

RR Lyrae Pulsation Theory

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Abstract. RR Lyrae stars play an important role as distance indicators and stellar population tracers. In this context the construction of accurate pulsation models is crucial to understand the observed properties and to constrain the intrinsic stellar parameters of these pulsators. The physical mechanism driving pulsation in RR Lyrae stars has been known since the middle of the 20th century and many efforts have been performed during the last few decades in the construction of more and more refined pulsation models. In particular, nonlinear pulsation models including a nonlocal time-dependent treatment of convection, such as the ones originally developed in Los Alamos in the seventies, allow us to reproduce all the relevant observables of stellar radial pulsation and to establish accurate relations and methods to constrain the intrinsic stellar properties and the distances of these variables. The most recent results on RR Lyrae pulsation obtained through these kinds of models will be presented and a few still debated problems will be discussed.

Keywords: variable stars, RR Lyrae, stellar pulsation, stellar distances

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INTRODUCTION

RR Lyrae are low mass central He burning stars belonging to the Horizontal Branch (HB) evolutionary phase in the Hertzsprung-Russell (HR) diagram. They are important tracers of the oldest stellar populations and the knowledge of their properties can provide relevant constraints to several important aspects of stellar evolution and cosmology. RR Lyrae are also important distance indicators for Pop. II systems through the absolute visual magnitude versus metallicity ($M_V(RR) - [Fe/H]$) relation but also thanks to the existence of a Period-Luminosity (PL) correlation in the Near Infrared bands. On this basis, the theoretical predictions through accurate pulsation models of the observed properties of RR Lyrae stars are of crucial importance for our understanding of many relevant astronomical issues. RR Lyrae are characterized by a variety of observed behaviours. As well known, with a period ranging from ~ 0.3 to 1.0 d and a pulsation amplitude in the V band always smaller than 2 mag, they are divided into two main subgroups: RRab pulsating in the fundamental mode and RRc pulsating in the first overtone mode. The first class shows asymmetric light curves with the pulsation amplitude decreasing with the period, while the latter class is characterized by shorter periods, smaller amplitudes and more symmetric light curves than the RRab. In the period-amplitude diagram (Bailey diagram) RRab stars have a linear behaviour, whereas first overtone pulsators form a bell-shaped sequence. They are located on the HB, with an absolute magnitude in the V banding ranging from ~ 0 to ~ 1 mag, and their distribution in color in a given globular cluster reflects the distribution in mass resulting from the mass loss phenomenon in the previous red giant

branch (RGB) evolutionary phase. RR Lyrae in Galactic Globular Clusters (GGCs) show a phenomenon known as the Oosterhoff Dichotomy dating back to Oosterhoff (1939). According to this phenomenon GGCs are separated into two groups: the Oosterhoff I (OoI) with average period of RRab ($\langle P_{ab} \rangle$) close to 0.55 d, a larger number of RRab than RRc and intermediate metallicity; the Oosterhoff II (OoII) with $\langle P_{ab} \rangle$ around 0.65 d, similar number of RRab and RRc and low metallicity. Accurate pulsation models should be able to reproduce all these properties as well as the absolute visual magnitude versus metallicity relation and the PL correlation in the NIR bands, but also additional observational features such as the behaviour of mixed mode pulsators and the Blazhko effect (see the review by G. Kovacs, this volume). In this paper we present a review of RR Lyrae pulsation models and summarize some relevant results obtained from nonlinear convective radial hydrocodes. In Section 2 we briefly review the physical mechanism driving pulsation in RR Lyrae stars; in Section 3 we summarize the theoretical efforts developed by several authors to interpret the RR Lyrae pulsation properties from a linear non adiabatic approach to nonlinear convective models; in Section 4 we discuss some significant results concerning RR Lyrae with implications for stellar astrophysics and the distance scale calibration; in Section 5 a number of still open questions are mentioned and some future perspectives are outlined. The conclusions close the paper.

RR LYRAE PULSATION MECHANISMS

Since the pioneering investigations by Eddington (1926) and Zhevakin (1959) it was clear that the physical phenomenon driving pulsation in variable stars lying in the classical pulsation instability strip, including RR Lyrae stars, was a valve mechanism mainly efficient in the second ionization region of He (30,000-60,000 K). This *k* mechanism was later confirmed by Baker & Kippenhahn (1962) and Cox (1963), whereas Christy (1962) first demonstrated that the HeI and H ionization layers also play a role in driving pulsation, as subsequently confirmed by Bono & Stellingwerf (1994). The need for a non-adiabatic theory to model the κ mechanism and to treat the problem of stellar stability is evident. Indeed from a linear adiabatic approach only the pulsation periodicity can be predicted.

FROM LINEAR NON-ADIABATIC TO NONLINEAR CONVECTIVE MODELS

The development of accurate pulsation models for RR Lyrae stars has known a tremendous improvement during the last few decades, with the inclusion of more and more sophisticated physical and numerical ingredients and their continuous update.

