Study on Instability of Anomalous Viscosity Disk with Vertical Structure

Yao Zhengrong, Li Xiaoqing

(School of Physical Science and Technology, Nanjing Normal University, 210097, Nanjing, China)

Abstract: It is well known that accretion disks are hydrodynamic disks with anomalous viscosity. An accretion disk with vertical structure is studied by using a new anomalous viscosity. There are two kinds of acoustic modes, O-mode and I-mode, in the inner disk. Because of the anomalous viscosity, the I-mode is always unstable in the inner disk. The vertical structure must be taken into account when the perturbations are high-frequency.

Key words: accretion disks, anomalous viscosity, instability, vertical structure

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具有垂向结构的反常粘滞吸积盘的不稳定性研究

姚峥嵘,李晓卿

(南京师范大学物理科学与技术学院,210097,江苏,南京)

[摘要] 天体物理吸积盘是具有反常粘滞的流体盘. 利用一种全新的反常粘滞,主要研究了具有垂向结构的吸积盘. 在盘的内区发现两种声速模:O-mode,I-mode. 由于反常粘滞的存在,导致 I-mode 在盘的内区是始终不稳定的. 另外,在讨论高频不稳定模时,必须注意到垂向结构是不可忽略的.

[关键词] 吸积盘,反常粘滞,不稳定性,垂向结构

0 Introduction

As a powerful and attractive theoretical model, accretion disks (AD) are widely believed to be sources of high-energy radiation for X-ray binaries and active galactic nuclei (AGNs). Since the standard thin disk model was constructed, in the early 1970s[1], the stability of the disk has become an important area in the theory of AD. This is because a lot of quasi-periodic variations in X-ray binaries and AGNs are believed to be related to the instabilities of the disk. Shortly after the publication of Shakura & Sunyaev 's work, the inner region of the standard thin disk was found to be secularly and thermally unstable[2,3]. The rapid growth of these unstable modes may result in the breakdown of the thin accretion disk configuration. Subsequently, much research was done concerning the instabilities. Except the thermal mode and the viscous mode, there are two acoustic modes in AD[4-7], the O-mode (propagating outward) and the I-mode (propagating inward). The O-mode is unstable throughout the disk, white the I-mode is unstable in the outer disk but stable in the inner disk, and the instability growth rate increases with smaller wavelengths λ[5].

The standard thin disk model is usually assumed that the disk is geometrically thin, $H(r) \ll r$ (H is the thickness of a disk and r is the radius). It involves averaging over the z coordinate of the three-dimensional hy-

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Biography: Yao Zhengrong, born in 1980, graduate student, majored in accretion disks. E-mail: maylight528@163.com

drodynamic equations, assuming a number of additional conditions to be fulfilled[1,6]. However, the thin disk model imposes a restriction on wavelength, $\lambda \gg h$ (h is the half-thickness of a disk), and therefore it is necessary to consider AD z-structure for correct treatment when $\lambda \leqslant h$[8].

It is well known that AD are hydrodynamic disks with anomalous turbulent viscosity, the viscosity of matter in disk results in a factor of more than 10^8 amplification to the microscopic viscosity. But we know little about turbulent viscosity, and usually we take standard α model: $\nu = \alpha C_s H$[1]. The anomalous viscosity has disturbed the astrophysicists for a long time, until recently it is resolved by Li & Zhang[9]. Their work has shown that the magnetic fields self-generated by the transverse plasmas are modulationally unstable in the Lyapunov sense, leading to a self-similar collapse of the magnetic flux and resulting in local magnetic structures. Highly spatially intermittent flux is responsible for generating the anomalous viscosity. Therefore, large-scale magnetic fields heat the outer parts of the disk, while small-scale intermittent flux gives rise to viscosity with radius r and temperature T dependence. It is these two magnetic processes that determine the stationary structures of AD.

In this paper we investigate the dynamics of acoustic perturbations taking into account the z-structure of a disk with anomalous viscosity. The main question is the existence of high-frequency instabilities in the inner disk.

