On geometric potentials in nanomechanical circuits

Alexander V. Chaplik[1] and Robert H. Blick[2]

[1] Institute of Semiconductor Physics, RAS, Siberian Branch, 630090 Novosibirsk, Russia.

[2] Electrical and Computer Engineering, University of Wisconsin-Madison, 1415 Engineering Drive, WI 53706, USA.

(April-2, 2002)

Abstract

We demonstrate the formation of confinement potentials in suspended nanostructures induced by the geometry of the devices. We then propose a setup for measuring the resulting geometric phase change of electronic wave functions in such a mechanical nanostructure. The device consists of a suspended loop through which a phase coherent current is driven. Combination of two and more geometrically induced potentials can be applied for creating mechanical quantum bit states.

03.65.Vf,85.35.Ds,85.85.+j

Quantum mechanics in curved-linear manifolds has been elaborated for some time [1]. The propagation of waves in curved waveguides can be 'translated' for quantum particles into a Hamiltonian consisting of the kinetic energy operator and a resulting potential energy, which is of pure geometric origin. The ability to build nanostructures with a three-dimensional relief allows the realization of low-dimensional electronic systems possessing a mechanical degree of freedom [2]. This is exemplified in recent work by Prinz et al. [3] and Schmidt and Eberl [4], who demonstrated how to realize rolled-up semiconductor films with a radis of curvature $R \approx 100$ nm. Thus it is worth while studying the influence of geometrical potentials on phase coherently propagating particles in curved low-dimensional electron systems. This will induce a phase shift in the electronic wave function corresponding to Berry's phase [5].

For the case of a two-dimensional electron gas, fixing it leads to a geometrical potential of the form

$$U = -\frac{\hbar^2}{8m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)^2 \qquad (1)$$

where m is the effective mass and R_1, R_2 are the principal curvature radii of the surface in the point where the electron resides. The geometric potential is always attractive and independent of the electric charge of a particle, similar to gravitation. Furthermore, it is of purely quantum origin, i.e. it vanishes for the limit $\hbar \to 0$.

If one of the radii tends to infinity we obtain a cylindrical surface. Particularly, this is the case when electrons are confined to a quantum wire having the shape of a plane curve. The Schrödinger equation for such a curved linear 1D system reads (see Fig. 1 (a))

$$\hat{H}\psi = \left[-\frac{\hbar^2}{2m}\frac{d^2}{ds^2} + \frac{\hbar^2}{8mR^2(s)}\right]\psi = E\psi; \qquad (2)$$

where s is the length of the arc of the wire counted from an arbitrary origin and $1/R(s)$ is the local curvature. A straight forward derivation of Eq. (2) is given in [6]. It is further shown in this work that a wire of the shape of an Archimedes spiral gives the geometric potential with an asymptotic behavior resulting in Coulomb's law

$$1/R^2(s) \quad 1/s: \tag{3}$$

Hence, there is an infinite set of bound states corresponding to the localization of electrons at the origin of the spiral.

If one starts from a quantum wire consisting of two straight lines conjugated by an arc of a circumference ('open book'-shape) the corresponding geometrical potential is a rectangular potential well (see Fig. 1(b)) with a width R and a depth $h^2/8mR^2$. Here R is the radius of the circumference and is the conjugating arc (angle between two rectilinear parts of the wire). There exists one and only one bound state for $<$ in such a system and its energy is given by

$$E_o = \frac{h^2}{8mR^2} \left(1 \quad \frac{16z^2}{{}^2} \right) ; \tag{4}$$

where $z(\ )$ is the root of $\cos z = 4z/$ between 0 and $/2$ (see Fig. 1(b)). For example, for a U-shaped wire ($=$) with $R = 100$ Å and an electron mass in GaAs $m = 0.07\, m_0$, we find a binding energy of $E_o = 4$ K. Whereas for the conjugation of two perpendicular straight lines, i.e. $= /2$, we obtain $E_o = 3$ K. The phase of the wave function in the quasiclassical regime is then given by the integral

$$\frac{1}{h}\int_0^s P\, ds' = \frac{1}{h}\int_0^s \sqrt{2m(E_F \quad U(s'))}\, ds': \tag{5}$$

For a wire with a small curvature the relation for the Fermi energy and the geometrical potential is $E_F \quad h^2/8mR^2$. The total phase shift after passing through the conjugation then is $= /8k_F R$ where $k_F R \quad 1$ with $k_F = P_F/h$. In the opposite limit, i.e. $k_F R \quad 1$ the integral in Eq. (5) gives $= /2$.

The most realistic case of course is given in the limit $k_F R \quad 1$. A mechanically deformable quantum interferometer will be able to sense such a deformation, provided its sensitivity to the phase shifts exceeds the value $/8k_F R$. Such a mechanical quantum interferometer (MQUI) can be realized by suspending a two-dimensional electron gas in a thin membrane. While the electron gas usually is 10 nm thin, the total membrane thickness will be around 90 nm. This leaves the minimal radius of curvature at $R \approx 500$ nm, giving a confinement potential for the lowest state of $E_o \approx 1.6$ mK. As Prinz et al. [3] have shown smaller curvature radii can be achieved. Especially, considering surface bound two-dimensional electron gases in InAs heterostructures should allow reaching the regime $R \approx 10$ nm, effectively leading to temperatures in the range of ≈ 4 K.

