

Black-Holes, Duality and Supersymmetry¹

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In order to study the discrepancy between the supersymmetry bound and the extremality bound for rotating black holes, the effect of duality transformation on the class of stationary, axially symmetric string backgrounds, called the TNbh, is considered. It is shown that the Bogomol'nyi bound is invariant under those duality transformations that transform the TNbh into itself, meaning that duality does not constrain the angular momentum in such a way as to reconcile the aforementioned bounds. A physical reason for the existence of the discrepancy is given in terms of superradiance.

1 Introduction

As is well known, rotating black holes (bh) become extremal before they become supersymmetric, which is in harrowing contrast to non-rotating bh's which become supersymmetric at the same moment they reach extremality, thus giving support to the Cosmic Censorship Hypothesis [1]. This discrepancy can be summarized by stating that the angular momentum does enter the extremality bound, typically in the combination $M^2 - J^2$, M being the mass and J the angular momentum of the bh, whereas it does not enter the Bogomol'nyi (supersymmetry) bound. This discrepancy is even more surprising in view of the fact that in the presence of only NUT charge (that is, for some stationary, non-static, cases) both bounds still coincide; the NUT charge squared must simply be added to the first member in the two bounds [2]. On the other hand, it is also known that some T-duality transformations seem to break spacetime supersymmetry making it non-manifest [3]. These two facts could perhaps give rise to a scenario in which extremal Kerr-Newman black holes (which are not supersymmetric) could be dual to some supersymmetric configuration. At the level

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of the supersymmetry bounds one would see the angular momentum transforming under a non-supersymmetry-preserving duality transformation into a charge that does appear in the supersymmetry bound (like the NUT charge). In this way, the constraints imposed by supersymmetry on the charges would equally constrain the angular momentum.

Although this scenario has been disproven by the calculations [4]² the transformation of black-hole charges and the corresponding Bogomol'nyi bounds under general string duality transformations remains an interesting subject on its own right and its study should help us gain more insight into the physical space-time meaning of duality.

In a previous work [5] a systematic analysis was made of the behavior of asymptotic charges under T-duality [6, 7] for four-dimensional non-rotating black holes³. In order to clarify the questions posed in the foregoing paragraphs, their results are extended, by essentially widening the class of metrics considered before to stationary and axially symmetric metrics and by including more non-trivial fields, thus enlarging the group of asymptotic-behavior-preserving transformations. Therefore, we will define the asymptotic behavior considered (“TNbh”) and will discuss the subgroup that preserves it (the “ADS”). One finds that the charges fit into natural multiplets under the ADS and that the Bogomol'nyi bound can be written as an invariant of this subgroup. This was to be expected since duality transformations in general respect unbroken supersymmetries [3], but since duality in general transforms conserved charges that appear in the Bogomol'nyi bound into non-conserved charges (associated to primary scalar hair) that in principle do not enter, the consistency of the picture will require the inclusion of those non-conserved charges into the generalized Bogomol'nyi bound.

2 The Derivation of the Duality Transformations

For simplicity we are going to consider a consistent truncation of the four-dimensional heterotic string effective action, to the lowest order in α' , including the metric, dilaton and two-form field plus two Abelian vector fields. This truncation is, however, rich enough to contain solutions with 1/4 of the supersymmetries unbroken [1, 8]. An extra reason for considering two abelian vector fields, is that it is the smallest theories, whose solution allows for the construction, through duality, of the most general bh solution in $N = 4$, $D = 4$ SUGRA.

²In fact, the angular momentum is part of a set of charges that transform amongst themselves under duality and never appear in the Bogomol'nyi bound.

³Since some of the objects studied are singular, as opposed to black holes, the name black hole will be used in a generalized sense for (usually point-like) objects described by asymptotics such that a mass, angular momentum etc. can be assigned to them.