1 Anomalous Magnetic Viscosity

To investigate the structure of the disk, we must know the anomalous viscosity η_m. The anomalous viscosity has disturbed astrophysicists for a long time and is a rather difficult problem. A solution was suggested recently by Li & Zhang[9]. It is known that the transverse plasmon fields are modulationally unstable in the Lyapunov sense, leading to a self-similar collapse of the magnetic flux. Such collapsing magnetic flux instability is analyzed in the case of kinetic plasma physics. Based on Vlasov equations and Maxwell equations, the collapsing feature of the self-generated magnetic field from transverse plasmons is investigated on rather small scale of the motion or electric current in accretion disks; as a result, the anomalous viscosity due to the intermittent flux occurs.

Here, we briefly introduce the calculation processes of the magnetic kinematics viscosity. The magnetic viscous stress tensor by the self-generated spatially intermittent flux is

$$t_{ij}^{m} = \frac{\langle \delta B_i \delta B_j - \frac{1}{2}\delta_{ij}(\delta \boldsymbol{B})^2 \rangle}{4\pi} \tag{1}$$

The work done on the volume d$\boldsymbol{r}$, per unit time, by the stress is $\left(-\frac{\partial t_{ij}^m}{\partial x_j} v_i\right)\mathrm{d}\boldsymbol{r}$, which contributes to viscous dissipation, resulting in the change of entropy within the volume:

$$\dot{S} = \int \frac{1}{T}\left(-\frac{\partial t_{ij}^m}{\partial x_j} v_i\right)\mathrm{d}\boldsymbol{r} = \int t_{ij}^m \frac{V_{ij}}{T}\mathrm{d}\boldsymbol{r} \tag{2}$$

where

$$V_{ij} \equiv \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \tag{3}$$

This corresponds to the linear relation of the ‘flow’ t_{ij}^m and the ‘force’ $-\frac{V_{ij}}{T}$[10]:

$$t_{ij}^m = \gamma_{ij;lk}\frac{V_{lk}}{T} = \eta_{ij;lk} V_{lk} \tag{4}$$

or

$$t_{ij}^m = \eta_m\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial v_k}{\partial x_k}\right),$$

where $\eta_{ij;lk} = \eta_m\left(\delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl} - \frac{2}{3}\delta_{ij}\delta_{lk}\right)$, and η_m is magnetic viscosity of the intermittent flux defined through the kinetic coefficient as $\eta_m=\eta_{lk;lk}(l\neq k$, no summation over repeated $lk)$.

In the accretion disks, the shear stress tensor V_{lk} has the dominant $r\varphi$-component and then (4) becomes

$$t_{ij}^m=\eta_{ij;r\varphi}\frac{1}{2}\left(\frac{\partial v_\varphi}{\partial r}-\frac{v_\varphi}{r}\right)=\eta_{ij;r\varphi}\frac{1}{2}r\frac{\partial\Omega(r)}{\partial r} \tag{5}$$

with $\Omega=\frac{v_\varphi}{r}$. Furthermore, it yields from (1) and (5), that

$$\frac{|\langle\delta B_r\delta B_\varphi\rangle|}{4\pi}=\eta_m\frac{1}{2}r\left|\frac{\partial\Omega(r)}{\partial r}\right| \tag{6}$$

It is assumed that self-generated magnetic fields by the transverse modes are statistically isotropic on the scales of interest:

$$\frac{1}{3}\langle(\delta\boldsymbol{B})^2\rangle\approx\langle\delta B_r\delta B_\varphi\rangle \tag{7}$$

In this case, (5) and (7), combined with a self-similar solution for the collapse magnetic flux[9]

$$\frac{\delta\boldsymbol{B}_{\max}^2}{8\pi}=\frac{16}{9}\mu c^2\rho\frac{(|\boldsymbol{E}_0|^2)^{\frac{4}{3}}}{\alpha^{\frac{2}{3}}},$$

mean that the magnetic kinematics viscosity ν can be read as

$$\nu=7\times10^{-12}\frac{c^2}{r\left|\frac{\partial\Omega(r)}{\partial r}\right|}T_0^{\frac{2}{3}}\left(\frac{T_e}{T_0}\right)^{\frac{2}{3}}\left(\frac{\overline{W}_0^P}{10^{-5}}\right)^{\frac{4}{3}}(\mathrm{cm^2/s}) \tag{8}$$

2 Model and Basic Equations

Now we consider an axisymmetric differentially rotating gas disk in the gravitational field of a mass M. Without including self-gravity and relativistic effects, and adopting cylindrical coordinates we have:

$$\Psi_{(r,z)}=-\frac{GM}{(r^2+z^2)^{\frac{1}{2}}}\approx-\frac{GM}{r}+\frac{1}{2}\Omega_k^2z^2 \tag{9}$$

