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Long distance regularization in chiral perturbation theory with decuplet fields

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Abstract

We investigate the use of long distance regularization in $SU(3)$ baryon chiral perturbation theory with decuplet fields. The one-loop decuplet contributions to the octet baryon masses, axial couplings, S-wave nonleptonic hyperon decays and magnetic moments are evaluated in a chirally consistent fashion by employing a cutoff to implement long distance regularization. The convergence of the chiral expansions of these quantities is improved compared to the dimensionally regularized version which indicates that the propagation of Goldstone bosons over distances smaller than a typical hadronic size, which is beyond the regime of chiral perturbation theory but included by dimensional regularization, is removed by use of a cutoff.

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1 Introduction

The holy grail of particle/nuclear physics is to be able to make direct contact between experimental reality and the QCD Lagrangian,

$$\mathcal{L}_{QCD} = -\frac{1}{2}\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} + \bar{q}(i\not{D} - m)q, \tag{1}$$

with

$$iD_\mu = i\partial_\mu - gA_\mu, \qquad G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - gA_\mu \times A_\nu, \tag{2}$$

which is presumed to underlie them. At the very highest energies the validity of asymptotic freedom permits this confrontation via perturbative QCD, since hard scattering is able to access the basic quark/gluon structure of hadronic systems. However, at low energies the quark/gluon degrees of freedom in terms of which $\mathcal{L}_{QCD}$ is written are hidden within hadronic structure and any solution is inherently nonperturbative. In order to make progress we must therefore

i) resort to models;

ii) attempt an "exact" solution via lattice gauge techniques;

iii) exploit the (broken) chiral symmetry of Eq. (1) via chiral perturbation theory.

The use of models has obvious limitations in that, despite being able to represent data, the size and scope of possible corrections is difficult to assess. Lattice gauge theory, while promising, is presently limited at least as far as reasonably precise calculations are concerned to two and three point functions, while experiments obviously do not have such limitations. The use of chiral perturbative methods[1] on the other hand has been enormously successful over a broad range of applications in the case of Goldstone bosons and their weak and electromagnetic interactions at low energy[2]. In the case of baryons, things are not so simple. For one thing, since the mass of the nucleon — M — is comparable to the chiral scale — $\Lambda_\chi \sim 4\pi F_\pi$ — retention of strict power counting requires an additional expansion in terms of $1/M$ which must be accomplished by a Foldy-Wouthuysen transformation or equivalent technique. This method is called heavy baryon chiral perturbation theory (HBχpt)[3] and has been applied to a large variety of baryonic and baryon-meson interactions[4]. However, things are not as simple as in the

meson case. On the one hand, there exist reactions such as low energy pion photoproduction and Compton scattering wherein there has been a generally very successful interplay of near threshold experiments and HBχpt predictions. However, there have also been failures and this is due to convergence problems in the chiral expansion. This slow convergence is due for one thing to the feature that in the baryon sector the expansion is linear in $(q, m_\pi)/\Lambda_\chi$ rather than quadratic as in the case of the mesons, but it also seems to have a deeper origin. In any case, even more serious problems arise when one attempts to extend this formalism to chiral $SU(3)$ by inclusion of kaon and eta degrees of freedom. In this case, since the leading chiral corrections generally go as the meson mass to a positive power (with or without an accompanying logarithm) any corrections from kaons or etas are quite substantial and often end up destroying the generally good agreement between experiment and the venerable and successful predictions of $SU(3)$. Examples of this phenomenon include

i) baryon masses, wherein the success of the simple Gell-Mann-Okubo mass formula at first order in $SU(3)$ breaking[5] picks up very large corrections proportional to m_P^3 at one loop in HBχpt[6];

ii) axial couplings in semileptonic hyperon decays, wherein the success of the lowest order $SU(3)$ representation in terms of F,D couplings[7] is modified subtantially by $\mathcal{O}(m_P^2 \log m_P^2)$ effects at one loop[8];

iii) S-wave amplitudes in nonleptonic hyperon decays, wherein the remarkable success of the lowest order current algebra/PCAC results which utilize a simple f,d parameterization[9] are modified at one loop by $\mathcal{O}(m_P^2 \log m_P^2)$ effects[8];

iv) baryon magnetic moments, wherein the basic success of a simple $SU(3)$ parameterization and its simple quark model extension[10] is modified by linear terms in m_P which arise at one loop[11].

Of course, these are not fatal problems and can be cured by counterterm corrections at higher order in the chiral expansion. The exact way in which this occurs has been seen in $\mathcal{O}(p^4)$ calculations which have been done in the case of the mass terms and of the magnetic moments[12, 13]. However, to the extent that such corrections are large they indicate that the series which describes the chiral corrections is of the generic form

$$\text{Amp} = A_0(1 - 1 + 1 - 1 + \ldots) \tag{3}$$

which is *not* what one looks for in an *effective* chiral expansion.

In previous papers we have suggested a solution to this dilemma — a way to make the effective field theory more effective[14]. The issue in any effective field theory treatment is to make sure that all the relevant scales are accounted for and we have argued that in the conventional HBχpt analysis this is *not* the case, in that the size of the baryon is not included. This can be seen for one thing from the feature that in the usual dimensional regularization (dim reg) prescription, by which chiral loops are regularized, contributions from high and low energy scales are weighted in the same fashion. (It should be noted that this same equal weighting has been shown to lead to other difficulties in understanding the OPE scaling of operators — *e.g.* operators which are purportedly of higher dimension contribute at lower order than expected on dimensional grounds[15].) On the other hand *physics* considerations tell us that, while components of the chiral Lagrangian which are large compared to the baryon size can be trusted to be determined by the underlying chiral symmetry of the problem, this is certainly not true for pieces which are small compared to the baryon radius and which are certainly modified by hadronic structure. Our solution is to include some sort of form factor in such calculations, which is unity at large distances (*i.e.* $r >> R_B$) but which vanishes for distances short compared to the baryon radius. For calculational simplicity we have employed a dipole form factor

$$F(q^2) = \left(\frac{\Lambda^2}{\Lambda^2 - q^2}\right)^2 \tag{4}$$

with a cutoff parameter in the range 300 MeV $< \Lambda <$ 600 MeV, but the precise shape of such a function is irrelevant — any function which has the right physics should suffice. For example, we have shown that the same physics results from use of lattice size as a regulator[16]. The purpose of the cutoff function is to remove the model-dependent short distance portions of the loop integrals which are not suppressed in dim reg. For obvious reasons we have called this procedure "long distance regularization" (LDR) and have demonstrated that it is guaranteed to isolate the basic chiral physics of any baryonic process in as model-independent a fashion as possible without the unphysical equal weighting of long and short distance physics. In particular we have shown explicitly how the power law dependences on Λ which arise in this procedure can be absorbed into phenomenologically-determined counterterms, while the remaining chiral corrections do not appreciably modify

the successful lowest order $SU(3)$ predictions provided that the cutoff is chosen in the range given above. Also, we have found that there exists only weak dependence on the cutoff Λ. If one worked to all chiral orders, all dependence could be absorbed by the LECs and the theory would not depend on the value of Lambda. Since we work at finite order, the residual dependence is expected to occur beyond the order we are working, so that the dependence on Λ should be rather mild.

That this idea makes sense physically is also indicated by the fact that it matches nicely onto successful and chirally well motivated model-dependent approaches such as the cloudy bag model[17] or a recent Bethe-Salpeter approach to baryon structure[18]. For example, in the dynamical model advocated in the latter picture, the chiral self-energy corrections to the baryon masses are of the form

$$\Sigma_N(M_N) \propto \int_0^\infty dk \left[\frac{k^4 u^2(k)}{\omega^2(k)} + \frac{32}{25} \frac{k^4 u^2(k)}{\omega(k)(M_\Delta - M_N - \omega(k)} \right], \tag{5}$$

where $\omega(k) = \sqrt{\vec{k}^2 + m_\pi^2}$, which has an identical form to that found in the LDR picture provided that we make the identification

$$F(k^2) = u^2(k).$$

Now in the dynamical model employed in Ref. [18] the form of the function $u(k)$ is generated dynamically and there is no freedom to choose the shape. Nevertheless, what is found matches nicely onto our assumed dipole form provided one picks a cutoff in the vicinity $\Lambda \sim 500$ MeV. Likewise similar effects are found in any sort of cloudy bag model approach, as argued by Leinweber et. al.[19].

A further reason for and application of such calculations was advocated by the Adelaide group[19],[20],[21], who emphasize that such realistic chiral baryon corrections are just what is needed in order to extrapolate state of the art lattice calculations, which are done for quark masses (and therefore pion masses) considerably heavier than the values given experimentally down to realistic values. In particular, the use of a functional form with m_π-dependence motivated by LDR ideas was shown to lead to successful extraction of nucleon masses and magnetic moments from lattice evaluations with m_π in the 600 MeV range.

But isn't HBχpt independent of regularization scheme? Yes, it is true that any physical observable is independent of which scheme is chosen, but a

discussion of convergence involves more than just the total observable. The contribution to any observable from a particular chiral order is not itself an observable, and yet the comparison among chiral orders is what determines the rate of convergence in HBχpt. In dim reg, parameters in the Lagrangian acquire a dependence on the regularization scale which exactly cancels the scale dependence in the loops, thus making observables independent of the unphysical dim reg scale. LDR works the same way, using the cutoff instead of the dim reg scale. However an important advantage of LDR, in contrast to dim reg, is that it does not ignore loop contributions that contain positive powers of the cutoff. The Lagrangian parameters which exactly cancel these contributions are not all at the same chiral order, so different numerical choices for the cutoff change the relative sizes of the various orders in the chiral expansion of an observable. Since dim reg keeps only the logarithmic divergences the relative sizes of chiral orders are fixed in that scheme, and the inherent choice made by the dim reg method unfortunately betrays a poor convergence in many observables as has already been noted. The present work indicates that, when the LDR cutoff is fixed to its appropriate phenomenological value, the LDR chiral expansion is superior.

