Superfluidity in neutron stars and cold atoms

Achim Schwenk

TRIUMF, 4004 Wesbrook Mall, Vancouver, BC, Canada, V6T 2A3
Department of Physics, University of Washington, Seattle, WA 98195-1560

Abstract. We discuss superfluidity in neutron matter, with particular attention to induced interactions and to universal properties accessible with cold atoms.

Superfluidity plays a central role in strongly-interacting many-body systems. Nuclear pairing shows striking trends in neutron-proton asymmetric systems [1]. The β decay of the two-neutron halo in ^{11}Li is suppressed due to pairing [2] similar to neutrino emission in neutron star cooling [3]. Ultracold atoms exhibit vortices and superfluid characteristics in thermodynamic and spectroscopic properties [4].

The physics of dilute Fermi gases with large scattering lengths is universal, independent of atomic or nuclear details. For neutrons the scattering length is also large, $a_{nn} = -18.5 \pm 0.3$ fm, and therefore cold atom experiments constrain low-density neutron matter. For instance, for two spin states with equal populations, the S-wave superfluid pairing gap of resonant gases of ^{6}Li atoms, ^{40}K atoms or neutrons is given by $\Delta/\varepsilon_F = \zeta$, where $\varepsilon_F = k_F^2/(2m)$ is the Fermi energy and ζ is a universal number.

For relative momenta $k \lesssim 2\,\text{fm}^{-1}$, nucleon-nucleon (NN) interactions are well constrained by the existing scattering data [5]. In Fig. 1, we show superfluid pairing gaps in neutron matter obtained by solving the BCS gap equation with a free spectrum. At low densities (in the crust of neutron stars), neutrons form a $^1S_0$ superfluid. At higher densities, the S-wave interaction is repulsive and neutrons pair in the $^3P_2$ channel (with a small coupling to $^3F_2$ due to the tensor force). Fig. 1 demonstrates that the $^1S_0$ BCS gap is practically independent of nuclear interactions, and therefore strongly constrained by the NN phase shifts [6]. This includes a very weak cutoff dependence for the class of low-momentum interactions $V_{\text{low}\,k}$ [5] with sharp or sufficiently narrow smooth regulators with $\Lambda > 1.6\,\text{fm}^{-1}$. The model dependence for larger momenta shows up prominently in Fig. 1 for the $^3P_2-^3F_2$ gaps at Fermi momenta $k_F > 2\,\text{fm}^{-1}$ [7].

Polarization effects ("induced interactions") due to particle-hole screening and vertex corrections are crucial for superfluidity. They lead to a reduction of the S-wave gap, which is significant $[(4e)^{-1/3} \approx 0.45]$ even in the perturbative $k_F a$ limit [8]:

$$\frac{\Delta}{\varepsilon_F} = \frac{8}{e^2} \exp\left\{ \left(\quad + \quad + \quad + \cdots \right)^{-1} \right\} = (4e)^{-1/3} \frac{8}{e^2} \exp\left\{ \frac{\pi}{2k_F a} + \mathcal{O}(k_F a) \right\}.$$

This reduction is due to spin fluctuations, which are repulsive for spin singlet pairing and overwhelm attractive density fluctuations. In finite systems, the spin and density
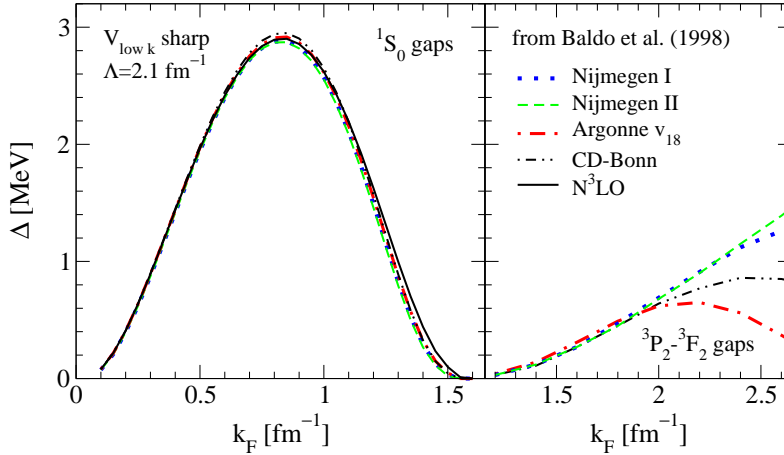
FIGURE 1. The $^1S_0$ (left) and $^3P_2-^3F_2$ (right) superfluid pairing gaps $\Delta \equiv \Delta(k_F)$ versus Fermi momentum k_F, based on various charge-dependent NN interactions at the BCS level. The results are for low-momentum interactions $V_{\text{low}k}$ with $\Lambda = 2.1\,\text{fm}^{-1}$ [6] (left) or taken from Baldo *et al.* [7] (right).

response differs. In nuclei with cores, the low-lying response is due to surface vibrations. Consequently, induced interactions may be attractive, since the spin response is weaker.

