Randomness in Classical Mechanics and Quantum Mechanics

Igor V. Volovich
Steklov Mathematical Institute
Gubkin St.8, 119991 Moscow, Russia
email: volovich@mi.ras.ru

Abstract

The Copenhagen interpretation of quantum mechanics assumes the existence of the classical deterministic Newtonian world. We argue that in fact the Newton determinism in classical world does not hold and in classical mechanics there is fundamental and irreducible randomness. The classical Newtonian trajectory does not have a direct physical meaning since arbitrary real numbers are not observable. There are classical uncertainty relations: $\Delta q > 0$ and $\Delta p > 0$, i.e. the uncertainty (errors of observation) in the determination of coordinate and momentum is always positive (non zero).

A "functional" formulation of classical mechanics was suggested. The fundamental equation of the microscopic dynamics in the functional approach is not the Newton equation but the Liouville equation for the distribution function of the single particle. Solutions of the Liouville equation have the property of delocalization which accounts for irreversibility. The Newton equation in this approach appears as an approximate equation describing the dynamics of the average values of the position and momenta for not too long time intervals. Corrections to the Newton trajectories are computed. An interpretation of quantum mechanics is attempted in which both classical and quantum mechanics contain fundamental randomness. Instead of an ensemble of events one introduces an ensemble of observers.

1 Introduction

In classical mechanics the motion of a point body is described by the trajectory in the phase space, i.e. the values of the coordinates and momenta as functions of time, which are solutions of the equations of Newton or Hamilton, [1].

However, this mathematical model is an idealization of the physical process, rather far separated from reality. The physical body always has the spatial dimensions, hence a mathematical point gives only an approximate description of the physical body. The mathematical notion of a trajectory does not have direct physical meaning, since it uses arbitrary real numbers, i.e. infinite decimal expansions, while the observation is only possible, in the best case, of rational numbers, and even them only with some error. Therefore, we suggest a "functional" approach to classical mechanics, which is not starting from Newton's equation, but with the Liouville equation. This approach can help to explain the infamous time irreversibility problem, see, for example [2] - [8].

The conventional widely used concept of the microscopic state of the system at some moment in time as the point in phase space, as well as the notion of trajectory and the microscopic equations of motion have no direct physical meaning, since arbitrary real numbers not observable (observable physical quantities are only presented by rational numbers, cf. the discussion of concepts of space and time in [9] - [14]).

In the functional approach [15] the physical meaning is attached not to a single trajectory but only to a "beam" of trajectories, or the distribution function on phase space. Individual trajectories are not observable, they could be considered as "hidden variables", if one uses the quantum mechanical notions.

The fundamental equation of the microscopic dynamics of the functional probabilistic approach is not Newton's equation, but a Liouville equation for distribution function. It is well known that the Liouville equation is used in statistical mechanics for description of the motions of gas. Let us stress that we shall use the Liouville equation even for description of a single particle in the empty space.

There are many discussions of the time irreversibility problem, i.e. the problem of how to explain the irreversible behaviour of macroscopic systems from the time-symmetric microscopic laws. The problem has been discussed by Boltzmann, Poincaré, Bogolyubov, Feynman and many other authors, [2] - [8]. Landau and Lifshiz wrote about the principle of increasing entropy [4]: "Currently it is not clear whether the law of increasing entropy can be in principle derived from classical mechanics." Landau speculated that to explain the second law of thermodynamics one has to use quantum mechanical measurement arguments.

Although the Liouville equation is symmetric in relation to the reversion of time, but his solutions have the property of *delocalization*, that, generally speaking, can be interpreted as a manifestation of irreversibility. It is understood that if at some moment in time the distribution function describes a particle, localized to a certain extent, then over time the degree of localization decreases, there is the spreading of distribution function. Delocalization takes place even for a free particle in infinite space, where there is no ergodicity and mixing. In a sense, the functional formulation of microscopic dynamics is irreversible in time. Thus the contradiction between microscopic reversibility and macroscopic irreversibility of the dynamics disappears, since both microscopic and macroscopic dynamics in the proposed

approach are irreversible.

In the functional approach to classical mechanics we do not derive the statistical or chaotic properties of deterministic dynamics, but we suggest that the Laplace's determinism at the fundamental level is absent not only in quantum, but also in classical mechanics.

We show that Newton's equation in the proposed approach appears as an approximate equation describing the dynamics of the average values of coordinates and momenta for not too long time. We calculate corrections to Newton's equation.

