Michelson interferometry with quantum noise reduction

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A Michelson interferometer with noise reduction to sub-shot noise levels is proposed and realized. Multiple measurements of a single signal beam are taken and the quantum property of light plays an essential role in the principle underlying this interferometry. The method makes use of the coherent state of light and requires only a simple modification to the standard Michelson interferometer. The surface fluctuation spectra of liquids are measured using this method down to a few orders of magnitude below the shot noise level. The spectrum derived from hydrodynamical considerations agrees well with the observed results for water. However, for oil, slight deviations are seen at high frequencies ($\gtrsim 1\,\mathrm{MHz}$), perhaps indicating its more complex underlying physics. The measurement requires a relatively low light power and a short time, so that it has a wide range of applicability.

Interferometry is a most precise tool for detecting small displacements and hence is used in a very broad range of areas in physics, from microscopic spectroscopy such as ours, structural measurements of optical elements to astrophysics[1] and arguably the most sensitive measurement, the attempts to detect gravitational waves[2, 3]. Surface thermal fluctuations have been measured with interferometry previously, on mirrors using high power lasers[4] and on liquid drop surfaces attached to fiber tips[5]. Spectral properties of surface waves on simple liquids have also been measured by using them as gratings [6, 7]. However this approach is difficult to apply to dissipative liquids such as oil, since they do not create well defined waves that act effectively as gratings. Spectra of surface inclination fluctuations have been measured[8–11] using the surface as an optical lever. Also, exceptionally large surface fluctuations due to low surface tension have been observed using other methods[12].

In this work, we measure surface thermal fluctuation spectra of simple liquids over a wide frequency range ($1\,\mathrm{kHz} \sim 40\,\mathrm{MHz}$) and down to few orders of magnitude below the shot noise level, using Michelson interferometry. While these spectra reflect fundamental physics principles of liquids and are interesting from a physics perspective[6, 7, 13], the spectra we obtain have not been done so previously, to our knowledge. The main reason for this is that these fluctuations are small and are buried under the shot noise. It is usually believed that the shot noise sets the limit for the signal-to-noise ratio that can be obtained under normal circumstances[2, 3, 14], with the exception being measurements involving sub-Poissonian photon statistics. We show both theoretically and experimentally that the standard Michelson interferometer, when combined with the quantum nature of light can achieve signal-to-noise ratio unlimited by the shot noise. In essence, while both thermal fluctuations and shot noise are random, the former are classical in their origin and the latter has fundamentally quantum nature and their difference enables us to separate these two. By being able to analyze the thermal fluctuation spectra to such precision, their theoretical understanding can be examined in detail. The traditional hydrodynamical description of simple liquids works well for water, but some deviations from it for liquids such as oil are observed at high frequencies.

We now summarize briefly the general principle behind the noise reduction employed here, which is not limited to optical measurements nor to shot noise[10, 11]: In analyzing weak signals, if the signal has a definite periodicity, we can accumulate data in accordance with the period to suppress the noise. However, in cases where there is no definite periodicity, it is difficult to separate out random signals, such as thermal fluctuations, from random noise. In fact, given a single measurement $D_1 = S + N_1$ of a random signal, there is no way, even in principle, to distinguish the signal S from the noise N_1, which inevitably occurs. Shot noise is a typical example of such random noise. To overcome this obstacle, we perform an additional measurement of the same signal $D_2 = S + N_2$ at the same time, whose noise N_2 is independent of N_1. Then, by taking the correlation of $D_{1,2}$ and averaging over time, we obtain

$$\langle \overline{\tilde{D}_1 \tilde{D}_2} \rangle \longrightarrow \langle |\tilde{S}|^2 \rangle \qquad (\mathcal{N} \to \infty) \quad , \tag{1}$$

where $\mathcal{N}$ is the number of averagings and tildes denote Fourier transforms[10, 11]. While the above principle is simple, the crucial point is to arrange multiple measurements of the same signal in such a way as to ensure that the noise in them are uncorrelated.