G is the gravitational constant, $\Omega_k=\sqrt{\frac{GM}{r^3}}$ is the Keplerian angular velocity. If the disk is Keplerian, we can get

$$f_\varphi=-\frac{3}{2r^2}\frac{\partial}{\partial r}(r^2\rho\nu\Omega_k) \tag{10}$$

where ρ is the volume density of matter in the disk. According to α model[1], we take the new anomalous viscosity[9]

$$\nu=\nu_i\left(\frac{r}{r_i}\right)^{\frac{3}{2}}\left(\frac{T}{T_i}\right)^{\frac{2}{3}} \tag{11}$$

where the subscript 'i' signifies the values taken at the inner edge of the disk.

We use the axisymmetric hydrodynamic equations with cylindrical coordinates. The equations of motion and continuity have the form

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}-\frac{\partial\Psi}{\partial r} \tag{12}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}=\frac{1}{\rho}f_\varphi \tag{13}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}-\frac{\partial\Psi}{\partial z} \tag{14}$$

$$\frac{\partial\rho}{\partial t}+\frac{\partial(r\rho u)}{r\partial r}+\frac{\partial(\rho w)}{\partial z}=0 \tag{15}$$

where P is the pressure, $\boldsymbol{V}=(u,v,w)$ is the velocity.

We take Khoperskov & Khrapov 's method[8], add the thermal equation to the system of Eqs. (12)—(15)

as

$$\frac{\mathrm{d}S}{\mathrm{d}t}=\frac{Q}{T} \tag{16}$$

where S is the entropy, Q defines the sources of heat.

When we investigate the equilibrium of the disk, we still adopt the model developed by Khoperskov & Khrapov[8]. It is assumed that the equilibrium velocity in the thin disk has only r and φ components: $\boldsymbol{V}_0=(U_0, V_0, 0)$, and usually we have $\left|\frac{U_0}{V_0}\right|\approx\alpha\left(\frac{h}{r}\right)^2$. The pressure and density have the form

$$P_0(r,z)=P_0(r)F(z)^a \tag{17}$$

$$\rho_0(r,z)=\rho_0(r)F(z)^b \tag{18}$$

where $F(z)=1-\frac{z^2}{h^2}, a=\frac{n}{n-1}, b=\frac{1}{n-1}$, n is the polytropic index. If we assume that $P_0(r)\propto r^{-\frac{3}{2}}, \rho_0(r)\propto r^{-\frac{1}{2}}$, we can get $T\propto r^{-1}$, considering the ideal-gas equation of state. Now we can obtain from (10) taking into account (11):

$$f_\varphi=-\frac{3\rho}{2r}\Omega_k\nu\left(\frac{4}{3}+\frac{r}{\rho}\frac{\partial\rho}{\partial r}\right) \tag{19}$$

With these conditions and assumptions the equilibrium equations are defined by:

$$\frac{V_0^2}{r}=\frac{1}{\rho_0}\frac{\partial P_0}{\partial r}+\frac{\partial\Psi}{\partial r} \tag{20}$$

$$U_0\frac{\partial(rV_0)}{r\partial r}=-\frac{5\Omega_k\nu}{4r} \tag{21}$$

$$\frac{\partial P_0}{\partial z}=-\rho_0\frac{\partial\Psi}{\partial z} \tag{22}$$

Using Eqs. (17),(18) and (22), the relation

$$C_s^2(r,z)=\frac{\gamma P_0}{\rho_0}=\frac{\gamma P_0(r)}{\rho_0(r)}F(z)=\frac{\gamma\Omega_k^2h^2}{2a}F(z) \tag{23}$$

defines the adiabatic sound speed (γ is the adiabatic index). And from now on, we choose $n=\gamma=5/3$, so $a=5/2$, $b=3/2$. Then we obtain the equilibrium velocities:

$$V_0(r,z)=r\Omega(r,z)=\sqrt{r^2\Omega_k^2-\frac{3}{2}\frac{C_s^2}{\gamma}}\approx r\Omega_k \tag{24}$$

$$U_0(r,z)=-\frac{5\Omega_k\nu}{2V_0}\approx-\frac{5\nu}{2r} \tag{25}$$

3 Linear Analysis

In the framework of the standard linear analysis the pressure, density and velocity are represented as:

$$u=U_0(r,z)+\tilde{u}(r,z,t),\ v=V_0(r,z)+\tilde{v}(r,z,t),\ w=\tilde{w}(r,z,t),$$
$$P=P_0(r,z)+\tilde{P}(r,z,t),\ \rho=\rho_0(r,z)+\tilde{\rho}(r,z,t).$$

