

# Flavour Singlet Mesons in full QCD on the Lattice

K. Schilling<sup>a</sup>, H. Neff<sup>a\*</sup>, N. Eicker<sup>a†</sup>, Th. Lippert<sup>a</sup>, J.W. Negele<sup>b</sup>

<sup>a</sup>Fachbereich Physik, Bergische Universität, D-42097 Wuppertal, Germany

<sup>b</sup>Center for Theoretical Physics, MIT, 77 Massachusetts Ave, Cambridge, MA 02139, USA

We apply spectral methods to compute the OZI-rule suppressed loop-loop correlators in the pseudoscalar meson flavour singlet channel. Using SESAM configurations (obtained with two degenerate sea quark flavours on  $16^3 \times 32$  lattices at  $\beta = 5.6$ , with standard Wilson action), we find for the first time clear evidence for mass plateau formation in the  $\eta'$  channel of this theory. As a consequence, we observe a clear signal of a mass gap persistent under chiral extrapolation. This sets the stage for a more realistic two-channel approach, where partially quenched strange quarks would be included, in addition to the  $u, d$  sea quarks.

## 1. INTRODUCTION

The large empirical mass gap between the  $\eta'$  and  $\pi$  masses,  $m_0^2 = m_{\eta'}^2 - m_\pi^2$ , is commonly believed as being due to the  $U_A(1)$  anomaly of the flavour singlet axial vector current. In this picture, the large  $\eta'$ -meson mass is driven by quark-antiquark interactions mediated by topological charge fluctuations in the QCD vacuum. In the  $N_c = \infty$ -limit of QCD this scenario is put forward through the famous Witten-Veneziano [1] formula (WVF) that models  $m_0^2$  in terms of the topological susceptibility, from quenched QCD. The pitfalls of a lattice evaluation of the Witten-Veneziano formula have been elucidated in Ref. [2].

In principle an *ab initio*, direct verification of the  $U_A(1)$  origin of flavour singlet pseudoscalar masses *beyond the large  $N_c$ -approximation* can be accomplished by simulating unquenched QCD on the lattice, but conclusive results on the mass gap have been impeded to date [3], mainly due to severe signal-to-noise problems in the analysis of the flavour singlet correlator [4–6].

As shown in Fig. (1), the correlator in question,

$$C_{\eta'}(\Delta t) = \langle 0 | \mathcal{O}^\dagger(\Delta t) \mathcal{O}(t=0) | 0 \rangle \quad (1)$$

contains a loop-loop (fermionically ‘disconnected’) contribution,  $D$ . This OZI-suppressed

\*since Oct 1, 2001: IASA, Athens, Greece

†since Dec 1, 2001: ParTec AG, Karlsruhe

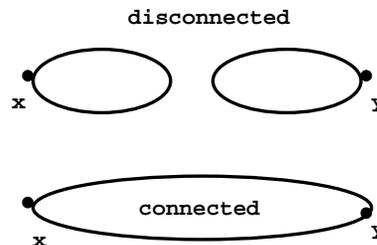


Figure 1. The two contributions to the  $\eta'$  propagator *in the full* QCD vacuum: the connected part with running through valence quark lines and the ‘disconnected’ piece where the valence quark lines annihilate instead of connecting source  $x$  and sink  $y$  at time slices  $t = 0$  and  $t = \Delta t$ .

piece arises from the Wick contractions within each of the two light quark bilinears

$$\mathcal{O} = \bar{q}\gamma_5 q \quad (2)$$

and was evaluated in the literature by use of stochastic estimator techniques (SET), to cover its entire volume dependence. As a net result the  $\eta'$  propagator  $C_{\eta'}(\Delta t)$  is computed as the *numerical difference* between its connected and disconnected pieces:

$$C_{\eta'}(\Delta t) = C(\Delta t) - 2D(\Delta t) \xrightarrow{\Delta t \rightarrow \infty} \exp(-m_{\eta'} \Delta t), \quad (3)$$

and the  $\eta'$ -mass is to be extracted from its asymptotic behaviour. Since the large mass  $m_{\eta'}$  cor-