In order to extend this work via application of other techniques such as $1/N_c$ methods, it is also important to extend such calculations by inclusion of the decuplet intermediate state, since decuplet and octet states become degenerate in this limit. Indeed, since the mass difference between the nucleon and decuplet vanishes it is no longer clear that the lowest excited state is that of the nucleon plus Goldstone boson. The interplay between the chiral and $1/N_c$ limits is thus a subtle and interesting one[22], and it is imperative to extend our previous LDR methods to include the case of the decuplet. In order to be specific, we shall examine the same four problems (outlined above) that were considered in our previous discussion and demonstrate that inclusion of decuplet states can be handled in a parallel fashion to that done with the octet. In particular, we will see that any power divergences (possibly accompanied by logarithms) involving the cutoff can be absorbed into phenomenologically-determined constants so that the remaining chiral corrections are small and do not destroy the traditional $SU(3)$ fits to these processes.

In the next section then we show how to perform the dipole integrals relevant to decuplet inclusion and demonstrate that they have the expected behavior in the chiral ($m_P << \Lambda$) and heavy quark ($m_P >> \Lambda$) limits. In section III we apply these results to the specific problems discussed above,

and in section IV we draw conclusions and set directions for future work.

2 Integrals

In this section we will review the decuplet integrals both in dim reg and in LDR. We begin with the integral which appears in the analysis of the baryon masses

$$\int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{k_i k_j}{(k_0 - \Delta + i\epsilon)(k^2 - m^2 + i\epsilon)} = -i\delta_{ij} \frac{I(m,\Delta)}{4(d-1)\pi^2}. \tag{6}$$

The dim reg result is

$$I(m,\Delta) = \frac{\Delta}{2}\left(\frac{3}{2}m^2 - \Delta^2\right)\left[2\tilde{L} + \ln\frac{m^2}{\mu^2}\right] - (m^2 - \Delta^2)\left(\frac{\Delta}{2} + G(m,\Delta)\right) \tag{7}$$

with

$$G(m,\Delta) = \begin{cases} (\Delta^2 - m^2)^{1/2} \ln\left(\frac{\Delta}{m} - \sqrt{\frac{\Delta^2}{m^2} - 1}\right), & \text{for } \Delta > m \\ (m^2 - \Delta^2)^{1/2} \arccos\left(\frac{\Delta}{m}\right), & \text{for } \Delta < m \end{cases} \tag{8}$$

and

$$\tilde{L} = \mu^{d-4}\left(\frac{1}{d-4} - \frac{1}{2}[\ln(4\pi) + 1 - \gamma]\right), \tag{9}$$

where μ is the regularization scale. The average octet-decuplet mass splitting Δ has a value of $\Delta = 231$ MeV and does not vanish in the chiral limit of vanishing quark masses. The appearance of the mass scale Δ destroys the strict chiral counting scheme also in dim reg which has already been spoilt in the cutoff scheme by introducing the scale Λ. Since this splitting is only slightly larger than the pion mass but considerably smaller than the kaon and eta masses, one expects the excitations of the decuplet to play an important role in $SU(3)$ $HB\chi$pt.

The integral contains a divergent piece $\tilde{L}$ which has both mass-dependent and -independent pieces. The latter ones are removed by redefining the common octet mass in the chiral limit, while the mass-dependent divergences can be absorbed into explicitly chiral symmetry breaking counterterms at second chiral order, cf. Ref. [6]. After renormalization the final result is finite and any dependence on the regularization scale μ as given by the logarithm

of Eq. (7) is compensated by counterterms of the same chiral order. This reflects the fact that chiral symmetry is preserved by dim reg. Albeit dim reg maintains chiral invariance, it fails in separating the short and long distance components of the integral as was illustrated in Ref. [14]. We would expect, *e.g.*, the long distance portion of the integral $I(m,\Delta)$ to be smaller for larger meson masses. However, in the limit of a large mass m compared to Δ and the cutoff Λ,

$$I(m,\Delta) \stackrel{m\gg\Lambda,\Delta}{\longrightarrow} -\frac{\pi}{2}m^3 + \ldots, \tag{10}$$

the function $I(m,\Delta)$ is found to have an m^3 dependence which implies that the pion will contribute much less than its heavier kaon and eta counterparts and is therefore in contradistinction to our intuitive expectation. An implicit short distance contribution is carried along also if dim reg is employed.

In Ref. [14] it was demonstrated that the long distance component of the integrals is isolated — and chiral invariance is simultaneously preserved — by using LDR. Both an exponential cutoff and a simple dipole regulator $\Lambda^4/(\Lambda^2-k^2)^2$ were employed. The latter one is more convenient for our purposes, since it enables the loop integration to be carried out in simple analytic form. However, the specific shape of the cutoff is irrelevant as long as it is chirally invariant — a consistent chiral expansion can be carried out to the order we are working.

The introduction of a dipole cutoff in Eq. (6) yields[23]

$$\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{k_ik_j}{(k_0-\Delta+i\epsilon)(k^2-m^2+i\epsilon)}\left(\frac{\Lambda^2}{\Lambda^2-k^2}\right)^2 = -i\delta_{ij}\frac{I_\Lambda(m,\Delta)}{12\pi^2} \tag{11}$$

with

$$\begin{aligned} I_\Lambda(m,\Delta) \;=\; & \frac{\Lambda^4}{(\Lambda^2-m^2)^2}\left[\frac{\Delta}{4}(\Lambda^2-m^2)+\Delta\left(\Delta^2-\frac{3}{2}m^2\right)\ln\left(\frac{\Lambda}{m}\right)\right. \\ & -\sqrt{\Lambda^2-\Delta^2}\left(\frac{\Lambda^2}{2}+\Delta^2-\frac{3m^2}{2}\right)\arccos\left(\frac{\Delta}{\Lambda}\right) \\ & \left.-(m^2-\Delta^2)G(m,\Delta)\right]. \end{aligned} \tag{12}$$

This time we obtain in the large m limit

$$I_\Lambda(m,\Delta) \stackrel{m\gg\Lambda,\Delta}{\longrightarrow} -\frac{\pi}{2}\frac{\Lambda^4}{m} + \ldots \tag{13}$$

which is in conformity with our expectations since now the contributions from pions are more important than those from kaons or etas. LDR seems to be suited to disentangle the long distance part of the integral we are interested in from the spurious high energy portion. As we will see in the next section it also preserves chiral invariance for the cases considered here and provides an alternative regularization scheme for HBχpt.

It is instructive to consider the opposite limit of the integral I_Λ: for large values of the cutoff we obtain

$$\begin{aligned} I_\Lambda(m,\Delta) &= -\frac{\pi\Lambda^3}{4}+\frac{3}{4}\Delta\Lambda^2-\frac{\pi}{8}\Lambda(3\Delta^2-2m^2)+\frac{5}{6}\Delta^3-\frac{m^2\Delta}{4} \\ &\quad +\Delta\left(\Delta^2-\frac{3}{2}m^2\right)\ln\left(\frac{\Lambda}{m}\right)-(m^2-\Delta^2)G(m,\Delta) \\ &\quad +O(1/\Lambda). \end{aligned} \tag{14}$$

One observes cubic, quadratic, linear and logarithmic divergences which can be absorbed by appropriate counterterms, see Sec. 3. The nonanalytic terms in the quark masses, on the other hand, reduce to the dim reg result.

We now turn to the Feynman integrals of the remaining cases. In the analysis of S-wave hyperon decays the relevant heavy baryon integral is

$$\int\frac{\mathrm{d}^dk}{(2\pi)^d}\frac{k_ik_j}{(k_0-\Delta+i\epsilon)^2(k^2-m^2+i\epsilon)}=-i\delta_{ij}\frac{3J(m,\Delta)}{16(d-1)\pi^2}. \tag{15}$$

In dim reg we obtain

$$J(m,\Delta)=\left(m^2-2\Delta^2\right)\left[2\tilde{L}+\ln\frac{m^2}{\mu^2}\right]+\frac{2}{3}\left(m^2+\Delta^2\right)+4\Delta G(m,\Delta), \tag{16}$$

whereas the LDR version is given by

$$\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{k_ik_j}{(k_0-\Delta+i\epsilon)^2(k^2-m^2+i\epsilon)}\left(\frac{\Lambda^2}{\Lambda^2-k^2}\right)^2=-i\delta_{ij}\frac{J_\Lambda(m,\Delta)}{16\pi^2} \tag{17}$$

with

$$\begin{aligned} J_\Lambda(m,\Delta) &= \frac{\Lambda^4}{(\Lambda^2-m^2)^2}\left[\Lambda^2-m^2+\left(m^2-2\Delta^2\right)\ln\left(\frac{m^2}{\Lambda^2}\right)\right. \\ &\quad -\frac{2\Delta}{\sqrt{\Lambda^2-\Delta^2}}\left(\Lambda^2+m^2-2\Delta^2\right)\arccos\left(\frac{\Delta}{\Lambda}\right) \\ &\quad \left.+4\Delta G(m,\Delta)\right]. \end{aligned} \tag{18}$$

In the limit of a large cutoff the integral simplifies to

$$J_\Lambda(m,\Delta) \stackrel{m,\Delta\ll\Lambda}{\longrightarrow} \Lambda^2+m^2+[m^2-2\Delta^2]\ln\left(\frac{m^2}{\Lambda^2}\right)-\pi\Delta\Lambda+2\Delta^2 \\ +4\Delta G(m,\Delta) \quad +O(1/\Lambda) \tag{19}$$

which are the nonanalytic pieces of the dim reg result plus a polynomial in m^2. In the opposite limit of large meson masses we obtain

$$J_\Lambda(m,\Delta) \stackrel{m\gg\Lambda,\Delta}{\longrightarrow} \frac{\Lambda^4}{m^2}\left[-1+\ln\left(\frac{m^2}{\Lambda^2}\right)-\frac{2\Delta}{\sqrt{\Lambda^2-\Delta^2}}\arccos\left(\frac{\Delta}{\Lambda}\right)\right] \tag{20}$$

and again our intuitive expectations are met.