In the linear approximation ($|\tilde{y}|\ll|y_0|$) and after neglecting some higher order perturbed terms, we obtain linear perturbed equations as follows:

$$\frac{\partial\tilde{u}}{\partial t}+U_0\frac{\partial\tilde{u}}{\partial r}+\tilde{u}\frac{\partial U_0}{\partial r}-\frac{2V_0}{r}\tilde{v}=-\frac{1}{\rho_0}\frac{\partial\tilde{P}}{\partial r}+\frac{\tilde{\rho}}{\rho_0^2}\frac{\partial P_0}{\partial r} \tag{26}$$

$$\frac{\partial\tilde{v}}{\partial t}+U_0\frac{\partial(r\tilde{v})}{r\partial r}+\tilde{u}\frac{\partial(rV_0)}{r\partial r}=\frac{3}{2}\Omega_k\nu\left(-\frac{1}{\rho_0}\frac{\partial\tilde{\rho}}{\partial r}+\frac{\tilde{\rho}}{\rho_0^2}\frac{\partial\rho_0}{\partial r}\right) \tag{27}$$

$$\frac{\partial\tilde{w}}{\partial t}+U_0\frac{\partial\tilde{w}}{\partial r}=-\frac{1}{\rho_0}\frac{\partial\tilde{P}}{\partial z}-g\frac{\tilde{\rho}}{\rho_0} \tag{28}$$

$$\frac{\partial\tilde{\rho}}{\partial t}+\frac{\partial[r(U_0\tilde{\rho}+\rho_0\tilde{u})]}{r\partial r}+\frac{\partial(\rho_0\tilde{w})}{\partial z}=0 \tag{29}$$

where $g\equiv\frac{\partial\Psi}{\partial z}=-\frac{1}{\rho_0}\frac{\partial P_0}{\partial z}$.

As we study only the dynamics of acoustic instability, we set $Q=0$[8]. The thermal equation becomes

$$\frac{\partial\tilde{S}}{\partial t}+U_0\frac{\partial\tilde{S}}{\partial r}+\tilde{u}\frac{\partial S_0}{\partial r}+\tilde{w}\frac{\partial S_0}{\partial z}=0 \tag{30}$$

where $S_0=c_V\ln\left(\frac{P_0}{\rho_0^\gamma}\right)$ is the entropy of the gas at equilibrium. On the other hand

$$\tilde{S}=\tilde{S}(\tilde{P},\tilde{\rho})=c_V\frac{\tilde{P}}{P_0}-c_P\frac{\tilde{\rho}}{\rho_0} \tag{31}$$

where c_V and c_P are the specific heats at constant density and pressure[8]. We set $c_V\approx c_P$, then obtain from (30):

$$\frac{1}{P_0}\frac{\partial\tilde{P}}{\partial t}-\frac{1}{\rho_0}\frac{\partial\tilde{\rho}}{\partial t}+\frac{U_0}{P_0}\left(\frac{\partial\tilde{P}}{\partial r}-\frac{\tilde{P}}{P_0}\frac{\partial P_0}{\partial r}\right)-\frac{U_0}{\rho_0}\left(\frac{\partial\tilde{\rho}}{\partial r}-\frac{\tilde{\rho}}{\rho_0}\frac{\partial\rho_0}{\partial r}\right)-\frac{2\tilde{u}}{3r}=0 \tag{32}$$

