Color Glass Condensate and Glasma

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Abstract

In this talk, I review the Color Glass Condensate theory of gluon saturation, and its application to the early stages of heavy ion collisions.

Keywords: QCD, Gluon saturation, Heavy ion collisions

1. Color Glass Condensate

In heavy ion collisions, the vast majority of the produced particles have a transverse momentum of at most a couple of GeV's. This implies that they are produced from incoming partons that carry a very small fraction x of the longitudinal momentum of the projectiles. Furthermore, this momentum fraction decreases with the center of mass energy of the collision. At RHIC energy, the typical value of x is of the order of 10^{-2} for the bulk of particle production, and of order $x \sim 4.10^{-4}$ at the LHC. However, at such small values of x, the gluon density in a proton or nucleus becomes large which leads to a new phenomenon known as *gluon saturation*, due to non-linear recombinations among the gluons. Gluon saturation is characterized by an x-dependent momentum scale, the saturation momentum $Q_s(x)$, that delimitates the domain in momentum where saturation takes place. This domain is represented in the figure 1.

Having identified this region where nonlinear gluon interactions become important, one faces now the question of how to compute physical observables reliably in this regime where one collides projectiles made of a very large number of constituents. The main novelty in the saturated regime, compared to the dilute one, is that the typical particle production processes involve multiple gluon interactions, as shown in the figure 2. The standard perturbative techniques based on Feynman diagrams are well suited to handle the dilute situation, but they become impractical in the saturated regime

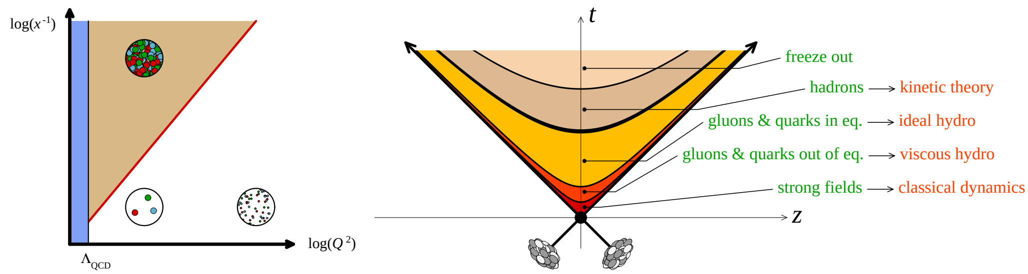

Figure 1: Left: Saturation domain in the x, Q^2 plane. Right: Stages of a heavy ion collision.

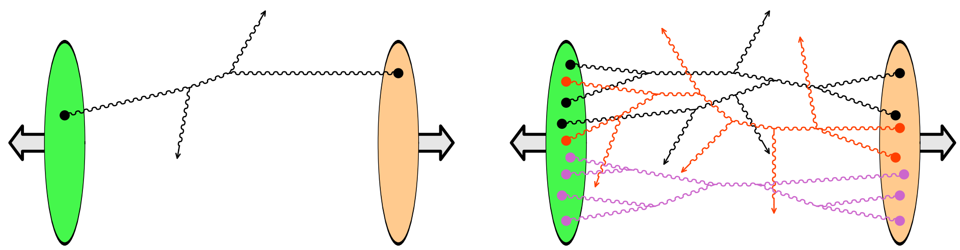

Figure 2: Left: Gluon production in the dilute regime. Right: Saturated regime.

because an infinite number of graphs would contribute at each order in g^2. In addition, in order to compute observables in this dense regime, we also need to know the probability to find these multigluon states in the wavefunctions of the incoming nuclei.

The Color Glass Condensate is an effective description of QCD in the saturation regime. In this effective theory, one divides the degrees of freedom (see the figure 3) into fast partons with longitudinal momentum $k^+ > \Lambda^+$, and slow partons with $k^+ < \Lambda^+$ that have a significant dynamical evolution over the time-scales of the reaction of interest [1]. Thanks to Lorentz time

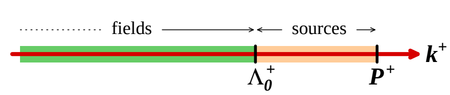

Figure 3: Separation of the degrees of freedom in the CGC.

