

Dynamical moment of inertia and quadrupole vibrations in rotating nuclei

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The contribution of quantum shape fluctuations to inertial properties of rotating nuclei has been analysed within the self-consistent one-dimensional cranking oscillator model. It is shown that in even-even nuclei the dynamical moment of inertia calculated in the mean field approximation is equivalent to the Thouless-Valatin moment of inertia calculated in the random phase approximation *if and only if* the self-consistent conditions for the mean field are fulfilled.

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A description of rotational states is one of the oldest, yet not fully solved, problem in nuclear structure physics. While various microscopic models based on the cranking approach [1,2] describe reasonably well the kinematical moment of inertia $\mathcal{J}^{(1)} = -(dE/d\Omega)/\Omega$ for a finite angular frequency Ω , there is still a systematic deviation of the dynamical moment of inertia $\mathcal{J}^{(2)} = -d^2E/d\Omega^2$ (E is the energy in the rotating frame) from the experimental data at high spins [3]. Since the moments of inertia are the benchmarks for microscopic models of collective motion in nuclei, the understanding of the source of the discrepancy becomes a challenge for a many body theory of finite Fermi systems.

The pairing correlations introduced in nuclear physics by Bogoliubov [4] and Bohr, Mottelson and Pines [5] improved the description of the inertial nuclear properties, especially in the low spin region [6]. It was conjectured that at high spins the Coriolis and centrifugal forces break Cooper pairs from the pairing condensate and cause the transition from the superfluid to the normal (unpaired) fluid [7]. As a consequence, the rigid body (kinematical) moment of inertia should be reached at the normal phase. However, even for superdeformed nuclei where the pairing condensate is expected to be strongly quenched, the moment of inertia is usually lower than the rigid body value.

In fact, the conjecture was based on the analogy between moment of inertia of a rotating nucleus and magnetic susceptibility of a macroscopic superconductor under a magnetic field. Therefore, the conjecture may lose its validity for finite Fermi systems like nuclei, while remaining correct for an infinite number of particles. For example, the deviation from the rigid body value could be partially explained due to shell effects [8]. In present paper we demonstrate that correlations caused by shape oscillations of a system are another important ingredient which is missing in all state of the art calculations

of the moments of inertia for rotating nuclei. We focus our analysis upon the dynamical moment of inertia, since the $\mathcal{J}^{(2)}$ contains more information about different properties of the system due to the obvious relation $\mathcal{J}^{(2)} = \mathcal{J}^{(1)} + \Omega d\mathcal{J}^{(1)}/d\Omega$.

The first attempt to take into account the contribution of the quadrupole oscillations to the correlation energy and to the moment of inertia within the cranking+random phase approximation (RPA) approach [9] suffered from an inconsistency between the mean field and the residual interaction. In addition, the calculations were done in a restricted configuration space (only 3 shells have been considered). For a time the solution of the problem was postponed, since there was no practical recipe to calculate the contribution of the correlation energy, even in a restricted configuration space. Using the integral representation method developed recently in [10], a full energy E_{RPA} can be calculated in the RPA order [1,11] with a high accuracy and with minimal numerical efforts. Consequently, this energy can be used to calculate the dynamical moments of inertia $\mathcal{J}^{(2)}$. On the other hand, in literature it is stated that the dynamical moment of inertia should be equivalent to the Thouless-Valatin moment of inertia [12]. Since the realistic application of the Thouless-Valatin theory requires the self-consistent solution of the mean field and the RPA equations, until now this point is not clarified.

To understand the role of shape oscillations upon the value of the moment of inertia and to calculate the Thouless-Valatin moment of inertia in a simple but still realistic model, we use a self-consistent cranked triaxial harmonic oscillator model as a rotating mean field. The self-consistent residual interaction constructed according to the recipe [13] is added to describe the collective excitations in a rotating system. Since all shells are mixed, we go beyond the approximation used in [9] (for a cranking harmonic oscillator see also [14]). Notice that the contribution of the pairing vibrations to the correlation energy aside of the one from the shape vibrations is also important (see [15,16] and references there). However, there are some open problems with the choice of the self-consistent pairing interaction. Therefore, the combined effect of the both types of vibrations is beyond the scope of the present investigation and we leave this problem for the future.