The short-wave approximation in the radial direction ($kr\gg1$, k is the radial wave number) allows writing the solution as:

$$\tilde{y}(r,z,t)\propto\exp(\mathrm{i}kr-\mathrm{i}\omega t) \tag{33}$$

where ω is frequency. Taking into account (33), the perturbed equations become

$$\left(\sigma+\frac{U'}{6}\right)\tilde{u}+2\Omega_k\tilde{v}-\frac{\mathrm{i}k}{\rho_0}\tilde{P}-\frac{3C_s^2}{2\gamma\rho_0 r}\tilde{\rho}=0 \tag{34}$$

$$\frac{1}{2}\Omega_k\tilde{u}-(\sigma-U')\tilde{v}+\frac{3\Omega_k\nu}{2\rho_0 r}\left(\frac{1}{2}+\mathrm{i}kr\right)\tilde{\rho}=0 \tag{35}$$

$$\sigma\tilde{w}-\frac{g}{\rho_0}\tilde{\rho}=0 \tag{36}$$

$$\frac{\rho_0}{r}\left(\frac{1}{2}+\mathrm{i}kr\right)\tilde{u}+\frac{\partial\rho_0}{\partial z}\tilde{w}-\left(\sigma-\frac{5U'}{6}\right)\tilde{\rho}=0 \tag{37}$$

$$\frac{2}{3r}\tilde{u}+\frac{\sigma-\frac{3U'}{2}}{\rho_0 C_s^2/\gamma}\tilde{P}-\frac{\sigma-\frac{U'}{2}}{\rho_0}\tilde{\rho}=0 \tag{38}$$

where $\sigma=\mathrm{i}(\omega-kU_0)$, $U'=U_0/r$, $U_0=-\frac{5\nu}{2r}$.

By setting the determinants of the coefficients in Eqs. (34)—(38) equal to zero, we get a dispersion equation

$$\begin{aligned}
&\sigma^5-\frac{19}{6}U'\sigma^4+\left\{\left[\frac{109}{36}U'^2+\Omega_k^2+\frac{3\Omega_k^2z^2}{F(z)h^2}+\frac{1}{5}\Omega_k^2F(z)h^2k^2-\frac{3}{20}\frac{\Omega_k^2F(z)h^2}{r^2}\right]-\left[\frac{4k}{15r}\Omega_k^2F(z)h^2\right]\mathrm{i}\right\}\sigma^3\\
&+\left\{\left[\frac{3\Omega_k^2\nu}{r^2}\left(\frac{1}{4}-k^2r^2\right)-\frac{7}{3}U'\Omega_k^2-\frac{1}{10}U'\Omega_k^2F(z)h^2k^2-\frac{7U'\Omega_k^2z^2}{F(z)h^2}-\frac{3U'\Omega_k^2F(z)h^2}{8r^2}-\frac{47}{72}U'^3\right]\right.\\
&\left.+\left[\frac{3k}{r}\Omega_k^2\nu-\frac{38k}{45r}U'\Omega_k^2F(z)h^2\right]\mathrm{i}\right\}\sigma^2\\
&+\left\{\left[\frac{3\Omega_k^4z^2}{F(z)h^2}+\frac{5}{4}U'^2\Omega_k^2-\frac{9}{8r^2}U'\Omega_k^2\nu+\frac{9}{2}U'\Omega_k^2\nu k^2-\frac{5}{24}U'^4+\frac{1}{10}U'^2\Omega_k^2F(z)h^2k^2\right.\right.\\
&\left.\left.+\frac{13}{4}\frac{U'^2\Omega_k^2z^2}{F(z)h^2}-\frac{9}{40r^2}U'^2\Omega_k^2F(z)h^2\right]+\left[\frac{2k}{5r}\Omega_k^4z^2-\frac{7k}{18r}U'^2\Omega_k^2F(z)h^2-\frac{9k}{2r}U'\Omega_k^2\nu\right]\mathrm{i}\right\}\sigma\\
&+\left\{\left[\frac{3}{4}\frac{U'^3\Omega_k^2z^2}{F(z)h^2}-\frac{9}{2}\frac{U'\Omega_k^4z^2}{F(z)h^2}\right]-\left(\frac{2k}{5r}U'\Omega_k^4z^2\right)\mathrm{i}\right\}=0
\end{aligned}$$

