Hysteresis behavior in current-driven stationary resonance induced by nonlinearity in the coupled sine-Gordon equation

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Recently novel current-driven resonant states characterized by the π-phase kinks were proposed in the coupled sine-Gordon equation. In these states hysteresis behavior is observed with respect to the application process of current, and such behavior is due to nonlinearity in the sine term. Varying strength of the sine term, there exists a critical strength for the hysteresis behavior and the amplitude of the sine term coincides with the applied current at the critical strength.

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Introduction. Recently the coupled sine-Gordon equation has been intensively studied numerically and analytically as a model of THz electromagnetic wave emission from intrinsic Josephson junctions (IJJs). [1, 2] In current-driven emission without external magnetic field, novel resonant states characterized by the π-phase kinks were proposed for fairly large surface impedance Z, [3, 4] and the present author showed [5] that such states were stationary for any Z with large enough current.

In these resonant states hysteresis behavior is observed. Strong emission which clearly breaks the Ohm's law only takes place in the current-increasing process, and the law almost holds in the current-decreasing process. Such dynamical behavior is due to nonlinearity of the system, namely the sine term in the present equation. The main question of the present Letter is how such nonlinearity affects on hysteresis behavior in the current-driven resonance. May infinitesimal nonlinearity cause such behavior, or may there exist a critical value of it?

In order to resolve this question, we introduce an artificial parameter γ in the equation and verify the strength of nonlinearlity continuously. As the parameter γ decreases from the value in the original equation ($\gamma = 1$), width of hysteresis also decreases, and there seems to exist a nonvanishing critical value γ_c with vanishing hysteresis.

Model and formulation. When the capacitive coupling is not taken into account, IJJs are described by the coupled sine-Gordon equation, [6]

$$\partial_{x'}^2 \psi_l = (1 - \zeta\Delta^{(2)}) \left(\partial_{t'}^2 \psi_l + \beta\partial_{t'}\psi_l + \sin\psi_l - J'\right), \quad (1)$$

with the layer index l and the operator $\Delta^{(2)}$ defined in $\Delta^{(2)} X_l \equiv X_{l+1} - 2X_l + X_{l-1}$. Quantities are scaled as

$$x' = x/\lambda_c,\ t' = \omega_p t,\ J' = J/J_c;\ \omega_p = c/\left(\sqrt{\epsilon_c}\lambda_c\right), \quad (2)$$

with the penetration depth along the c axis λ_c, the plasma frequency in each layer ω_p, and the critical current J_c. Using material parameters of $Bi_2Sr_2CaCu_2O_8$ given in Ref. [7], we result in a large inductive coupling $\zeta = 4.4 \times 10^5$, and $\epsilon_c = 10$ and $\beta = 0.02$ are taken.

Neglecting temperature fluctuations and assuming homogeneity along the y axis, we have the two-dimensional formula (1). Following our previous study, [5] width of the junction is chosen as $L_x = 86\mu$m, and the periodic boundary condition (PBC) along the c axis is considered. Then, plasma velocity of the stationary state automatically coincides with that of light in IJJs, which corresponds to the case with infinite number of junctions, though actual number of junctions N still affects on physical properties even in the PBC, especially on response to the in-plane magnetic field. [8] Here we concentrate on the simplest case $N = 4$, which takes spatial inhomogeneity of the superconducting phase into account.

Since direct evaluation of electromagnetic wave emission from edges of a thin sample to vacuum is quite complicated, we use a simplified version [9] of the dynamical boundary condition, [10] where effects outside of the sample are only included in the relation between dynamical parts of the rescaled electric and magnetic fields, $\tilde{E}_l' = \mp Z\tilde{B}_l'$, with rescaled quantities E_l' and B_l' related with ψ_l as $\partial_{t'}\psi_l = E_l'$ and $\partial_{x'}\psi_l = (1 - \zeta\Delta^{(2)})B_l'$, respectively. Here we take $Z = 30$, which gives strong enough emission close to the optimal value. [5] The sample along the x axis is divided into 80 numerical grids, and calculations are based on the RADAU5 ODE solver. [11] In order to investigate effect of nonlinearity in the sine term, Eq. (1) is slightly modified as

$$\partial_{x'}^2 \psi_l = (1 - \zeta\Delta^{(2)}) \left(\partial_{t'}^2 \psi_l + \beta\partial_{t'}\psi_l + \gamma\sin\psi_l - J'\right), \quad (3)$$

and the parameter γ is controlled hereafter.