To measure surface fluctuations, laser light is shone on the surface and the reflected light is used as a signal for measurement. The quantum property of light plays a critical role in our experiment; the beam splitter randomly partitions the photons from the single signal beam to one of the two detection systems (Fig. 1(a)). The photons giving rise to the shot noise is random, when a coherent light source is used. Therefore, the noise in the two detectors are uncorrelated and is eliminated when we compute their correlation, Eq. (1). It is important to note that splitting the beam by itself does not guarantee the independence of the noise in the two measurements. Had we used a squeezed light source, for instance, the photons in the two measurements would have been correlated, so that the noise could not have been eliminated by using the correlation of detector

measurements. Therefore, our approach is in contrast to those that use squeezed light sources to obtain sub-shot noise measurements[3, 15, 16]. Using squeezed light sources, a reduction of the shot noise by a factor of two have been achieved and a light source squeezing factor close to 20 has been attained[17].
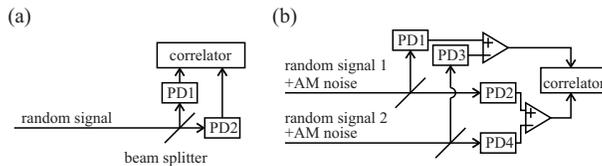

FIG. 1: (a)By taking the correlation of two photodetector (PD1,2) measurements, we separate the signal from the uncorrelated random noise. (b)To eliminate AM noise that inevitably occur, we additionally use the differential detection of random signal 1 and 2, by adding PD3,4. Random signals 1,2 can either be phase inverted or uncorrelated.

The signals obtained from the photodetectors (Fig. 1(a)), in practice, additionally contain amplitude modulation (AM) noise from the light source. This is common to the two measurements and hence will not be eliminated by taking their correlation. To reduce this noise, we further employ differential detection (Fig. 1(b)).
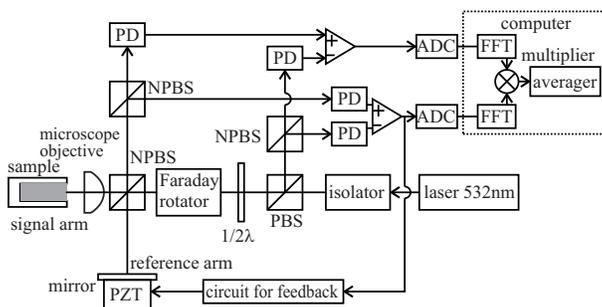

FIG. 2: Experimental setup that incorporates noise reduction due to correlation and differential detection shown in Fig. 1. NPBS: nonpolarizing beam splitter.

Our Michelson interferometry setup that incorporates these principles is shown in Fig. 2. The light source for this experiment is a solid-state diode-pump laser (SAPPHIRE SF-532-50, Coherent) with a wavelength of λ=532 nm arranged so that the power is 500 μW each at the sample and at the reference mirror. A typical measurement we show below requires a few seconds of measurement time. To implement the differential detection, a Faraday rotator is used for separating out the light reflected back from the sample at the polarizing beam splitter (PBS). The lengths of the signal and reference arms in the Michelson interferometer are adjusted to be equal within $1\,\mu$m, since a larger difference increases frequency modulation noise effects. To obtain maximal sensitivity in detecting sample surface displacements, the reference arm length is adjusted dynamically by a piezo-electric transducer (PZT) with feedback. In the Michelson interferometer, the light at the reference mirror and that at the sample need to have the same properties or the destructive interference within the beam spot will reduce the signal significantly. Light reflected by the reference mirror and the sample is detected by four photodiodes (S5973 Hamamatsu Photonics, Japan). The signal currents obtained through differential detection are digitized by an analog to digital converter (ADC, 8 bit, 125 Ms/s, PicoScope 5203, Pico Technology). Their Fourier transforms and the correlation in Eq. (1) are calculated using a computer.