The mean field part of the many-body Hamiltonian (Routhian) in the rotating frame is given by

$$H = \sum_{i=1}^N (h_0 - \Omega l_x)_i = H_0 - \Omega L_x \quad (1)$$

where the single-particle triaxial harmonic oscillator Hamiltonian h_0 is aligned along its principal axes and reads

$$h_0 = \frac{1}{2m} \vec{p}^2 + \frac{m}{2} (\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2). \quad (2)$$

The eigenmodes and the total energy of the mean field Hamiltonian Eq.(1) are well known [17–19],

$$\omega_{\pm}^2 = \frac{1}{2} (\omega_y^2 + \omega_z^2 + 2\Omega^2 \pm [(\omega_y^2 - \omega_z^2)^2 + 8\Omega^2(\omega_y^2 + \omega_z^2)]^{1/2}) \quad (3)$$

$$E_{MF} = \hbar \left(\omega_x \sum_x + \omega_+ \sum_+ + \omega_- \sum_- \right). \quad (4)$$

Here, $\sum_k = \langle \sum_j^N (n_k + 1/2)_j \rangle$ and $n_k = a_k^\dagger a_k$ ($k = x, +, -$) where a_k^\dagger, a_k are the oscillator quanta operators. The lowest levels are filled from the bottom, which give the ground state energy in the rotating frame. The Pauli principle is taken into account such that two particles occupy one level. The minimisation of the total energy Eq.(4) with respect to all three frequencies, subject to the volume conservation condition $\omega_x \omega_y \omega_z = \omega_0^3$, yields the self-consistent condition [20,21] for a finite rotational frequency

$$\omega_x^2 \langle x^2 \rangle = \omega_y^2 \langle y^2 \rangle = \omega_z^2 \langle z^2 \rangle. \quad (5)$$

It should be pointed out that the condition Eq.(5) provides generally the absolute minima in comparison with the local minima obtained from the condition of the *isotropic velocity distribution* [18,19]

$$\sum_x \omega_x = \sum_+ \omega_+ = \sum_- \omega_- \quad (6)$$

at large rotational frequency.

To analyse the contribution of the quadrupole shape oscillations we add to the mean field Hamiltonian Eq.(1) the self-consistent interaction resulting from small angular rotations around the x -, y -, z - axes and small variations of two the intrinsic shape parameters ε and γ [13]. Consequently, the total Hamiltonian can be expressed as

$$H_{\text{RPA}} = H_0 - \Omega L_x - \frac{\kappa}{2} \sum_{\mu=-2}^2 Q_\mu^\dagger Q_\mu = \tilde{H} - \Omega L_x. \quad (7)$$

The effective interaction restores the rotational invariance of the Hamiltonian H_0 such that now $[\tilde{H}, L_i] = 0$ ($i = x, y, z$) in the RPA order. The self-consistency condition Eq.(5) fixes the quadrupole strength $\kappa = \frac{4\pi}{5} \frac{m\omega_0^2}{\langle r^2 \rangle}$. Here a mean value $\langle r^2 \rangle = \langle \bar{x}^2 + \bar{y}^2 + \bar{z}^2 \rangle$ and quadrupole operators $Q_\mu = r^2 \bar{Y}_{2\mu}$ are expressed

in terms of the double-stretched coordinates $\bar{q}_i = \frac{\omega_i}{\omega_0} q_i$, ($q_i = x, y, z$). We remind that the self-consistent residual interaction does not affect the equilibrium deformation obtained from the minimisation procedure.