The action of interest, in the string frame⁴, reads

$$S = \int d^4x \sqrt{|\hat{g}|} e^{-\hat{\phi}} \left[R(\hat{g}) + \hat{g}^{\hat{\mu}\hat{\nu}} \partial_{\hat{\mu}} \hat{\phi} \partial_{\hat{\nu}} \hat{\phi} - \frac{1}{12} \hat{H}_{\hat{\mu}\hat{\nu}\hat{\rho}} \hat{H}^{\hat{\mu}\hat{\nu}\hat{\rho}} - \frac{1}{4} \hat{F}^I_{\hat{\mu}\hat{\nu}} \hat{F}^{I\hat{\mu}\hat{\nu}} \right], \quad (1)$$

where $I = 1, 2$ sums over the Abelian gauge fields $\hat{A}^I_{\hat{\mu}}$ with standard field strengths, and the two-form field strength is

$$\hat{H}_{\hat{\mu}\hat{\nu}\hat{\rho}} = 3\partial_{[\hat{\mu}} \hat{B}_{\hat{\nu}\hat{\rho}]} - \frac{3}{2} \hat{F}^I_{[\hat{\mu}\hat{\nu}} \hat{A}^I_{\hat{\rho}]}. \quad (2)$$

As announced in the Introduction, it will be assumed that the metric has a timelike and a rotational spacelike isometry. The former is physically associated to the stationary character of the spacetime and the latter to the axial symmetry. Since they commute one can find two adapted coordinates, in this case the time t and the angular variable φ , such that the background does not depend on them. This then implies that the theory can be reduced dimensionally. Using the standard technique [9, 10] the resulting dimensionally reduced, Euclidean, action turns out to be⁵

$$S = \int d^2x \sqrt{|g|} e^{-\phi} \left[R(g) + g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi + \frac{1}{8} \text{Tr} \partial_{\mu} \mathcal{M} \partial^{\mu} \mathcal{M}^{-1} - \frac{1}{4} W^i_{\mu\nu} (\mathcal{M}^{-1})_{ij} W^{j\mu\nu} \right],$$

where $\mu\nu = 2, 3$ and $\alpha, \beta = 0, 1$. The two-dimensional fields are the metric $g_{\mu\nu}$, six vector fields $\mathcal{K}^i_{\mu} = (K^{(1)\alpha}_{\mu}, K^{(2)}_{\alpha\mu}, K^{(3)I}_{\mu})$ with the standard Abelian field strengths $W^i_{\mu\nu}$ ($i = 1, \dots, 6$) and a bunch of scalars $G_{\alpha\beta}, \hat{B}_{\alpha\beta}, \hat{A}^I_{\alpha}$ that appear combined in the 6×6 matrix \mathcal{M}_{ij} . They are given by

$$\begin{aligned} G_{\alpha\beta} &= \hat{g}_{\alpha\beta}, & \phi &= \hat{\phi} - \frac{1}{2} \log |\det G_{\alpha\beta}|, \\ K^{(1)\alpha}_{\mu} &= \hat{g}_{\mu\beta} (G^{-1})^{\beta\alpha}, & C_{\alpha\beta} &= \frac{1}{2} \hat{A}^I_{\alpha} \hat{A}^I_{\beta} + \hat{B}_{\alpha\beta}, \\ g_{\mu\nu} &= \hat{g}_{\mu\nu} - K^{(1)\alpha}_{\mu} K^{(1)\beta}_{\nu} G_{\alpha\beta}, & K^{(3)I}_{\mu} &= \hat{A}^I_{\mu} - \hat{A}^I_{\alpha} K^{(1)\alpha}_{\mu}, \\ K^{(2)}_{\alpha\mu} &= \hat{B}_{\mu\alpha} + \hat{B}_{\alpha\beta} K^{(1)\beta}_{\mu} + \frac{1}{2} \hat{A}^I_{\alpha} K^{(3)I}_{\mu}, \end{aligned} \quad (3)$$

and

$$(\mathcal{M}_{ij}) = \begin{pmatrix} G^{-1} & -G^{-1}C & -G^{-1}A^T \\ -C^T G^{-1} & G + C^T G^{-1}C + A^T A & C^T G^{-1}A^T + A^T \\ -AG^{-1} & AG^{-1}C + A & \mathbb{I}_2 + AG^{-1}A^T \end{pmatrix}, \quad (4)$$

⁴Our signature is $(-, +, +, +)$. All hatted symbols are four-dimensional and so $\hat{\mu}, \hat{\nu} = 0, 1, 2, 3$. The relation between the four-dimensional Einstein metric $\hat{g}_{E\hat{\mu}\hat{\nu}}$ and the string-frame metric $\hat{g}_{\hat{\mu}\hat{\nu}}$ is $\hat{g}_{E\hat{\mu}\hat{\nu}} = e^{-\hat{\phi}} \hat{g}_{\hat{\mu}\hat{\nu}}$.

