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Translation-invariant models for non-commutative gauge fields

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Abstract

Motivated by the recent construction of a translation-invariant renormalizable non-commutative model for a scalar field [1], we introduce models for non-commutative $U(1)$ gauge fields along the same lines. More precisely, we include some extra terms into the action with the aim of getting rid of the UV/IR mixing.

1 Introduction

Non-commuting space-time coordinates naturally appear in various approaches to quantum gravity, e.g. see the reviews [2]. Field theories on non-commutative space generally suffer from a new class of problematic infrared divergences which have the same degree as the usual ultraviolet divergences at the perturbative level. This phenomenon is commonly referred to as UV/IR mixing, see [2] and references therein. Recently, this problem could be overcome within certain models of scalar field theories. The first of these models, which was introduced by Grosse and Wulkenhaar [3], is the ϕ^4 theory supplemented by an oscillator term in the Euclidean x-space action: this model has been proved to be renormalizable to all orders of perturbation theory by different methods [4]. Since the oscillator term breaks the translational invariance, Gurau, Magnen, Rivasseau and Tanasa [1] recently introduced another renormalizable model in which the oscillator term in x-space is replaced in the Euclidean momentum space action by a $1/\tilde{k}^2$ term (with $\tilde{k}^2 = \tilde{k}^\mu \tilde{k}_\mu$ and $\tilde{k}_\mu = \theta_{\mu\nu} k^\nu$, where $\theta_{\mu\nu}$ are the non-commutativity parameters for the Euclidean space-time coordinates.) This term is motivated by the fact that the 1-loop self-energy of the standard non-commutative ϕ^4 model has a quadratic IR divergence which is proportional to $1/\tilde{k}^2$: the new term in the momentum space action yields a dressed propagator at 1-loop level involving a similar contribution. (As a matter of fact, such a term had already been considered earlier in connection with a resummation procedure [5, 6].)

The deformation matrix $(\theta_{\mu\nu})$ can be (and is) assumed to have the simple form

$$(\theta_{\mu\nu}) = \theta \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \qquad \text{with} \ \ \theta \in \mathbf{R}.$$

The action of Gurau et al. [1] is given in Euclidean momentum space by

$$S = \int d^4k \left[\frac{1}{2} k_\mu \phi k^\mu \phi + \frac{1}{2} m^2 \phi\phi + \frac{a}{2} \frac{1}{\theta^2 k^2} \phi\phi + \frac{\lambda}{4!} \phi \star \phi \star \phi \star \phi \right], \tag{1}$$

or, more explicitly [7],

$$S[\hat{\phi}] = \int d^4k \left[\frac{1}{2} \hat{\phi}(-k) \left(k^2 + m^2 + \frac{a}{\theta^2 k^2} \right) \hat{\phi}(k) + \frac{\lambda}{4!} \mathcal{F} \left(\phi \star \phi \star \phi \star \phi \right)(k) \right],$$

where $a > 0$ and where $\hat{\phi} \equiv \mathcal{F}\phi$ denotes the Fourier transform of ϕ. This leads to the improved propagator

$$G^{\phi\phi}(k) = \frac{1}{k^2 + m^2 + \frac{a}{\theta^2 k^2}}, \tag{2}$$

which has a "damping" behaviour for vanishing momentum:

$$\lim_{k\to 0} G^{\phi\phi}(k) = 0.$$

As in the Grosse-Wulkenhaar model, the UV/IR mixing is avoided due to a mixing of long and short scales. As we already mentioned, the model defined by the action (1) is translation-invariant and it has been proved to be renormalizable to all orders [1].

