Study on Vertical Structure of Thin Accretion Disks with Anomalous Viscosity

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Abstract: The stationary vertical structure of a thin disk around a black hole is considered with the anomalous magnetic viscosity[1]. These stationary vertical structures, including distributions of the pressure, temperature, radiative energy flux, mass density and radial velocities with different viscosities, are numerically examined. It is shown that the consideration of anomalous viscosity is necessary as all the physical variables are more close to the real conditions.

Key words: anomalous viscosity, accretion disks, vertical structure

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利用反常粘滞研究薄吸积盘的垂向结构

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［摘要］　利用反常粘滞[1]研究了黑洞周围薄吸积盘的静态垂向结构.数值模拟得到了垂向结构上压力,温度,径向能流,物质密度和不同粘滞下径向速度的分布,发现考虑反常粘滞使各个物理量的分布更接近实际情况.

［关键词］　反常粘滞,吸积盘,垂向结构

0 Introduction

The two-dimensional ($r-z$, in a cylindrical coordinate system) structure of thin stationary Keplerian disk is usually studied by separating the radial from the vertical direction. In this approximation, the radial structure of the disk is deduced in terms of vertically averaged quantities, like the surface density Σ[2,3]. Hydrostatic vertical models[2,4] are then constructed to obtain the variation of density and temperature with height. An implicit assumption in these models is a strictly Keplerian rotation profile that does not vary with height, i. e., $\Omega = \Omega(r)$. These disk models are then referred to as hydrostatic thin accretion disk models.

It is well known that astrophysical accretion disks are hydrodynamic ones with anomalous viscosity, the viscosity of matter in disk results in a factor of more than 10^8 amplification to the microscopic viscosity. If we can't understand anomalous viscosity, we can only get a kind of phenomenological disk structure. This structure may greatly deviate from the actual structure. A recent work[1] has shown that the magnetic fields self-generated by the transverse plasmons are modulationally unstable in the Lyapunov sense, leading to a self-similar collapse of the magnetic flux and resulting in local magnetic structures. Highly spatially intermittent flux is responsible for generating the anomalous viscosity. Therefore, large-scale magnetic fields heat the outer parts of the disk, while small-scale intermittent flux gives rise to viscosity with r and T dependence. It is these two magnetic processes that de-

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termine the stationary structures of the accretion disks.

1 Basic Equations for a Thin Disk

The full set of equations governing the vertical structure of a Keplerian disk without magnetic field[5] is

$$\frac{\partial p}{\partial z} = -\rho\Omega^2 z,$$

$$\frac{\partial F}{\partial z} = \frac{9}{4}\rho\nu\Omega^2,$$

$$F = -\frac{16\sigma T^3}{3\kappa\rho}\frac{\partial T}{\partial z},$$

$$\frac{\rho u_r}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r^2\Omega) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho\nu r^3\frac{\mathrm{d}\Omega}{\mathrm{d}r}\right).$$

Our notation is standard: p is the pressure, ρ is the mass density, Ω is the Keplerian value, i. e. $\Omega = \left(\frac{GM}{r^3}\right)^{1/2}$, F is the radiative energy flux, T is the temperature, ν is the kinematic viscosity, σ is the Stefan-Boltzmann constant, and κ is the Rosseland mean opacity.

These equations must be supplemented by constitutive relationa specifying the equation of state, the opacity, the viscosity. We will adopt the ideal-gas equation of state, $p = \frac{\kappa_B\rho T}{\mu m_H}$, where κ_B is Boltzmann constant, μ is mean molecular mass, and m_H is the mass of the hydrogen atom, and a generic power-law opacity, $\kappa = C_\kappa\rho^x T^y$, where C_κ, x, and y are constants. Here we focus on the Thomson scattering opacity[5] ($x = y = 0, C_\kappa \approx 0.33\ \mathrm{cm^2 g^{-1}}$).

The viscosity is less certain before, and has usually been chosen the form of standard prescription[5] $\rho\nu = \frac{\alpha p}{\Omega}$, where α is a dimensionless constant. This is the well-known alpha viscosity prescription[2]. It is equivalent to assuming that the viscous stress scales with the pressure of disk. In this paper we will choose a new form of viscosity which is introduced in the next section.