Linear non-adiabatic models

Several authors computed linear non-adiabatic radial models of RR Lyrae stars, both in a radiative regime (e.g. Cox 1963; Castor 1971; van Albada & Baker 1971; Iben & Huchra 1971; Glasner & Buchler 1993; Dékány et al. 2008) and including convection (e.g. Tuggle & Iben 1973; Baker & Gough 1979; Xiong 1982; Xiong et al. 1998). These models were able to predict the topology of the RR Lyrae instability strip. In particular radiative linear non-adiabatic models provided only the location of the blue edge (see e.g. Iben & Huchra 1971), whereas the convective ones were able to predict also an approximate evaluation of the effective temperature of the red edge (Tuggle & Iben 1973; Xiong et al. 1998) because it is the quenching effect of convection on pulsation that determines the occurrence of the red edge. A very important result of linear non adiabatic convective models was obtained by van Albada & Baker (1971), who predicted linear relations (logarithmic scale) between the pulsation period and the stellar mass, luminosity and effective temperature for both the fundamental and the first overtone mode. These relations establish an important link between pulsational and evolutionary parameters and represented the milestone of any study aimed at investigat-

ing the evolutionary behaviour of RR Lyrae through the analysis of their pulsation characteristics. In a subsequent paper (van Albada & Baker 1973) they predicted the efficiency of an *hysteresis mechanism* in determining the pulsation mode in the OR region: RR Lyrae that enter the instability strip from the blue keep pulsating in the first overtone mode, while the ones that come from the red continue to pulsate in the fundamental mode. This idea implies that the morphology of the evolutionary tracks determines the pulsation mode in the OR region, and was at the basis of several subsequent papers aiming at explaining the Oosterhoff dichotomy (see e.g. Bono et al. 1995 and references therein). A time-dependent generalization of the mixing-length theory was adopted by Baker & Gough (1979) in the construction RR Lyrae nonadiabatic models. These authors studied the effect of including the interaction between pulsation and convection on their stability analysis. A similar approach was followed more recently by Xiong et al. (1998) on the basis of a non-local and time-dependent treatment of convection.

Nonlinear models

When the small oscillation approximation is released and the fundamental equations of the pulsating stellar envelope are not linearized, we are in the position to predict the full amplitude behaviour of models at their limit cycle. In particular nonlinear models are able to provide information not only on the pulsation period and the instability strip boundaries, but also on the pulsation amplitudes and on the morphological characteristics of the variation along a pulsation cycle of luminosity, radius, radial velocity, effective temperature, surface gravity. This is true also for radiative models even if, without including convection, no information on the red boundary of the instability strip can be obtained and there are also difficulties in matching the observed light curve amplitudes and morphologies (Kovacs & Kanbur 1998, Feuchtinger 1999a). In order to properly reproduce all the observables of radial pulsation nonlinear convective models are required. Since the early 80's several authors included convection in their nonlinear hydrocodes (Stellingwerf 1982, 1984; Gehmeyr 1992, 1993; Bono & Stellingwerf 1994; Feuchtinger 1999a; Szabò, Kollath & Buchler 2004). Stellingwerf was the first to introduce a non-local time-dependent treatment of convection in RR Lyrae nonlinear pulsation models, based on the treatment of the transport equation by Castor (1968). Several subsequent investigations of RR Lyrae properties, that provided relevant results for the use of these stars as distance indicators and stellar population tracers, were based on refinements and updates of Stellingwerf's original code

(Bono & Stellingwerf 1994; Bono et al. 1997, 2003; Marconi et al. 2003; Di Criscienzo et al. 2004; Marconi & Clementini 2005; Marconi & Degl'Innocenti 2007; Marconi et al. 2009 in preparation). On the other hand the hydrocode developed by Gehmeyr (1992) was based on an adaptive grid approach and on a nonlinear convective treatment and was able to determine precisely the red edge position; Feuchtinger (1999a) used the Vienna nonlinear convective code (Feuchtinger 1999b) to predict accurate theoretical light and radial velocity curves, as well as to reproduce the Fourier parameters; finally Szabò et al. (2004) performed a consistent modeling of pulsation and evolutionary properties through an amplitude equation formalism and were able to reproduce the observed slope of the fundamental blue edge, as well as the behaviour of double mode RR Lyrae. On this basis, we can conclude that nonlinear convective radial models are necessary to reproduce all the relevant pulsation observables, apart from a number of additional phenomena that require the inclusion of nonradial pulsation modes and/or the introduction of new physics. These are for example the Blazhko effect (see the review by G. Kovacs, this volume), the nonradial pulsation of RR Lyrae stars (see e.g. the contribution by A. Cox, this volume), the role of magnetic fields (see e.g. the contribution by K. Kolenberg, this volume), as well as the formation and propagation of shock waves (see e.g. the contributions by Chadid and Paparo, this volume).

SIGNIFICANT RESULTS CONCERNING RR LYRAE

Many important results have been obtained during the last few decades from nonlinear convective models of RR Lyrae stars, with relevant implications for both the distance scale problem and for the study of stellar populations. In this section we will concentrate on three important topics, namely the topology of the RR Lyrae instability strip and of the ensuing Bailey diagram, the pulsation stellar mass determination from the study of RR Lyrae properties, and the calibration of the RR Lyrae distance scale.

Topology of the RR Lyrae instability strip

Nonlinear convective pulsation models allow us to predict the complete topology of the instability strip for both fundamental and first overtone modes. Fig. 1 reports the location of the first overtone blue edge (FOBE) the fundamental blue edge (FBE), the first overtone red edge (FORE) and the fundamental red edge (FRE) in the HR diagram, as obtained on the basis of our nonlinear

convective pulsation models (Bono, Caputo, Marconi 1995; but see also Bono et al. 1997) for $Z=0.0001$ and $Y=0.24$.

