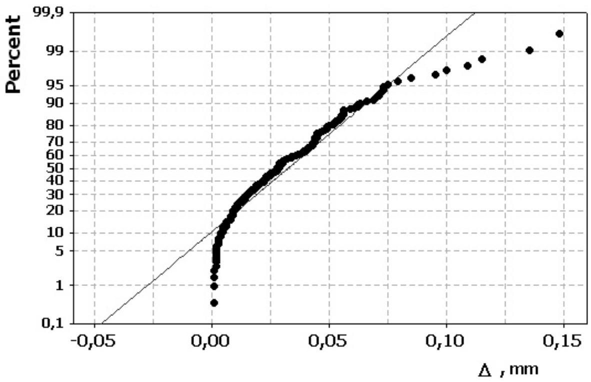

Fig. 3. Parameter Δ – normal probability plot

In the meaning of the Anderson-Darling test (p-value = 0,131) [12] there is no basis for rejection of the hypothesis that it is the Weibull's distribution with the following parameters: shape parameter α = 1.24, scale parameter β = 0.034.

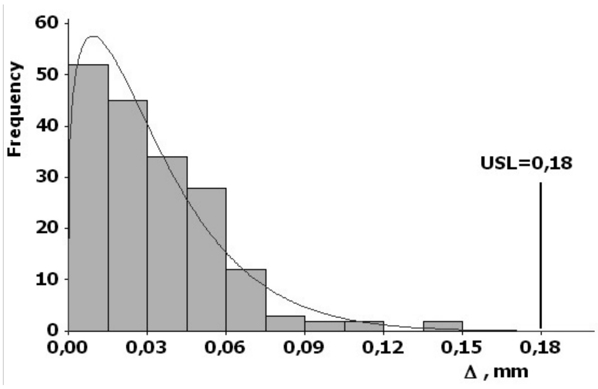

Fig. 4. Parameter Δ – empirical distribution with fit Weibull theoretical distribution (shape α = 1.24, scale β = 0.034)

In connection with it Cpk capability indice have been determined according to adequate expression presented in Table 2 (Eq. 13). To determine the values $x_{0.00135}$, $x_{0.5}$, $x_{0.99865}$ there are used both the Clements' method and the Exact method basing on knowledge of density function. The results are shown in Table 4.

Table 4.
Results – capability indices

	$x_{0.5}$	$x_{0.99865}$	Cpk
Clements' Method	0.0270	0.1593	1.150
Exact Method	0.0260	0.1605	1.149

5. Discussion of results

Two applied methods of determination of the values $x_{0.00135}$, $x_{0.5}$, $x_{0.99865}$ and Cpk index give very close results. Therefore in case of the analyzed distribution the approximate and easier-counting Clements' method brings sufficiently correct results. The value of Cpk index achieved in analysis is not unfortunately an evidence of meeting the client's expectations (the required minimal value of Cpk index determined by the client was 1.33).

6. Summary

The estimation of process capability is one of the basic tasks of the statistical process control (SPC). The values of Cp, Cpk indices are very precise information on a process potential relating to the client's expectations.

Correct determination of Cp, Cpk indices values by counting requires identification of a distribution shape, at least as a general settlement whether it is a normal distribution or not. If it is a normal distribution, for the estimation of Cp, Cpk we can use a simple-counting classic approach that is based on the rule of three standard deviations. If it is not a normal distribution, the application of a classic approach leads to wrong results. In such a case we should use either the Clements' method or the more difficult by counting Exact method basing on the density function describing the distribution.

It is worth adding that there are also, but used not so often, other methods of estimation of Cp, Cpk capability indices in case of distributions unlike the normal one, e.g. method of mirror reflection [16].

Statistical process control methods (SPC) and especially estimation of a process capability show opportunities of practical application of statistics in aspect of the analysis of technological processes. It justifies among others the need or even necessity of education of mathematical statistics methods in technical faculties in universities.

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