Two light sources and two sets of detectors can also be used to obtain two independent measurements and extract signals at sub-shot noise levels from their correlation. This was the approach used for inclination fluctuation spectra of surfaces[10, 11]. In this work, by using a single light source, we make minimal modifications to the classic Michelson interferometer, essentially by just adding an additional beam splitter at the photodetector, to attain sub-shot noise measurements. This leads to a simple elegant setup. Using a single light source is not only simpler, but has an important practical advantage; to achieve maximal sensitivity in the interferometer, the path lengths need to be adjusted according to the wavelength of the light source, which is difficult to attain for multiple wavelengths simultaneously.

The spectral function of thermal surface fluctuations for a simple liquid is determined from hydrodynamical considerations by its density ρ, surface tension σ and viscosity η and is

$$P(k,\omega) = \frac{k_{\mathrm{B}}T}{\pi} \frac{ku^2}{\rho\omega^3} \, \mathrm{Im} \left[(1-iu)^2 + y - \sqrt{1-2iu} \right]^{-1}, \tag{2}$$

where $u \equiv \rho\omega/(2\eta k^2), y \equiv \rho\sigma/(4\eta^2 k)$[18]. Here, k, ω are the wave number and the angular frequency of the surface wave. Through Michelson interferometry, we measure fluctuations perpendicular to the liquid surface

and its spectrum can be computed analogously to the inclination fluctuation spectra[10, 11] as

$$S_{\mathrm{h}}(f) = 2 \int_0^{\infty} dk\, k\, e^{-b^2 k^2 / 8} P(k, 2\pi f) \quad , \tag{3}$$

Here, $f = \omega/(2\pi)$ and b is the beam diameter. Surface waves with wavelengths larger than the sample size are cut off so that the effects of gravity can be ignored. Also, surface waves with wavelengths smaller than the beam spot size is suppressed due to averaging. In the measurements below, the size of the surface sample is $2.2\,\mathrm{mm}$ in diameter and $b = 0.96\,\mu\mathrm{m}$.
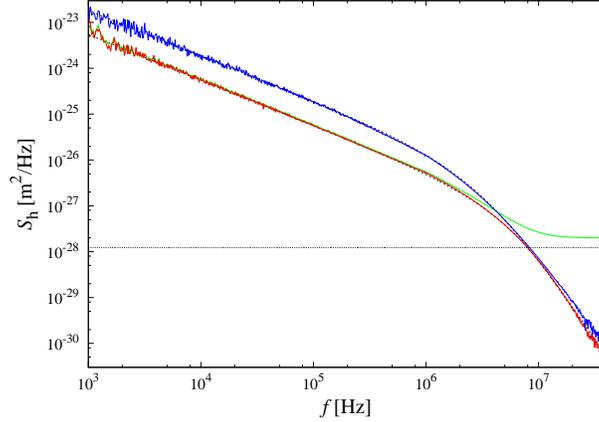

FIG. 3: (Color online) Experimentally observed surface height fluctuation spectra for water (red) and ethanol (blue). Water has smaller fluctuations. Respective theoretical spectra are also shown (black, dashed), which agree with the experimental results and are almost invisible. For comparison, observed data for a single differential detection without using the correlation Eq. (1) is shown for water (green) which is clearly dominated by the shot noise at higher frequencies. The theoretical value for the shot noise level, Eq. (4), is also indicated (black, dotted).
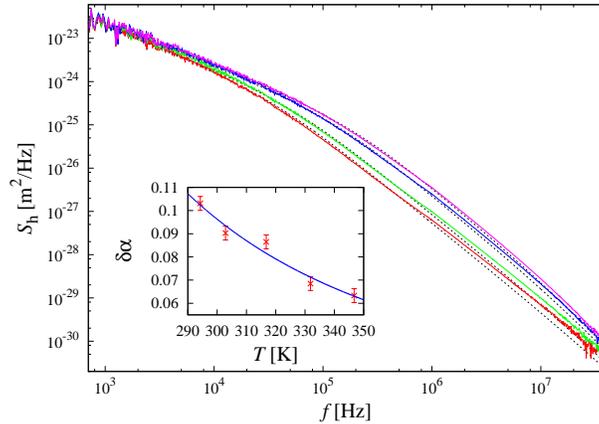