Before proceeding further, we briefly spell out how the term $1/k^2$ looks like in x-space. In 4 dimensions, the function $1/x^2$ is invariant under Fourier transformation (up to a factor), hence the term $1/k^2$ in the action (1) can be rewritten as a non-local term:

$$\int d^4k\, \hat{\phi}(-k)\frac{1}{k^2}\hat{\phi}(k) \propto \int d^4x \int d^4x'\, \phi(x)\frac{1}{(x-x')^2}\phi(x') \equiv \int d^4x\, \phi\frac{1}{\Box}\phi\,. \qquad (3)$$

Here, the last expression is the usual short-hand notation used in physics, where the symbol $1/\Box$ denotes the Green function G associated to the differential operator $\Box \equiv \partial^\mu\partial_\mu = \partial_1^2 + \cdots + \partial_4^2$:

$$\Box G(x) = \delta^{(4)}(x)\,, \qquad \text{with}\;\; G(x) = \frac{\text{const.}}{x^2}\,. \qquad (4)$$

As expected, matters are more complicated in gauge field theories. Although there have been several suggestions as to how to handle the UV/IR mixing [8, 9], the corresponding models have some drawbacks. The one introduced by Slavnov [8] relies on a constraint which reduces the degrees of freedom of the gauge field whereas the ones involving an oscillator-type term [9] (in analogy to the scalar field model of Grosse and Wulkenhaar) break the translational invariance. Accordingly, the goal of the present letter is to put forward some ideas for generalizing the procedure of Gurau et al. in view of constructing a renormalizable and translation-invariant model for $U(1)$ gauge fields in 4 dimensional non-commutative Euclidean space.

2 New gauge field model I

The quadratic IR divergence of a non-commutative $U(1)$ gauge theory is known to be of the form

$$\Pi^{\text{IR}}_{\mu\nu} \propto \frac{\tilde{k}_\mu\tilde{k}_\nu}{(\tilde{k}^2)^2}\,, \qquad (5)$$

and to be independent of the chosen gauge fixing (see references [10]). This expression motivated the authors of reference [6] to introduce the following gauge invariant term into their action (in connection with a resummation procedure):

$$\int d^4x\, \widetilde{F} \star \frac{1}{(\widetilde{D}^2)^2} \star \widetilde{F}\,. \tag{6}$$

Here,

$$\begin{aligned} \widetilde{F} &= \theta^{\mu\nu} F_{\mu\nu}\,, \qquad &&\text{with } \; F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}g\, [A_\mu \stackrel{\star}{,} A_\nu]\,, \\ \widetilde{D}^2 &= \widetilde{D}^\mu \star \widetilde{D}_\mu\,, \qquad &&\text{with } \; \widetilde{D}_\mu = \theta_{\mu\nu} D^\nu\,, \end{aligned} \tag{7}$$

hence $\frac{1}{\widetilde{D}^2} \star \widetilde{F} = \frac{1}{\theta^2}\frac{1}{D^2} \star \widetilde{F}$. The expression $\frac{1}{D^2} \star \widetilde{F} \equiv Y$ is to be understood as a formal power series in the gauge field A_μ which may be determined recursively as follows. First, note that

$$\begin{aligned} \widetilde{F} &= D^2 \star \frac{1}{D^2} \star \widetilde{F} = D^2 Y = \partial^\mu (D_\mu Y) - \mathrm{i}g\, [A^\mu \stackrel{\star}{,} D_\mu Y] \\ &= \Box Y - \mathrm{i}g\partial^\mu\, [A_\mu \stackrel{\star}{,} Y] - \mathrm{i}g\, [A^\mu \stackrel{\star}{,} \partial_\mu Y] + (\mathrm{i}g)^2\, [A^\mu \stackrel{\star}{,} [A_\mu \stackrel{\star}{,} Y]]\,. \end{aligned} \tag{8}$$

By applying $\frac{1}{\Box} \equiv \Box^{-1}$ (i.e. the Green function of the operator $\Box$, see equation (4)) to this relation, we find

$$Y = \frac{1}{\Box}\widetilde{F} + \frac{\mathrm{i}g}{\Box}\partial^\mu\, [A_\mu \stackrel{\star}{,} Y] + \frac{\mathrm{i}g}{\Box}\, [A^\mu \stackrel{\star}{,} \partial_\mu Y] - \frac{(\mathrm{i}g)^2}{\Box}\, [A^\mu \stackrel{\star}{,} [A_\mu \stackrel{\star}{,} Y]]\,. \tag{9}$$