The intersection region between the fundamental and the first overtone instability strips is called OR region and the different population of this region by fundamental and first overtone pulsators, as triggered by the already mentioned *hysteresis mechanism* is at the basis of the explanation of the Oosterhoff dichotomy suggested by current pulsation models (see Bono et al. 1995, 1997 for details). According to this explanation, the OR region is populated by RR_{ab} in OoI GCs and by RR_c in OoII GCs, so that the average period of RR_{ab} is shorter in OoI than in OoII. This different population of the OR region is also evident from inspection of Fig. 2, showing the comparison in the Bailey diagram of RR Lyrae belonging to the two prototype GGCs of the OoI and OoII classes, namely M3 and M15. The middle part of this plot, roughly corresponding to the OR region, is populated by high amplitude RR_{ab} in M3 and by the decreasing branch of the RR_c *bell* in M15.

In other terms M15 lacks high amplitude F pulsators and M3 lacks the decreasing branch of the FO *bell* in the OR region. The theoretical Bailey diagram based on our nonlinear convective models nicely reproduces the observed linear behaviour of fundamental pulsators, providing the basis for the derivation of Period-Luminosity-Amplitude relations (see Di Criscienzo, Marconi, Caputo 2004 and Bono, Caputo & Di Criscienzo 2007 and references therein) that can in principle be used as an additional theoretical tool to infer RR Lyrae distances (see below). This is shown in Fig. 3 for a sample of GGCs with a significant number of RR Lyrae (see Di Criscienzo et al. 2004 for details).

Stellar pulsation mass determination

A *pulsational* mass estimate, that is independent of stellar evolution predictions, can be obtained either using double mode pulsators or through the model fitting of observed light (and radial velocity) curves. The first method is based on the use of the Petersen diagram (see Petersen 1978 for the first application to RR Lyrae stars), reporting the ratio between the first overtone and the fundamental period as a function of the fundamental period (in logarithmic scale). Many fundamental papers were based on the comparison between linear non-adiabatic model predictions and observations in the Petersen diagram, as a tool to constrain the stellar mass of field and cluster RR Lyrae (see e.g. Cox, Hodson & King 1980; Cox, Hodson & Clancy 1983). At the time these *pulsational* masses were found to be in good agreement with stellar evolution estimates. However, with the update of

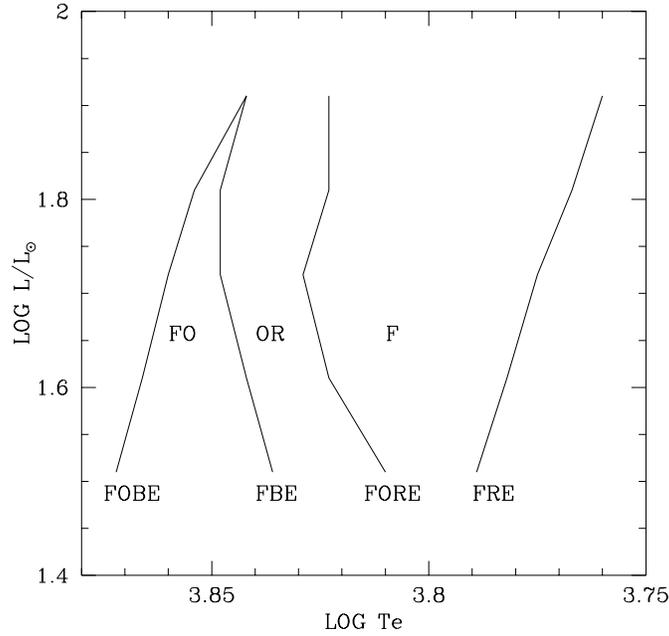


FIGURE 1. Topology of the fundamental and first overtone instability strip for RR Lyrae models with $Z=0.0001$, $Y=0.24$ and $M=0.65 M_{\odot}$.

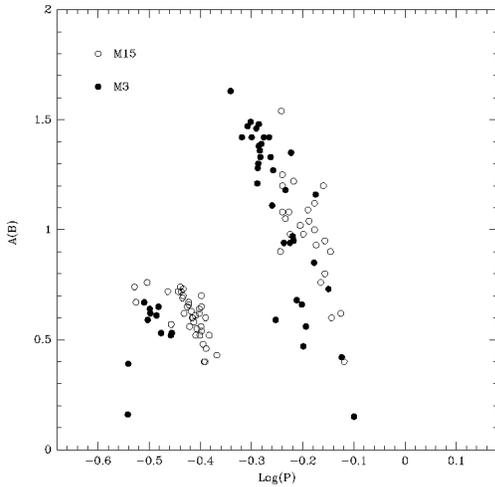


FIGURE 2. Comparison in the Bailey diagram between the RR Lyrae belonging to the two prototype GGCs of the OoI and OoII classes, namely M3 (filled circles) and M15 (open circles).

the input physics in evolutionary computations, a *mass discrepancy* problem appeared with *pulsational* masses resulting to be systematically smaller than the evolutionary ones. This discrepancy was solved when the input physics of pulsation models was also updated, in particular with the adoption of Livermore opacity tables (see

Cox 1991 and Bono et al. 1996 for details). Moreover Bono et al. (1996) found that using nonlinear convective models the comparison with observations in the Petersen diagram can be used not only to derive the stellar mass but also to constrain the luminosity level.