FIG. 4: (Color online) Experimentally observed surface fluctuation spectra for oil at various temperatures (red, green, blue, magenta from lower to higher temperatures). Fluctuations are larger at higher temperatures. Their respective theoretical spectra, Eq. (3), are also shown (black, dashed). (Inset)Temperature dependence of the deviation from theory of the fall off at high frequencies ($\gtrsim 1\,\mathrm{MHz}$).

We compare the measured spectra against their theoretical predictions, Eq. (3), for water and ethanol in Fig. 3. The physical properties of water, ethanol are well known for a given temperature and are $(\rho\,[\mathrm{kg/m^3}], \sigma\,[\mathrm{kg/s^2}], \eta\,[\mathrm{kg/(m \cdot s)}], T[\mathrm{K}]) = (1.0 \times 10^3, 7.5 \times 10^{-2}, 1.7 \times 10^{-3}, 275), (0.80 \times 10^3, 2.3 \times 10^{-2}, 1.7 \times 10^{-3}, 278)$, respectively. Given the temperature, these properties and the spectral function $P(k, \omega)$ are completely determined. The integrated spectrum $S_{\mathrm{h}}(f)$, Eq. (3), is uniquely determined by those properties and the beam size. Water and ethanol are cooled to suppress evaporation which can cause the sample surface to leave the focal plane. For water surface fluctuations, the theoretical formula derived from hydrodynamic considerations agree essentially perfectly with the observed spectrum. These measurements confirm the theoretical spectrum over a wide frequency range. For ethanol, good agreement is seen, except for the slightly slower fall off at frequencies above $10\,\mathrm{MHz}$. We come back to this issue for the case of oil surface fluctuations.

Oil (Olympus immersion oil AX9602) surface thermal fluctuation spectra were obtained at various temperatures and compared to the theoretical spectra Eq. (3) in Fig. 4. There is a qualitative difference from the spectra of water and ethanol surface fluctuation spectra in Fig. 3, due to the high viscosity of the liquid. In particular, the spectral density decays much more slowly than that of water and oil at higher frequencies. For the oil, the temperature dependence of σ, η are not known so that they had to be deduced from the spectra and were found to be $(\rho\,[\text{kg/m}^3], \sigma\,[\text{kg/s}^2], \eta\,[\text{kg/(m}\cdot\text{s)}], T[\text{K}]) = (0.92\times10^3, 3.2\times10^{-2}, 0.15, 294), (0.91\times10^3, 3.1\times10^{-2}, 0.096, 303),$ $(0.89\times10^3, 2.9\times10^{-2}, 0.037, 332), (0.88\times10^3, 2.7\times10^{-2}, 0.025, 347).$ σ, η are determined to ten percent from the spectra. For oil, we see that the fluctuation spectrum changes with the temperature, as it should. Most of this change is caused by the decrease in the viscosity with higher temperatures, making the fluctuations larger.