The quantity Y can be determined from this equation up to an arbitrary order:

$$\begin{aligned} Y^{(0)} &= \frac{1}{\Box}\widetilde{F}\,, \\ Y^{(1)} &= \frac{1}{\Box}\widetilde{F} + \frac{\mathrm{i}g}{\Box}\partial^\mu \left[A_\mu \stackrel{\star}{,} \frac{1}{\Box}\widetilde{F}\right] + \frac{\mathrm{i}g}{\Box}\left[A^\mu \stackrel{\star}{,} \partial_\mu \frac{1}{\Box}\widetilde{F}\right] - \frac{(\mathrm{i}g)^2}{\Box}\left[A^\mu \stackrel{\star}{,} \left[A_\mu \stackrel{\star}{,} \frac{1}{\Box}\widetilde{F}\right]\right]\,, \end{aligned} \tag{10}$$

and so on.

Next, we define the BRST transformations of the gauge field A_μ, the ghost c, the anti-ghost $\bar{c}$ and the Lagrange multiplier B as usual:

$$\begin{aligned} &sA_\mu = D_\mu c \equiv \partial_\mu c - \mathrm{i}g\, [A_\mu \stackrel{\star}{,} c]\,, \qquad && s\bar{c} = B, \\ &sc = \mathrm{i}g c \star c, && sB = 0, \\ &s^2 \varphi = 0 \quad \text{for } \; \varphi \in \{A_\mu, c, \bar{c}, B\}\,. \end{aligned} \tag{11}$$

The s-variation of A_μ implies $s\widetilde{F} = \mathrm{i}g \left[c \stackrel{\star}{,} \widetilde{F}\right]$, from which it follows (as we will now show) that

$$s\left(\frac{1}{\widetilde{D}^2} \star \widetilde{F}\right) = \mathrm{i}g \left[c \stackrel{\star}{,} \frac{1}{\widetilde{D}^2} \star \widetilde{F}\right]\,. \tag{12}$$

Indeed, for a field Φ transforming as $\tilde{F}$, i.e.

$$s\Phi = \mathrm{i}g\,[c \stackrel{\star}{,} \Phi]\ , \tag{13}$$

the field $D^2\Phi$ also transforms covariantly: $s(D^2\Phi) = \mathrm{i}g\,[c \stackrel{\star}{,} D^2\Phi]$. From

$$s(D^2\Phi) = (sD^2)\,\Phi + D^2(s\Phi)$$

and the previous transformation law, we obtain the operatorial relation

$$(sD^2)\bullet = -\mathrm{i}g\left[D^2 c \stackrel{\star}{,} \bullet\right] - 2\mathrm{i}g\,[D^\mu c \stackrel{\star}{,} D_\mu \bullet]\ . \tag{14}$$

By applying the s-operator to $\Phi = D^2 \star \frac{1}{D^2} \star \Phi$,

$$s\Phi = (sD^2) \star \left(\frac{1}{D^2} \star \Phi\right) + D^2 \star s\left(\frac{1}{D^2} \star \Phi\right)\ ,$$

we can deduce the transformation law of $\frac{1}{D^2} \star \Phi$:

$$s\left(\frac{1}{D^2} \star \Phi\right) = \frac{1}{D^2} \star (s\Phi) - \frac{1}{D^2} \star (sD^2) \star \left(\frac{1}{D^2} \star \Phi\right)\ .$$

Substitution of (13) and (14) into this relation leads to the conclusion that $\frac{1}{D^2} \star \Phi$ transforms in the same manner as Φ,

$$s\left(\frac{1}{D^2} \star \Phi\right) = \mathrm{i}g\left[c \stackrel{\star}{,} \frac{1}{D^2} \star \Phi\right]\ , \tag{15}$$

whence the result (12).