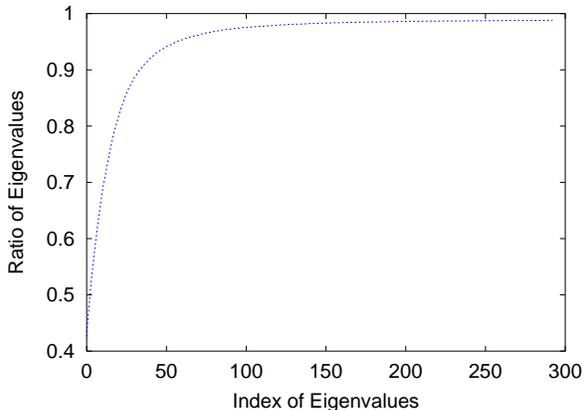


Figure 2. The net variation of low eigenmodes of  $Q$  over the sea quark mass range reached by the SESAM simulation, as expressed in the spread of eigenmode ratios,  $R = |\lambda_i(0.1575)/\lambda_i(0.1560)|$ , with the eigenvalues ordered according to increasing moduli.

responds to a strong numerical cancellation between the positive terms  $C$  and  $D$ , the calamity of previous LQCD attempts was to establish a window in  $\Delta t$  with unambiguous single exponential decay, before signals disappear under the noise level.

Barring brute force methods it appears that important progress in the field could be achieved by

1. avoiding SET and/or
2. accomplish precocious ground state dominance in the flavour singlet channel.

With this motivation, we report here on some recent methodological studies of ours [7,8] where we readdress the unsettled problem of the direct  $\eta'$  signal in LQCD and explore the possibility of spectral representations of quark propagators in superseding the SET treatment of  $D(\Delta t)$  [4–6]. Such approach is of course in lign with the expectation that chiral physics should be controlled by the low modes of the Dirac-Wilson operator. The point to find out is whether in the sea quark mass range of state-of-the-art *full* QCD simula-

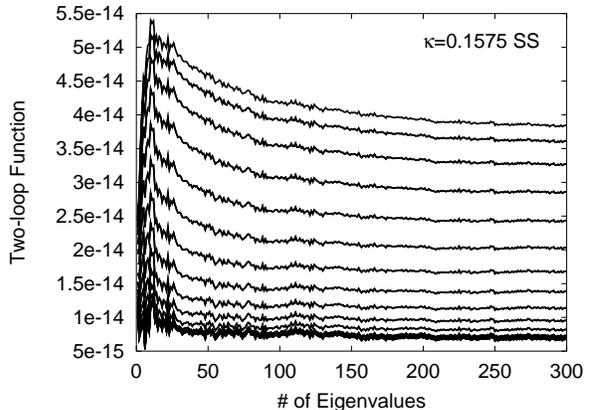


Figure 3. The spectral cutoff dependence of the disconnected contribution,  $D^l(\Delta t)$ , for various time separations  $\Delta t = 1, \dots, 16$  (top to bottom), at the lightest SESAM sea quark mass.

tions – like the SESAM simulation [9] – a reasonably small number of low eigenmodes actually suffices to saturate the fermion loop expansions with appropriate degree of accuracy.

## 2. The spectral approach

Since the Wilson-Dirac matrix,  $M$ , is non-normal at non-zero lattice spacings, it is preferable to expand in the eigenspectrum of its Hermitian equivalent

$$Q^\dagger = Q := \gamma_5 M . \quad (4)$$

$Q$  has been shown to provide the optimal eigenvalue basis in the sense of the singular value decomposition [10].

For our practical benchmarking we use 200 (195) SESAM lattices of size  $16^3 \times 32$  with  $\kappa = 0.1575$  (0.1560) [9,11]. A detailed account of our analysis can be found in Ref. [8].

The eigenmodes of  $Q$  can be determined by means of Lanczos methods. In Fig. (2) the 300 lowest eigenvalues are plotted to show their response under the SESAM variation in sea quark mass. The characteristic range of this response in eigenvalue space suggests that a truncated eigen-

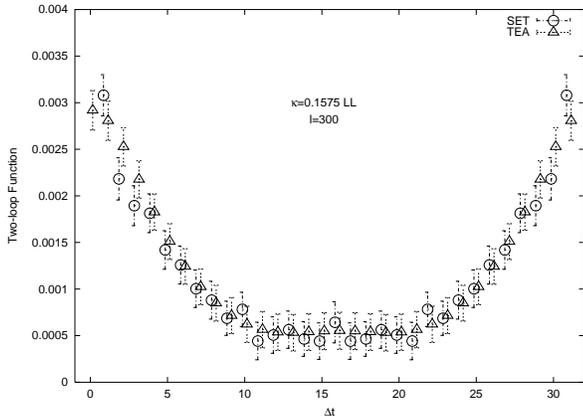


Figure 4. The disconnected contribution to the  $\eta'$ -correlator from its spectral representation with cutoff  $l = 300$ , at the smallest sea quark mass of the SESAM simulation.

mode approach (TEA), based on some *few hundred low modes*, might indeed encompass the infrared physics of interest.