An interpretation of quantum mechanics is attempted in which both classical and quantum mechanics contain fundamental randomness. Instead of an ensemble of events one introduces an ensemble of observers.

In the next section the fundamentals of the functional formulation of classical and quantum mechanics are presented. Section 3 deals with the free movement of particles and Newton's equation for the average coordinates. Comparison with quantum mechanics is discussed in Section 4. General comments on the Liouville and Newton equations are given in section 5. Corrections to the Newton equation for a nonlinear system are calculated in Section 6. The dynamics of the classical and quantum particle in a box and their interrelationships are summarized in section 7.

2 States and Observables in Functional Mechanics

2.1 Classical mechanics

An *exact* derivation of the coordinate and momentum can not be done, not only in quantum mechanics, where there is the Heisenberg uncertainty relation, but also in classical mechanics. Always there are some errors in setting the coordinates and momenta. There are classical uncertainty relations: $\Delta q > 0$ and $\Delta p > 0$, i.e. the uncertainty (errors of observation) in the determination of coordinate and momentum is always positive (non zero). The concept of arbitrary real numbers, given by the infinite decimal series, is a mathematical idealization, such numbers can not be measured in the experiment.

Consider the motion of a classical particle along a straight line in the potential field. The general case of many particles in the 3-dimensional space is discussed below. Let (q, p) be coordinates on the plane $\mathbb{R}^2$ (phase space), $t \in \mathbb{R}$ is time. The state of a classical particle at time t will be described by the function $\rho = \rho(q, p, t)$, it is the density of the probability that the particle at time t has the coordinate q and momentum p.

In [16] it is given a construction of the probability density function starting from the directly observable quantities, i.e., the results of measurements, which are rational numbers.

2.2 Classical and quantum mechanics

Note that the description of a mechanical system with the help of probability distribution function $\rho = \rho(q, p, t)$ does not necessarily mean that we are dealing with a set of identically prepared ensemble of particles. Usually in probability theory one considers an ensemble of events and a sample space [20]. But we can use the description with the function $\rho = \rho(q, p, t)$ also for individual bodies, such as planets in astronomy (the phase space in this case

the 6-dimensional). In this case one can think on the "ensemble" of different astronomers which observe the planet. It might be that there is only one "intelligent" observer and an "ensemble" of different scenario of behaviour of a given object in such a way that one can deal with an individual quantum phenomenon. From this point of view there is no difference between "Einstein's moon" and "Heisenberg's electron". Actually, it is implicitly always dealt with the function $\rho = \rho(q, p, t)$ which takes into account the inherent uncertainty in the coordinates and momentum of the body.

The wave function in quantum mechanics $\psi = \psi(q, t)$ or the density operator actually depends not only from time t and position q but also from other parameters such as the form of the potential field and the length of the box as well as from the mass, charge and the Planck constant. Denote these parameters by ξ. Some of these parameters can be called the "contextual" variables. We have the wave function $\psi = \psi(q, t; \xi)$. In the functional formulation of quantum mechanics we introduce a distribution $\sigma(\xi)$ which describes the uncertainty in the derivation of these parameters. To get observed quantities we have to evaluate the average value of ψ or $|\psi|^2$ with $\sigma(\xi)$. More discussions of functional quantum mechanics will be presented in a separate work. Note that similar distribution $\sigma(\xi)$ we have to introduce already in classical functional mechanics.

The specific type of function ρ depends on the method of preparation of the state of a classical particle at the initial time and the type of potential field. When $\rho = \rho(q, p, t)$ has sharp peaks at $q = q_0$ and $p = p_0$, we say that the particle has the approximate values of coordinate and momentum q_0 and p_0.

In the functional approach to classical mechanics the concept of precise trajectory of a particle is absent, the fundamental concept is a distribution function $\rho = \rho(q, p, t)$ and δ-function as a distribution function is not allowed.

We assume that the continuously differentiable and integrable function $\rho = \rho(q, p, t)$ satisfies the conditions:

$$\rho \geq 0, \quad \int_{\mathbb{R}^2} \rho(q, p, t) dq dp = 1, \ t \in \mathbb{R}\,. \tag{1}$$

The motion of particles in the functional approach is not described directly by the Newton (Hamilton) equation. Newton's equation in the functional approach is an approximate equation for the average coordinates of the particles, and for non-linear dynamics there are corrections to the Newton equations.