Using the transformation from p_i, q_i variables to the quanta a_k^\dagger, a_k [19], all matrix elements are calculated analytically. We solve the RPA equation of motion for the generalised coordinates \mathcal{X}_λ and momenta \mathcal{P}_λ

$$[H_{\text{RPA}}, \mathcal{X}_\lambda] = -i\omega_\lambda \mathcal{P}_\lambda, \quad [H_{\text{RPA}}, \mathcal{P}_\lambda] = i\omega_\lambda \mathcal{X}_\lambda, \quad (8)$$

$$[\mathcal{X}_\lambda, \mathcal{P}_\lambda] = i\delta_{\lambda, \lambda'}.$$

where ω_λ are the RPA eigen-frequencies in the rotating frame and the associated phonon operators are $O_\lambda = (\mathcal{X}_\lambda - i\mathcal{P}_\lambda)/\sqrt{2}$. Here $\mathcal{X}_\lambda = \sum_s X_s^\lambda \hat{f}_s$, $\mathcal{P}_\lambda = i \sum_s P_s^\lambda \hat{g}_s$ are bilinear combinations of the quanta a_k^\dagger, a_k such that $\langle [\hat{f}_s, \hat{g}_{s'}] \rangle = V_s \delta_{s, s'}$ where quantities V_s are proportional to different combinations of \sum_i ($i = x, +, -$). Further, $\langle \dots \rangle$ means the averaging over mean field states. Since the mean field violates the rotational invariance, among the RPA eigen-frequencies there exist two spurious solutions. One solution with zero frequency is associated with the rotation around the x -axes, since $[H, L_x] = 0$. The other "spurious" solution at $\omega \equiv \Omega$ corresponds to a collective rotation, since $[H, L_\pm] = [H, L_y \pm iL_z] = \mp \Omega L_\pm$ [22]. The Hamiltonian Eq.(7) possesses the signature symmetry, i.e. $[R_x, H_{\text{RPA}}] = 0$ ($R_x = e^{-i\pi \hat{L}_x}$), such that it decomposes into positive and negative signature terms

$$H_{\text{RPA}} = H(+)+H(-) \quad (9)$$

which can be separately diagonalized [22–24]. The negative signature Hamiltonian contains the rotational mode and the vibrational mode describing the wobbling motion [23,25]. We focus on the positive signature Hamiltonian. It contains the zero-frequency mode defined by

$$[H(+), \phi_x] = \frac{-iL_x}{\mathcal{J}_{TV}}, \quad [\phi_x, L_x] = i \quad (10)$$

and allows one to determine the Thouless-Valatin moment of inertia \mathcal{J}_{TV} [26]. Here, the angular momentum operator $L_x = \sum_s l_s^x \hat{f}_s$ and the canonically conjugated angle $\phi_x = i \sum_s \phi_s^x \hat{g}_s$ are expressed via \hat{f}_s and \hat{g}_s which obey the condition $R_x \hat{d}_s R_x^{-1} = \hat{d}_s$ ($\hat{d}_s = \hat{f}_s$ or \hat{g}_s). Solving Eqs.(10) for the Hamiltonian $H(+)$,

$$H(+)= \sum_{k=x,+, -} \hbar \omega_k (a_k^\dagger a_k + 1/2) - \frac{\kappa}{2} (Q_0^2 + Q_1^{(+)^2} + Q_2^{(+)^2}) \quad (11)$$

where

$$Q_0 = \sqrt{\frac{5}{16\pi}} (2\bar{z}^2 - \bar{x}^2 - \bar{y}^2) = \sqrt{\frac{5}{16\pi}} \sum_s q_s^0 \hat{f}_s \quad (12)$$

$$Q_1^{(+)} = \sqrt{\frac{15}{4\pi}} \bar{y}\bar{z} = \sqrt{\frac{15}{4\pi}} i \sum_s q_s^1 \hat{g}_s \quad (13)$$

$$Q_2^{(+)} = \sqrt{\frac{15}{16\pi}}(\bar{x}^2 - \bar{y}^2) = \sqrt{\frac{15}{16\pi}} \sum_s q_s^2 \hat{f}_s \quad (14)$$

we obtain the expression for the Thouless-Valatin moment of inertia

$$\mathcal{J}_{TV} = \mathcal{J}_{\mathcal{I}} + \frac{[2S_{x0}S_{x2}S_{02} - S_{x0}^2(S_{22} - \frac{1}{\kappa_2}) - S_{x2}^2(S_{00} - \frac{1}{\kappa_0})]}{[(S_{00} - \frac{1}{\kappa_0})(S_{22} - \frac{1}{\kappa_2}) - S_{02}^2]} \quad (15)$$