Several terms in the equations have been omitted on the grounds that the disk is thin and the solution should be stationary on the dynamical timescale (although not necessary on the viscous timescale). The radial pressure gradient, the vertical variation of radial gravity, and the inertial terms associated with the meridional flow have been neglected as usual. However, enough terms have been retained to determine the profile of radial velocity in the absence of a magnetic field.

The case of $\frac{\partial(\rho\nu)}{\partial r}$ is more problematic. This viscous term is retained in the angular momentum equation because it partially determines the radial velocity, which in turn causes radial advection of magnetic flux and also affects the shape of the field lines, at least when the field is weak. This term may be approximated[5] by arguing that $\rho\nu \approx \frac{\bar{\nu}\Sigma}{H}f\left(\frac{z}{H}\right)$ in the neighborhood of radius under consideration, where $\bar{\nu}\Sigma = \int_{-H}^{H}\rho\nu\mathrm{d}z$ is the vertical integrated dynamic viscosity, H is the semi-thickness, and f is an undetermined dimensionless function.

Under this assumption,

$$\frac{\partial\ln(\rho\nu)}{\partial\ln r} = \frac{\partial\ln(\bar{\nu}\Sigma)}{\partial\ln r} - \left[1 + \frac{\partial\ln(\rho\nu)}{\partial\ln z}\right]\frac{\partial\ln H}{\partial\ln r}$$

the vertical derivative is available as part of local solution, while $\frac{\partial\ln(\bar{\nu}\Sigma)}{\partial\ln r}$ and $\frac{\partial\ln(H)}{\partial\ln r}$ appear as additional dimensionless parameters, which can be estimated from the well-known behavior of the steady, nonmagnetized solution[2]. In the limit of a nonmagnetized disk, this prescription predicts the radial velocity in the disk as

$$u_r = -\frac{3\nu}{2r}\left\{1 + 2\frac{\partial\ln(\bar{\nu}\Sigma)}{\partial\ln r} - 2\left[1 + \frac{\partial\ln(\rho\nu)}{\partial\ln z}\right]\frac{\partial\ln H}{\partial\ln r}\right\}.$$

The dependent variables may be taken as p, T, F, from which the variables ρ and u_r are algebraically determined. The solution should be symmetrical about the mid-plane, such that $F=0$ at $z=0$.

At the upper surface, the solution should properly be matched to an atmospheric model that the photosphere $z=H$ where the gas becomes optically thin. $T=T_s=T_{\rm eff}$, $F=F_s=\sigma T_s^4$, where the subscript "s" denotes a surface value.

Treating the above as a dimensional analysis, define characteristic physical units

$$U_H = \Sigma^{1/3}\Omega^{-5/6}\left(\frac{\mu m_H}{\kappa}\right)^{-2/3}\left(\frac{16\sigma}{3C_\kappa}\right)^{-1/6},$$

$$U_\rho = \frac{\Sigma}{U_H},\ U_p = \Sigma\Omega^2 U_H,\ U_F = \Sigma\Omega^3 U_H^2,$$

$$U_T = \left(\frac{\mu m_H}{\kappa}\right)\Omega^2 U_H^2,\ U_\nu = \Omega U_H^2,\ U_{u_r} = \Omega U_H.$$

Note that the above expression for U_H can be obtained from the condition

$$U_F = \frac{16\sigma U_T^3}{3C_\kappa U_\rho^x U_T^y U_\rho}\frac{U_T}{U_H},$$

which is a dimensional analysis of the definition of the radiative flux and here we focus on the Thomson scattering opacity ($x=y=0, C_\kappa \approx 0.33\ \mathrm{cm^2 g^{-1}}$). There are two small dimensionless parameters in the problem,

$$\varepsilon = \frac{U_H}{r},\ \delta = \frac{U_F}{\sigma U_T^4},$$

Evidently ε is a characteristic measure of the angular semi-thickness of the disk, while δ is an inverse measure of the total optical thickness.