In the calculation of the baryon axial couplings another integral enters in addition to Eq. (15), cf. Ref. [24],

$$\int\frac{\mathrm{d}^dk}{(2\pi)^d}\frac{k_ik_j}{(k_0-\Delta+i\epsilon)(k_0+i\epsilon)(k^2-m^2+i\epsilon)}=-i\delta_{ij}\frac{\bar{J}(m,\Delta)}{4(d-1)\pi^2}. \tag{21}$$

In dim reg the integral reads

$$\begin{aligned}\bar{J}(m,\Delta) &= \frac{1}{\Delta}\left[I(m,\Delta)-I(m,0)\right] \\ &= \frac{1}{2}\left(\frac{3}{2}m^2-\Delta^2\right)\left[2\tilde{L}+\ln\frac{m^2}{\mu^2}\right] \\ &\quad -(m^2-\Delta^2)\left(\frac{1}{2}+\frac{1}{\Delta}G(m,\Delta)\right)+\frac{\pi}{2}\frac{m^3}{\Delta}.\end{aligned} \tag{22}$$

For the LDR version,

$$\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{k_ik_j}{(k_0-\Delta+i\epsilon)(k_0+i\epsilon)(k^2-m^2+i\epsilon)}\left(\frac{\Lambda^2}{\Lambda^2-k^2}\right)^2=-i\delta_{ij}\frac{\bar{J}_\Lambda(m,\Delta)}{12\pi^2}, \tag{23}$$

we obtain

$$\begin{aligned}\bar{J}_\Lambda(m,\Delta) &= \frac{\Lambda^4}{(\Lambda^2-m^2)^2}\left[\frac{1}{4}(\Lambda^2-m^2)+\left(\Delta^2-\frac{3}{2}m^2\right)\ln\left(\frac{\Lambda}{m}\right)\right. \\ &\quad -\sqrt{\frac{\Lambda^2}{\Delta^2}-1}\left(\frac{\Lambda^2}{2}+\Delta^2-\frac{3m^2}{2}\right)\arccos\left(\frac{\Delta}{\Lambda}\right) \\ &\quad \left.-\frac{1}{\Delta}(m^2-\Delta^2)G(m,\Delta)+\frac{\pi}{4\Delta}\left(\Lambda^3-3m^2\Lambda+2m^3\right)\right].\end{aligned} \tag{24}$$

This yields, in the limit $\Lambda \to \infty$,

$$\begin{aligned}\bar{J}_\Lambda(m,\Delta) &= \frac{3}{4}\Lambda^2 - \frac{3\pi}{8}\Lambda\Delta + \frac{5}{6}\Delta^2 - \frac{m^2}{4} + \frac{\pi}{2}\frac{m^3}{\Delta} \\ &\quad + \left(\Delta^2 - \frac{3}{2}m^2\right)\ln\left(\frac{\Lambda}{m}\right) - \frac{1}{\Delta}(m^2-\Delta^2)G(m,\Delta) \\ &\quad + O(1/\Lambda), \end{aligned} \tag{25}$$

and for large meson masses

$$\bar{J}_\Lambda(m,\Delta) \stackrel{m\gg\Lambda,\Delta}{\longrightarrow} \frac{3\Lambda^4}{4m^2}\left[1 + \ln\left(\frac{m^2}{\Lambda^2}\right) + 2\sqrt{\frac{\Lambda^2}{\Delta^2}-1}\arccos\left(\frac{\Delta}{\Lambda}\right) - \pi\frac{\Lambda}{\Delta}\right]. \tag{26}$$

Finally, we consider the integral involved in the calculation of the magnetic moments. It reads

$$\int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{k_i k_j}{(k_0 - \Delta + i\epsilon)(k^2 - m^2 + i\epsilon)^2} = -i\delta_{ij}\frac{3K(m,\Delta)}{16(d-1)\pi^2} \tag{27}$$

where

$$K(m,\Delta) = \Delta\left[2\tilde{L} + \ln\frac{m^2}{\mu^2}\right] - \frac{\Delta}{3} - 2G(m,\Delta). \tag{28}$$

In LDR one obtains

$$\int \frac{\mathrm{d}^4 k}{(2\pi)^4}\frac{k_i k_j}{(k_0 - \Delta + i\epsilon)(k^2 - m^2 + i\epsilon)^2}\left(\frac{\Lambda^2}{\Lambda^2 - k^2}\right)^2 = -i\delta_{ij}\frac{K_\Lambda(m,\Delta)}{16\pi^2} \tag{29}$$

with

$$\begin{aligned}K_\Lambda(m,\Delta) &= \frac{\Lambda^4}{(\Lambda^2-m^2)^3}\left[\frac{2}{3}\Delta[\Lambda^2 - m^2] + \Delta\left[m^2 + \Lambda^2 - \frac{4}{3}\Delta^2\right]\ln\left(\frac{m^2}{\Lambda^2}\right)\right. \\ &\quad + 2\left[m^2 + \frac{1}{3}\Lambda^2 - \frac{4}{3}\Delta^2\right]\sqrt{\Lambda^2-\Delta^2}\arccos\left(\frac{\Delta}{\Lambda}\right) \\ &\quad \left. - \frac{2}{3}\left[m^2 + 3\Lambda^2 - 4\Delta^2\right]G(m,\Delta)\right]. \end{aligned} \tag{30}$$

The small and large mass limits of the LDR form are given by

$$K_\Lambda(m,\Delta) \stackrel{m,\Delta\ll\Lambda}{\longrightarrow} \Delta\ln\left(\frac{m^2}{\Lambda^2}\right) + \frac{\pi}{3}\Lambda + \frac{2}{3}\Delta - 2G(m,\Delta) \quad + O(1/\Lambda) \tag{31}$$

and

$$K_\Lambda(m,\Delta) \stackrel{m\gg\Lambda,\Delta}{\longrightarrow} \frac{\pi}{3}\frac{\Lambda^4}{m^3} + \ldots \tag{32}$$

This discussion of the relevant integrals in the analysis of the baryon masses, axial couplings, S-wave hyperon decays, and magnetic moments has revealed that LDR is well suited to separating the long distance physics from the high energy portion of the integrals, whereas dim reg includes large contributions from short distance physics. With LDR as a regularization technique for handling loop integrals, we expect to ameliorate problems which have arisen in previous calculations where dim reg produced large loop effects.

3 Decuplet contributions

We now turn to the calculation of the decuplet contributions in the four processes discussed above and give numerical results. In the present investigation we will restrict ourselves to the presentation of loop integrals with an internal decuplet field. (For an evaluation of the octet baryon loops the reader is referred to Ref. [14].)

Up to linear order in the derivative expansion the effective Lagrangian in the heavy baryon formulation is given by

$$\begin{aligned}\mathcal{L}_{\phi BT} &= i\langle\bar{B}[v\cdot D,B]\rangle + D\langle\bar{B}S_\mu\{u^\mu,B\}\rangle + F\langle\bar{B}S_\mu[u^\mu,B]\rangle \\ &\quad -i\bar{T}_\mu v\cdot DT^\mu + \Delta\bar{T}_\mu T^\mu + \frac{\mathcal{C}}{2}\left(\bar{T}_\mu u^\mu B + \bar{B}u^\mu T_\mu\right) \\ &\quad +H\bar{T}_\mu S_\nu u^\nu T^\mu + h_c\bar{T}_\mu h_+ T^\mu,\end{aligned} \tag{33}$$

where $\langle\ldots\rangle$ denotes the trace in flavor space and we have suppressed flavor indices for the decuplet terms. The first three terms contain the kinetic parts of the octet baryon fields $B=(N,\Lambda,\Sigma,\Xi)$[5] and the axial-vector coupling of the mesons to the octet baryons. The covariant derivative D_μ for the baryon fields enters in combination with the four-velocity v_μ and $2S_\mu = i\gamma_5\sigma_{\mu\nu}v^\nu$ is the Pauli-Lubanski spin vector. For the coupling constants D and F we use the values in the $SU(6)$ limit, $D=0.75, F=0.5$, which yield $g_A=1.25$. The Goldstone bosons are summarized in

$$u_\mu = iu^\dagger\nabla_\mu U u^\dagger, \qquad U = u^2 = \exp\Big(\frac{i}{F_\pi}\sum_j \lambda_j\phi_j\Big), \tag{34}$$

[5]We work in the isospin limit of equal up and down quark masses.

with ∇_μ being the covariant derivative of the meson fields and $F_\pi = 92.4$ MeV the pion decay constant. A mass term is still present for the decuplet fields $T = (\Delta, \Sigma^*, \Xi^*, \Omega)$ after transforming to the heavy mass formulation due the octet-decuplet mass splitting Δ which does not vanish in the chiral limit. In the Feynman rules the mass splitting Δ is contained in the decuplet propagator

$$\frac{i}{v \cdot l \ - \Delta + \ i\epsilon} \left(v_\mu v_\nu - g_{\mu\nu} - \frac{4}{d-1} S_\mu S_\nu \right) \tag{35}$$

in d dimensions. The appearance of the mass scale Δ destroys in the case of dim reg the one–to–one correspondence between meson loops and the expansion in small momenta and quark masses. No further complications arise in our case since the strict chiral counting scheme has already been spoilt by introducing the scale Λ.