Here, the term $\mathcal{J}_{\mathcal{I}}$ corresponds to the Inglis moment of inertia

$$\mathcal{J}_{\mathcal{I}} = \sum_s \frac{(l_s^x)^2 V_s}{E_s}. \quad (16)$$

The second term in Eq.(15) is a contribution of the quadrupole residual interaction in the cranking model. In the cranking harmonic oscillator it consists of the terms which have the following structure

$$S_{xm} = \sum_s \frac{l_s^x q_s^m V_s}{E_s}, \quad S_{nm} = \sum_s \frac{q_s^n q_s^m V_s}{E_s}, \quad n, m = 0, 2 \quad (17)$$

where E_s are the energies of particle-hole excitations: $E_1 = 2\hbar\omega_+$, $E_2 = 2\hbar\omega_-$, $E_3 = 2\hbar\omega_x$, $E_4 = \hbar\omega_+ + \hbar\omega_-$ and $E_5 = \hbar\omega_+ - \hbar\omega_-$. We also introduced the following notations: $\kappa_0 = \frac{5}{16\pi}\kappa$ and $\kappa_2 = \frac{15}{16\pi}\kappa$.

The above results are the starting point for our numerical analysis. To take into account shell effects we consider two systems with number of particles $A = 20, 64$ ($N = Z$). For $\hbar\Omega = 0$ MeV the global minimum occurs for a prolate shape and for a near oblate triaxial shape for $A = 20$ and 64 , respectively [21]. If we trace the configurations which characterise the ground states, with increasing rotational frequency both systems become oblate. At this point the moment of inertia vanishes, since there is no a kinetic energy associated with such a rotation.

In order to compare various moments of inertia, i.e. the Thouless-Valatin, Eq.(15), the Inglis, Eq.(16), and $\mathcal{J}_{MF}^{(2)} = -d^2 E_{MF}/d\Omega^2$ with $\mathcal{J}_{RPA}^{(2)} = -d^2 E_{RPA}/d\Omega^2$, we calculate the RPA correlation energy $E_{corr}^{RPA} = \frac{1}{2}(\sum_\lambda \omega_\lambda - \sum_s E_s)$ which includes the positive and negative signature contributions. In Figs.1,2 the results of calculations for different moments of inertia are presented for $A = 20$ and 64 , respectively. For our knowledge this is a first numerical demonstration of the equivalence between the dynamical moment of inertia $\mathcal{J}_{MF}^{(2)}$ calculated in the mean field approximation and the Thouless-Valatin moment of inertia \mathcal{J}_{TV} calculated in the RPA. For the both systems the Inglis moment of inertia $\mathcal{J}_{\mathcal{I}}$ is smaller than the \mathcal{J}_{TV} and $\mathcal{J}_{MF}^{(2)}$ and has a different rotational dependence. While the Inglis moment of inertia

characterises the collective properties of non-interacting fermions, the dynamical moment of inertia reflects the changes in the rotating self-consistent mean field due to an inter-nucleon interaction. As it was pointed out in [27], the volume conservation condition, used as a constraint in the mean field calculations, can be interpreted as a Hartree approximation applied to an interaction that involves the sum of one-, two- etc forces.