The second method to infer a *pulsational* estimate of RR Lyrae mass is the model fitting of observed light and, when available, radial velocity curves. The fact that nonlinear convective models allow us to predict accurate variations of all the relevant quantities along a pulsation cycle implies that the comparison between theoretical and observed variations represents a powerful tool to constrain the intrinsic stellar parameters including the mass. The model fitting of observed light curves was applied for the first time to a Cepheid in the LMC by Wood, Arnold & Sebo (1997), whereas the first application to a RR Lyrae pulsator, the Galactic field variable U Comae, was presented by Bono, Castellani & Marconi (2000). As shown in Fig. 4, in the case of U Comae, the obtained best fit model was able to accurately reproduce the light curves in the UBV filters, and also the poor sampled K band curve, for stellar parameters, including the mass (and the distance), in good agreement with the literature and the evolutionary prescriptions.

A second example showing the successful simultaneous fit of light and radial velocity curves was the case of CM Leo (see Di Fabrizio et al. 2002). The comparisons between the data and the best fit model are shown

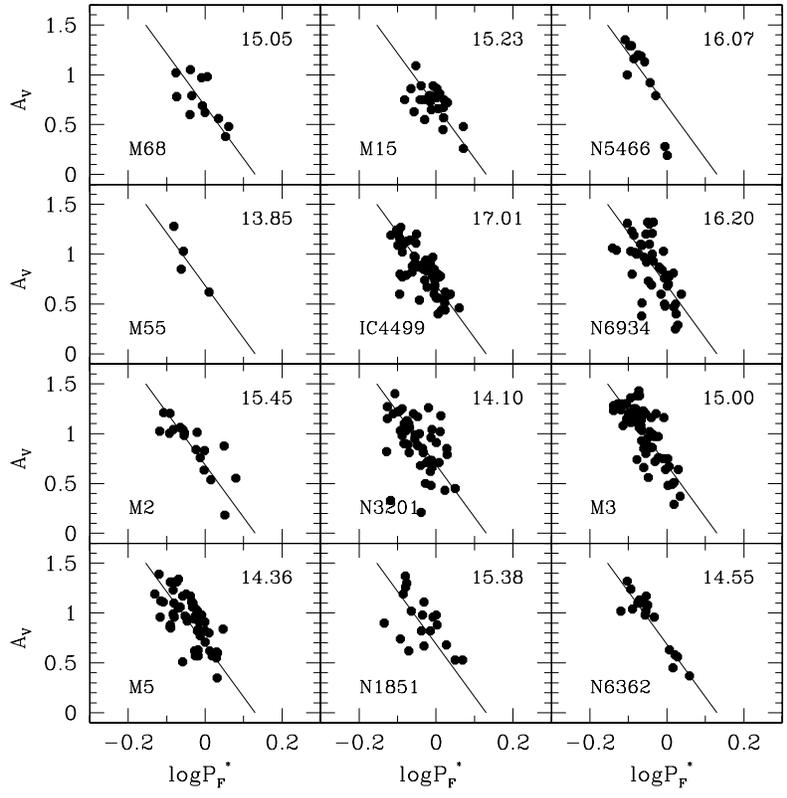


FIGURE 3. Application of a Period-Luminosity-Amplitude relation for fundamental pulsators at $l/H_p = 1.5$ to the RR Lyrae in a sample of GGCs. The solid lines are the predicted relations at constant mass. The derived apparent distance moduli are given in each panel.

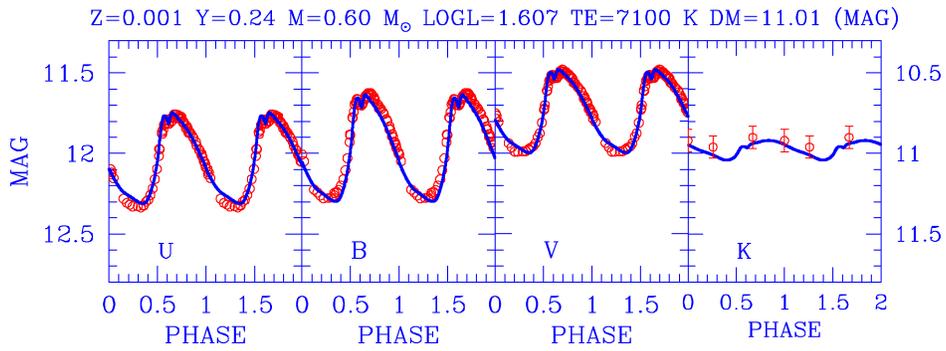


FIGURE 4. Model fitting of U Comae multi-band light curves (see Bono et al. 2000 for details.)