⁵Following [11] we have set the resulting 2-dimensional vector fields to zero. Other choices could lead to a 2 dimensional cosmological constant.

A being the 2×2 matrix with entries $\hat{A}^I{}_\alpha$. If B stands for the 2×2 scalar matrix $(\hat{B}_{\alpha\beta})$, then the 2×2 scalar matrix C is given by

$$C = \frac{1}{2}A^T A + B. \quad (5)$$

The matrix \mathcal{M} satisfies $\mathcal{M}\mathcal{L}\mathcal{M}\mathcal{L} = \mathbb{I}_6$, with

$$\mathcal{L} \equiv \begin{pmatrix} 0 & \mathbb{I}_2 & 0 \\ \mathbb{I}_2 & 0 & 0 \\ 0 & 0 & \mathbb{I}_2 \end{pmatrix}. \quad (6)$$

From Eq. (3) it is obvious that the dimensionally reduced action, is invariant under the global transformations given by

$$\mathcal{M} \rightarrow \Omega \mathcal{M} \Omega^T, \quad \mathcal{K}^i{}_\mu \rightarrow \Omega^i{}_j \mathcal{K}^j{}_\mu, \quad (7)$$

iff the transformation matrices Ω satisfy the identity $\Omega \mathcal{L} \Omega^T = \mathcal{L}$, showing that the action is invariant under $O(2,4)$, the classical duality group.

The $N = 4, d = 4$ supergravity equations of motion are actually invariant under S-duality [12], although this isn't obvious from Eq. (1). In order to write Eq. (1) in a manifest S-duality invariant form one goes over to the Einstein frame and Hodge dualizes the 3-form \hat{H} , imposing that it satisfies the Bianchi identity, so that the the number of degrees of freedom is not changed. The action then reads

$$S = \int d^4x \sqrt{|g_E|} \left[\hat{R}(\hat{g}_E) - \frac{1}{2} (\Im \hat{\lambda})^{-2} \partial_{\hat{\mu}} \hat{\lambda} \partial^{\hat{\mu}} \bar{\hat{\lambda}} + \frac{1}{4} \hat{F}^I \star \tilde{\hat{F}}^I \right]. \quad (8)$$

where we have defined

$$\hat{\lambda} \equiv \hat{a} + ie^{-\hat{\phi}}, \quad \tilde{\hat{F}}^I \equiv e^{-\hat{\phi}} \star \hat{F}^I + \hat{a} \hat{F}^I, \quad \partial_{\hat{\mu}} \hat{a} = \frac{1}{3! \sqrt{|\hat{g}_E|}} e^{-2\hat{\phi}} \hat{\epsilon}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} \hat{H}^{\hat{\nu}\hat{\rho}\hat{\sigma}}. \quad (9)$$

The S-duality transformations then take the form

$$\hat{\lambda}' = \frac{a\hat{\lambda} + b}{c\hat{\lambda} + d}, \quad \begin{pmatrix} \tilde{\hat{F}}^{I'} \\ \hat{F}^{I'} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \tilde{\hat{F}}^I \\ \hat{F}^I \end{pmatrix}, \quad (10)$$

with $ad - bc = 1$, meaning that S-duality is $Sl(2, \mathbb{R})$. $Sl(2, \mathbb{R})$ is generated by three types of transformations: rescalings $\hat{\lambda}' = a^2 \hat{\lambda}$, continuous shifts of the axion $\hat{\lambda}' = \hat{\lambda} + b$, and the discrete transformation $\hat{\lambda}' = -1/\hat{\lambda}$.