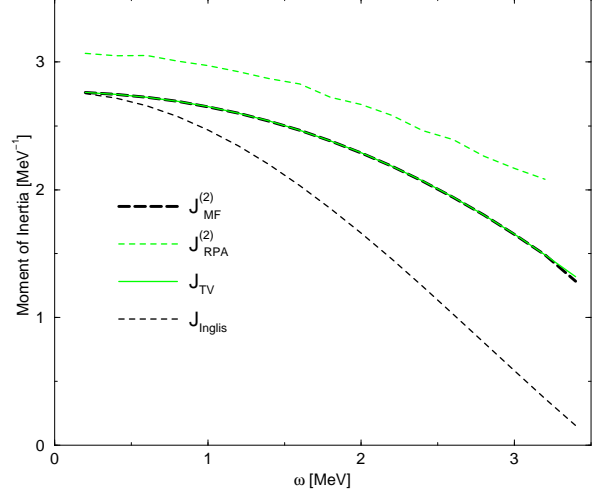


FIG. 1. Moments of inertia for $N = Z = 10$ system as a function of the rotational frequency $\omega \equiv \Omega$. The definitions of different moments of inertia are given in the text.

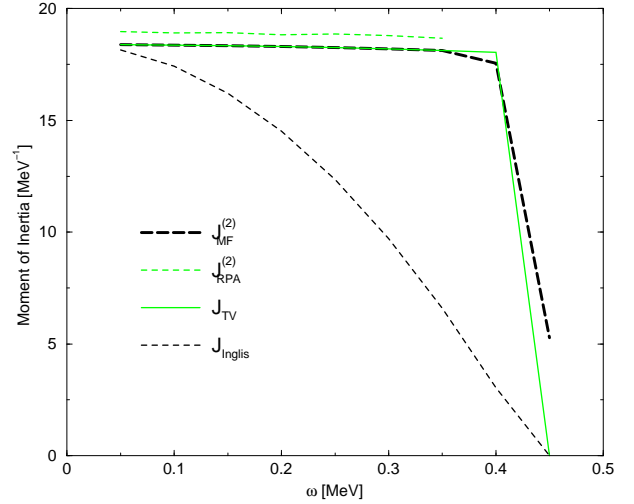


FIG. 2. As in Fig.1 for $N = Z = 32$ system

The sharp drop of all moments of inertia in Fig.2 is caused by the onset of the oblate shape where the collective rotation does not exist. For $A = 64$ the onset of the oblate deformation occurs at a smaller rotational frequency in contrast to the one for the system $A = 20$.

The dynamical moment of inertia $\mathcal{J}_{RPA}^{(2)}$ is larger than the Thouless-Valatin moment of inertia. However, from our calculations it follows that the contribution of the RPA ground state correlations decreases with an increase of the number of particles. The difference between the $\mathcal{J}_{RPA}^{(2)}$ and the \mathcal{J}_{TV} is due to the following reason. The Inglis moment of inertia is smaller than the Thouless-Valatin (or $\mathcal{J}_{MF}^{(2)}$) value, since the \mathcal{J}_{TV} contains the effect of the residual particle-hole interaction. On the other hand, the Thouless-Valatin moment of inertia manifests the rotational dependence of the residual interaction. Thus, we may speculate that inclusion of the phonon interaction could help to reproduce the behaviour of the $\mathcal{J}_{RPA}^{(2)}$ which characterises the rotational dependence of the phonon-phonon interaction.

In summary, using the self-consistent cranking harmonic oscillator model, we have numerically proved the equivalence of the dynamical moment of inertia calculated in the mean field approximation to the Thouless-Valatin moment of inertia calculated in the RPA. Our result is a consequence of the self-consistent condition Eq.(5) which minimises the expectation value of the mean field Hamiltonian, Eq.(1). This condition is equivalent to the stability condition of collective modes in the RPA [28], i.e. ω_λ to be real and non-negative, and has been used to calculate different moments of inertia. The rotational dependence of the both dynamical moments of inertia, $\mathcal{J}_{MF}^{(2)}$ and $\mathcal{J}_{RPA}^{(2)}$, is similar, however, the $\mathcal{J}_{RPA}^{(2)}$ is larger than the $\mathcal{J}_{MF}^{(2)}$ due to the contribution of the ground state correlations. This difference between the moments of inertia is less important for heavy systems.

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