The sixth term in Eq. (33) is responsible for strong decays of the decuplet into a ground state baryon and a pseudoscalar meson. The pertinent coupling constant $|\mathcal{C}| = 1.5 \pm 0.3$ can be determined from the decay $\Delta \to N\pi$. The seventh term describes the axial-vector coupling of the Goldstone bosons to the decuplet fields, whereas the last term induces weak nonleptonic decay of the decuplet fields. We have defined $h_+ = u^\dagger h u$ where $h_b^a = \delta_2^a \delta_b^3$ is the weak transition matrix. The weak coupling h_c has been determind from a fit to the nonleptonic hyperon decays in Ref. [25], $h_c = (-0.10 \pm 0.19) \times 10^{-7}$ GeV. Although in that work only the leading nonanalytic terms of the chiral loops were taken into account using dim reg, the light quark masses m_u, m_d were set to zero, and the octet-decuplet mass splitting was neglected, we will use the resulting value for h_c due to the lack of any more general study in the literature. The parameter H can, in principle, be determined from a fit to the baryon axial currents as has been done in Ref. [24] up to one-loop order. However, the value for H depends strongly on the chosen cutoff scale and differs considerably from the value in dim reg. One obtains $H = 3.0 \pm 5.0$ in the cutoff scheme, whereas in the case of dim reg a fit yields $H = -3.5$, and thus no reliable estimate for the parameter H can be given in chiral perturbation theory. An independent estimate for H has recently been obtained in large N_c QCD[26] where the authors obtain values in the range $H = -2.25 \ldots -0.66$ from performing different fits to the octet baryon axial currents.

3.1 Baryon masses

The decuplet contributions to the octet baryon masses in dim reg involve the integral given in Eq. (6),

$$\delta M_B = \frac{\mathcal{C}^2}{24\pi^2 F_\pi^2} \sum_{\phi=\pi,K,\eta} \alpha_B^\phi I(m_\phi, \Delta), \tag{36}$$

with the coefficients

$$\begin{array}{llllll} \alpha_N^\pi = 4, & \alpha_N^K = 1, & \alpha_N^\eta = 0 & \alpha_\Sigma^\pi = 2/3, & \alpha_\Sigma^K = 10/3, & \alpha_\Sigma^\eta = 1, \\ \alpha_\Lambda^\pi = 3, & \alpha_\Lambda^K = 2, & \alpha_\Lambda^\eta = 0 & \alpha_\Xi^\pi = 1, & \alpha_\Xi^K = 3, & \alpha_\Xi^\eta = 1. \end{array} \tag{37}$$

By using the following inputs, $m_\pi = 138\text{MeV}, m_K = 495\text{MeV}$ and $m_\eta = \sqrt{(4m_K^2 - m_\pi^2)/3} = 566$ MeV which is the value from the Gell-Mann—Okubo mass relation for the pseudoscalar mesons, $|\mathcal{C}| = 1.5$, the regularization scale $\mu = 1$ GeV, omitting the piece proportional to $\tilde{L}$ and setting $d = 4$ we obtain the dim reg results in Table 1 for the sum of octet and decuplet contributions. Strictly speaking, the d-dependence of the decuplet propagator and the definition of the integral I in Eq. (6) will yield additional analytic contributions in the $d \to 4$ limit due to the divergent piece $\tilde{L}$. However, these are numerically unimportant and can always be removed by appropriate counterterms so that our conclusions remain unchanged for the four processes discussed in this work. In order to keep the formulae and the discussion as simple as possible, we therefore drop the $\tilde{L}$ term and perform the limit $d \to 4$ afterwards.

For comparison we also present in Table 2 the contributions of the octet loops only. In both of the dim reg cases we observe large mass shifts which upset the convergence of the chiral series.

We now repeat the calculation in the LDR scheme by employing the integral from Eq. (11) for the decuplet contributions.[6] Before producing numerical results, however, it is necessary to confirm that LDR preserves chiral invariance and is suited for evaluating loop integrations in HBχpt. To this end, we utilize the large cutoff expansion of the integral in Eq. (14),

$$\delta M_B = \frac{\mathcal{C}^2}{24\pi^2 F_\pi^2} \sum_{\phi=\pi,K,\eta} \alpha_B^\phi \left(-\frac{\pi\Lambda^3}{4} + \frac{3}{4}\Delta\Lambda^2 - \frac{\pi}{8}\Lambda(3\Delta^2 - 2m_\phi^2) \right.$$

[6] For the explicit expressions for the octet loop contributions see Ref. [14] and also Ref. [23].

	Dim.	Λ=300	Λ=400	Λ=500	Λ=600
N	-0.46	-0.04	-0.03	-0.00	0.02
Σ	-1.48	-0.02	0.02	0.07	0.13
Λ	-1.02	-0.03	0.00	0.04	0.10
Ξ	-1.83	-0.01	0.04	0.10	0.18

Table 1: Sum of octet and decuplet contributions to the octet baryon masses in GeV both in dimensional regularization and for various values of the cutoff Λ in MeV.

	Dim.	Λ=300	Λ=400	Λ=500	Λ=600
N	-0.31	0.02	0.03	0.05	0.07
Σ	-0.62	0.03	0.05	0.08	0.12
Λ	-0.69	0.03	0.06	0.09	0.13
Ξ	-1.03	0.04	0.08	0.12	0.17

Table 2: Octet contributions to the octet baryon masses in GeV both in dimensional regularization and for various values of the cutoff Λ in MeV.

$$+\Delta\left(\Delta^2-\frac{3}{2}m_\phi^2\right)\ln\Lambda+\ldots\Big)\,, \tag{38}$$

where the ellipsis denotes terms which remain finite as $\Lambda\to\infty$. The quark mass independent pieces can be absorbed into a renormalization of the octet baryon mass in the chiral limit, M_0, by defining

$$M_0^r=M_0+\frac{5\mathcal{C}^2}{24\pi^2F_\pi^2}\Big[-\frac{\pi}{4}\Lambda^3+\frac{3}{4}\Delta\Lambda^2-\frac{3\pi}{8}\Lambda\Delta^2+\Delta^3\ln\Lambda+\pi\Delta^3\Big]. \tag{39}$$

Note that we have also absorbed a constant term $\pi\Delta^3$ which does not depend on the cutoff scale Λ for reasons to be explained after Eq. (43). The renormalization of M_0 already occurs in dim reg since the introduction of the mass scale Δ spoils the strict chiral power counting[6]. In order to absorb the mass-dependent components in Eq. (38), one must introduce explicitly chiral symmetry breaking counterterms which are proportional to the quark masses. They are given by

$$\mathcal{L}_\chi=b_D\langle\bar{B}\{\chi_+,B\}\rangle+b_F\langle\bar{B}[\chi_+,B]\rangle+b_0\langle\bar{B}B\rangle\langle\chi_+\rangle. \tag{40}$$

The quark mass matrix $\mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$ enters in the combination $\chi_+ = 2B_0(u^\dagger \mathcal{M} u^\dagger + u\mathcal{M}u)$ with $B_0 = -\langle 0|\bar{q}q|0\rangle/F_\pi^2$ the order parameter of the spontaneous symmetry violation. The low-energy constants b_D and b_F are responsible for the splitting of the baryon octet masses at leading order in symmetry breaking. By redefining

$$b_D^r = b_D + \frac{1}{2}c, \quad b_F^r = b_F - \frac{5}{12}c, \quad b_0^r = b_0 - \frac{7}{6}c \tag{41}$$

with

$$c = \frac{\mathcal{C}^2}{24\pi^2 F_\pi^2}\left(\frac{\pi}{4}\Lambda - \frac{3}{2}\Delta \ln \Lambda\right). \tag{42}$$

one can absorb the mass-dependent pieces in Eq. (38) into the low-energy constants. This proves the chiral invariance of the cutoff procedure and shows that the final physics is independent of Λ if performed to all orders since one can remove the Λ dependence by renormalizing the couplings of the counterterms.