If $f = f(q, p)$ is a function on phase space, the average value of f at time t is given by the integral

$$\overline{f}(t) = \int f(q, p)\rho(q, p, t) dq dp\,. \tag{2}$$

In a sense we are dealing with a random process $\xi(t)$ with values in the phase space.

2.3 Basic equation for a single particle

Motion of a point body along a straight line in the potential field will be described by the equation

$$\frac{\partial \rho}{\partial t} = -\frac{p}{m}\frac{\partial \rho}{\partial q} + \frac{\partial V(q)}{\partial q}\frac{\partial \rho}{\partial p}\,. \tag{3}$$

Here $V(q)$ is the potential field and mass $m > 0$.

Equation (3) looks like the Liouville equation which is used in statistical physics to describe a gas of particles but here we use it to describe a single particle.

If the distribution $\rho_0(q,p)$ for $t = 0$ is known, we can consider the Cauchy problem for the equation (3):

$$\rho|_{t=0} = \rho_0(q,p)\,. \tag{4}$$

Let us discuss the case when the initial distribution has the Gaussian form:

$$\rho_0(q,p) = \frac{1}{\pi ab} e^{-\frac{(q-q_0)^2}{a^2}} e^{-\frac{(p-p_0)^2}{b^2}}\,. \tag{5}$$

At sufficiently small values of the parameters $a > 0$ and $b > 0$ the particle has coordinate and momentum close to the q_0 and p_0. For this distribution the average value of the coordinates and momentum are:

$$\overline{q} = \int q\rho_0(q,p)dqdp = q_0\,,\quad \overline{p} = \int p\rho_0(q,p)dqdp = p_0\,, \tag{6}$$

and dispersion

$$\Delta q^2 = \overline{(q-\overline{q})^2} = \frac{1}{2}a^2,\quad \Delta p^2 = \overline{(p-\overline{p})^2} = \frac{1}{2}b^2\,. \tag{7}$$

3 Free Motion

Consider first the case of the free motion of the particle when $V = 0$. In this case the equation (3) has the form

$$\frac{\partial\rho}{\partial t} = -\frac{p}{m}\frac{\partial\rho}{\partial q} \tag{8}$$

and the solution of the Cauchy problem is

$$\rho(q,p,t) = \rho_0(q - \frac{p}{m}t, p)\,. \tag{9}$$

Using expressions (5), (9),

$$\rho(q,p,t) = \frac{1}{\pi ab}\exp\{-\frac{(q-q_0-\frac{p}{m}t)^2}{a^2} - \frac{(p-p_0)^2}{b^2}\}\,, \tag{10}$$

we get the time dependent distribution of coordinates:

$$\rho_c(q,t) = \int \rho(q,p,t)dp = \frac{1}{\sqrt{\pi}\sqrt{a^2 + \frac{b^2t^2}{m^2}}}\exp\{-\frac{(q-q_0-\frac{p_0}{m}t)^2}{(a^2+\frac{b^2t^2}{m^2})}\}\,, \tag{11}$$

while the distribution of momenta is

$$\rho_m(p,t) = \int \rho(q,p,t)dq = \frac{1}{\sqrt{\pi}b}e^{-\frac{(p-p_0)^2}{b^2}}. \tag{12}$$

Thus, for the free particle the distribution of the particle momentum with the passage of time does not change, and the distribution of the coordinates change. There is, as one says in quantum mechanics, the spreading of the wave packet. From (11) it follows that the dispersion Δq^2 increases with time:

$$\Delta q^2(t) = \frac{1}{2}(a^2 + \frac{b^2 t^2}{m^2})\,. \tag{13}$$

Even if the particle was arbitrarily well localized (a^2 is arbitrarily small) at $t = 0$, then at sufficiently large times t the localization of the particle becomes meaningless, there is a *delocalization* of the particle.

What role can play the Newton equation in the functional approach? The average coordinate for the free particle in the functional approach satisfies the Newton equation. Indeed, the average coordinate and momentum for the free particles have the form

$$\overline{q}(t) = \int q\rho_c(q,t)dq = q_0 + \frac{p_0}{m}t\,, \quad \overline{p}(t) = \int p\rho_m(p,t)dp = p_0\,. \tag{14}$$

Note that in the functional mechanics the Newton equation for the average coordinates is obtained only for the free particle or for quadratic Hamiltonians with a Gaussian initial distribution function. For a more general case there are corrections to Newton's equations, as discussed below.