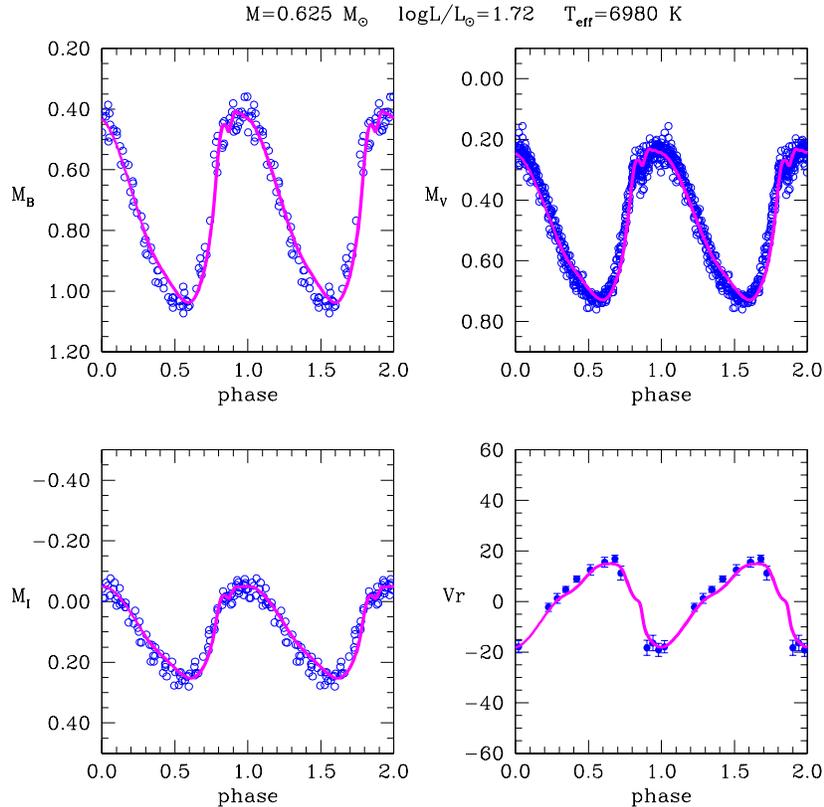


FIGURE 5. Model fitting of light and radial velocity curves for the field RR Lyrae CM Leo (see Di Fabrizio et al. 2002 for details.)

in Fig. 5. Again the best fit model was found to have stellar parameters consistent with evolutionary predictions. More recently the method has been applied also to cluster RR Lyrae. In Marconi & Degl’Innocenti (2007) we presented the first application to a GGC performing the model fitting of the light curves of a sample of RR Lyrae in M3. The stellar masses of the selected pulsators were constrained and found to be in agreement with the evolutionary predictions. At the same time the resulting distance modulus is in very good agreement with independent estimates in the literature. The application of the model fitting technique to more peculiar GGCs, namely NGC2419 and NGC6441, is in progress (Di Criscienzo et al. 2009 in preparation; Marconi & Clementini 2009 in preparation).

Calibration of the RR Lyrae distance scale

As already noted, RRLyrae are the most important primary distance indicators for Pop.II systems. Several results have been obtained from the theoretical point of view through the calibration of different methods. First

of all, the model fitting technique discussed above is a powerful tool to infer individual RR Lyrae distances through the comparison between the apparent magnitudes and the best fit model intrinsic luminosity. The results of the application of this method to a sample of RR Lyrae in the LMC are shown in Fig. 6, for the fundamental and first overtone pulsators respectively (see Marconi & Clementini 2005). The resulting distance modulus is 18.54 ± 0.02 mag and is in excellent agreement with the values obtained through the model fitting of other classes of pulsating stars in the LMC, namely two samples of Classical Cepheids (18.53 ± 0.05 mag Bono, Castellani, Marconi 2002; 18.54 ± 0.02 mag Keller & Wood 2002, 2006) and a δ Scuti pulsator (18.48 ± 0.15 mag McNamara, Clementini & Marconi 2007).

A method to infer cluster RR Lyrae distances is based on the comparison between theory and observations in the M_V versus $\log P$ plane (Caputo 1997; Caputo et al. 2000; Di Criscienzo et al. 2004). By matching the ob-

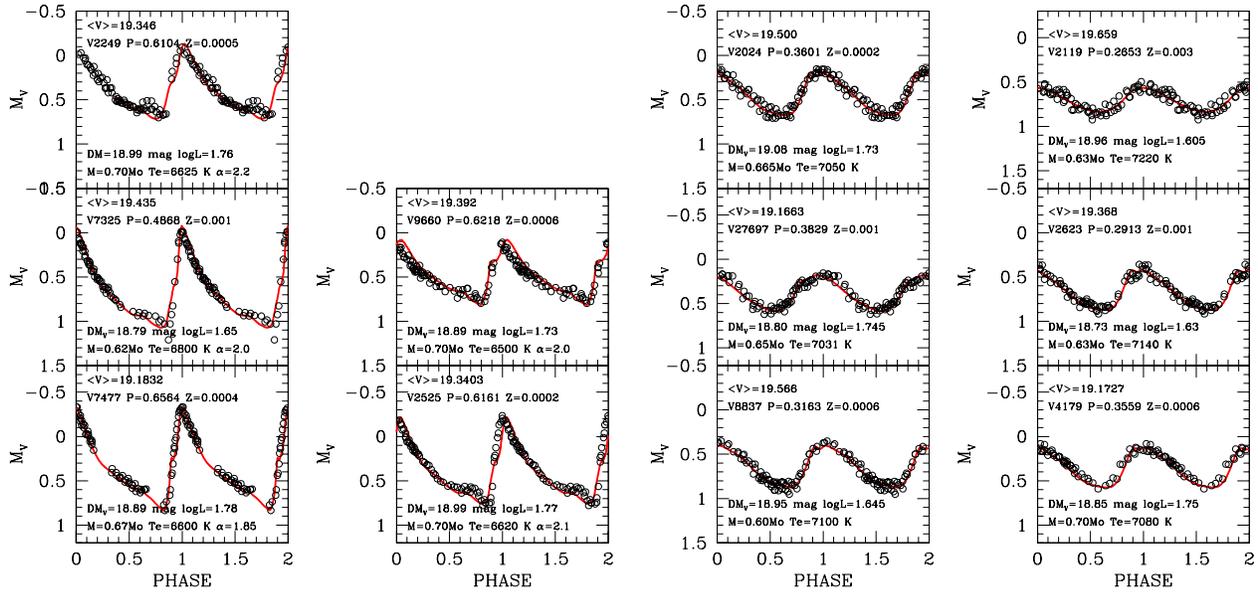


FIGURE 6. Model fitting of the V band light curves for a sample of 5 RRab (left panels) and 6 RRc (right panels) in the LMC.