Having convinced ourselves from the chiral invariance of the cutoff procedure we can now vary the cutoff in the phenomenologically relevant range 300 MeV $< \Lambda <$ 600 MeV. But we first remove the asymptotic mass-independent components of the function I_Λ by defining

$$I_\Lambda^r = I_\Lambda + \frac{\pi}{4}\Lambda^3 - \frac{3}{4}\Delta\Lambda^2 + \frac{3\pi}{8}\Lambda\Delta^2 - \Delta^3 \ln \Lambda - \pi\Delta^3 \tag{43}$$

since these effects can be absorbed into M_0 and yield misleading indications about the size of the nonanalytic effects in the large cutoff limit. As mentioned before, we have also subtracted a constant term $\pi\Delta^3$ which ensures that the original integral I is not altered much numerically for physically relevant values of the cutoff, 300 MeV $\leq \Lambda \leq$ 600 MeV. Its exact value is not of importance, it has just been chosen in such a way that it compensates to a large extent the contribution from the power and log divergences in Eq. (43); in fact, for $\Lambda \simeq 450$ MeV one obtains $I_\Lambda^r \approx I_\Lambda$ in Eq. (43). This renormalization procedure allows one both to reduce the Λ-dependence of the integrals even for larger values of the cutoff Λ and to conserve approximately the numerical value of the original integral.

The long distance decuplet contribution to the octet baryon masses is then given by

$$\delta M_B = \frac{\mathcal{C}^2}{24\pi^2 F_\pi^2} \sum_{\phi=\pi,K,\eta} \alpha_B^\phi I_\Lambda^r(m_\phi, \Delta) \tag{44}$$

	Dim.	Λ=300	Λ=400	Λ = 500	Λ=600
b_D[GeV^{-1}]	0.447	0.061	0.055	0.047	0.036
b_F[GeV^{-1}]	-0.97	-0.19	-0.17	-0.15	-0.12
$\bar{M}_0$[GeV]	2.71	1.22	1.18	1.13	1.06
$\chi^2/d.o.f.$	0.24	0.56	0.51	0.46	0.42

Table 3: Baryon mass fits with octet plus decuplet loop contributions and calculated with meson couplings having their $SU(6)$ values — D=0.75, F=0.50, $C = -1.5$.

and the corresponding numerical results are shown in Tables 1 and 2. Clearly, the convergence of the chiral series seems to be under control. Moreover, what matters for $SU(3)$ breaking are differences in the loop contributions to the different octet baryon masses, since a constant effect can be absorbed into the common octet mass M_0. This difference is quite small for the cutoff version, whereas the $SU(3)$ breaking effects are significantly larger in dim reg due to the inclusion of spurious short distance physics.

It is also important to consider the quality of the HBχpt fit to experimental data. To accomplish this, we combine the experimental uncertainty of each observable with an overall theoretical uncertainty arising from our neglect of $O(p^4)$ terms (and beyond) in HBχpt. In particular we determine the theoretical uncertainty from the assumption that the expansion parameter of HBχpt is 30%, which is between m_π/m_N and m_K/m_Σ. If our assumption is valid, then our fits will have $\chi^2/d.o.f.$ of order unity or smaller and the HBχpt fits will then be considered successful.

In the case of the baryon masses, which begin at $O(1)$ in HBχpt, our theoretical uncertainty is (1GeV)$\times$(30%)^4, and a very good fit can be found both in lowest order and with chiral loop corrections. The former is simply the traditional $SU(3)$ fit and yields a $\chi^2/d.o.f. = 0.61$ using the parameters

$$b_D = 0.066 \text{ GeV}^{-1}, \;\; b_F = -0.209 \text{ GeV}^{-1}, \;\; \bar{M}_0 = 1.20 \text{ GeV},$$

while with the inclusion of octet plus decuplet (octet only) loops we find the results given in Table 3 (Table 4) respectively. Notice that all of these mass fits satisfy our success criterion.

	Dim.	Λ=300	Λ=400	Λ = 500	Λ=600
b_D[GeV^{-1}]	0.008	0.068	0.070	0.072	0.075
b_F[GeV^{-1}]	-0.61	-0.20	-0.19	-0.17	-0.15
$\bar{M}_0$[GeV]	1.81	1.17	1.14	1.11	1.08
$\chi^2/d.o.f.$	0.07	0.58	0.56	0.53	0.51

Table 4: Baryon mass fits with octet only loop contributions and calculated with meson couplings having their $SU(6)$ values — D=0.75, F=0.50.

3.2 Axial couplings

Next, we consider the decuplet contributions to the octet baryon axial couplings which have the form

$$\begin{aligned} g_A[ij] &= -\frac{C^2}{24\pi^2 F_\pi^2} \sum_{\phi=\pi,K,\eta} \Big(-\frac{4}{3}\rho_{ij}^{\phi} \bar{J}(m_\phi,\Delta) \\ &\quad +\frac{5}{6} H\, \sigma_{ij}^{\phi} J(m_\phi,\Delta) + \frac{3}{2}\gamma_{ij}(\epsilon_i^{\phi}+\epsilon_j^{\phi}) J(m_\phi,\Delta)\Big) . \end{aligned} \tag{45}$$

The last term with the coefficients ϵ_i^{ϕ} takes into account the contributions from intermediate decuplet states to the octet baryon Z–factors. The coefficients read

$$\begin{aligned}
&\rho^{\pi}_{pn} = \tfrac{8}{3}(D+F), && \rho^{K}_{pn} = D+\tfrac{1}{3}F, && \rho^{\eta}_{pn} = 0;\\
&\rho^{\pi}_{\Lambda\Sigma^-} = \tfrac{1}{3\sqrt{6}}(D+6F), && \rho^{K}_{\Lambda\Sigma^-} = \tfrac{1}{\sqrt{6}}(\tfrac{8}{3}D+4F), && \rho^{\eta}_{\Lambda\Sigma^-} = \tfrac{1}{\sqrt{6}}D;\\
&\rho^{\pi}_{\Xi^0\Xi^-} = -\tfrac{1}{3}(D-F), && \rho^{K}_{\Xi^0\Xi^-} = \tfrac{1}{3}(D+5F), && \rho^{\eta}_{\Xi^0\Xi^-} = \tfrac{1}{3}(D+3F);\\
&\rho^{\pi}_{p\Lambda} = -\tfrac{3}{2\sqrt{6}}(\tfrac{11}{3}D+F), && \rho^{K}_{p\Lambda} = -\tfrac{3}{2\sqrt{6}}(D+F), && \rho^{\eta}_{p\Lambda} = 0;\\
&\rho^{\pi}_{\Lambda\Xi^-} = \tfrac{3}{2\sqrt{6}}(\tfrac{1}{3}D-F), && \rho^{K}_{\Lambda\Xi^-} = \tfrac{3}{2\sqrt{6}}(D-F), && \rho^{\eta}_{\Lambda\Xi^-} = \tfrac{1}{\sqrt{6}}D\\
&\rho^{\pi}_{n\Sigma^-} = \tfrac{1}{3}(D+5F), && \rho^{K}_{n\Sigma^-} = \tfrac{1}{6}(D+5F), && \rho^{\eta}_{n\Sigma^-} = -\tfrac{1}{6}(D-3F);\\
&\rho^{\pi}_{\Sigma^0\Xi^-} = \tfrac{1}{3\sqrt{2}}(2D+F), && \rho^{K}_{\Sigma^0\Xi^-} = \tfrac{1}{6\sqrt{2}}(15D+13F), && \rho^{\eta}_{\Sigma^0\Xi^-} = \tfrac{1}{2\sqrt{2}}(D+F);\\
&\rho^{\phi}_{\Sigma^+\Xi^0} = \sqrt{2}\rho^{\phi}_{\Sigma^0\Xi^-}.
\end{aligned} \tag{46}$$

$$
\begin{aligned}
&\sigma^{\pi}_{pn} = 10/9, && \sigma^{K}_{pn} = 2/9, && \sigma^{\eta}_{pn} = 0,\\
&\sigma^{\pi}_{\Lambda\Sigma^-} = 2/(3\sqrt{6}), && \sigma^{K}_{\Lambda\Sigma^-} = 1/(3\sqrt{6}), && \sigma^{\eta}_{\Lambda\Sigma^-} = 0,\\
&\sigma^{\pi}_{\Xi^0\Xi^-} = 1/18, && \sigma^{K}_{\Xi^0\Xi^-} = -2/9, && \sigma^{\eta}_{\Xi^0\Xi^-} = -1/6,\\
&\sigma^{\pi}_{p\Lambda} = -2/\sqrt{6}, && \sigma^{K}_{p\Lambda} = -1/\sqrt{6}, && \sigma^{\eta}_{p\Lambda} = 0,\\
&\sigma^{\pi}_{\Lambda\Xi^-} = 1/\sqrt{6}, && \sigma^{K}_{\Lambda\Xi^-} = 1/\sqrt{6}, && \sigma^{\eta}_{\Lambda\Xi^-} = 0,\\
&\sigma^{\pi}_{n\Sigma^-} = -2/9, && \sigma^{K}_{n\Sigma^-} = -1/9, && \sigma^{\eta}_{n\Sigma^-} = 0,\\
&\sigma^{\pi}_{\Sigma^0\Xi^-} = 2/(9\sqrt{2}), && \sigma^{K}_{\Sigma^0\Xi^-} = 7/(9\sqrt{2}), && \sigma^{\eta}_{\Sigma^0\Xi^-} = 1/(3\sqrt{2}),\\
&\sigma^{\phi}_{\Sigma^0\Xi^-} = \sqrt{2}\sigma^{\phi}_{\Sigma^0\Xi^-}.
\end{aligned}
\tag{47}
$$

$$
\begin{aligned}
&\gamma_{pn} = D + F, && \gamma_{\Lambda\Sigma^-} = 2D/\sqrt{6}, && \gamma_{\Xi^0\Xi^-} = D - F,\\
&\gamma_{p\Lambda} = -(D+3F)/\sqrt{6}, && \gamma_{\Lambda\Xi^-} = -(D-3F)/\sqrt{6}, && \gamma_{n\Sigma^-} = D - F,\\
&\gamma_{\Sigma^0\Xi^-} = (D+F)/\sqrt{2}, && \gamma_{\Sigma^+\Xi^0} = D + F.
\end{aligned}
\tag{48}
$$

$$
\begin{aligned}
&\epsilon^{\pi}_{N} = 1, && \epsilon^{K}_{N} = 1/4, && \epsilon^{\eta}_{N} = 0, && \epsilon^{\pi}_{\Sigma} = 1/6, && \epsilon^{K}_{\Sigma} = 5/6, && \epsilon^{\eta}_{\Sigma} = 1/4\\
&\epsilon^{\pi}_{\Lambda} = 3/4, && \epsilon^{K}_{\Lambda} = 1/2 && \epsilon^{\eta}_{\Lambda} = 0, && \epsilon^{\pi}_{\Xi} = 1/4, && \epsilon^{K}_{\Xi} = 3/4, && \epsilon^{\eta}_{\Xi} = 1/4.
\end{aligned}
\tag{49}
$$

The numerical results for the finite pieces can be found in Tables 5 and 6. We have used the value $H = -2.0$ which was obtained in Ref. [26].