Hysteresis in the original equation. First, the *I-V* curve of the original equation (1) is displayed in Figs. 1(a) and 1(b). As long as the fundamental and first harmonic modes are observed, large hysteresis is quite apparent. In the current-increasing process (Fig. 1(a)), the current rapidly increases as the voltage approaches the value corresponding to the cavity resonance point given by the ac Josephson relation (broken lines),

$$V = \phi_0 f = \phi_0 \frac{c}{\sqrt{\epsilon_c}} \frac{n}{2L_x} \approx 1.14\ n\ [\text{mV}], \quad (4)$$

with the number of nodes n(= 1: the fundamental mode). In the vicinity of emission peaks, the voltage exceeds the

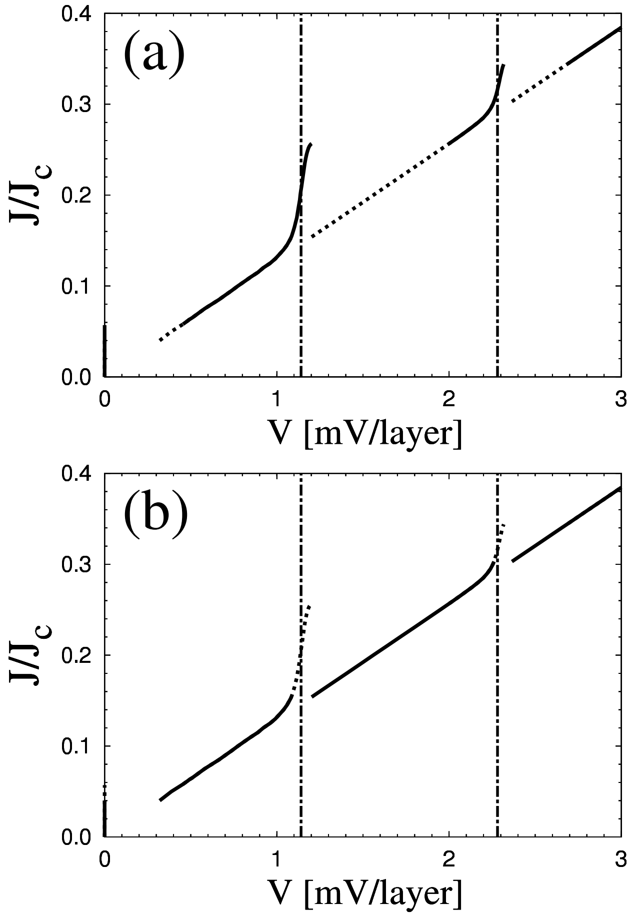

FIG. 1: *I-V* curve (solid line) for the (a) current-increasing and (b) current-decreasing processes. Dotted line stands for the curve in another figure, and the solid-dotted lines denote the voltages corresponding to the cavity resonance points (4).