In terms of the eigenstates of  $Q$ ,

$$Q|\psi_i\rangle = \lambda_i|\psi_i\rangle, \quad (5)$$

the quark propagator can readily be computed, from any source  $z$  to every sink  $z'$ :

$$M^{-1}(z, z') = \sum_{i=1}^l \frac{\gamma_5 |\psi_i(z)\rangle \langle \psi_i(z')|}{\lambda_i \langle \psi_i | \psi_i \rangle}. \quad (6)$$

From this expression it is straightforward to compute the loop-loop correlator of Fig. (1) in spectral representation,  $T$ ; it should be dominated by a small number of modes at large enough values of  $\Delta t$ . This is indeed reflected in Fig. (3) where we display the  $l$ -dependence of the disconnected contribution to the  $\eta'$ -propagator,  $D^l$ , the family of curves representing the set of different time slices,  $\Delta t = 1, \dots, 16$  (from top to bottom). While we find a few tens of modes to bear out the long range (in  $\Delta t$ ) features of  $D$ , a few hundred modes appear to saturate its short range structure under the SESAM conditions. This finding is also

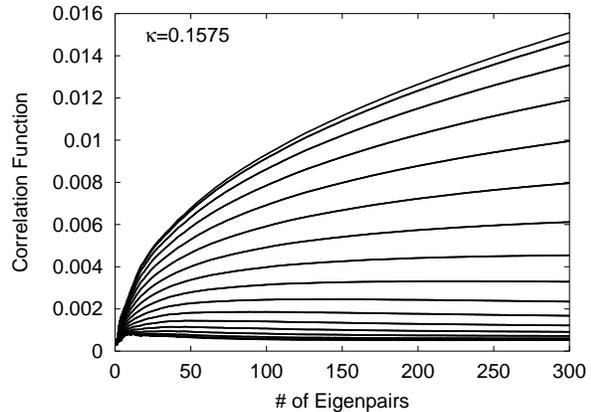


Figure 5. The analogue to Fig. (3), but for the connected contribution,  $C(\Delta t)$ , to Eq. (3).

visualized in Fig. (4) where we plotted  $D^{300}(\Delta t)$  together with the previous SET result [6] and find them to agree nicely within their errors.

For the connected piece of the  $\eta'$ -correlator,  $C$ , the viability of TEA is less evident as can be seen from Fig. (5). Even with a cutoff  $l = 300$ , saturation at large  $\Delta t$  is not fulfilled in detail, not to speak of the region of small time separations, where excited state contributions are not covered at all by the 300 lowest modes. This is of course not an obstacle for our present purposes, since this part of the correlator can easily be treated by standard methods. In fact, at this point, we go one step beyond and *exploit the high accuracy* of  $C$  as achieved from iterative solvers: we replace  $C$  by its ground state contribution,  $C_\pi^g$  (as obtained from a single exponential fit at large time separations, see Fig. (6)),  $C \rightarrow C_\pi^g$ . This then leads us to the final form of the projected  $\eta'$ -correlator

$$\tilde{C}_{\eta'}(\Delta t) = C_\pi^g(\Delta t) - 2T(\Delta t). \quad (7)$$

in terms of the two loop spectral approximation,  $T$ :

$$T(\Delta t) = \text{tr}|_{t=0} Q^{-1} \text{tr}|_{\Delta t} Q^{-1}, \quad (8)$$

with  $N_f = 2$  being the number of active sea quark flavours and  $Q^{-1}$  taken from Eq. (6).