served blue edge¹ of the instability strip with the theoretical one, as based on an extensive set of nonlinear convective pulsation models, we obtain a direct estimate of the apparent distance modulus for each investigated cluster and, by subtracting this value to the apparent mean magnitude level of the RR Lyrae, also an estimate of the absolute RR Lyrae magnitude (and of the true distance modulus for each investigated GGC, see Di Criscienzo et al. 2004 for details). As a result of the application of this technique to a sample of GGCs containing RR Lyrae (stistically significant samples) we obtain a $M_V(RR) - [Fe/H]$ relation that is not linear over the whole observed metallicity range but with a slope that gets steeper as the metallicity increases and evidence of a change of the slope around $[Fe/H] = -1.5$ as shown in Fig. 7 (see also Caputo et al. 2000).

The individual distance moduli obtained from the fitting of the blue edge in the M_V versus $\log P$ plane are in excellent agreement with the values inferred from application of the theoretical period-Wesenheit relations in the BV bands based on the same set of pulsation models (see Di Criscienzo et al. 2004 for details). Finally, current pulsation models are able to reproduce the already quoted correlation between the period and the absolute magnitude in the near-infrared bands and in particular in the K ($2.2 \mu\text{m}$) band (Bono et al. 2001, 2002, 2003), providing

useful constraints on the PL(K)-based RR Lyrae distance scale. These theoretical predictions show that the metallicity affects the PL(K) relation: RR Lyrae models are found to obey to a $\log P - M_K - [Fe/H]$ relation rather than to a $\log P - M_K$ relation. The application of these model results to the data for the prototype RR Lyr gives a distance value in excellent agreement with the HST astrometric determination, as shown in Fig. 8 (Bono et al. 2002, 2003), whereas the application to a sample of well studied field RR Lyrae confirms the existence of a strict correlation between the period, the K band magnitude, and the metallicity, as shown in Fig. 9. Moreover, correcting the apparent V magnitudes with the distances obtained from the K band theoretical analysis, we obtain a $M_V(RR) - [Fe/H]$ relation consistent with the one obtained from the fitting of the blue edge in M_V versus $\log P$ plane (see Bono et al. 2003 for details).

SOME STILL OPEN PROBLEMS IN RR LYRAE PULSATION MODELS

Even if the discussion presented in the previous section indicates that current nonlinear convective pulsation models are able to nicely reproduce many of the observed characteristics of RR Lyrae and to provide relevant constraints to important issues of stellar astrophysics, there are a number of still debated problems and residual uncertainties. As stated above, on the basis of the current theoretical scenario we find that the $M_V(RR) - [Fe/H]$

¹ The red edge is less reliable due to its well known sensitivity to the efficiency of convection and to the uncertainties still affecting the treatment of turbulent convection.

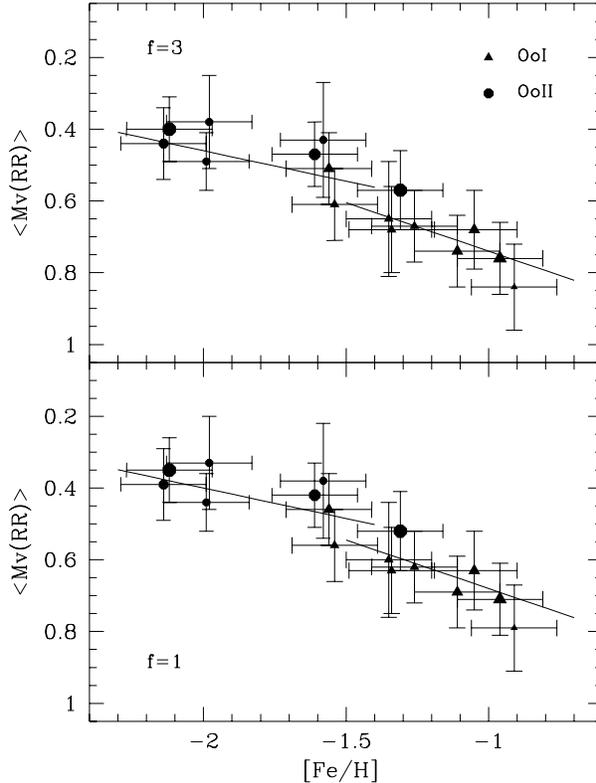


FIGURE 7. $M_V(RR) - [Fe/H]$ relations for two assumptions on the α element enhancement, as obtained from application of the FOBE fitting technique to a sample of GGCs containing rich samples of RR Lyrae (see Di Criscienzo et al. 2004 for details.)

relation is not linear over the whole observed metallicity range. This prediction is confirmed by evolutionary models (see e.g. Cassisi et al. 1998, Demarque et al. 2000, Catelan, Pritzl & Smith 2004) and there are also observational cases that support this prediction (see e.g. the results obtained by Rey et al. 2000 for the RR Lyrae in Omega Cen). However we find a clear evidence of linearity of the $M_V(RR) - [Fe/H]$ relation for example in the LMC (Clementini et al. 2003). The true nature of this correlation and its link with the Oosterhoff dichotomy phenomenon definitely deserves further theoretical attention. Another open problem is the modeling of light curves for cool RR Lyrae. In several cases, both in the Galactic field and in GGCs, we have found that models are unable to properly reproduce the morphology of the light curves close to the red edge of the instability strip. Given the higher sensitivity to uncertainties affecting the convective transfer, of the pulsation properties of RR Lyrae in the red part of the strip, these results support the idea that further improvements in the treatment of turbulent convection are required.