Then introduce dimensionless variables according to

$$z = z_* U_H,\ \rho = \rho_* U_\rho,\ H = H_* U_H,\ F = F_* U_F,$$
$$p = p_* U_p,\ T = T_* U_T,\ \nu = \nu_* U_\nu,\ u_r = u_{r*} U_{u_r}.$$

The dimensionless equations of vertical structure are then

$$\frac{\mathrm{d}p_*}{\mathrm{d}z_*} = -\rho_* z_*,\ \frac{\mathrm{d}F_*}{\mathrm{d}z_*} = \frac{9}{4}\rho_*\nu_*,\ F_* = -\frac{16T_*^3}{3\rho_*}\frac{\mathrm{d}T_*}{\mathrm{d}z_*},\ p_* = \rho_* T_*,$$

$$u_{r*} = -\frac{3}{2}\varepsilon\nu_*(1+2D_{\nu\Sigma}) + 3\varepsilon\rho_* D_H\frac{\partial(\rho_*\nu_* z_*)}{\partial z_*}.$$

where

$$D_{\nu\Sigma} = \frac{\partial\ln(\bar{\nu}\Sigma)}{\partial\ln r},\quad D_H = \frac{\partial\ln H}{\partial\ln z}.$$

The boundary conditions are

$$F_*(0) = 0,\ p_*(H_*) = \frac{1}{8}\delta H_*,\ \delta F_*(H_*) = T_*(H_*)^4.$$

Finally, the definition of the surface density Σ requires

$$\int_{-H_*}^{H_*}\rho_*\mathrm{d}z_* = 1.$$

These equations have a unique solution that must be obtained numerically. When the solution is found, the optical depth at the midplane can be determined from

$$\tau_c = \frac{2}{3} + \int_0^H \kappa\rho\mathrm{d}z = \frac{2}{3} + \frac{16}{3}\int_0^{H_*}\rho_*^{1+x}T_*^y\mathrm{d}z_*.$$

2 Anomalous Magnetic Viscosity

To investigate the structure of the disk, we must know the anomalous viscosity η_m. The anomalous viscosity has disturbed astrophysicists for a long time and is a rather difficult problem. A solution has been suggested recently by Li and Zhang[1]. It is known that the transverse plasmon fields are modulationally unstable in the Lyapunov sense, leading to a self-similar collapse of the magnetic flux. Such a collapsing magnetic flux instability is analyzed in the case of kinetic plasma physics. Based on Vlasov equations and Maxwell equations, the collapsing feature of the self-generated magnetic field from transverse plasmons is investigated on a rather small scale of the motion or electric current in accretion disks. As a result, the anomalous viscosity occurs due to the intermittent flux.

Here, we briefly introduce the calculation processes of the magnetic kinematic viscosity. The magnetic viscous stress tensor by the self-generated spatially intermittent flux is

$$t_{ij}^{m} = \frac{\langle \delta B_i \delta B_j - \frac{1}{2}\delta_{ij}(\delta \boldsymbol{B})^2 \rangle}{4\pi} \tag{1}$$

The work done on the volume $d\boldsymbol{r}$, per unit time, by the stress is $-(\partial t_{ij}^{m}/\partial x_j)v_i d\boldsymbol{r}$, which contributes to viscous dissipation, resulting in the change of entropy with the volume:

$$\dot{S} = \int \frac{1}{T}\left(-\frac{\partial t_{ij}^{m}}{\partial x_i}v_i\right)d\boldsymbol{r} = \int t_{ij}^{m}\frac{V_{ij}}{T}d\boldsymbol{r},$$

where

$$V_{ij} \equiv \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right).$$

This corresponds to the linear relation of the 'flow' t_{ij}^{m} and the 'force' $-\frac{V_{ij}}{T}$ [6]:

$$t_{ij}^{m} = \gamma_{ij;lk}\frac{V_{lk}}{T} = \eta_{ij;lk}V_{lk} \tag{2}$$

or

$$t_{ij}^{m} = \eta_m\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial v_k}{\partial x_k}\right)$$

with

$$\eta_{ij;lk} = \eta_m(\delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl} - \frac{2}{3}\delta_{ij}\delta_{lk}) \tag{3}$$

where η_m is magnetic viscosity of the intermittent flux defined through the kinetic coefficient as $\eta_m = \eta_{lk;lk}(l \neq k$, no summation over repeated $lk)$.

In the accretion disks, the shear stress tensor V_{lk} has the dominant $r\varphi$-component and then (2) becomes

$$t_{ij}^{m} = \eta_{ij;r\varphi}\frac{1}{2}\left(\frac{\partial v_\varphi}{\partial r} - \frac{v_\varphi}{r}\right) = \eta_{ij;r\varphi}\frac{1}{2}r\frac{\partial \Omega(r)}{\partial r} \tag{4}$$

with $\Omega = v_\varphi/r$.