4 Comparison with Quantum Mechanics

Compare the evolutions of Gaussian distribution functions in functional classical mechanics and in quantum mechanics for the motion of particles along a straight line. The scene of work for the functional classical mechanics is $L^2(\mathbb{R}^2)$ (or $L^1(\mathbb{R}^2)$), and for quantum mechanics - $L^2(\mathbb{R}^1)$.

The Schrodinger equation for a free quantum particle on a line reads:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}\,. \tag{15}$$

Here $\psi = \psi(x,t)$ is the wave function and $\hbar$ is the Planck constant. The density of the distribution function for the Gaussian wave function has the form

$$\rho_q(x,t) = |\psi(x,t)|^2 = \frac{1}{\sqrt{\pi}\sqrt{a^2 + \frac{\hbar^2 t^2}{a^2 m^2}}}\exp\{-\frac{(x - x_0 - \frac{p_0}{m}t)^2}{(a^2 + \frac{\hbar^2 t^2}{a^2 m^2})}\}\,. \tag{16}$$

We find that the distribution functions in functional classical and in quantum mechanics (11) and (16) coincide, if we set

$$a^2 b^2 = \hbar^2\,. \tag{17}$$

If the condition (17) is satisfied then the Wigner function $W(q,p,t)$ for ψ corresponds to the classical distribution function (10) , $W(q,p,t) = \rho(q,p,t)$.

Gaussian wave functions on the line are coherent or compressed states. The compressed states on the interval are considered in [17].

5 Liouville Equation and the Newton Equation

In the functional classical mechanics the motion of a particle along the stright line is described by the Liouville equation (3). A more general Liouville equation on the manifold Γ with coordinates $x = (x^1, ..., x^k)$ has the form

$$\frac{\partial \rho}{\partial t} + \sum_{i=1}^{k} \frac{\partial}{\partial x^i}(\rho v^i) = 0\,. \tag{18}$$

Here $\rho = \rho(x,t)$ is the density function and $v = v(x) = (v^1, ..., v^k)$ - vector field on Γ. The solution of the Cauchy problem for the equation (18) with initial data

$$\rho|_{t=0} = \rho_0(x) \tag{19}$$

might be written in the form

$$\rho(x,t) = \rho_0(\varphi_{-t}(x))\,. \tag{20}$$

Here $\varphi_t(x)$ is a phase flow along the solutions of the characteristic equation

$$\dot{x} = v(x)\,. \tag{21}$$

A system in the phase space $\mathbb{R}^{2N}$ with coordinates $q = (q_1, ..., q_N)$, $p = (p_1, ..., p_N)$ is described by the Liouville equation for the function $\rho = \rho(q,p,t)$

$$\frac{\partial \rho}{\partial t} = \sum_i (\frac{\partial V(q)}{\partial q_i}\frac{\partial \rho}{\partial p_i} - \frac{p_i}{m_i}\frac{\partial \rho}{\partial q_i})\,. \tag{22}$$

Here summation goes on $i = 1, ..., N$. The characteristics equations for (22) are Hamilton's equations

$$\dot{q}_i = \frac{\partial H}{\partial p_i},\ \dot{p}_i = -\frac{\partial H}{\partial q_i}\,, \tag{23}$$

where the Hamiltonian is

$$H = \sum_i \frac{p_i^2}{2m_i} + V(q)\,. \tag{24}$$

Emphasize here again that the Hamilton equations (23) in the current functional approach to the mechanics do not describe directly the motion of particles, and they are only the characteristic equations for the Liouville equation (22) which has a physical meaning. The Liouville equation (22) can be written as

$$\frac{\partial \rho}{\partial t} = \{H, \rho\}\,, \tag{25}$$

where the Poisson bracket

$$\{H, \rho\} = \sum_i (\frac{\partial H}{\partial q_i}\frac{\partial \rho}{\partial p_i} - \frac{\partial H}{\partial p_i}\frac{\partial \rho}{\partial q_i})\,. \tag{26}$$

6 Corrections to Newton's Equations

In section 3, it was noted that for the free particle in the functional approach to classical mechanics the averages coordinates and momenta satisfy the Newton equations. However, when there is a nonlinear interaction, then in functional approach corrections to the Newton's equations appear.