dilation, the fast partons are essentially frozen during the collision process, and it is sufficient to know what color current they carry. Therefore, their description in the CGC is simplified into a distribution of static color currents $J^\mu = \delta^{\mu +}\rho(x^-, \boldsymbol{x}_\perp)$ (this is for a projectile moving in the $+z$ direction). On the contrary, this approximation does not work for the slow degrees of freedom, and therefore we must keep describing them as conventional gauge fields A^μ. Due to the separation in longitudinal momentum between these two types of degrees of freedom, their coupling is eikonal, $A_\mu J^\mu$. Thus, the effective Lagrangean of the CGC is:

$$\mathcal{L} = \underbrace{-\frac{1}{2}\,\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu}}_{\text{gluon dynamics}} + \underbrace{(J_1^\mu + J_2^\mu)}_{\text{fast partons}}\; A_\mu \; .$$

This description would not be complete without a way to specify the color charge density $\rho(x^-, \boldsymbol{x}_\perp)$. This distribution depends on the precise configuration of the fast partons (e.g. their localization in the transverse plane at the time of the collision), which is not known. Thus, it should be considered as a stochastic quantity, with a probability distribution $W_{\Lambda^+}[\rho]$. The distribution $W_{\Lambda^+}[\rho]$ plays in the CGC the same role as partons distributions in conventional low density perturbative QCD.

The CGC effective theory introduces a cutoff Λ^+ that separates the fast and the slow degrees of freedom. However, this cutoff is unphysical and one should ensure that physical observables do not depend upon it. It turns out that for this to be true, the probability distribution $W_{\Lambda^+}[\rho]$ must evolve with Λ^+ according to the JIMWLK equation [2],

$$\frac{\partial W_{\Lambda^+}}{\partial \ln(\Lambda^+)} = \mathcal{H}\;\; W_{\Lambda^+} \; , \quad \mathcal{H} = \frac{1}{2}\int\limits_{\boldsymbol{x}_\perp, \boldsymbol{y}_\perp} \frac{\delta}{\delta\alpha(\boldsymbol{y}_\perp)}\eta(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)\frac{\delta}{\delta\alpha(\boldsymbol{x}_\perp)} \; ,$$

where $-\boldsymbol{\partial}_\perp^2\,\alpha(\boldsymbol{x}_\perp) = \rho(1/\Lambda^+, \boldsymbol{x}_\perp)$. Effectively, the JIMWLK equation resums to all orders the powers of $\alpha_s \ln(\Lambda^+)$ (or, equivalently, powers of $\alpha_s \ln(1/x)$), and its kernel $\eta(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)$ includes all corrections in the color source ρ. When approximated to low density by expanding the kernel to lowest order ρ, one recovers the BFKL equation that describes the evolution to low x of non-integrated gluon distribution of a dilute hadron.

2. Just before the collision: factorization

A high energy heavy ion collision can be divided in several successive stages, as shown in the figure 1. The CGC applies to the description of the wavefunctions of the incoming projectiles, to the collision itself, and to a brief time after the collision, of the order of $\tau \sim Q_s^{-1}$. Since the subsequent stages are usually described by some form of relativistic fluid dynamics, one of the CGC goals is to provide the value of the energy momentum tensor at some initial time τ_0. But more fundamentally, since the CGC provides a QCD-based description of the early stages of heavy ion collisions, it is also the framework to be used in order to justify the applicability of hydrodynamics.

In the CGC, the energy-momentum tensor admits an expansion in powers of the strong coupling g^2. However, because the color sources are strong in the saturated regime, this expansion starts by a term in g^{-2},

$$T^{\mu\nu} = \frac{Q_s^4}{g^2}\left[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots\right] .$$

The coefficients $c_{0,1,2,\cdots}$ in this expansion are themselves infinite series in $g\rho$ (which is parametrically of order one in the saturated regime), and therefore each order corresponds to an infinite series of Feynman diagrams.

At leading order, one can prove that the sum of the corresponding set of diagrams can be expressed in terms of classical solutions of the Yang-Mills equations [3],

$$T^{\mu\nu}_{\text{LO}} = \frac{1}{4} g^{\mu\nu}\, \mathcal{F}^{\lambda\sigma}\mathcal{F}_{\lambda\sigma} - \mathcal{F}^{\mu\lambda}\mathcal{F}^{\nu}{}_{\lambda}\ , \quad \underbrace{\left[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}\right] = J^\nu}_{\text{Yang}-\text{Mills equation}}\ , \quad \lim_{t\to-\infty} \mathcal{A}^\mu(t, \boldsymbol{x}) = 0\ .$$

These equations are non-linear and in general one cannot find analytic solutions. However, they have been solved numerically to obtain e.g. the initial energy density released in a collision [4].