3 TNbh Asymptotics

The asymptotic behavior of four-dimensional asymptotically flat metrics is completely characterized to first order in $1/r$ by only two charges: the ADM mass

M and the angular momentum J . Duality, however, transforms asymptotically flat metrics into non-asymptotically flat metrics which need different additional charges to be characterised asymptotically. One of them [5] is the NUT charge N and closure under duality forces us to consider it. With the aforementioned conditions on the four-dimensional metric it is always possible to choose coordinates such that the Einstein metric in the $t - \varphi$ subspace has the following expansion in powers of $1/r$:⁶

$$(\hat{g}_{E\alpha\beta}) = \begin{pmatrix} -1 + 2M/r & 2N \cos \theta + 2J \sin^2 \theta / r \\ 2N \cos \theta + 2J \sin^2 \theta / r & (r^2 + 2Mr) \sin^2 \theta \end{pmatrix}$$

We will assume the following behavior for the dilaton

$$e^{-\hat{\phi}} = 1 - 2\mathcal{Q}_d/r + 2\mathcal{W} \cos \theta / r^2 - 2\mathcal{Z}/r^2 + \mathcal{O}(r^{-3}), \quad (11)$$

where \mathcal{Q}_d is the dilaton charge and \mathcal{W} is a charge related to the angular momentum that will be forced upon us by S-duality. Observe that we have fixed the constant asymptotic value equal to zero using the same reasoning as Burgess *et.al.* [5], i.e. rescaling it away any time it arises. The time coordinate, when appropriate, will be rescaled as well, in order to bring the transformed Einstein metric to the above form (i.e. to preserve our coordinate (gauge) choice), but in a duality-consistent way. Sometimes it will also be necessary to rescale the angular coordinate φ in order to get a metric looking like (11). Conical singularities are then generically induced, and then the metric is not asymptotically TNbh in spite of looking like (11).

The objects we will consider will generically carry electric (\mathcal{Q}_e^I) and magnetic (\mathcal{Q}_m^I) charges with respect to the Abelian gauge fields $\hat{A}_{\hat{\mu}}^I$. Since we allow also for angular momentum, they will also have electric (\mathcal{P}_e^I) and magnetic (\mathcal{P}_m^I) dipole momenta. This implies for the two-dimensional scalar matrix A the following asymptotic behavior⁷

$$\hat{A}^I_t = -2\mathcal{Q}_e^I/r + 2\mathcal{P}_e^I \cos \theta / r^2, \quad \hat{A}^I_\varphi = -2\mathcal{Q}_m^I \cos \theta - 2\mathcal{P}_m^I \sin^2 \theta / r. \quad (12)$$

Electric dipole momenta appear at higher order in $1/r$ and it is not strictly necessary to consider them from the point of view of T-duality, since it will not interchange them with any of the other charges we are considering and that appear at lower orders in $1/r$. However, S-duality will interchange the electric and magnetic dipole momenta and we cannot in general ignore them⁸.

⁶We will only write those terms in the asymptotic expansion that will actually be used, although more general terms are possible.

⁷One could include constant parts in the gauge fields. Usually they can be absorbed by a gauge transformation, but in doing dimensional reduction the gauge group gets broken, giving the constant parts an invariant meaning. See [4, 5] for a more elaborated discussion.

⁸The different behavior of T- and S-duality is due to the fact that T-duality acts on the potential's components whereas S-duality acts on the field strengths

The two-index form will have the usual charge \mathcal{Q}_a . Closure under duality again demands the introduction of a new extra parameter (“charge”) that we denote by \mathcal{F} and which will interchange with J under duality. The asymptotic expansion is, then

$$\hat{B}_{t\varphi} = \mathcal{Q}_a \cos \theta + \mathcal{F} \sin^2 \theta / r . \quad (13)$$

Using the definition in Eq. (9) for the pseudoscalar \hat{a} we can find its, allowing for a constant value at infinity \hat{a}_0 which will be set to zero in the initial configuration, asymptotic expansion to be

$$\hat{a} = \hat{a}_0 + 2\mathcal{Q}_a/r - 2\mathcal{F} \cos \theta / r^2 , \quad (14)$$

showing that \mathcal{Q}_a is the standard axion charge [8].

The class of asymptotic behavior just described, determined by the twelve charges $M, J, N, \mathcal{Q}_a, \mathcal{F}, \mathcal{Q}_d, \mathcal{Q}_e^I, \mathcal{Q}_m^I, \mathcal{P}_m^I$ will henceforth be referred to as *TNbh asymptotics*.