The renormalization group (RG) provides a systematic tool to reduce a physical system to a simpler, equivalent problem focusing on relevant degrees of freedom. Following Shankar [9], we have applied the RG to neutron matter, restricting the effective interaction to low-lying states in the vicinity of the Fermi surface [10]. Starting from the low-momentum interaction $V_{\text{low}k}$ [5], we solve a one-loop RG equation in the particle-hole channels ("phRG") that includes contributions from successive ph momentum shells. The RG builds up many-body effects similar to the two-body parquet equations, and efficiently includes induced interactions on superfluidity beyond the perturbative result.

The phRG results for the $^1S_0$ gap are shown in Fig. 2. We find a factor $3-4$ reduction to a maximal gap $\Delta \approx 0.8\,\text{MeV}$. At the larger densities, the dotted band indicates the uncertainty due to an approximate self-energy treatment in [10]. For the lowest densities, the phRG is consistent with the dilute result $\Delta/\Delta_0 = (4e)^{-1/3}$. This is similar to the GFMC calculations of Carlson *et al.* [12] for cold atoms in the unitary regime, which are also consistent with the extrapolated dilute result to a good approximation. On the lower side of Fig. 2, there are differences between neutron matter and unitary gases: For $k_F \approx 0.4\,\text{fm}^{-1}$, one has $k_F r_e \approx 1$ (with effective range r_e), and pairing is weaker, $\Delta/\varepsilon_F \approx 0.1$. For these densities, neutron matter is close to the unitary regime, but theoretically simpler due to an appreciable effective range [13]. Note that the (low-order) CBF results of [14] do not include long-range polarization effects, and therefore are close to the BCS gap at low densities.

The RG approach is widely used in condensed matter physics to study the interference of different instabilities, especially in the context of the 2d Hubbard model. Similar competing instabilities are present in color superconductivity at intermediate densities. Here, the RG method seems ideal to resolve the zoo of possible phases.

Non-central spin-orbit and tensor interactions are crucial for $^3P_2-^3F_2$ superfluidity. Without a spin-orbit interaction, neutrons would form a $^3P_0$ superfluid instead. The
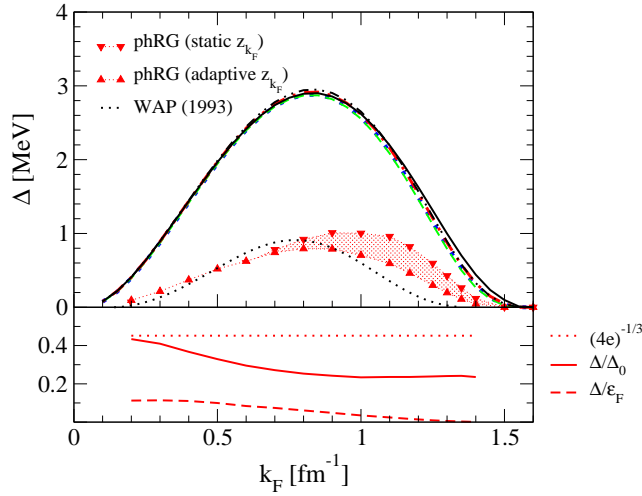
FIGURE 2. Top panel: Comparison of the $^1S_0$ BCS gap to the results including polarization effects through the phRG, for details see [10], and to the results of Wambach *et al.* [11]. Lower panel: Comparison of the full superfluid gap Δ to the BCS gap Δ_0 and to the Fermi energy ε_F.

first perturbative calculation of non-central induced interactions shows that $^3P_2$ gaps below $10\,\text{keV}$ are possible (while $\langle V_{\text{ind}}\rangle/\langle V_{\text{low}\,k}\rangle < 0.5$) [15]. This arises from a repulsive induced spin-orbit interaction due to the mixing with the (large) spin-spin interaction. Our result impacts the cooling of neutron stars [3] and would imply that core neutrons are only superfluid at late times ($t \sim 10^5\,\text{yrs}$).

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