Consider the motion of a particle along the line in the functional mechanics. Average value $\overline{f}$ of the function on the phase space $f = f(q,p)$ at time t is given by the integral (2)

$$\overline{f}(t) =< f(t) >= \int f(q,p)\rho(q,p,t)dqdp\,. \tag{27}$$

Here $\rho(q,p,t)$ has the form (20)

$$\rho(q,p,t) = \rho_0(\varphi_{-t}(q,p))\,. \tag{28}$$

By making the replacement of variables, subject to the invariance of the Liouville measure, we get

$$< f(t) >= \int f(q,p)\rho(q,p,t)dqdp = \int f(\varphi_t(q,p))\rho_0(q,p)dqdp\,. \tag{29}$$

Let us take

$$\rho_0(q,p) = \delta_\epsilon(q-q_0)\delta_\epsilon(p-p_0)\,, \tag{30}$$

where

$$\delta_\epsilon(q) = \frac{1}{\sqrt{\pi}\epsilon}e^{-q^2/\epsilon^2}\,, \tag{31}$$

$q \in \mathbb{R}$, $\epsilon > 0$.

Let us show that in the limit $\epsilon \to 0$ we obtain the Newton (Hamilton) equations:

$$\lim_{\epsilon\to 0} < f(t) >= f(\varphi_t(q_0,p_0))\,. \tag{32}$$

Proposition 1. *Let the function $f(q,p)$ in the expression (27) be continous and integrable, and ρ_0 has the form (30). Then*

$$\lim_{\epsilon\to 0}\int f(q,p)\rho(q,p,t)dqdp = f(\varphi_t(q_0,p_0))\,. \tag{33}$$

Proof. Functions $\delta_\epsilon(q)$ form a δ-sequence in $D^{'}(\mathbb{R})$ [18]. Hence we obtain

$$\lim_{\epsilon\to 0}\int f((q,p))\rho(q,p,t)dqdp = \lim_{\epsilon\to 0}\int f(\varphi_t(q,p))\delta_\epsilon(q-q_0)\delta_\epsilon(p-p_0) = f(\varphi_t(q_0,p_0))\,, \tag{34}$$

that was required to prove.

Now calculate the corrections to the solution of the equation of Newton. In functional mechanics consider the equation, see (3) ,

$$\frac{\partial\rho}{\partial t} = -p\frac{\partial\rho}{\partial q} + \lambda q^2\frac{\partial\rho}{\partial p}\,. \tag{35}$$

Here λ is a small parameter and we set the mass $m = 1$. The characteristic equations have the form of the following Hamilton (Newton) equations:

$$\dot{p}(t) + \lambda q(t)^2 = 0\,,\ \dot{q}(t) = p(t)\,. \tag{36}$$

Solution of these equations with the initial data $q(0) = q,\ \dot{q}(0) = p$ for small t has the form

$$(q(t), p(t)) = \varphi_t(q,p) = (q + pt - \frac{\lambda}{2}q^2t^2 + ...,\ p - \lambda q^2 t + ...) \tag{37}$$

Use the asymptotic expansion $\delta_\epsilon(q)$ in $D^{'}(\mathbb{R})$ for $\epsilon \to 0$, compare [7, 19]:

$$\delta_\epsilon(q) = \delta(q) + \frac{\epsilon^2}{4}\delta^{''}(q) + ...\,, \tag{38}$$

then for $\epsilon \to 0$ we obtain corrections to the Newton dynamics:

$$< q(t) >= \int (q + pt - \frac{\lambda}{2}q^2t^2 + ...)[\delta(q - q_0) + \frac{\epsilon^2}{4}\delta^{''}(q - q_0) + ...] \tag{39}$$

$$\cdot[\delta(p - p_0) + \frac{\epsilon^2}{4}\delta^{''}(p - p_0) + ...]dqdp = q_0 + p_0t - \frac{\lambda}{2}q_0^2t^2 - \frac{\lambda}{4}\epsilon^2t^2\,.$$

Denoting the Newton solution

$$q_{\rm Newton}(t) = q_0 + p_0t - \frac{\lambda}{2}q_0^2t^2\,,$$

we obtain for small ϵ, t and λ:

$$< q(t) >= q_{\rm Newton}(t) - \frac{\lambda}{4}\epsilon^2t^2\,. \tag{40}$$

Here $-\frac{\lambda}{4}\epsilon^2t^2$ is the correction to the Newton solution received within the functional approach to classical mechanics with the initial Gaussian distribution function. If we choose a different initial distribution we get correction of another form.