Consider now the following action for the $U(1)$ gauge field A_μ in 4 dimensional non-commutative Euclidean space:

$$\begin{aligned}
\Gamma^{(0)} &= S_{\mathrm{inv}} + S_{\mathrm{gf}}\,, \\
S_{\mathrm{inv}} &= \int d^4x \left[\frac{1}{4} F^{\mu\nu} \star F_{\mu\nu} + \frac{\beta}{4}\left(\frac{1}{\widetilde{D}^2} \star \widetilde{F}\right) \star \left(\frac{1}{\widetilde{D}^2} \star \widetilde{F}\right)\right]\ , \\
S_{\mathrm{gf}} &= s \int d^4x\, \bar{c} \star \left[\left(1 + \frac{\gamma}{\Box\widetilde{\Box}}\right) \partial^\mu A_\mu - \frac{1}{2} B\right] \\
&= \int d^4x \left[B \star \left(1 + \frac{\gamma}{\Box\widetilde{\Box}}\right) \partial^\mu A_\mu - \frac{1}{2} B \star B - \bar{c} \star \left(1 + \frac{\gamma}{\Box\widetilde{\Box}}\right) \partial^\mu D_\mu c\right].
\end{aligned} \tag{16}$$

Here, β and γ are constants, and the term parametrized by γ has been introduced in order to improve the IR behaviour in the ghost sector. (For $\gamma \to 0$, one recovers the Feynman gauge expression.) Furthermore, $\widetilde{\Box} = \widetilde{\partial}^\mu \widetilde{\partial}_\mu$ and $\widetilde{\partial}_\mu = \theta_{\mu\nu}\partial^\nu$.

The action $\Gamma^{(0)}$ is invariant under the BRST transformations (11), (12). Its bilinear part S_{bil} yields the following equations of motion for the free fields:

$$
\begin{aligned}
0 &= \frac{\delta S_{\text{bil}}}{\delta A^\nu} = -\left(\Box\delta_{\nu\mu} - \partial_\nu\partial_\mu\right) A^\mu + \frac{\beta}{\widetilde{\Box}^2}\widetilde{\partial}_\nu\widetilde{\partial}_\mu A^\mu - \left(1 + \frac{\gamma}{\Box\widetilde{\Box}}\right)\partial_\nu B\,, \\
0 &= \frac{\delta S_{\text{bil}}}{\delta B} = \left(1 + \frac{\gamma}{\Box\widetilde{\Box}}\right)\partial^\mu A_\mu - B\,, \\
0 &= \frac{\delta S_{\text{bil}}}{\delta \bar{c}} = -\left(1 + \frac{\gamma}{\Box\widetilde{\Box}}\right)\Box c\,. \qquad (17)
\end{aligned}
$$

This leads to the following propagators in momentum space:

$$
\begin{aligned}
G^A_{\mu\nu}(k) &= \frac{1}{k^2}\left(\delta_{\mu\nu} + \frac{k_\mu k_\nu}{k^2} - \frac{k_\mu k_\nu}{k^2\left(1 + \frac{\gamma}{k^2\tilde{k}^2}\right)^2} - \beta\frac{\tilde{k}_\mu\tilde{k}_\nu}{(\tilde{k}^2)^2(k^2 + \frac{\beta}{\tilde{k}^2})}\right)\,, \\
G^{\bar{c}c}(k) &= \frac{1}{k^2 + \frac{\gamma}{\tilde{k}^2}}\,. \qquad (18)
\end{aligned}
$$

Since the gauge field propagator $G^A_{\mu\nu}$ involves an overall factor $\frac{1}{k^2}$, it is not damped for $k \to 0$ and one may argue that it does not sufficiently mix long and short scales. If one takes this issue of "mixing" more seriously, one is led to the alternative model presented in the next section.