At next to leading order, the energy-momentum tensor starts being sensitive to the slow gluon fields of the CGC effective theory, via a loop correction. It is convenient to consider only one small slice of these slow modes, located just below the original cutoff Λ_0^+ of the CGC, as shown in the figure 4. The

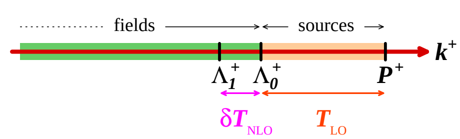

Figure 4: Slice of slow modes at NLO.

contribution of this slice can be calculated at leading log accuracy [5],

$$\delta T^{\mu\nu}_{\rm NLO} = \left[\ln\left(\frac{\Lambda_0^+}{\Lambda_1^+}\right)\mathcal{H}_1 + \ln\left(\frac{\Lambda_0^-}{\Lambda_1^-}\right)\mathcal{H}_2\right] T^{\mu\nu}_{\rm LO}\;,$$

where $\mathcal{H}_{1,2}$ are the JIMWLK Hamiltonians of the two projectiles. Thanks to their form that does not mix the two projectiles, these logarithms appear to be intrinsic properties of the wavefunctions of the two nuclei. Therefore, they can be absorbed by defining a new CGC effective theory that has its cutoff at the lower values Λ_1^+,

$$\Big\langle \boldsymbol{T}_{\rm LO} + \delta\boldsymbol{T}_{\rm NLO}\Big\rangle_{\Lambda_0} = \Big\langle \boldsymbol{T}_{\rm LO}\Big\rangle_{\Lambda_1}\;,\quad W_{\Lambda_1^\pm} \equiv \left[1 + \ln\left(\frac{\Lambda_0^\pm}{\Lambda_1^\pm}\right)\;\mathcal{H}_{1,2}\right] W_{\Lambda_0^\pm}\;,$$

provided the distribution of sources in the new effective theory is defined as prescribed by the JIMWLK equation. This process can be iterated, until the cutoffs become smaller than the physically relevant scales. The outcome of this procedure is the following formula for the energy-momentum tensor,

$$\left\langle T^{\mu\nu}(\tau,\eta,\boldsymbol{x}_\perp)\right\rangle_{\rm LLog} = \int \big[D\rho_1\,D\rho_2\big]\;W_1\big[\rho_1\big]\;W_2\big[\rho_2\big]\;T^{\mu\nu}_{\rm LO}(\tau,\boldsymbol{x}_\perp)\;,$$

which resums all these logarithms. Note that at this level of accuracy, the rapidity dependence of the left hand side comes entirely from the JIMWLK evolution of the distributions W of the two projectiles. Besides its usefulness in heavy ion collisions, this factorization is important because it establishes a bridge between nucleus-nucleus collisions and other reactions such as electron-nucleus collisions, where distributions W with the same JIMWLK evolution also appear. Let us also stress that this factorization also applies to any sufficiently inclusive observables. In particular, the correlation between the components of the energy-momentum tensor measured at different points in space [5] reads

$$\begin{aligned}&\left\langle T^{\mu_1\nu_1}(\tau,\eta_1,\boldsymbol{x}_{1\perp})\cdots T^{\mu_n\nu_n}(\tau,\eta_n,\boldsymbol{x}_{n\perp})\right\rangle_{\rm LLog} =\\ &\qquad = \int \big[D\rho_1\,D\rho_2\big]\;W_1\big[\rho_1\big]\;W_2\big[\rho_2\big]\;T^{\mu_1\nu_1}_{\rm LO}(\tau,\boldsymbol{x}_{1\perp})\cdots T^{\mu_n\nu_n}_{\rm LO}(\tau,\boldsymbol{x}_{n\perp})\;.\end{aligned}$$

One sees that at leading log accuracy, all these correlations come from the W's (since the integrand is a product of n independent factors). This formula predicts a correlation in rapidity over a range of order $\Delta\eta \sim \alpha_s^{-1}$, since this is the rapidity variation necessary to produce a change in the W's.

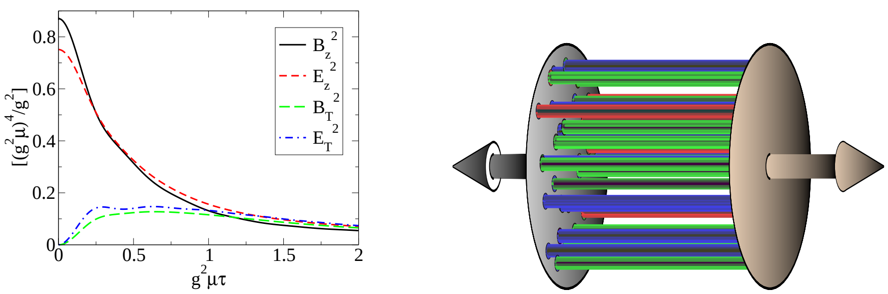

Figure 5: Left: field components evaluated by solving numerically the Yang-Mills equations (from [6]). Right: longitudinal color flux tubes.