4 Numerical Results and Discussions

If we set $W=\omega/\Omega_k$ (W is dimensionless frequency), $K=hk$ (K is dimensionless wave number), $R=r/r_i$, $Z=z/h$, $D=h/r=0.05$, and $N=\frac{\nu}{r^2\Omega_k}=\frac{\nu_i}{r_i^2\Omega_{ki}}R^{\frac{1}{3}}=\frac{2}{5}\left|\frac{U_{0i}}{V_{0i}}\right|R^{\frac{1}{3}}\approx\frac{2}{5}\alpha D^2R^{\frac{1}{3}}$, the dimensionless dispersion equation is

$$\begin{aligned}
&\Delta^5-\frac{19}{6}\left(-\frac{5}{2}N\right)\Delta^4+\Big\{\Big[\frac{109}{36}\left(-\frac{5}{2}N\right)^2+1+\frac{3Z^2}{F(Z)}+\frac{1}{5}K^2F(Z)-\frac{3}{20}D^2F(Z)\Big]\\
&-\Big[\frac{4}{15}KDF(Z)\Big]\mathrm{i}\Big\}\Delta^3+\Big\{\Big[3N\left(\frac{1}{4}-\frac{K^2}{D^2}\right)-\frac{7}{3}\left(-\frac{5}{2}N\right)-\frac{1}{10}K^2F(Z)\left(-\frac{5}{2}N\right)\\
&-7\frac{Z^2\left(-\frac{5}{2}N\right)}{F(Z)}-\frac{3}{8}D^2F(Z)\left(-\frac{5}{2}N\right)-\frac{47}{72}\left(-\frac{5}{2}N\right)^3\Big]+\Big[3\frac{K}{D}N-\frac{38}{45}KDF(Z)\left(-\frac{5}{2}N\right)\Big]\mathrm{i}\Big\}\Delta^2\\
&+\Big\{\Big[3\frac{Z^2}{F(Z)}+\frac{5}{4}\left(-\frac{5}{2}N\right)^2-\frac{9}{8}\left(-\frac{5}{2}N^2\right)+\frac{9K^2}{2D^2}\left(-\frac{5}{2}N^2\right)-\frac{5}{24}\left(-\frac{5}{2}N\right)^4+\frac{1}{10}K^2F(Z)\left(-\frac{5}{2}N\right)^2\\
&+\frac{13}{4}\frac{Z^2}{F(Z)}\left(-\frac{5}{2}N\right)^2-\frac{9}{40}D^2F(Z)\left(-\frac{5}{2}N\right)^2\Big]+\Big[\frac{2}{5}KDZ^2-\frac{7}{18}KDF(Z)\left(-\frac{5}{2}N\right)^2\\
&-\frac{9K}{2D}\left(-\frac{5}{2}N^2\right)\Big]\mathrm{i}\Big\}\Delta+\Big\{\Big[\frac{3}{4}\frac{Z^2}{F(Z)}\left(-\frac{5}{2}N\right)^3-\frac{9}{2}\frac{Z^2}{F(Z)}\left(-\frac{5}{2}N\right)\Big]-\Big[\frac{2}{5}KDZ^2\left(-\frac{5}{2}N\right)\Big]\mathrm{i}\Big\}=0,
\end{aligned}$$

where $\Delta=\frac{\sigma}{\Omega_k}=\mathrm{i}\left(W+\frac{5K}{2D}N\right)$, $F(Z)=1-Z^2$. Because $\alpha<1$ [1], we take $N=0.4\times10^{-4}R^{\frac{1}{3}}$. We investigate the instability of the inner disk with z-structure, so we choose $R=2$, $Z=0$, 0.5, 0.9. We use $W=-\mathrm{i}\Delta-\frac{5KN}{2D}$ get the dimensionless frequency, and a positive imaginary part of W means that the mode is unstable. The O-mode (propagating outward) has a negative real part of W, while the I-mode (propagating inward) has a positive one.

At each point we chosen, we find two kinds of acoustic modes: O-mode and I-mode, and they are all unstable (Fig. 1). Wu & Yang[5] found that the I-mode was unstable in the outer disk, but turned stable in the inner disk. So we change R to 1. Here we just show the situation when $Z=0$ (Fig. 2). The propagating properties