Finally we still lack in the literature a systematic investigation of the effect of a possible He enhancement

on the properties of RR Lyrae models. Very long period RR Lyrae variables have been observed in peculiar GGCs for which He enhancement has been invoked (e.g. NGC6441, NGC6388, see Busso et al. 2007; D’Antona & Ventura 2007).

Model computations for Y higher than 0.30 are in progress but we show in Fig. 10 and 11 some preliminary results. From these plots we notice that moving from $Y=0.30$ to $Y=0.38$ the pulsation amplitudes decrease significantly and the light curve morphology can also be modified as exemplified by the first overtone case (Fig. 9, right panel). These results imply that the study of the pulsation properties of the RR Lyrae contained in these peculiar clusters might in principle be used to constrain the occurrence of He enhancement.

CONCLUSIONS

RR Lyrae are very important standard candles and tracers of the oldest stellar populations. This implies that we need accurate pulsation models to interpret their observed characteristics. Only with nonlinear pulsation

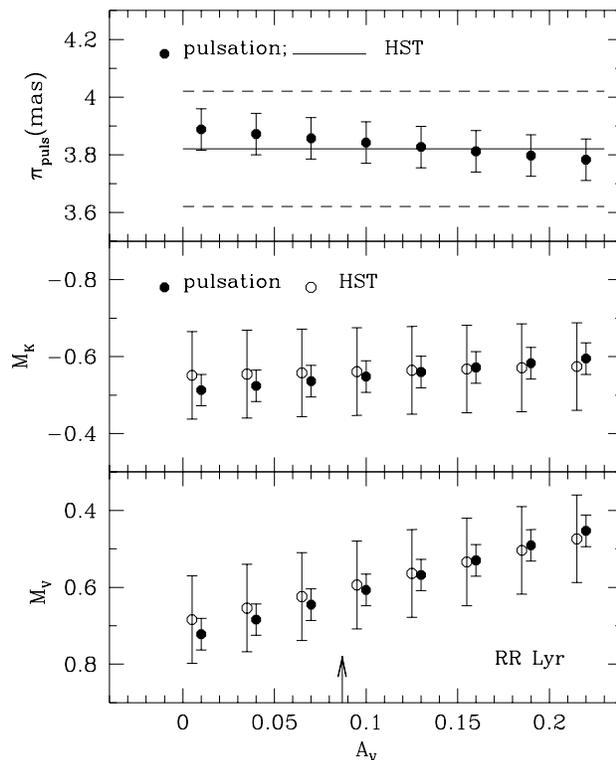


FIGURE 8. Comparison between the theoretical pulsation parallax based on the calibration of the K band PL and the astrometric HST result (upper panel). Similar comparisons for the K and V absolute magnitudes are also shown (middle and lower panels).

models including a non-local time-dependent treatment of convection we are able to reproduce all the relevant observables of radial pulsation. Important results have been obtained from these models and new techniques have been devised to exploit their predictive capabilities. However there are a number of physical phenomena that either require the modeling of nonradial modes in addition to the radial ones or need the inclusion of additional physical inputs. Moreover, even in the context of the above discussed nonlinear convective radial models, there are some still open problems that deserve further theoretical investigation. In particular to obtain a successful modeling of the light and radial velocity curves for RR Lyrae located close to the red edge of the instability strip we most likely need a more sophisticated treatment of turbulent convection.

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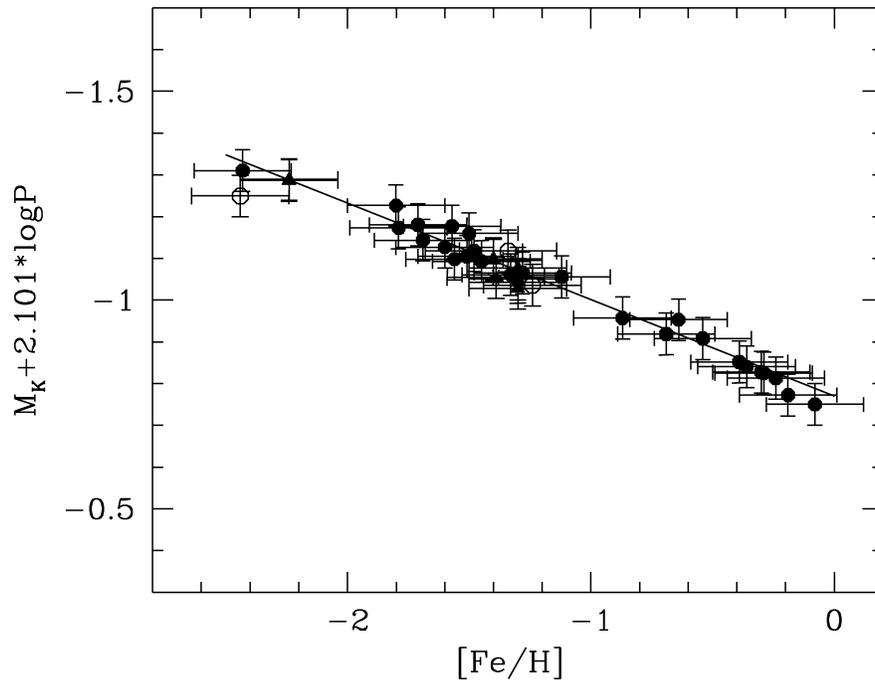