Furthermore, it yields from (1), (3) and (4), that

$$\frac{1}{4\pi}|\langle \delta B_r \delta B_\varphi \rangle| = \eta_m \frac{1}{2}r\left|\frac{\partial \Omega(r)}{\partial r}\right|.$$

It is assumed that self-generated magnetic fields by the transverse model are statistically isotropic on the scales of interest:

$$\frac{1}{3}\langle(\delta \boldsymbol{B})^2\rangle \approx \langle \delta B_r \delta B_\varphi \rangle \tag{5}$$

In this case, (4)(5), combined with a self-similar solution for the collapse magnetic flux[1]

$$\frac{\delta \boldsymbol{B}_{\max}^2}{8\pi} = \frac{16}{9}\mu c^2 \rho \frac{(|\boldsymbol{E}_0|^2)^{4/3}}{\alpha^{2/3}},$$

mean that the magnetic kinematic viscosity ν_m can be read as

$$\nu_m = 7 \times 10^{-12}\frac{c^2}{r|\partial \Omega(r)/\partial r|}T_0^{2/3}\left(\frac{T_e}{T_0}\right)^{2/3}\left(\frac{\overline{W}_0^P}{10^{-5}}\right)^{4/3}(\mathrm{cm}^2\mathrm{s}^{-1}).$$

For accretion disks, the magnetic kinematic viscosity ν_m would result in a factor of more than 10^8 amplification to microscopic viscosity ν_μ [1].

3 Numerical Investigation and Discussion

The dimensionless ODEs are integrated from the photosphere $z_* = H_*$ to the midplane $z_* = 0$. The dependent variables are p_*, T_*, F_*. The values of $H_*, F_*(H_*)$ are guessed and then adjusted by Newton-Raphson iteration to match the symmetry conditions on the midplane.

The form of the solution for p_*, T_*, and F_* depends only on the dimensionless parameter δ (for a given opacity law). If u_{r*} is required, there are further dependence on $D_{\nu\Sigma} = 0$ and $D_H = 21/20$ [2]. For the purposes of illustration, we consider a location at 1 000 Schwarzschild radii from a black hole of mass 10 $M_\odot$, i. e, $r = 2.95 \times 10^9$ cm. For a surface density $\Sigma = 10^4$ g/cm^2, and assuming $\mu = 0.6$, we find illustrative values $U_H = 6.34 \times 10^7$ cm, $U_\rho = 1.58 \times 10^{-4}$ g/cm^3, $U_p = 3.27 \times 10^5$ N, $U_T = 1.50 \times 10^6$ K, $U_F = 4.70 \times 10^{10}$ J cm^{-2}s^{-1}. Then $\varepsilon = 2.15 \times 10^{-2}$ and $\delta = 1.62 \times 10^{-3}$.

The numerically determined solution is shown from Fig. 1 to Fig. 4 (the dash-dot lines refer to the old form of viscosity, the continuous lines refer to the new form of viscosity). It has a dimensionless photospheric height of $H_* = 1.70$ and an optical depth at the midplane of $\tau_c = \frac{2}{3} + \frac{8}{3}\delta^{-1}$. Thus, our illustrative values correspond to $H/r = 0.0364$ and $\tau_c = 1\,650$, representing a geometrically thin and optically thick disk.