In order to prove the chiral invariance of the cutoff procedure, we extract the mass-independent divergent components of the integrals as Λ goes to infinity, cf. Eqs. (19) and (25), which are compensated by the axial-vector couplings D and F. By defining

$$
\begin{aligned}
D^r &= D + \frac{\mathcal{C}^2}{16\pi^2 F_\pi^2}\left(\frac{4}{3}\left[\frac{4}{3}D + 2F\right]\bar{J}^{\rm div}_{\Lambda}(\Delta) - \frac{5}{18}[H + 9D]J^{\rm div}_{\Lambda}(\Delta)\right)\\
F^r &= F + \frac{\mathcal{C}^2}{16\pi^2 F_\pi^2}\left(\frac{40}{27}D\bar{J}^{\rm div}_{\Lambda}(\Delta) - \frac{5}{18}\left[\frac{5}{3}H + 9F\right]J^{\rm div}_{\Lambda}(\Delta)\right)
\end{aligned}
\tag{50}
$$

with

$$
\begin{aligned}
\bar{J}^{\rm div}_{\Lambda}(\Delta) &= \frac{3}{4}\Lambda^2 - \frac{3\pi}{8}\Lambda\Delta + \Delta^2\ln\Lambda + 2\Delta^2\\
J^{\rm div}_{\Lambda}(\Delta) &= \Lambda^2 + 4\Delta^2\ln\Lambda - \pi\Delta\Lambda + 2\pi\Delta^2
\end{aligned}
\tag{51}
$$

	Dim.	Λ=300	Λ=400	Λ=500	Λ=600
$g_A(\bar{p}n)$	0.63	0.00	0.10	0.19	0.27
$g_A(\bar{p}\Lambda)$	-0.83	-0.06	-0.15	-0.24	-0.32
$g_A(\bar{\Lambda}\Sigma^-)$	0.29	-0.03	0.03	0.07	0.12
$g_A(\bar{n}\Sigma^-)$	-0.14	-0.08	-0.05	-0.02	-0.01
$g_A(\bar{\Lambda}\Xi^-)$	0.57	0.09	0.11	0.14	0.16
$g_A(\bar{\Sigma}^0\Xi^-)$	0.83	0.03	0.13	0.22	0.30

Table 5: The octet and decuplet contributions to the octet baryon axial couplings both in dimensional regularization and for various values of the cutoff Λ in MeV calculated with meson couplings at their $SU(6)$ values — D=0.75, F=0.50, $C = -1.5$.

	Dim.	Λ=300	Λ=400	Λ=500	Λ=600
$g_A(\bar{p}n)$	0.92	0.20	0.28	0.37	0.45
$g_A(\bar{p}\Lambda)$	-0.95	-0.18	-0.27	-0.36	-0.45
$g_A(\bar{\Lambda}\Sigma^-)$	0.62	0.12	0.18	0.23	0.29
$g_A(\bar{n}\Sigma^-)$	0.19	0.04	0.05	0.07	0.09
$g_A(\bar{\Lambda}\Xi^-)$	0.44	0.08	0.12	0.17	0.21
$g_A(\bar{\Sigma}^0\Xi^-)$	1.14	0.21	0.31	0.42	0.53

Table 6: Octet contributions to the octet baryon axial couplings both in dimensional regularization and for various values of the cutoff Λ in MeV calculated with meson couplings at their $SU(6)$ values — D=0.75, F=0.50.

we are able to remove the divergences. Similar to the case of the baryon masses we have also included constant terms Δ^2 and $2\pi\Delta^2$ which compensate effects of the divergent pieces, *e.g.*, $J^{\rm div}_{\Lambda=380{\rm MeV}} \approx 0$, i.e. the numerical value of the original integral is approximately maintained for the cutoff range discussed here. The mass-dependent divergences can also be renormalized by counterterms of higher chiral orders, however the cumbersome calculation involves numerous new counterterms[27] and is not presented here for brevity.

These asymptotic components of the integrals contain only short distance physics and should therefore be removed from our final result. Thus we insert the renormalized functions

$$\bar{J}^r_\Lambda(\Delta) = \bar{J}_\Lambda(\Delta) - \bar{J}^{\rm div}_\Lambda(\Delta), \qquad J^r_\Lambda(\Delta) = J_\Lambda(\Delta) - J^{\rm div}_\Lambda(\Delta) \tag{52}$$

into the expressions for the axial couplings

$$g_A[ij] = -\frac{C^2}{24\pi^2 F_\pi^2} \sum_{\phi=\pi,K,\eta} \Big(-\frac{4}{3}\rho_{ij}^\phi \bar{J}_\Lambda^r(m_\phi,\Delta) + \frac{5}{6} H\, \sigma_{ij}^\phi J_\Lambda^r(m_\phi,\Delta) + \frac{3}{2}\gamma_{ij}(\epsilon_i^\phi + \epsilon_j^\phi) J_\Lambda^r(m_\phi,\Delta)\Big) \tag{53}$$

which yields the numerical results given in Tables 5 and 6 ($H = -2.0$). Different choices for H do not change any of our conclusions, although the actual values for the decuplet contributions are altered. But these changes are not significant since the contributions from the H-term in Eq. (53) are smaller than the remaining decuplet portions and the octet counterparts which do not depend on the size of H. In fact, for $\Lambda = 600$ MeV the contributions from the H-term are negligible. We conclude that the contributions from the loops are smaller than in the dim reg version and indicate that the convergence of the chiral series is under control.

To check the quality of the HBχpt fits to experiment, the method proposed in Subsection 3.1 indicates a theoretical uncertainty of $(30\%)^3$ due to truncation of all HBχpt terms beyond $O(p^3)$. A simple lowest order $SU(3)$ fit, which yields

$$D = 0.80, \;\; F = 0.46, \;\; \chi^2/d.o.f. = 0.4,$$

is certainly a successful fit. The results of including octet plus decuplet (octet only) loop contributions are shown in Table 7 (Table 8) respectively. Notice that LDR maintains the successful fit when loops are included, since $\chi^2/d.o.f. \sim 1$, but dim reg produces a significantly poorer fit. For a complete calculation, there are extra counterterms that should be added and in dim reg these will need to be large in order to cancel the large loop effects indentified here. In LDR, these subleading counterterms will be small and no dramatic cancelations are required.

3.3 S-wave hyperon decays

There exist seven nonleptonic hyperon decays: $\Sigma^+ \to n\pi^+$, $\Sigma^+ \to p\pi^0$, $\Sigma^- \to n\pi^-$, $\Lambda \to p\pi^-$, $\Lambda \to n\pi^0$, $\Xi^- \to \Lambda\pi^-$, and $\Xi^0 \to \Lambda\pi^0$. Isospin symmetry of the strong interactions reduces the number of independent amplitudes to four which we choose to be $\Sigma^+ \to n\pi^+$, $\Sigma^+ \to p\pi^0$, $\Lambda \to n\pi^0$, and $\Xi^0 \to \Lambda\pi^0$.

	Dim.	Λ=300	Λ=400	Λ = 500	Λ=600
D	0.78	0.85	0.77	0.73	0.70
F	-0.27	0.42	0.42	0.40	0.39
$\chi^2/d.o.f.$	2.2	0.8	1.0	1.4	1.8

Table 7: Axial coupling fits with octet and decuplet loop contributions. The weak axial decuplet parameter H was chosen to have the value -2.0, as given in Ref. [26].

	Dim.	Λ=300	Λ=400	Λ = 500	Λ=600
D	0.55	0.70	0.68	0.65	0.63
F	0.31	0.41	0.39	0.37	0.36
$\chi^2/d.o.f.$	3.5	0.5	0.8	1.1	1.4

Table 8: Axial coupling fits with octet loop contributions.