3. Just after the collision: Glasma fields

Immediately after the collision, the chromo- $\boldsymbol{E}$ and $\boldsymbol{B}$ fields have only longitudinal components [6], forming flux tubes along the collision axis (see the figure 5). This configuration of color fields has been named the *glasma*. The typical transverse size of a flux tube is of order Q_s^{-1}, and the color fields are correlated over α_s^{-1} units of rapidity in the longitudinal direction.

This particular topology of the post-collision color fields has several consequences, among which a peculiar form of the energy-momentum tensor (see section 4), the fact that the multiplicity distribution is a negative binomial [7], and the existence of a non-zero topological density $F\widetilde{F}$, possibly at the origin of observable CP violating effects [8]. But the most direct and striking consequence of these structures, when taken as initial conditions for hydrodynamical expansion, is that they lead to the formation of the so-called *ridge* correlations, a structure in the 2-hadron spectrum which is elongated in $\Delta\eta$ and narrow in $\Delta\phi$ (see the left plot of the figure 6). By examining the causal relation between two particles separated in rapidity (right part of the figure 6), one can see that the process responsible for producing a correlation between these particles must have taken place at early times,

$$t_{\rm correlation} \le t_{\rm freeze\ out}\ \ e^{-\frac{1}{2}|\eta_A-\eta_B|}\ .$$

Since the color fields produced at early times in the CGC formalism are correlated over rapidity intervals of order $\alpha_s^{-1} \gg 1$, they provide a natural explanation for the rapidity dependence of the ridge [9]. The strength of the 2-particle correlation is controlled by $(Q_sR)^{-2}$ –the area of one flux tube relative to the total transverse area– since the two particles must come from

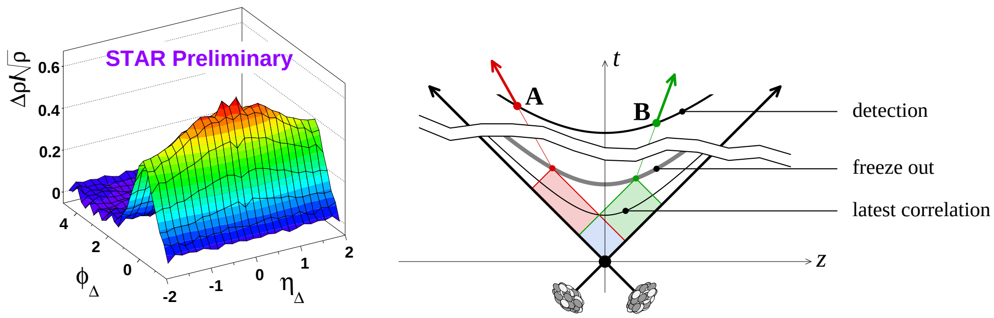

Figure 6: Left: STAR result on 2-hadron correlations. Right: causal relationship between two produced particles.

the same tube to have been produced by the same coherent field (see the figure 7, left panel). The azimuthal dependence is produced at a later stage, by the radial hydrodynamical flow, that collimates the azimuthal angles of the two particles in the direction of the radial velocity (figure 7, right panel).

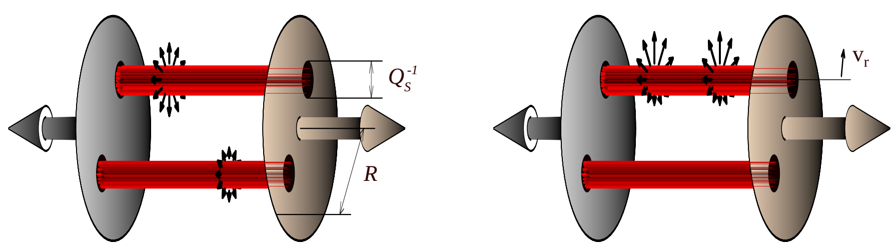

Figure 7: Left: particle emitted from distinct tubes are uncorrelated. Right: collimation due to the radial flow.