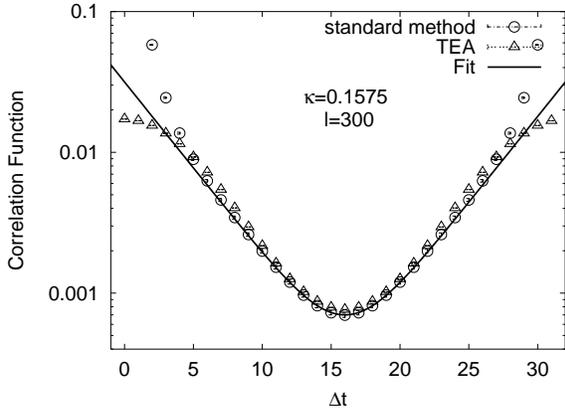


Figure 6. The one loop correlator computed by standard linear solver techniques (labeled ‘standard method’) and TEA with spectral cutoff  $l = 300$ , at the lightest SESAM sea quark mass. Also shown is the ground state projection (marked ‘Fit’).

This enables us to carry out a standard effective local  $\eta'$ -mass analysis, which is much more sensitive than looking at correlators or ratios thereof [5]. As a result we find an effective mass plot (see Fig. (7)) with striking mass plateau formation which extends over the entire interval  $1 \leq \Delta t \leq 9$ , for the smallest sea quark mass. This precocious ground state dominance in the  $\eta'$  channel provides us with hitherto unobserved accuracy for the  $\eta'$ -mass estimate, over the entire range of sea quark masses of the SESAM QCD simulation!

Note from Fig. (7) that the flavour-nonsinglet mass (marked ‘ $\pi$ ’) is clearly resolved from the  $\eta'$ -mass plateau, with increasing mass gap as the quark mass is lowered, at our value of lattice spacing,  $a^{-1}(m_\rho) = 2.30$  GeV.

Since TEA works better with decreasing sea quark masses and Lanczos methods do not deteriorate with growing condition numbers, TEA is expected to be by far superior to SET in the deep chiral regime of tomorrow’s simulations where linear solvers will tend to become unstable.

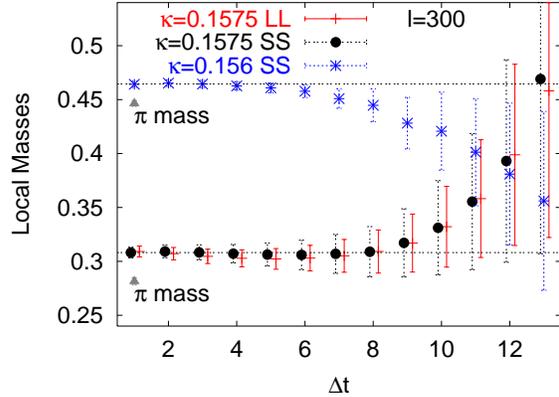


Figure 7. Effective  $\eta'$ -mass in lattice units at the lightest ( $\kappa = 0.1575$ ) and heaviest ( $\kappa = 0.1560$ ) sea quark masses of the SESAM simulation, both with (SS) and without (LL) source and sink smearing [11].

### 3. Discussion and outlook

Our analysis of the  $\eta'$  signal in two-flavour QCD has led us to the reassuring result that spectral methods in conjunction with ground state projection on the connected contribution of the  $\eta'$ -correlator can resolve the  $\eta'$ - $\pi$  mass gap before the chiral and continuum extrapolations. In Fig. (8) we show that the lattice data are sufficient to sustain a chiral extrapolation as well. The figure demonstrates moreover that there is full consistency of TEA with SET data when being analysed in the same manner. Before continuum extrapolation, a linear fit of  $m_{\eta'}^2(m_{sea})$  works with reasonable  $\chi^2$  and results in  $M_{\eta'} = 307(15)$  MeV. This value is far away from the actual experimental number,  $m_{\eta'} = 957$  MeV; this does not come as a surprise, however, since the SESAM QCD vacuum configurations are missing the strange sea quark contributions. Therefore our pseudoscalar flavour singlet meson resembles more an (octet)  $\eta$  than the  $\eta'$  meson proper. Apart from the continuum extrapolation *t.b.d.*, it is therefore necessary to introduce strange quarks into the calculation, in order to make contact with

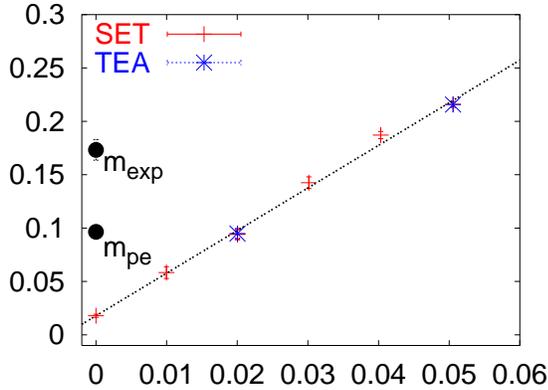


Figure 8. Demonstration of the quality of data: chiral extrapolation of  $m_{\eta'}^2$ , plotted *vs.* sea quark mass  $m_{\text{sea}}$ , all in lattice units.  $m_{\text{pe}}$  is a naive estimate from WVF (replacing  $m_0^2 \rightarrow 2/3m_0^2$  to accommodate to  $N_f = 2$ ).

real experiment.