The general form of the decay amplitude is given by

$$\mathcal{A} = \sqrt{Z_a Z_b}[A^{(\text{tree})} + A^{(\text{loop})}], \tag{54}$$

where Z_a and Z_b denote the Z-factors of the incoming and outgoing baryon, respectively, and the amplitude has been decomposed into tree and loop contributions. Let us focus on the decuplet loop contributions. They read

$$A^{(\text{loop})}(B^a \to \tilde{B}^b \pi^i) = \frac{\sqrt{2}\mathcal{C}^2 h_c}{16\pi^2 F_\pi^3} \sum_{\phi=\pi,K,\eta} \kappa(B_i^a)^\phi J(m_\phi, \Delta) \tag{55}$$

with the coefficients

$$\begin{aligned} &\kappa(\Sigma_0^+)^\pi = -\tfrac{1}{9\sqrt{2}}, \quad \kappa(\Sigma_0^+)^K = -\tfrac{1}{18\sqrt{2}}, \quad \kappa(\Lambda_0^0)^\pi = -\tfrac{1}{2\sqrt{3}}, \\ &\kappa(\Lambda_0^0)^K = -\tfrac{1}{4\sqrt{3}}, \quad \kappa(\Xi_0^0)^\pi = \tfrac{1}{4\sqrt{3}}, \qquad \kappa(\Xi_0^0)^K = \tfrac{1}{4\sqrt{3}}, \end{aligned} \tag{56}$$

and all the remaining $\kappa(B_i^a)^\phi$ being zero. In LDR the asymptotic mass-independent component of the integral $J_\Lambda(m, \Delta)$ can be absorbed by two counterterms at lowest order

$$\mathcal{L}_w = d_w \langle \bar{B}\{h_+, B\}\rangle + f_w \langle \bar{B}[h_+, B]\rangle. \tag{57}$$

A least-squares fit to the S-wave hyperon decays at tree level yields $d_w = 0.16 \times 10^{-7}$ GeV and $f_w = -0.41 \times 10^{-7}$ GeV[27]. The parameters d_w and

f_w contribute to $A^{(\mathrm{tree})}$ in the following way:

$$\begin{aligned}
A^{(\mathrm{tree})}(\Sigma^+ \to p\pi^0) &= \frac{1}{\sqrt{2}}\zeta(\Sigma_0^+) = -\frac{1}{2F_\pi}(d_w - f_w), \\
A^{(\mathrm{tree})}(\Lambda \to n\pi^0) &= \frac{1}{\sqrt{2}}\zeta(\Lambda_0^0) = \frac{1}{2\sqrt{6}F_\pi}(d_w + 3f_w), \\
A^{(\mathrm{tree})}(\Xi^0 \to \Lambda\pi^0) &= \frac{1}{\sqrt{2}}\zeta(\Xi_0^0) = \frac{1}{2\sqrt{6}F_\pi}(d_w - 3f_w), \\
A^{(\mathrm{tree})}(\Sigma^+ \to n\pi^+) &= \frac{1}{\sqrt{2}}\zeta(\Sigma_+^+) = 0.
\end{aligned} \tag{58}$$

The divergent pieces of the prefactor $\sqrt{Z_a Z_b}$ must be taken into account as well. We note that the asymptotic pieces for all decays are given by

$$\sqrt{Z_a Z_b}\Big|_{\mathrm{asy}} = -\frac{5\mathcal{C}^2}{32\pi^2 F_\pi^2}\left[\Lambda^2 - \pi\Delta\Lambda + 4\Delta^2 \ln\Lambda\right] \tag{59}$$

so that one can immediately absorb these components into a redefinition of d_w and f_w.

By defining the renormalized parameters

$$\begin{aligned}
d_w^r &= d_w + \frac{\mathcal{C}^2}{32\pi^2 F_\pi^2}(h_c - 5d_w)\left[\Lambda^2 - \pi\Delta\Lambda + 4\Delta^2 \ln\Lambda + 2\pi\Delta^2\right], \\
f_w^r &= f_w + \frac{5\mathcal{C}^2}{96\pi^2 F_\pi^2}(h_c - 3f_w)\left[\Lambda^2 - \pi\Delta\Lambda + 4\Delta^2 \ln\Lambda + 2\pi\Delta^2\right],
\end{aligned} \tag{60}$$

we are able to remove the short distance portion of both $A^{(\mathrm{loop})}$ and the Z-factors. The mass-dependent divergences can be removed in a similar way, but the calculation is rather involved and we refrain from presenting it here. (For the complete renormalization at one-loop order in dim reg see Refs. [27, 28].)

Having assured ourselves about the conservation of chiral invariance for the S-wave hyperon decays also, we calculate numerically the decuplet contributions to the decays by employing the formula

$$\delta\mathcal{A}(B^a \to \tilde{B}^b \pi^i) = \frac{\sqrt{2}\mathcal{C}^2}{16\pi^2 F_\pi^3} \sum_{\phi=\pi,K,\eta} \left[h_c \kappa (B_i^a)^\phi - \frac{1}{2}\zeta(B_i^a)(\epsilon_B^\phi + \epsilon_{\tilde{B}}^\phi)\right] J_\Lambda^r(m_\phi, \Delta). \tag{61}$$

The numerical results are given in Tables 9 and 10 where we used $d_w = 0.16 \times 10^{-7}$ GeV and $f_w = -0.41 \times 10^{-7}$ GeV. The convergence of the chiral series is again improved by using LDR. The results are afflicted with some uncertainty due to the appearance of a new parameter h_c which was obtained with large error bars from a fit to data using dim reg, $h_c = (-0.10 \pm 0.19) \times 10^{-7}$[25]. Changes in h_c affect the numerical results only slightly, in particular for a cutoff of $\Lambda = 600$ MeV the contributions from the h_c term in Eq. (61) are in general negligible. Our conclusions are therefore independent of the particular choice for h_c.

	Dim.	Λ=300	Λ=400	Λ=500	Λ=600
$\delta\mathcal{A}(\Lambda_0^0)$	1.12	0.42	-0.09	-0.53	-0.91
$\delta\mathcal{A}(\Xi_0^0)$	-3.75	-0.70	-0.15	0.30	0.70
$\delta\mathcal{A}(\Sigma_0^+)$	3.70	0.75	0.25	-0.16	-0.52

Table 9: Shown are the octet and decuplet contributions to the S-wave hyperon decays in units of 10^{-7} both in dimensional regularization and for various values of the cutoff Λ in MeV.

	Dim.	Λ=300	Λ=400	Λ=500	Λ=600
$\delta\mathcal{A}(\Lambda_0^0)$	-1.87	-0.32	-0.50	-0.68	-0.87
$\delta\mathcal{A}(\Xi_0^0)$	1.00	0.18	0.28	0.38	0.47
$\delta\mathcal{A}(\Sigma_0^+)$	-0.72	-0.12	-0.19	-0.26	-0.33

Table 10: Octet contributions to the S-wave hyperon decays in units of 10^{-7} both in dimensional regularization and for various values of the cutoff Λ in MeV.

The omission of $O(p^4)$ HBχpt contributions leads to a fractional theoretical uncertainty of $(30\%)^3 = 0.03$. Combining this with the experimental uncertainties leads to the following lowest order $SU(3)$ fit (here d_w, f_w are in units of 10^{-7} GeV):

$$d_w = 0.18, \;\; f_w = -0.41, \;\; \chi^2/d.o.f. = 1.2.$$

The effect of including chiral loop corrections with octet plus decuplet (octet only) states is shown in Table 11 (Table 12). In the case of LDR, the $\chi^2/d.o.f.$

	Dim.	Λ=300	Λ=400	Λ = 500	Λ=600
d_w	-1.65	0.26	0.22	0.20	0.18
f_w	0.55	-0.52	-0.42	-0.36	-0.32
$\chi^2/d.o.f.$	502	1.9	2.2	2.4	2.6

Table 11: S-wave hyperon decay fits with octet plus decuplet loop contributions and calculated with lowest order meson couplings having their $SU(6)$ values — D=0.75, F=0.50, $C = -1.5$. Here the decuplet weak decay parameter $h_c = -0.1 \times 10^{-7}$ was used, as recommended in Ref. [25], and d_w and f_w are in units of 10^{-7}.

	Dim.	Λ=300	Λ=400	Λ = 500	Λ=600
d_w	0.20	0.19	0.20	0.20	0.20
f_w	-0.28	-0.38	-0.36	-0.35	-0.34
$\chi^2/d.o.f.$	11	2.0	2.6	3.4	4.3

Table 12: S-wave hyperon decay fits with octet loop contributions and calculated with lowest order meson couplings having their $SU(6)$ values — D=0.75, F=0.50. d_w and f_w are in units of 10^{-7}.

values are close to being acceptable, and the fit becomes perfectly fine when the $O(p^3)$ counterterms, omitted here, are added in. In the dim reg case, addition of the decuplet loops severely worsens the fit and the missing counterterms must be assigned huge values if the fit is to be repaired. This is another signal of the breakdown of convergence for dim reg.

3.4 Magnetic moments

Finally, we turn to the calculation of the magnetic moments. The decuplet loop contributions to the octet baryon magnetic moments have been found to improve the convergence of the chiral series in dim reg [29]. They have the structure

$$\delta\mu_i = \frac{M_0 \mathcal{C}^2}{144\pi^2 F_\pi^2} \sum_{\phi=\pi,K} \lambda_i^\phi K(m_\phi, \Delta) \tag{62}$$

with the coefficients

$$\begin{array}{llll} \lambda_p^\pi = 4, & \lambda_p^K = -1, & \lambda_n^\pi = -4, & \lambda_n^K = -2, \\ \lambda_{\Sigma^+}^\pi = -1, & \lambda_{\Sigma^+}^K = 4, & \lambda_{\Sigma^-}^\pi = 1, & \lambda_{\Sigma^-}^K = 2, \\ \lambda_{\Sigma^0}^\pi = 0, & \lambda_{\Sigma^0}^K = 3, & \lambda_{\Xi^-}^\pi = 2, & \lambda_{\Xi^-}^K = 1, \\ \lambda_{\Xi^0}^\pi = -2, & \lambda_{\Xi^0}^K = -4, & \lambda_{\Lambda^-}^\pi = 0, & \lambda_{\Lambda^-}^K = -3, \\ \lambda_{\Lambda\Sigma^0}^\pi = 2\sqrt{3}, & \lambda_{\Lambda\Sigma^0}^K = \sqrt{3}. & & \end{array} \tag{63}$$

Omitting the terms in $K(m, \Delta)$ proportional to $\tilde{L}$ we obtain the numerical results in Tables 13 and 14, where we used $M_0 = 767$ MeV[12].