FIGURE 9. Absolute K magnitudes of field RR Lyrae corrected for the period dependence predicted by pulsation models as a function of metal content. The solid line is the theoretical $P - M_K - [Fe/H]$ relation projected onto a two dimensional plane.

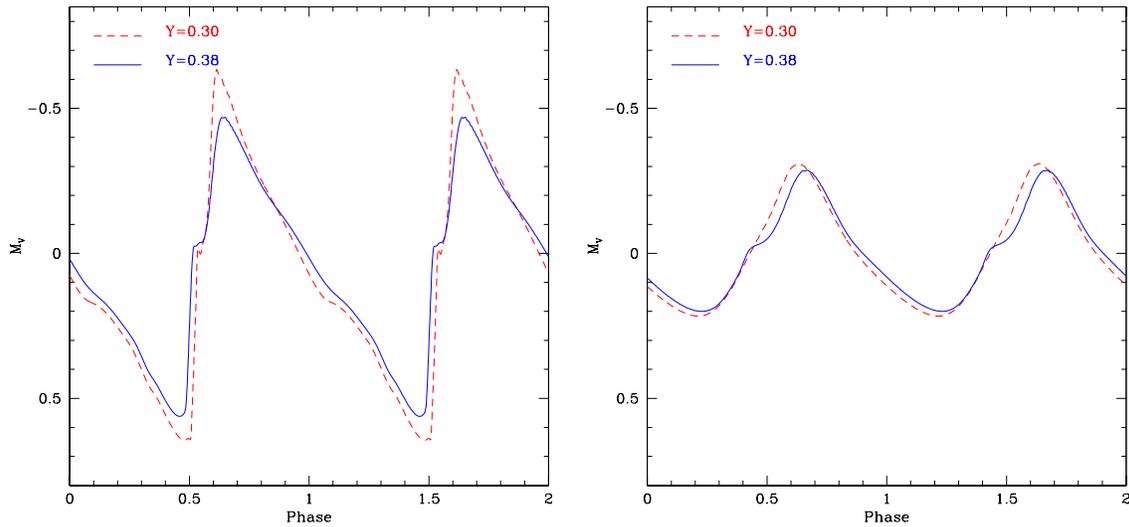


FIGURE 10. Comparison between the predicted light curves at $Y=0.30$ and the ones obtained at $Y=0.38$ for a fundamental (left panel) and first overtone (right panel) model with $Z = 0.001$, $M=0.60 M_{\odot}$, $\log L/L_{\odot} = 1.9$.

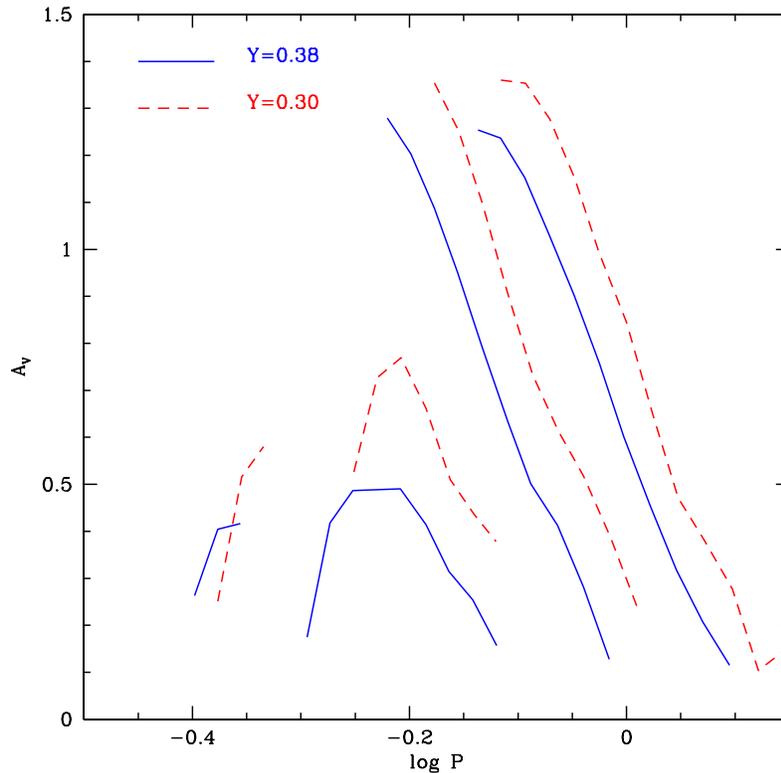


FIGURE 11. Effect of a variation of Y from 0.30 to 0.38 on the Bailey diagram topology. The other model characteristics are the same as in the previous figure.

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