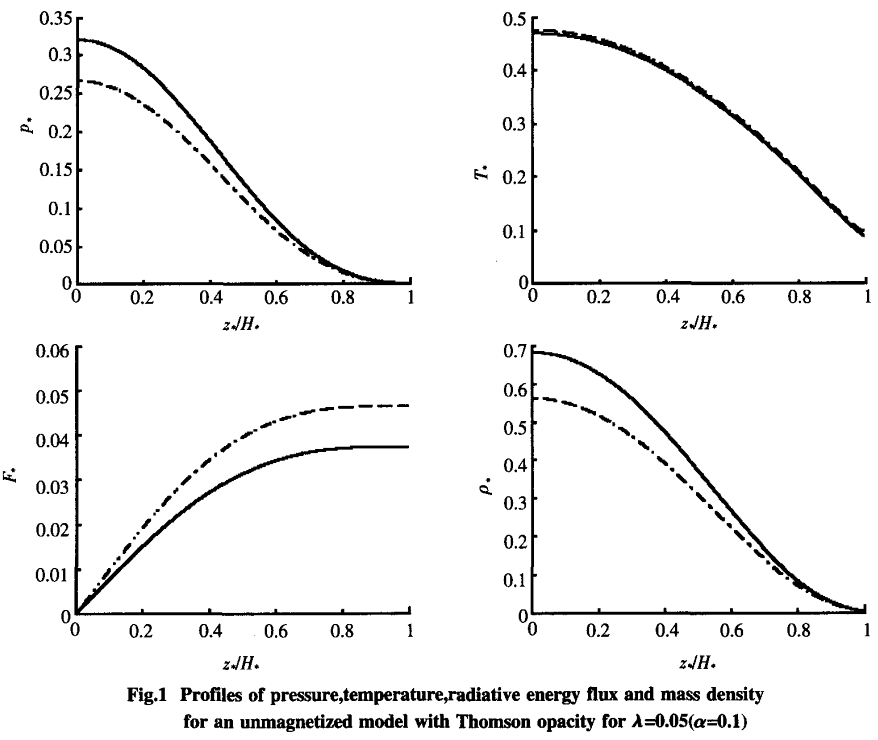

Fig.1 Profiles of pressure, temperature, radiative energy flux and mass density for an unmagnetized model with Thomson opacity for λ=0.05(α=0.1)

Now we choose the new form of anomalous viscosity,

$$\rho\nu = \frac{\alpha p}{\Omega} = \rho\lambda\left(\frac{r}{r_0}\right)^{3/2}\left(\frac{T}{T_0}\right)^{2/3},$$

we have

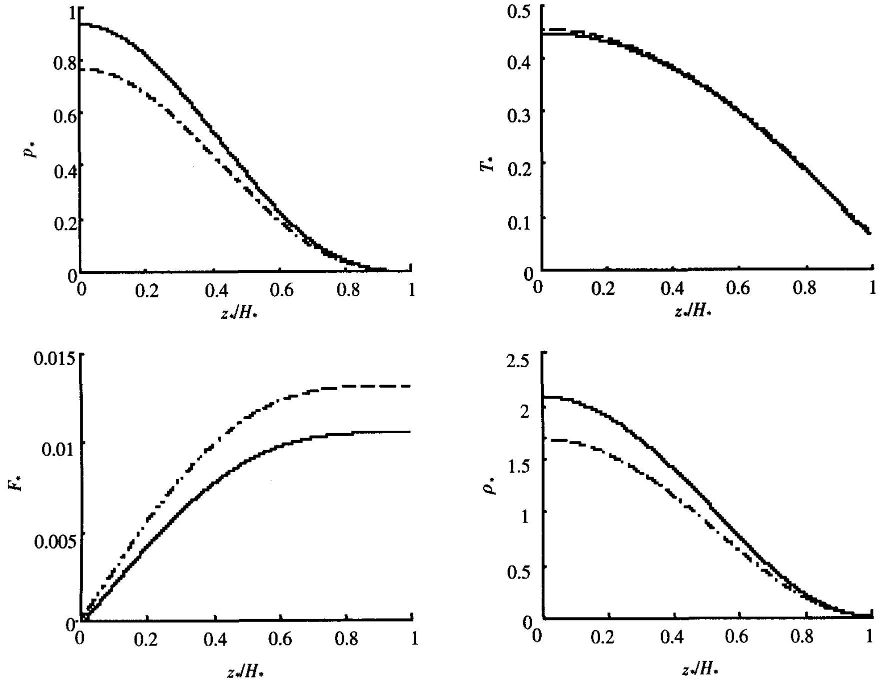

Fig.2 Profiles of pressure, temperature, radiative energy flux and mass density for an unmagnetized model with Thomson opacity for λ=0.005(α=0.01)