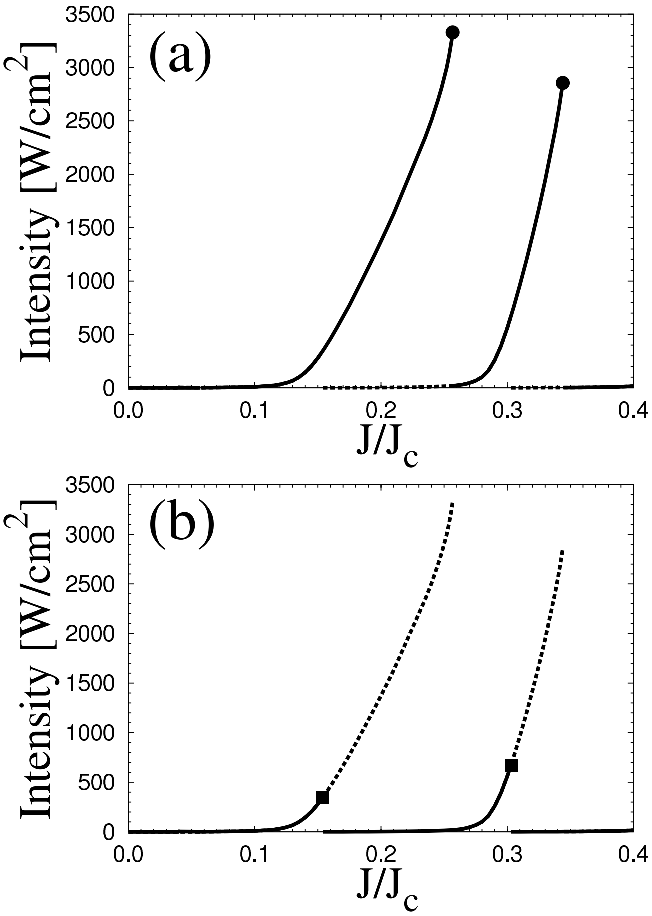

FIG. 2: Emission intensity versus current for the (a) current-increasing and (b) current-decreasing processes. Dotted line stands for the curve in another figure. Circles and squares represent the emission peak in each node and in the current-increasing and current-decreasing processes, respectively.

value given by Eq. (4), which assumes perfect cavity resonance. In experiments about 10% of frequency looks tunable in a single resonant branch by varying the current, [12, 13] which is consistent with this result. When a much larger value of Z is taken, the range of voltage becomes much smaller [3] during varying the current in the region of strong emission.

Emission intensity corresponding to the situations for Figs. 1(a) and 1(b) is plotted versus current in Figs. 2(a) and 2(b), respectively. Strong emission observed in the current-increasing process is quite reduced in the current-decreasing process. Especially in the fundamental mode, the maximum intensity in the current-decreasing process is almost one order smaller than that in the current-increasing process. In such a case emission is expected to be invisible when experimental noise is overloaded in the reverse (current-decreasing) process, which may represent "irreversible" emission in experiments.

Hysteresis in the equations with varying nonlinearity. Next, the *I-V* curve of the modified equation (3) is investigated. Here we concentrate on hysteresis behavior around the emission peaks in the fundamental ($n = 1$) mode, and only observe the $n = 1$ curves in the current-increasing process near the upper edges and the $n = 2$ curves in the current-decreasing process near the lower edges. In Fig. 3(a), parameter dependence of the upper and lower edges is visualized by various symbols for a wide range of parameters between $\gamma = 0.8$ and 0.1. As γ decreases, the width of hysteresis, namely the difference of the voltages at the both edges in the current-varying processes, becomes smaller and smaller. Hysteresis is observed up to $\gamma = 0.2$, while it is invisible at $\gamma = 0.1$.

Then, more precise measurement is made between $\gamma = 0.2$ and 0.1 as shown in Fig. 3(b). Although the upper edges of the $n = 1$ curve decrease monotonically, the lower edges of the $n = 2$ curve exhibit non-monotonic behavior. The lower edges decrease up to $\gamma = 0.17$, increase up to $\gamma = 0.15$, and decrease again for smaller γ. Hysteresis behavior is observed up to $\gamma = 0.16$, and from $\gamma = 0.15$ current at the upper and lower edges is the same, or the current-varying process becomes reversible.

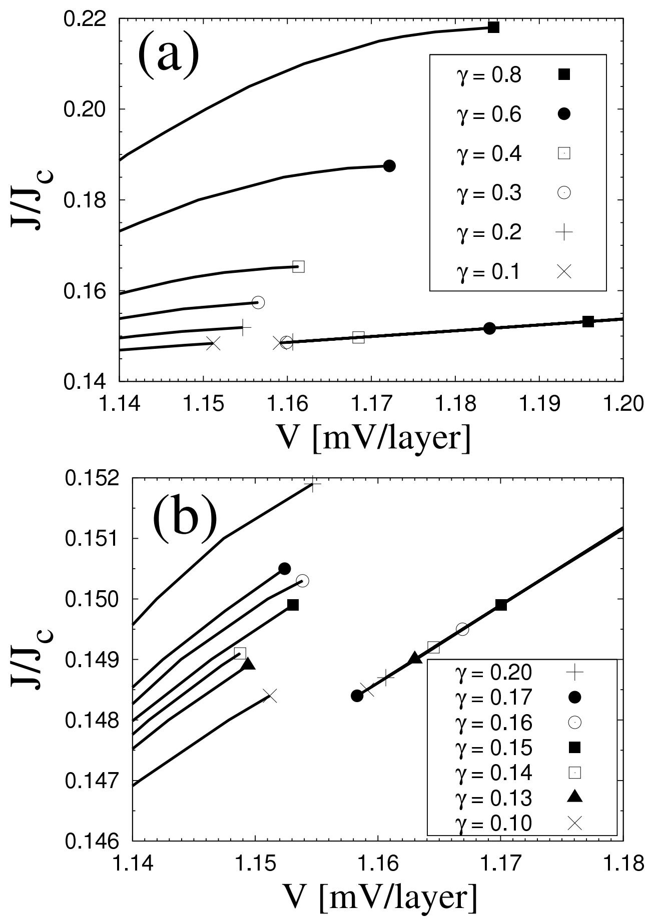

FIG. 3: *I*-*V* curve for various values of γ for (a) $\gamma = 0.8$ to 0.1 and (b) $\gamma = 0.2$ to 0.1 (expanded figure around the irreversible-reversible boundary). Edges of the $n = 1$ curve in the current-increasing process and the $n = 2$ curve in the current-decreasing process are visualized by various symbols.