We have proved

Proposition 2. *In the functional approach to mechanics the first correction at ϵ to the Newton dynamics for small t and λ for equation (36) has the form (40).*

Note that in the functional approach to mechanics instead of the usual Newton equation

$$m\frac{d^2}{dt^2}q(t) = F(q)\,, \tag{41}$$

where $F(q)$ is a force, we obtain

$$m\frac{d^2}{dt^2} < q(t) >=< F(q)(t) >\ . \tag{42}$$

The task of calculating the corrections at ϵ for Newton's equation for mean values is similar to the problem of calculating semiclassical corrections in quantum mechanics.

7 Dynamics of a Particle in a Box

Dynamics of collisionless continuous medium in a box with reflecting walls is considered by Poincare and in [8]. This studied asymptotics of solutions of Liouville equation. In functional approach to mechanics, we interpret the solution of the Liouville equation as described the dynamics of a single particle. Here we consider this model in the classical and also in the quantum version for the special case of Gaussian initial data. In particular we obtain that a single free particle in the box behaves like a gas with the Maxwell type distribution.

7.1 Dynamics of a classical particle in a box

Consider the motion of a free particle on the interval with the reflective ends. Using the method of reflections [18], the solution of the Liouville equation (8)

$$\frac{\partial \rho}{\partial t} = -\frac{p}{m}\frac{\partial \rho}{\partial q}$$

on the interval $0 \leq q \leq 1$ with the reflective ends we write as

$$\rho(q,p,t) = \sum_{n=-\infty}^{\infty} \left[\rho_0(q - \frac{p}{m}t + 2n, p) + \rho_0(-q + \frac{p}{m}t + 2n, -p)\right], \tag{43}$$

where it is assumed that the function ρ_0 has the Gaussian form (5).

One can show that for the distribution for coordinates $\rho_c(q,t) = \int \rho(q,p,t)dp$ one gets the uniform limiting distribution (pointwise limit): $\lim_{t\to\infty} \rho_c(q,t) = 1$. For the distribution of the absolute values of momenta $(p > 0)$ $\rho_a(p,t) = \rho_m(p,t) + \rho_m(-p,t)$, where

$$\rho_m(p,t) = \int_0^1 \rho(q,p,t)dq\,,$$

as $t \to \infty$ we get the distribution of the Maxwell type (but not the Maxwell distribution):

$$\lim_{t\to\infty} \rho_a(p,t) = \frac{1}{\sqrt{\pi}b}[e^{-\frac{(p-p_0)^2}{b^2}} + e^{-\frac{(p+p_0)^2}{b^2}}]\,.$$

7.2 Dynamics of a quantum particle in a box

The Schrodinger equation for free quantum particle on the interval $0 \leq x \leq 1$ with reflecting ends has the form

$$i\hbar\frac{\partial \phi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \phi}{\partial x^2} \tag{44}$$

with the boundary conditions $\phi(0,t) = 0, \quad \phi(1,t) = 0, \quad t \in \mathbb{R}$. Solution of this boundary problem can be written as follows:

$$\phi(x,t) = \sum_{n=-\infty}^{\infty} \left[\psi(x + 2n, t) - \psi(-x + 2n, t)\right],$$

where $\psi(x,t)$ is some solution of the Schrodinger equation. To get an observable quantity we have to compute the average value with the distribution $\sigma(\xi)$ (see Sect.2). Such average values will demonstrate an irreversible behaviour for large time. If we choose the function $\psi(x,t)$ in the form, corresponding to the distribution (16), then one can show that in the semiclassical limit for the probability density $|\phi(x,t)|^2$ the leading term is the classical distribution $\rho_c(x,t)$.

8 Conclusions

In this paper the functional formulation of classical mechanics is considered which is based not on the notion of an individual trajectory of the particle but on the distribution function on the phase space.

The fundamental equation of the microscopic dynamics in the functional approach is not the Newton equation but the Liouville equation for the distribution function of a single particle. Solutions of the Liouville equation have the property of delocalization which accounts for irreversibility. It is shown that the Newton equation in this approach appears as an approximate equation describing the dynamics of the average values of the positions and momenta for not too long time intervals. Corrections to the Newton equation are computed.

If we accept the functional approach to classical mechanics then both classical and quantum mechanics contain fundamental randomness. It requires a reconsideration of the usual interpretation(s) of quantum mechanics. Some remarks on these questions we made in Sect.2. Interesting problems related with applications of the functional mechanics to statistical mechanics, field theory, singularities in cosmology and black holes we hope to consider in further works.

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