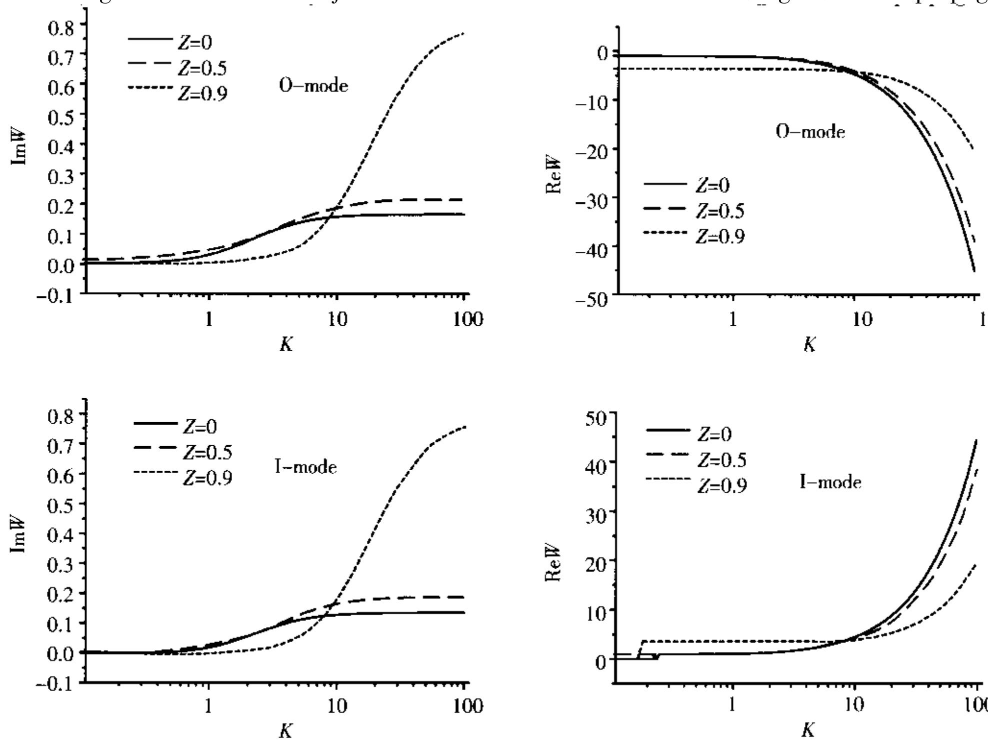

Fig.1 The relationship between dimensionless frequency W and dimensionless wave number K to unstable mode with different Z (R=2)

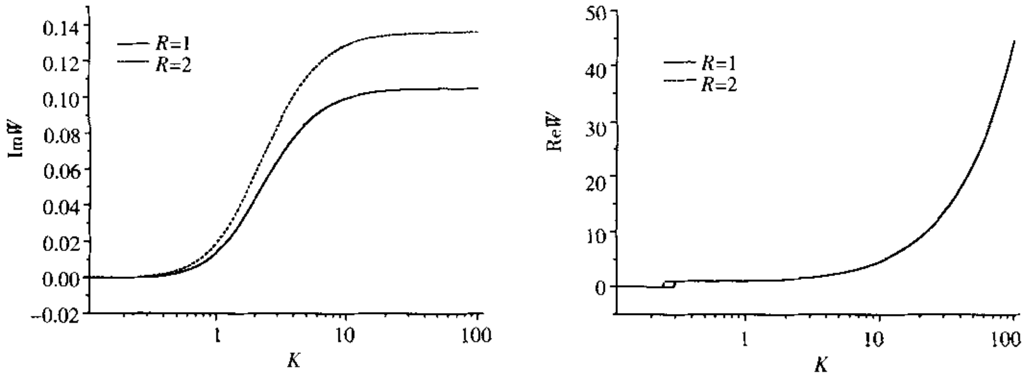

Fig.2 The relationship between dimensionless frequency W and dimensionless wave number K to I-mode with different R (Z=0)

have no change, and the I-mode is unstable, although with smaller growth rate. Wu & Yang[5] took the α model, so we think the anomalous viscosity[9] cause the I-mode always unstable in the inner disk.

In this paper, the main question is the existent of the high-frequency acoustic perturbations in the inner disk with non-homogeneous z-structure. So far, we have already found the high-frequency modes. From Fig. 1, we can clearly find that the vertical structure shouldn't be ignored when we discuss the short wavelength perturbations ($K>3$). But when $K<1$, the averaging over the z coordinate is still reasonable.

In summary, we investigate an accretion disk with non-homogeneous vertical structure and we take the new anomalous viscosity. We find two unstable acoustic modes in the inner disk: O-mode and I-mode. The I-mode is always unstable because of the anomalous viscosity. When we discuss the high-frequency perturbations, we must consider the vertical structure.

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