	Dim.	Λ=300	Λ=400	Λ=500	Λ=600
$\delta\mu_p$	-2.43	-0.71	-0.82	-0.91	-1.00
$\delta\mu_n$	1.36	0.45	0.46	0.46	0.47
$\delta\mu_\Lambda$	1.88	0.31	0.36	0.42	0.47
$\delta\mu_{\Sigma^+}$	-3.83	-0.79	-0.96	-1.11	-1.25
$\delta\mu_{\Sigma^0}$	-1.88	-0.31	-0.36	-0.42	-0.47
$\delta\mu_{\Sigma^-}$	0.07	0.18	0.24	0.28	0.31
$\delta\mu_{\Sigma\Lambda}$	-1.79	-0.44	-0.47	-0.50	-0.54
$\delta\mu_{\Xi^-}$	1.21	0.26	0.37	0.46	0.54
$\delta\mu_{\Xi^0}$	3.62	0.61	0.72	0.83	0.93

Table 13: Shown are the octet and decuplet contributions to the octet baryon magnetic moments both in dimensional regularization and for various values of the cutoff Λ in MeV.

Before comparing with the cutoff version, we prove the chiral invariance of the cutoff procedure. There exist two counterterms at lowest order[13]

$$\mathcal{L} = -\frac{i}{4M_0} b_6^D \langle \bar{B}[S^\mu, S^\nu]\{f_{\mu\nu}^+, B\}\rangle - \frac{i}{4M_0} b_6^F \langle \bar{B}[S^\mu, S^\nu][f_{\mu\nu}^+, B]\rangle \tag{64}$$

where

$$f_{\mu\nu}^+ = u^\dagger f_{\mu\nu} u + u f_{\mu\nu} u^\dagger = -2eQ[\partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu] + \mathcal{O}(\phi^2) \tag{65}$$

is the field strength tensor for the photon field $\mathcal{A}_\mu$ and $Q = \frac{1}{3}\mathrm{diag}(2, -1, -1)$ the quark charge matrix. By introducing the renormalized parameters

$$b_6^{D\,;\,r} \;=\; b_6^D - \frac{M_0 \mathcal{C}^2}{48\pi^2 F_\pi^2}(\pi\Lambda - 6\Delta \ln\Lambda - 3\pi\Delta),$$

	Dim.	Λ=300	Λ=400	Λ=500	Λ=600
$\delta\mu_p$	-2.29	-0.67	-0.81	-0.93	-1.03
$\delta\mu_n$	0.65	0.35	0.41	0.44	0.47
$\delta\mu_\Lambda$	1.31	0.25	0.32	0.39	0.44
$\delta\mu_{\Sigma^+}$	-3.16	-0.73	-0.91	-1.07	-1.21
$\delta\mu_{\Sigma^0}$	-1.31	-0.25	-0.32	-0.39	-0.44
$\delta\mu_{\Sigma^-}$	0.54	0.23	0.27	0.30	0.32
$\delta\mu_{\Sigma\Lambda}$	-1.18	-0.35	-0.43	-0.49	-0.54
$\delta\mu_{\Xi^-}$	1.56	0.31	0.39	0.47	0.54
$\delta\mu_{\Xi^0}$	2.70	0.51	0.65	0.78	0.90

Table 14: Octet contributions to the octet baryon magnetic moments both in dimensional regularization and for various values of the cutoff Λ in MeV.

$$b_6^{F\,;\,r} \;=\; b_6^F\,, \tag{66}$$

we are able to remove the asymptotic pieces of $K_\Lambda(m,\Delta)$. Our final result reads

$$\delta\mu_i = \frac{M_0\mathcal{C}^2}{144\pi^2 F_\pi^2}\sum_{\phi=\pi,K}\lambda_i^\phi K_\Lambda^r(m_\phi,\Delta) \tag{67}$$

with

$$K_\Lambda^r(m,\Delta) = K_\Lambda(m,\Delta) - \frac{\pi}{3}\Lambda + 2\Delta\ln\Lambda + \pi\Delta \tag{68}$$

where the constant piece $\pi\Delta$ compensates approximately the numerical effects of the Λ-dependent terms in the region 300 MeV $< \Lambda <$ 600 MeV, and the final results can be found in Tables 13 and 14. Again, the loop contributions yield smaller results in LDR than in dim reg with the only exception being $\delta\mu_{\Sigma^-}$ where there happen to be large cancelations between the octet and decuplet contributions in dim reg.

We can study the goodness of fit as done above for the other observables. In this case the HBχpt expressions begin at $O(p^2)$ so the relative theoretical error is $(30\%)^2$. A lowest order $SU(3)$ fit yields

$$b_6^D = 2.46,\;\; b_d^F = 1.76,\;\; \chi^2/d.o.f. = 2.3$$

while the effects of loop corrections with octet plus decuplet (octet only) states are shown in Table 15 (Table 16).

	Dim.	Λ=300	Λ=400	Λ = 500	Λ=600
b_6^d	6.15	3.27	3.35	3.43	3.51
b_6^F	3.67	2.23	2.35	2.44	2.53
$\chi^2/d.o.f.$	39	5.1	3.6	2.4	1.5

Table 15: Magnetic moment fits with octet and decuplet loop contributions and calculated with lowest order meson couplings having their $SU(6)$ values — D=0.75, F=0.50, $C = -1.5$.

	Dim.	Λ=300	Λ=400	Λ = 500	Λ=600
b_6^d	4.92	3.11	3.26	3.38	3.49
b_6^F	3.67	2.23	2.35	2.44	2.53
$\chi^2/d.o.f.$	23	5.0	4.0	2.9	2.0

Table 16: Magnetic moment fits with octet loop contributions and calculated with lowest order meson couplings having their $SU(6)$ values — D=0.75, F=0.50.

All of the $\chi^2/d.o.f.$ values are noticeably larger than unity, suggesting that higher orders of HBχpt are required. This is not too surprising since leading order for the magnetic moments is $O(p^2)$ rather than $O(p)$. It should be noted, however, that LDR maintains the moderate $\chi^2/d.o.f.$ that was obtained from the tree-level fit, whereas the huge loop effects in dim reg destroy the fit completely.

4 Conclusions

In this work we have investigated the use of long distance regularization, LDR, in baryon χpt with decuplet fields. For practical purposes we employed a dipole regulator as a cutoff, however the specific shape of the cutoff is irrelevant as long as it preserves chiral symmetry.

We extended our previous LDR methods to include the case of the decuplet by examining the loop contributions to the octet baryon masses, axial couplings, S-wave nonleptonic hyperon decays and magnetic moments. In each of these four cases we were able to show that all possible power divergences (modulo logs) in the cutoff could be removed by redefining the coupling constants of the Lagrangian. This verifies the chiral consistence of the cutoff procedure — a chiral expansion *can* be carried out to the order

we are working. By choosing a finite cutoff in the range 300 MeV $< \Lambda <$ 600 MeV, which corresponds to distances of about the baryon radius, we are able to separate the low momentum *long distance* physics from the high momentum *short distance* portion of the integrals. The key point here is that it is this long distance component, whose form is determined simply by the basic chiral symmetry of the underlying QCD Lagrangian, which can be trusted. Once one is dealing with interactions at a scale of order the size of the baryon or smaller, structure issues surely significantly modify the form and results of the loop integrals. Without a detailed model, we do not know the form that such changes take, so our approach is to use a cutoff scheme, wherein such short distance effects are suppressed. While this approach is a bit brute force in nature, the results are quite consistent with detailed model calculations such as found in a Bethe-Salpeter approach[18] or in the cloudy bag model[17]. In any case it is a general property that within this procedure chiral invariance is maintained but smaller loop effects are found than in the conventional dim reg method, leading in general therefore to less damage to the traditional $SU(3)$ fits to these processes. We studied the goodness of fit in each case and found that dim reg at one chiral loop often gave a substantially degraded fit, which was improved by the LDR procedure.

This work also shed some light on the interplay between the chiral and $1/N_c$ limits, since octet and decuplet fields become degenerate as $N_c \to \infty$. Indeed the success of $1/N_c$ arguments in other contexts *demands* the addition of decuplet effects to these and other baryonic applications. In general, we found that such inclusion could be handled straightforwardly and that the resulting changes to our previous octet-only loop results were relatively minor.

Moreover, it has been emphasized that LDR ideas are useful when extrapolating lattice data down to physical pion masses[19] and the present investigation may help to compare with such chiral extrapolations of lattice calculations which are usually done with a pion mass in the vicinity of 600 MeV or so. Our study is a first step towards comparing and combining these different techniques and will hopefully improve the communication between the involved communities of physicists.

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