4 Transformation of the Charges under Duality

As is well known [13], the T-duality group can be decomposed locally as $O(2, 4) \sim SO^\uparrow(2, 4) \otimes \mathbb{Z}_2^{(B)} \otimes \mathbb{Z}_2^{(S)}$, where $SO^\uparrow(2, 4)$ is the connected component containing the identity of $O(2, 4)$. One can show that $\mathbb{Z}_2^{(S)}$ can be taken to be generated by \mathbb{I}_6 and $-\mathbb{I}_6$, thus acting trivially, and that $\mathbb{Z}_2^{(B)}$ generates [14] the Buscher transformations [15].

The action of the T-duality group was discussed in [4], where it was shown that only a seven dimensional subgroup of $SO^\uparrow(2, 4)$ transforms TNbh into TNbh, the ADS, and that the charges transform linearly under the ADS. Actually, the ADS closes on sets of four charges and its effect on the charges can be described by a four dimensional matrix representation acting on three charge-vectors:

$$\vec{M} \equiv (M, \mathcal{Q}_d, \mathcal{Q}_e^1, \mathcal{Q}_e^2), \quad \vec{N} \equiv (N, \mathcal{Q}_a, \mathcal{Q}_m^1, \mathcal{Q}_m^2), \quad \vec{J} \equiv (J, \mathcal{F}, \mathcal{P}_m^1, \mathcal{P}_m^2), \quad (15)$$

which will be referred to, respectively, as *electric*, *magnetic* and *dipole* charge vectors. There is a fourth charge vector that contains the electric dipole momenta \mathcal{P}_e^I , the dilaton dipole-type charge \mathcal{W} and an unidentified geometrical charge which we denote by K

$$\vec{K} \equiv (K, \mathcal{W}, \mathcal{P}_e^1, \mathcal{P}_e^2) . \quad (16)$$

The presence of this fourth charge vector is required by S-duality, as will be explained later.

The generator of $\mathbb{Z}_2^{(B)}$ is not unique (it is an element of a coset group). Two obvious choices correspond to the Buscher transformations in the directions t

and φ . The Buscher transformation in the direction φ does not preserve TNbh asymptotics and so we will take as generator of $\mathbb{Z}_2^{(B)}$ the Buscher transformation in the direction t ,⁹ that we denote by τ , with matrix

$$\Omega_\eta(\tau) = \begin{pmatrix} +1 & \\ & -\mathbb{I}_5 \end{pmatrix}. \quad (17)$$

The effect of τ on all the charges can be expressed in terms of the same symmetric 4×4 matrix $\Omega_\tau^{(4)}$

$$\Omega_\tau^{(4)} = \begin{pmatrix} 0 & 1 & & \\ 1 & 0 & & \\ & & & \mathbb{I}_2 \end{pmatrix}, \quad (18)$$

acting on the charge vectors $\vec{M}, \vec{N}, \vec{J}$. The involutive property, that on the charges $\tau^2 = id$ is immediately apparent.

Looking at the action of the TNbh preserving transformations [4] one can see that they act as the group $O(1, 2)$ on the charges. This then not only means that the multiplets get broken up into a singlets and triplets, but also that T-duality leaves invariant the four dimensional metric $\eta^{(4)} = \text{diag}(+, +, -, -)$, which, as we will see, will be of some importance whilst discussing the transformations of the Bogomol'nyi bound under duality.

The transformation of the electric, magnetic, dilaton and axion charges under S-duality has been previously studied in Ref. [8, 16]. Here we are considering more charges and we are choosing initial configurations with vanishing asymptotic values of the axion and dilaton. In general, S-duality generates non-vanishing values of these constants and we will remove them by applying further S-duality transformations. The net effect [4] is that one allows only an $SO(2)$ subgroup of S-duality whose result, expressed in terms of the entries of the original $SL(2, \mathbb{R})$ matrix is