An MQUI is shown in Fig. 2: the interferometer basically consists of a ring shaped suspended membrane containing a two-dimensional electron gas. This geometry first allows to measure interference induced by a magnetic field applied perpendicular to the plane of the ring, i.e. 'classical' Aharonov-Bohm oscillations (see Fig. 2(a)). In this way the interferometer is to be calibrated. The diameter of the ring should be smaller than the phase coherence length L_ϕ, i.e. in this case 5 µm as found for two-dimensional electron gases will be sufficient. Deformation of the arms of the ring is facilitated by using gating electrodes beneath the ring, also indicated in Fig. 2(b). In order to avoid depletion of the electron gas the suspended heterostructure contains a highly n-doped GaAs back-layer. By gating the interferometer arms individually two flexing modes can be supported. Both modes are shown in Fig. 2(b). Since the potential shows a dependence of $\propto R^{-2}$ both will lead to the same wave function shift. Another technique for mechanically modulating the suspended electron ring is given by using surface acoustic waves [7]. The modulation amplitude is considerably increased in suspended membranes [8].

Intriguingly the combination of two curved sections connected by a thin wire w – denoted as a Ω-shaped element – will lead to a double quantum well potential, as demonstrated in Fig. 3(a). Depending on $R(s)$ and on w the two discrete states in the wells $E_o^{A,B}$ can communicate, i.e. a tunnel splitting of the order of $2\Delta E_o$ will occur. Thus this system represents a mechanical quantum bit (mqubit), whose communication is given by the exchange of phonons at an energy $2\Delta E_o = hf_{ph} = hc_a/w$, where c_a is the velocity of sound in the heterostructure and w is the length of the connecting element. Naturally, this scheme can be extended to a chain with N elements forming $N/2$ Ω-wire mqubits as shown in Fig. 3(b). A whole variety of different modes is available for information exchange between two mqubits (ii). Stretching and contracting the wire elements individually allows to steer inter and intra-m qubit communication. The overall information exchange is performed through low frequency phonon modes, e.g. indicated by dashed lines (iii). The energy of these modes is determined by c_a and the chain length. Sectioning into sub-chains the interaction of the m qubits organized in a hierarchy.

As another example we imagine to illuminate the U-shaped wires for generating electron-hole pairs forming excitons. Both electrons and holes and hence the excitons will be attracted to the bottom of the U-wires' geometric potentials. Since excitons do not obey the Pauli exclusion principle, the probability to capture more and more excitons will increase.

This work has been supported by the Deutsche Forschungsgemeinschaft (DFG) through the projects Bl/487-1& 3 and the Russian Foundation for Basic Research via grant 16377, by the program 'Physics of Solid Nanostructures' of the Russian Ministry of Science, and by the Sonderforschungsbereich (SFB 348) of the DFG. AVC thanks Jorg Kotthaus for the hospitality during the research visit to the Center for NanoScience. RHB likes to thank Herbert Walther for discussing the importance of vibronic modes for quantum computation in atom chains.

REFERENCES

[1] C.T. da Costa, Phys. Rev. A 23, 1982 (1981); H. Jensen and H. Koppe, Annals of Phys. 46, 586 (1971); N. Ogawa, K. Fujii, and A. Kobushukin, Progr. Theor. Phys. 83, 894 (1990).

[2] R.H. Blick, F.G. Monzon, W. Wegscheider, M. Bichler, F. Stern, and M.L. Roukes, Phys. Rev. B 62, 17103 (2000); J. Kirschbaum, E.M. Hohberger, R.H. Blick, W. Wegscheider, M. Bichler, Appl. Phys. Lett. 81, 280 (2002).

[3] V.Ya. Prinz, V.A. Seleznev, A.K. Gutakovsky, A.V. Chehovskiy, V.V. Preobrazhenskii, M.A. Putyato, and T.A. Gavrilova, Physica E 6, 828 (2000).

[4] Oliver G. Schmidt and Karl Eberl, Nature 410, 168 (2001).

[5] Michael V. Berry, Proc. R. Soc. London Ser. A 392, 45 (1982).

[6] A.I. Vedernikov and A.C. Chaplik, ZhETF 11, 448 (2000). In this speci c case the kinetic energy operator has the conventional form of a 1D Schrodinger equation, obtained by replacing 'x' by 's'. A curve is not possessing an internal geometry in contrast to the 2D-case and manifolds of higher dimension.

[7] A. Wixforth, J.P. Kotthaus, and G. Weimann, Phys. Rev. Lett. 56, 2104 (1986).

[8] F.W. Beil, A. Wixforth, and R.H. Blick, Physica E 13, 473 (2002).

Fig. 1: (a) Sketch of a single wire and the resulting geometrically induced confinement potentials. The parameter gives the degree of bending and E_o represents a bound electronic state in the resulting potential. (b) For = and = =2 square well potentials with different binding energies are obtained.

Fig. 2: Mechanical quantum interferometer: (a) electrons propagate phase coherently through the ring similar to an Aharonov-Bohm geometry. Flexing the arms of the ring (, ^{0}) a geometrical potential is formed which leads to an effective phase shift of the electronic wave function within the interferometer. (b) Two flexing modes can be distinguished (i) and (ii) both of which are leading to an identical phase shift.

Fig. 3: (a) A -wire forming a double quantum well potential: the discrete states in the two wells can interact depending on the connecting wire element length w. In case of tunneling a mechanical quantum bit (mqubit) is formed, i.e. the two discrete states are tunnel split by E_o. (b) Chain of -wire elements defining a circuit of ten coupled mechanical quantum bits (i). Communication between two mqubits is achieved by a variety of local deformations of the wires (ii). Parallel addressing of mqubit chains is possible through acoustic phonons propagating along the wire with the velocity of sound (iii) but at lower frequencies (indicated by dashed lines).