While the fluctuation spectra of liquids we study are usually regarded as well understood, the observed higher frequency ($\gtrsim 1\,\text{MHz}$) fall offs in the spectra for oil and ethanol are slightly slower than their theoretical predictions, especially at lower temperatures. This is possibly due to the more complex nature of the liquids that can not be explained just from the hydrodynamical considerations of simple liquids. The slower fall off of the spectrum has also been observed for solid materials and a gradual transition to such a dependence was seen for complex fluids[10]. We obtain the dependence $S_\text{h}(f) \sim f^\alpha$ for higher frequencies ($10^6 - 10^7\,\text{Hz}$) and its deviation $\delta\alpha$ from the theoretical value in Eq. (3). In Fig. 4(inset), the temperature dependence of $\delta\alpha$ is shown. The dependence can be reasonably well described by $\delta\alpha = C\exp(U/k_\text{B}T)$ with $U = 7\,\text{kJ/mol}$. This energy scale U is comparable to the latent heat for oil, which is consistent with more complex behavior, such as molecular interactions causing visible effects in the spectrum.

In Fig. 3, we also included the results from one differential measurement of water surface fluctuations, in which the shot noise level is clearly visible. Theoretically, the shot noise level in the spectrum is

$$e\frac{\lambda^2}{32\pi^2}\frac{1+r^2}{rI_\text{PD}}\Delta f \quad , \tag{4}$$

when the sensitivity in the interferometer is maximal. Here, r is the ratio of the reflectivities of the sample surface and the reference mirror, I_PD the signal photocurrent and e the electron charge. As can be seen in Fig. 3 and 4, shot noise is clearly eliminated when the correlation of the two differential measurements are taken. The observed shot noise level in Fig. 3 is roughly twice its theoretical value, Eq. (4). This indicates that the interferometer sensitivity is not at its theoretical maximum, whose most likely cause is the aberration of the objective lens.

Let us briefly describe the physical properties of the spectra Eq. (3): When $16\sqrt{2}\pi\eta^3 f/(\rho\sigma^2) \gtrsim 1$, the liquid can be regarded as being highly viscous so that any liquid is dissipative at high enough frequencies. In the frequency region we study, water, ethanol have low and oil has high viscosity. The spectra have qualitatively different f dependence for low and high viscosities, as explained below[10, 11]. In both cases, $S_\text{h}(f) \sim k_\text{B}T/(\sigma f)$ at low frequencies. For a liquid with low viscosity, the spectrum crosses over at $f \sim \sqrt{\sigma/(\rho b^3)}$ to $S_\text{h}(f) \sim k_\text{B}T\eta/(\rho^2 b^5 f^4)$. For a liquid with high viscosity, the spectrum crosses over at $f \sim \sigma/(\eta b)$ to $S_\text{h}(f) \sim k_\text{B}T/(\eta b f^2)$. While outside the region of our measurements, at even higher frequencies, $f \gtrsim \eta/(\rho b^2)$, the spectrum changes to $S_\text{h}(f) \sim k_\text{B}T\eta/(\rho^2 b^5 f^4)$. For both low and high viscosities, the behavior of $S_\text{h}(f)$ is dominated by σ at low frequencies and is independent of η since the dynamical time scale is relatively large. At high frequencies, the behavior is governed by η and is independent of σ, to leading order. These crossovers seen in the spectra are similar to those in the surface inclination fluctuation spectra and arise from the properties of $P(k, 2\pi f)$, Eq. (2)[10, 11]. However, the spectra differ qualitatively. In particular, $S_\text{h}(f)$ has a distinctive $1/f$ dependence at lower frequencies and is independent of the beam diameter b to leading order. Consequently, the height fluctuation measurements are more sensitive than the inclination fluctuation measurements at lower frequencies. This is natural since the inclination fluctuations due to longer wavelength fluctuations are smaller for a fixed beam size. At high frequencies, the cutoff frequency determined by b becomes important, since it is responsible for suppressing the shorter wavelength modes.