4. Matching to hydrodynamics

At times $\tau \gg Q_s^{-1}$, the standard description of the evolution of the fireball is via hydrodynamical expansion. However, a trivial consequence of the fact that the chromo- $\boldsymbol{E}$ and $\boldsymbol{B}$ fields are initially parallel to the collision axis in the glasma is that the energy-momentum tensor one obtains at leading order in g^2 in the CGC is of the form $T^{\mu\nu}_{\rm LO}(0^+,\eta,\boldsymbol{x}) = {\rm diag}\,(\epsilon,\epsilon,\epsilon,-\epsilon)$ where

ϵ is the energy density. The fact that the longitudinal pressure is negative is problematic for a smooth matching to hydrodynamics, because it means that the CGC energy-momentum tensor is quite far from the form for which quasi-ideal hydrodynamics is applicable (i.e. one where the spatial part of the tensor is not too far from being proportional to the identity).

It has been known for some time that corrections to the leading order CGC prediction suffers from secular divergences. Indeed, the solutions of classical Yang-Mills equations are usually unstable, as noticed in several works [10]. This implies that the next to leading order correction to the energy momentum tensor calculated in the CGC framework grows with time, and eventually becomes larger than the leading order. It is possible to resum a subset of these higher order corrections, that at each loop order picks up the most singular of these contributions. This resummation amount to superimposing a fluctuation to the initial condition at $\tau = 0^+$ for the classical glasma field [11],

$$T^{\mu\nu}_{\text{resummed}}(\tau,\eta,\boldsymbol{x}_\perp) = \int \big[D\rho_{{}_1}\, D\rho_{{}_2}\big]\big[Da\big]\; F[a]\; W_1[\rho_{{}_1}]\, W_2[\rho_{{}_2}]\; T^{\mu\nu}_{\text{LO}}[\,\underbrace{\mathcal{A}+a}_{\text{initial field}}\,]$$

with a Gaussian distribution $F[a]$ for this fluctuation. The effect of this resummation has not been fully investigated in the context of the CGC due to some technical difficulties that preclude a straightforward numerical evaluation of the previous formula. However, it is possible to devise a scalar toy model where the effect of such fluctuations can be studied. To keep things

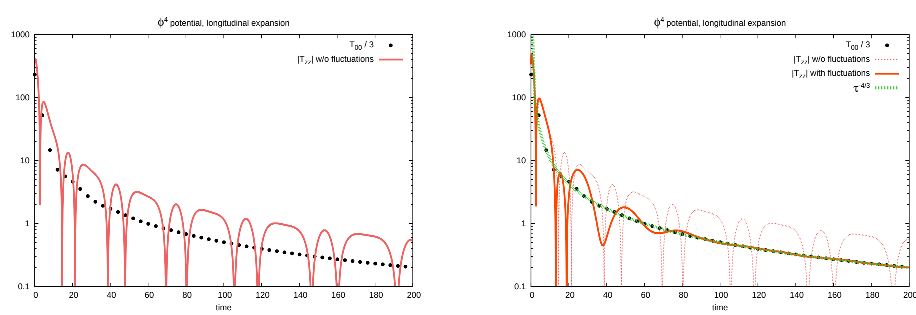

Figure 8: Left: evolution of ϵ and p without fluctuations of the initial field. Right: evolution with initial fluctuations.

extremely simple in this toy model, we discard any spatial dependence, so that the classical equation of motion reads

$$\ddot{\phi} + \frac{1}{\tau}\dot{\phi} - \boldsymbol{\nabla}_\perp^2\phi + V'(\phi) = 0 \;,$$

and we take a quartic potential $V(\phi) \sim \phi^4$. The term $\dot{\phi}/\tau$ is due to longitudinal expansion in the rapidity direction, as would be the case in a collision process. The time evolution is started at some some initial proper time $\tau = \tau_0$, and we average the energy-momentum tensor over Gaussian fluctuations of the initial field. We first did this toy calculation without fluctuations of the initial field (see the left plot of the figure 8) and there one sees that the pressure oscillates with time: there is no single-valued relationship between the energy density and the pressure, which means that there is no equation of state in this case. In the right plot of the figure 8, we show the effect of the initial field fluctuations: the oscillations of the pressure are now damped, and it relaxes towards $\epsilon/3$ – which is the equation of state expected for a scale invariant theory in four dimensions. Moreover, one also observes that the energy density decreases as $\tau^{-4/3}$, which is the expected behavior in hydrodynamics if the longitudinal pressure is $\epsilon/3$. Thus, in this toy model, it appears that the fluctuations of the initial field play a crucial role in the relaxation of the system towards hydrodynamical behavior.

Conclusions

The Color Glass Condensate provides a first principles framework to study the early stages of high energy nuclear collisions, that involve gluons in the saturated regime. It correctly describes the energy density released in such a collision, as well as important features of the correlations observed in the final state. In this context, the most pressing and challenging question for future work is arguably that of thermalization.

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