Discussions. These results suggest that the boundary between the irreversible and reversible behavior locates between $\gamma = 0.16$ and 0.15, but further precise calculation in search for the next digit of γ may not be productive. Instead, we point out that the current at $\gamma = 0.15$ is $J' = 0.150$ on the edges, or that the amplitude of the sine term coincides with the constant term at $\gamma = 0.15$ in the equation (3). This fact strongly suggests that $\gamma = 0.15$ is the irreversible-reversible boundary. Quite recently reversible THz wave emission from IJJs was reported, [14] which may be related with the present finding.

Introduction of the parameter γ can be regarded as modification of the critical current, namely $J_c \to \gamma J_c$. Then, $\gamma = J'$ means that the current J is equal to the modified critical current γJ_c, where superconductivity breaks down and the present model is not justified anymore. It is natural that hysteresis behavior of the model may also change there. In the original equation (1), the situation $J > J_c$ occurs in higher harmonic modes and similar behavior may also be observed. Numerical study along this direction is now in progress. [15]

Frequency of resonance f in Eq. (4) is derived from the linearized version of Eq. (1), [16] and the role of nonlinearity is to generate the π-phase kinks [3] and hysteresis behavior. The present result shows that the hysteresis behavior is also nontrivial. We consider this behavior is general and can be regarded as a prototype of hysteresis in harmonic oscillations induced by nonlinearity.

Summary. In the present Letter we numerically investigate characteristic behavior of novel current-driven resonant states in the coupled sine-Gordon equation. These states were proposed theoretically as strong emission states in THz electromagnetic wave emission from intrinsic Josephson junctions, and such emission depends on the application process of current. That is, hysteresis is observed in emission behavior driven by applied current. Such irreversible behavior is due to nonlinearity of the equation, and we vary the strength of the sine term.

As long as the fundamental resonant mode is observed, the maximum intensity of emission (and consequently the maximum value of the current) decreases monotonically as the strength of nonlinearity decreases, and at the nonvanishing critical value hysteresis behavior disappears and the amplitude of the sine term coincides with that of the applied rescaled current at the emission peak. From a physical point of view, superconductivity and hysteresis behavior vanish at the same time.

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[1] L. Ozyuzer *et al.*, Science **318**, 1291 (2007); see also K. Lee *et al.*, Phys. Rev. B **61**, 3616 (2000).
[2] K. Kadowaki *et al.*, Physica C **468**, 634 (2008).
[3] S. Lin and X. Hu, Phys. Rev. Lett. **100**, 247006 (2008).
[4] A. E. Koshelev, Phys. Rev. B **78**, 174509 (2008).
[5] Y. Nonomura, Phys. Rev. B **80**, 140506(R) (2009).
[6] T. Koyama and M. Tachiki, Solid State Commun. **96**, 367 (1995).
[7] M. Tachiki *et al.*, Phys. Rev. B **71**, 134515 (2005).
[8] Y. Nonomura, preprint.
[9] S. Lin, X. Hu, and M. Tachiki, Phys. Rev. B **77**, 014507 (2008).
[10] L. N. Bulaevskii and A. E. Koshelev, Phys. Rev. Lett. **97**, 267001 (2006), J. Supercond. Nov. Mag. **19**, 349 (2006).
[11] http://www.unige.ch/~hairer/software.html
[12] M. Tsujimoto *et al.*, Phys. Rev. Lett. **105**, 037005 (2010).
[13] T. M. Benseman *et al.*, Phys. Rev. B **84**, 064523 (2011).
[14] H. Minami *et al.*, Physica C **470**, S822 (2010).
[15] Y. Nonomura, in preparation.
[16] S. Sakai *et al.*, J. Appl. Phys. **73**, 2411 (1993), Phys. Rev. B **50**, 12905 (1994).