Since  $N_f = 3$  simulations are not feasible at this time the next step would be to consider both  $\eta$  and  $\eta'$  in a partially quenched setting with two degenerate light quarks  $n$  in the sea, and a strange valence quarks added. This opens the scenario of a coupled two channel approach of pseudoscalar singlet mesons in the  $n$  and  $s$  sectors, in terms of the quark flavour basis [12]

$$\begin{aligned} |\eta\rangle &= \cos\Phi |\eta_{nn}\rangle + \sin\Phi |\eta_{ss}\rangle \\ |\eta'\rangle &= -\sin\Phi |\eta_{nn}\rangle + \cos\Phi |\eta_{ss}\rangle \end{aligned} \quad (9)$$

with Eq. (3) being replaced by a  $2 \times 2$  correlator

$$C(t) = \begin{pmatrix} C_{nn}(t) - 2D_{nn}(t) & -\sqrt{2}D_{ns}(t) \\ -\sqrt{2}D_{sn}(t) & C_{ss}(t) - D_{ss}(t) \end{pmatrix}.$$

So far we have dealt only with the left hand upper corner of this matrix. The evaluation of the OZI-rule violating contributions in its remaining entries are expected to proceed along the lines described above, without any additional complication. The mixing problem then amounts to the solution of the eigenvalue problem [13]

$$C(t)C^{-1}(t_0)|i\rangle = \lambda_i(\Delta t)|i\rangle \quad \text{with } t > t_0 \quad (10)$$

with  $t - t_0 = \Delta t$ . The two eigenmodes coincide with the physical  $\eta'$  and  $\eta$  states. Their masses are to be extracted through the asymptotic relations

$$\lambda_i(\Delta t) \xrightarrow{\Delta t \rightarrow \infty} \exp(-m_i \Delta t). \quad (11)$$

Previously, the  $\eta$ - $\eta'$  mixing could only be addressed by perturbative modeling [4]. One would expect that the precocious mass plateau formation established in the  $nn$ -sector will persist in the presence of channel mixing from this eigenvalue problem. Work along this line is in progress [14].

**Acknowledgements** K.S. thanks A.G. Williams, A.C. Kalloniatis, W. Melnitchouk and their staff for the inspiring atmosphere of the 2001 Cairns Workshop on Lattice Hadron Physics. A.T. and H.N are supported by the EC Human Potential Programme, contracts HPRN-CT-2000-00145 and HPRN-CT-2000-00130.

## REFERENCES

1. E. Witten, Nucl. Phys. B156(1979) 269; G. Veneziano, Nucl. Phys. 159(1979) 213.
2. L. Giusti et al, hep-lat/0108009 and hep-lat/0110036.
3. ECFA report, F. Jegerlehner et al, ECFA/99/200 (1999).
4. UKQCD coll., C. McNeile et al, Phys. Lett. B491 (2000) 123.
5. CP-PACS coll, A. Ali Khan et al, Nucl. Phys. B83(2000)162.
6. SESAM coll., T. Struckmann et al, Phys. Rev. D63(2001)074503.
7. H. Neff et al, Phys.Rev.D64:114509,2002; K. Schilling et al, hep-lat/0110077.
8. H. Neff, PhD thesis, Wuppertal, 2001, WUB-DIS 2001-16, <http://www.theorie.physik.uni-wuppertal.de/comphys>.
9. SESAM coll., N. Eicker et al, Phys. Lett. B407(1997)290
10. I. Hip et al, hep-lat=0110155.
11. SESAM coll., N. Eicker et al, Phys. Rev. D59(1999)014509.
12. T. Feldmann et al, Phys. Rev. D58:114006 (1998).
13. M. Lüscher et al, Nucl. Phy. B339 (1990) 222.
14. H. Neff, A. Tsapalis et al, work in progress.