In this work, we have proposed and implemented the Michelson interferometer with noise reduction at sub-shot noise levels. We make multiple measurements of a single signal light. The shot noise in them are independent due to their quantum nature, hence the noise can be eliminated through their correlation and the signal below this noise level can be extracted. While both random, the distinct difference between the thermal fluctuations and the shot noise, which are classical and quantum in their origins, allow for their separation. It would be interesting to understand the behavior when the method is applied to quantum fluctuation measurements with shot noise. At present, some applications such as quantum cryptography and quantum random number generation exist, yet there have not been many cases where the quantum property of light has been of practical value[14] and it is satisfying to find it provide a practical advantage here. The measurement requires only a small light power, a relatively short time and a small sample surface ($500\,\mu\text{W}$, few seconds, diameter $1\,\mu\text{m}$, in this work). Our noise reduction can also be added to higher power measurements such as those in [4] to reduce the noise further. Longer measurement times can lead to higher resolution, more averaging and less statistical error. Interferometry is perhaps the most commonly used method for precision measurements and shot noise can often be the major limiting factor in its accuracy[2, 3, 14]. We believe that our method can be applied in various situations in which precision, low power or short measurement time is required.

Let us discuss the limitations of our approach: The approach is effective when the number of averagings $\mathcal{N} = \Delta f T$ is large, where Δf is frequency resolution in the spectrum and T is the total measurement time. The relative error in the spectrum is $1/\sqrt{\mathcal{N}}$. Δf needs not be the same across the spectrum and if a same relative resolution is used, this statistical error is smaller at higher frequencies. While this integration is effective for stationary signals, the situation is more subtle for transient signals such as gravitational waves radiated from a single event. In such situations, the restriction that the measurement time needs to be within the time when the signal is present can be demanding. The duration of the signal needs to be long enough compared to the inverse of the desired frequency resolution, to reduce the noise using the correlation of measurements.

[1] J.D. Monnier, Rep. Prog. Phys. 66, 789 (2003).
[2] M. Maggiore, "Gravitational waves", Oxford University Press, Oxford (2008); C.M. Caves et al, Rev. Mod. Phys. 52, 341 (1980); G. Cella, A. Giazotto, Rev. Sci. Inst. 82, 101101 (2011).
[3] LIGO Sci Collaboration, Nature Phys. 7, 962 (2011).
[4] K. Numata, M. Ando, K. Yamamoto, S. Otsuka, K. Tsubono, Phys. Rev. Lett. 91, 260602 (2003); E.D. Black et al, Phys. Lett. A 328, 1 (2004).
[5] T. Mitsui: Jpn. J. Appl. Phys. 43 (2005) 8345.
[6] R.H. Katyl, U. Ingard, Phys. Rev. Lett. 19, 64 (1967).
[7] D. Langevin, "Light scattering by liquid surfaces and complementary techniques", Marcel Dekker, New York (1992).
[8] T. Mitsui, Jpn. J. Appl. Phys. 47, 6563 (2008).
[9] A.Tay et al., Rev. Sci. Instrum. 79, 103107 (2008).
[10] T. Mitsui, K. Aoki, Phys. Rev. **E** 80, 020602(R)-1 (2009).
[11] T. Mitsui, K. Aoki, arXiv:1112.4260.
[12] H.J. Lauter, H. Godfrin, V.L.P. Frank, P. Leiderer, Phys. Rev. Lett 68, 2484 (1992); A. Vailati, M. Giglio, Nature 390, 262 (1997).
[13] M. von Schmoluchowski, Ann Physik 25, 225 (1908); L. Mandelstam, Ann. Physik 41, 609 (1913).
[14] See, for instance, M. Fox, "Quantum Optics", Oxford University Press, New York (2006).
[15] C.M. Caves, Phys. Rev. D23, 1693 (1981).
[16] K. McKenzie, B.C. Buchler, D.A. Shaddock, P.K. Lam, D.E. McClelland, Class. Quantum Grav. 21, S1037 (2004).
[17] H. Vahlbruch et al, Phys. Rev. Lett. 100, 033602 (2008); T. Eberle et al, Phys. Rev. Lett. 104, 251102 (2010).
[18] M.-A. Bouchiat, and J. Meunier, J. de Phys. 32, 561 (1971).