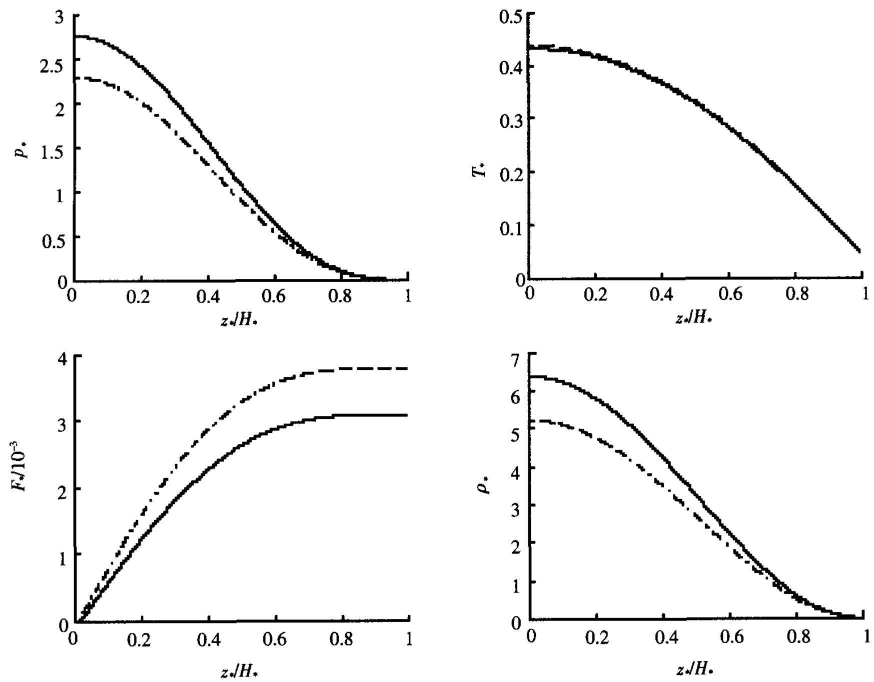

Fig.3 Profiles of pressure, temperature, radiative energy flux and mass density for an unmagnetized model with Thomson opacity for λ=0.000 5(α=0.001)

$$\lambda = \frac{\dfrac{\alpha p}{\Omega \rho}}{\left(\dfrac{r}{r_0}\right)^{3/2}\left(\dfrac{T}{T_0}\right)^{2/3}}.$$

The dimensionless form of the λ is as follows

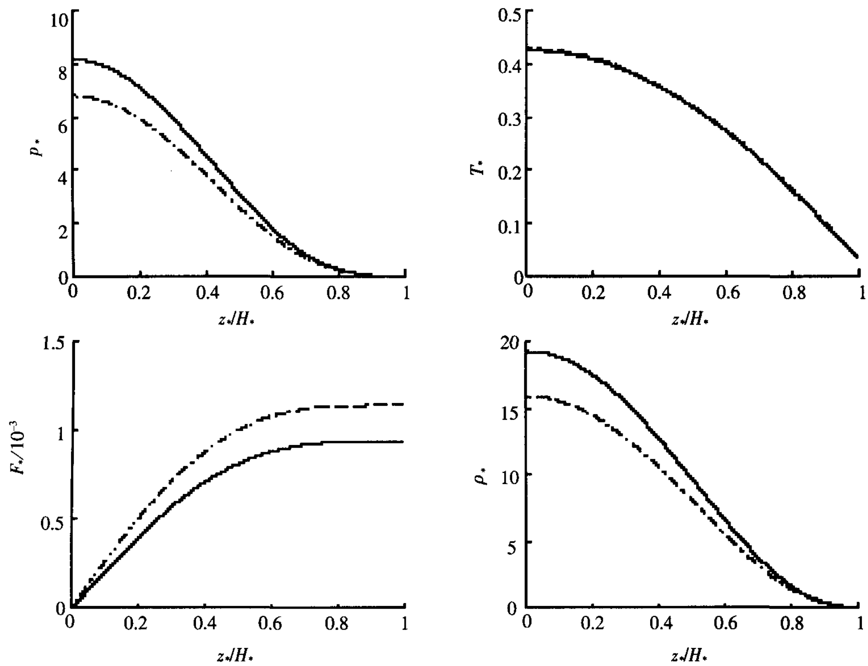

Fig.4 Profiles of pressure, temperature,radiative energy flux and mass density for an unmagnetized model with Thomson opacity for λ=0.000 05(α=0.000 1)

$$\tilde{\lambda} = \alpha T_*^{1/3},$$

so the form of the anomalous viscosity is then

$$\nu = \lambda\left(\frac{r}{r_0}\right)^{3/2}\left(\frac{T}{T_0}\right)^{2/3},$$

the dimensionless form of the ν is as follows

$$\nu_* = \tilde{\lambda} T_*^{2/3}.$$

In the calculation processes, we take $\tilde{\lambda} = 0.05, 0.005, 0.000\,5, 0.000\,05$ and it is equivalent to $\alpha = 0.1, 0.01, 0.001, 0.000\,1$, which satisfies the α-model and this is more reasonable in many models.