$$\begin{pmatrix} \mathcal{Q}_e^I{}' \\ \mathcal{Q}_m^I{}' \end{pmatrix} = \begin{pmatrix} \frac{d}{\sqrt{c^2+d^2}} & \frac{c}{\sqrt{c^2+d^2}} \\ \frac{-c}{\sqrt{c^2+d^2}} & \frac{d}{\sqrt{c^2+d^2}} \end{pmatrix} \begin{pmatrix} \mathcal{Q}_e^I \\ \mathcal{Q}_m^I \end{pmatrix}, \quad (19)$$

and similarly for the vector of dipole momenta $(\mathcal{P}_m^I, \mathcal{P}_e^I)$ and

$$\begin{pmatrix} \mathcal{Q}'_d \\ \mathcal{Q}'_a \end{pmatrix} = \begin{pmatrix} \frac{d^2-c^2}{\sqrt{c^2+d^2}} & \frac{2cd}{\sqrt{c^2+d^2}} \\ \frac{-2cd}{\sqrt{c^2+d^2}} & \frac{d^2-c^2}{\sqrt{c^2+d^2}} \end{pmatrix} \begin{pmatrix} \mathcal{Q}_d \\ \mathcal{Q}_a \end{pmatrix}, \quad (20)$$

and, analogously for the charge vector $(\mathcal{W}, \mathcal{F})$. Observe that the last $SO(2)$ transformation matrix is precisely the square of the former.

⁹Note that the transformation taking Buschers transformation in the φ direction into τ is not part of the ADS, which is as it ought to be.

It is now clear that the multiplet structure that we built for the T-duality transformations is not respected by S-duality: the last three components of the “electric” multiplet \vec{M} are rotated into the last three components of the “magnetic” multiplet \vec{N} and vice versa. The same happens with the multiplet \vec{K} defined in Eq. (16), whose last three components are rotated into those of the multiplet \vec{J} in exactly the same way, and vice versa (this is the reason why we introduced K and \vec{K} in the first place). To respect the T-duality multiplet structure and, at the same time incorporate the S-duality multiplet structure it is useful to introduce the complexified multiplets

$$\vec{\mathcal{M}} \equiv \vec{M} + i\vec{N} \quad \text{and} \quad \vec{\mathcal{J}} \equiv \vec{K} + i\vec{J}. \quad (21)$$

These two complex vectors transform under T-duality with exactly the same matrices as the real vectors and, under the above S-duality transformations with the complex matrix $\Sigma^{(4)} = \text{diag}(1, e^{2i\theta}, e^{i\theta}, e^{i\theta})$, with $\theta = \text{Arg}(d - ic)$, so

$$\vec{\mathcal{M}}' = \Sigma^{(4)} \vec{\mathcal{M}}, \quad \vec{\mathcal{J}}' = \Sigma^{(4)} \vec{\mathcal{J}}. \quad (22)$$

5 The Bogomol’nyi Bound and its Variation

In $N = 4$ supergravity there are two Bogomol’nyi (B.) bounds, of the form

$$M^2 - |Z_i|^2 \geq 0, \quad i = 1, 2, \quad (23)$$

where the Z_i ’s are the complex skew eigenvalues of the central charge matrix and are combinations of electric and magnetic charges of the six graviphotons. These two bounds can be combined into a single bound by multiplying them and then dividing by M^2 . One then gets a *generalized* B. bound

$$M^2 + \frac{|Z_1 Z_2|}{M^2} - |Z_1|^2 - |Z_2|^2 \geq 0. \quad (24)$$

In regular black-hole solutions the second term can be identified with scalar charges of “secondary” type. The identification is, actually (with zero value for the dilaton at infinity)

$$\frac{|Z_1 Z_2|}{M^2} = Q_d^2 + Q_a^2. \quad (25)$$

Inserting then the expressions for the central charges one obtains a generalized B. bound [8], which doesn’t take account of the NUT charge, meaning that the B. bound is valid for asymptotically flat spaces only. This problem can be overcome by the reasoning of Ref. [2] where it was observed that the NUT charge N does enter in the generalized B. bound. With our definitions the B. bound for asymptotically TNbh spaces takes the form