3 New gauge field model II

Considering that the scaling behaviour of the propagator (2) of Gurau et al. [1] ensures the IR finiteness of their model, we look for a BRST invariant action leading to a similar propagator for the $U(1)$ gauge field A_μ. Accordingly, we introduce the following action in 4 dimensional non-commutative Euclidean space:

$$
\begin{aligned}
\Gamma^{(0)} &= S_{\text{inv}} + S_{\text{gf}}\,, \\
S_{\text{inv}} &= \int d^4x \left[\frac{1}{4}F^{\mu\nu} \star F_{\mu\nu} + \frac{1}{4}F^{\mu\nu} \star \frac{1}{D^2\widetilde{D}^2} \star F_{\mu\nu}\right]\,, \\
S_{\text{gf}} &= s\int d^4x\, \bar{c} \star \left[\left(1 + \frac{1}{\Box\widetilde{\Box}}\right)\partial^\mu A_\mu - \frac{\alpha}{2}B\right] \\
&= \int d^4x \left[B \star \left(1 + \frac{1}{\Box\widetilde{\Box}}\right)\partial^\mu A_\mu - \frac{\alpha}{2}B \star B - \bar{c} \star \left(1 + \frac{1}{\Box\widetilde{\Box}}\right)\partial^\mu D_\mu c\right]. \qquad (19)
\end{aligned}
$$

Here, α is a real parameter and $\frac{1}{D^2\widetilde{D}^2} \star F_{\mu\nu}$ is again to be understood as a formal power series in the gauge field A_μ. The functional $\Gamma^{(0)}$ is invariant under the

BRST transformations (11) which imply

$$s\left(\frac{1}{D^2\widetilde{D}^2}\star F_{\mu\nu}\right) = \mathrm{i}g\left[c\stackrel{\star}{,}\frac{1}{D^2\widetilde{D}^2}\star F_{\mu\nu}\right]. \tag{20}$$

The bilinear part of the action now leads to the following equations of motion for the free fields:

$$\begin{aligned}
0 &= \frac{\delta S_{\text{bil}}}{\delta A^\nu} = -\left(1+\frac{1}{\Box\widetilde{\Box}}\right)\left(\Box\delta_{\nu\mu}-\partial_\nu\partial_\mu\right)A^\mu - \left(1+\frac{1}{\Box\widetilde{\Box}}\right)\partial_\nu B\,,\\
0 &= \frac{\delta S_{\text{bil}}}{\delta B} = \left(1+\frac{1}{\Box\widetilde{\Box}}\right)\partial^\mu A_\mu - \alpha B\,,\\
0 &= \frac{\delta S_{\text{bil}}}{\delta \bar{c}} = -\left(1+\frac{1}{\Box\widetilde{\Box}}\right)\Box c\,.
\end{aligned} \tag{21}$$

Hence, we get the following propagators in momentum space:

$$\begin{aligned}
G^A_{\mu\nu}(k) &= \frac{1}{k^2+\frac{1}{\tilde{k}^2}}\left(-\delta_{\mu\nu}+\frac{k_\mu k_\nu}{k^2}-\alpha\frac{k_\mu k_\nu}{k^2+\frac{1}{\tilde{k}^2}}\right),\\
G^{\bar{c}c}(k) &= \frac{1}{k^2+\frac{1}{\tilde{k}^2}}\,.
\end{aligned} \tag{22}$$

If one chooses the Landau gauge $\alpha = 0$ for the gauge parameter, then the gauge field propagator simplifies to

$$G^A_{\mu\nu}(k) = \frac{1}{k^2+\frac{1}{\tilde{k}^2}}\left(-\delta_{\mu\nu}+\frac{k_\mu k_\nu}{k^2}\right). \tag{23}$$

4 Concluding remarks

In the preceding sections, we introduced two natural models for non-commutative $U(1)$ gauge fields. These models are both BRST-invariant and translation-invariant, and they are devised for curing the UV/IR mixing problem. The second model has the advantage that the gauge field propagator has an improved "damping" behaviour for vanishing momentum. The question whether this property is sufficient for ensuring the renormalizability of the model obviously requires further and more involved investigations (work in progress).

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