For new viscosity , we recover solutions very similar to the the old viscosity. In the example shown in Fig. 1 to Fig. 4, the pressure in the midplane is larger, the temperature is lower and the surface of the disk is more luminous (cf. F_*).

The profile of radial velocity is of particular interest, since this will affect the advection of the magnetic field. When $\lambda = 0.01$, the radial velocity is positive on the midplane and becomes negative at the larger z. This result has been found previously by several authors[7]. When the anomalous viscosity is implied, the anomalous viscosity is smaller than the standard α prescription and this is more reasonable. It follows s that the pressure in the midplane increasing but the temperature do not change almost , so the density defined as $\rho_* = p_*/T_*$ in the midplane is increasing too. Large density can lead to the velocity decreasing. According to the continuum equation as follows

$$\Sigma r u_r = \text{constant},$$

where Σ is defined as $\Sigma = \int \rho \mathrm{d}z$. As the r is chosen in the disk, the increasing of ρ leads to the increasing of Σ , as a result , the radial velocity should decrease. This phenomenon is well conformed to the numerical investigation. From Fig. 5 to Fig. 8 (the dash-dot lines refer to the old form of viscosity, the continuous lines refer to the new form of viscosity), we find that the vertical velocity is caused by viscosity, when

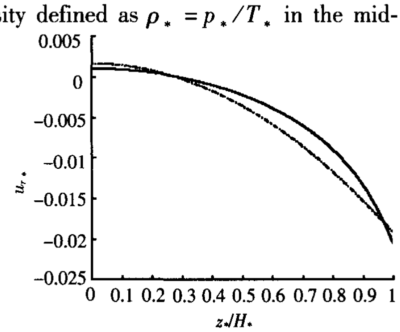

Fig.5 Distribution of the radial velocity with different viscosity for λ=0.05(α=0.1)

viscosity coefficient λ is from 0.05 to 0.000 05, it decreases rapidly in the midplane. In Fig. 9 (the dot line refers to $\lambda = 0.05$, the dash-dot line refers to $\lambda = 0.005$, the dash line refers to $\lambda = 0.000\,5$, the continuous line refers to $\lambda = 0.000\,05$), we see that when the viscosity is smaller, the radial velocity in the midplane is almost equal to zero. This means that when the viscosity is small enough, the matter flow into the disk parallel to the disk.

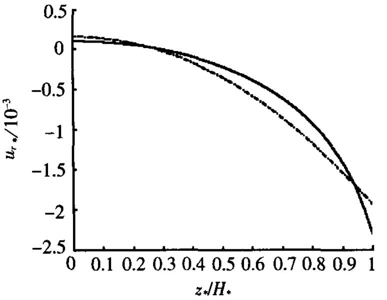

Fig.6 Distribution of the radial velocity with different viscosity for λ=0.005(α=0.01)

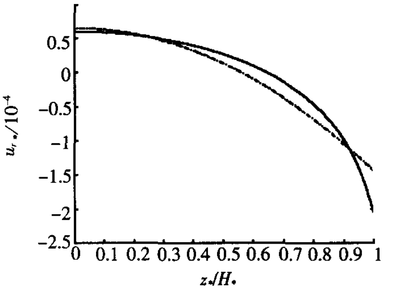

Fig.7 Distribution of the radial velocity with different viscosity for λ=0.000 5(α=0.001)

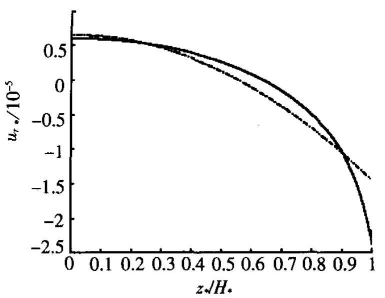

Fig.8 Distribution of the radial velocity with different viscosity for λ=0.000 05(α=0.000 1)

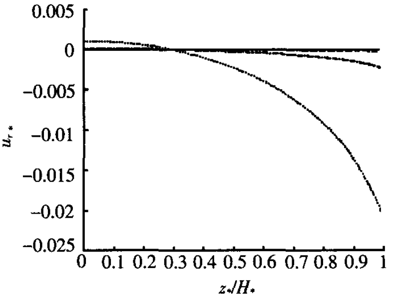

Fig.9 Distribution of the radial velocity with different values of λ(new form of viscosity ν)

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