$$M^2 + N^2 + Q_d^2 + Q_a^2 - Q_e^I Q_e^I - Q_m^I Q_m^I \geq 0. \quad (26)$$

To study the transformation properties of the B. bound under the physical TNbh asymptotics-preserving duality group it is convenient to use the diagonal metric of $SO(2, 2)$ $\eta^{(4)} = \text{diag}(1, 1, -1, -1)$ already introduced in section (4). Using this metric and the charge vectors defined in Eq. (21) the B. bound can be easily rewritten to the form

$$\vec{\mathcal{M}}^\dagger \eta^{(4)} \vec{\mathcal{M}} \geq 0. \quad (27)$$

In this form the B. bound of $N = 4, d = 4$ supergravity is manifestly $U(2, 2)$ -invariant. Observe that $U(2, 2) \sim O(2, 4)$, although it is not clear if this fact is a mere coincidence or it has a special significance. The T-duality piece of the ADS is an $O(1, 2)$ subgroup of the $O(2, 2)$ canonically embedded in $U(2, 2)$ and obviously preserves the B. bound. The S-duality piece of the ADS is a $U(1)$ subgroup diagonally embedded in $U(2, 2)$ through the matrices $\Sigma^{(4)}$ and obviously preserve the B. bound.

6 Conclusions

In the course of our argument we saw that duality acts linearly on the charges defining stationary, axially symmetric, possibly non-asymptotically flat string backgrounds, and that duality collects these charges in tetraplets, see Eq. (15, 16). It was further shown that the charges in the vector $\vec{\mathcal{J}}$ do not appear in the $D = 4, N = 4$ Bogomol'nyi bound and that neither T- nor S-duality changes this fact. It is not possible to constrain the values of any of the charges it (in particular J) by using duality and supersymmetry, as was suggested in the Introduction.

Although our reasoning is completely clear when we look at specific solutions one should be able to derive B. bounds including primary scalar charges using a Nester construction based on the supersymmetry transformation laws of the fermions of the supergravity theory under consideration. To be able to do this one has to be able to manage more general boundary conditions including the seemingly unavoidable naked singularities that primary hair implies.

The results discussed so far leave unanswered the question hinted at in the Introduction: why does the angular momentum appear in the definition of extremality (defining the borderline between a regular horizon and a naked singularity, with zero Hawking temperature) but not in the Bogomol'nyi bound (whose saturation guarantees absence of quantum corrections, as well as a “zero force condition”, allowing superposition of static solutions)? We hold this to be due to the fact that stationary (as opposed to static) black holes possess a specific decay mode, known in the bh literature as “superradiance” [17, 18], even visible classically by scattering waves off black holes. The way this appears is that the amplitude for reflected waves is greater than the corresponding incident amplitude, for low frequencies, up to a given frequency cutoff, $m\Omega_H$, depending on the angular momentum of the hole, and such that $\Omega_H(a = 0) = 0$. The

angular momentum of the hole decreases by this mechanism until a static configuration is reached. The physics underlying this process is similar to the one supporting Penrose's energy extraction mechanism, i.e. the fact that energy can be negative in the ergosphere. This, in turn, is a straightforward consequence of the fact that the Energy of a test particle is defined as $E = p.k$, where p is the momentum of the particle, and k is the Killing vector (which has spacelike character precisely in the ergosphere); and the product of a spacelike and a timelike vector does not have a definite sign.

Quantum mechanically this means that there are two competing mechanisms of decay for a rotating (stationary) black hole: spontaneous emission (the quantum effect associated to the superradiance), which is not thermal (and disappears when the angular momentum goes to zero) and Hawking radiation, which is thermal. The first is more efficient for massive black holes, but its width is never zero until the black hole has lost all its angular momentum.

This then shows that even if the black hole is extremal, it cannot be stable quantum mechanically as long as its angular momentum is different from zero. This argument taken literally would suggest that it is not possible to have BPS states with non zero angular momentum, unless they are such that no ergosphere exists. This is the case of the supersymmetric Kerr-Newman solutions which are singular and, therefore, do not have ergosphere. What is not clear is why supersymmetry signals the singular case as special and not the usual extremal Kerr-Newman black hole.¹⁰

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¹⁰This argument seems to be valid only in four dimensions though, since rotating charged black holes which are BPS states exist in five dimensions [19]. The existence of two Casimirs for the five-dimensional angular momentum seems to play an important role.

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