De Broglie wave, spontaneous emission and Planck's radiation law according to stochastic electrodynamics

O. A. Senatchin

Institute of Theoretical and Applied Mechanics,

SB Academy of Sciences, Institutskaia St. 4/1, Novosibirsk, Russia 630090

E-mail: olsenat@itam.nsc.ru and olsenat@mail.ru

The idea about a quantum nature of Planck's blackbody radiation law is deeply rooted in minds of most physicists. Einstein's work, in which the coefficients of spontaneous and induced emission were introduced, has always been regarded as a proof that quantum energy discreteness of an atom plays a crucial role in the derivation of this law. In our paper we avoid this standpoint. It may be shown that the de Broglie wave assigned to every material particle is a result of interaction of the particle with zero-point vibrations of electromagnetic ground field. The energetic spectrum of a harmonic oscillator is obtained from this fact within classical physics which coincides with the quantum result. Thus, it is explained here how the energy discreteness came into existence in stochastic electrodynamics (SED) — the classical electrodynamics with classical electromagnetic zero-point radiation. Next we reconsider the Einstein work from the viewpoint of SED and derive the Planck formula.

I. INTRODUCTION

Every time when we discuss the Planck radiation law, the quantum radiation theory arises in its monumentality before our mental vision. Its method of derivation of the probability coefficients for spontaneous and induced emission practically has not changed since the appearance of Dirac's fundamental work [1] in 1927. But is this method so flawless? Does an eye not catch any contradictions that can be resolved from the positions of modern physics? As early as 1939 V. L. Ginzburg [2] paid his attention to one of such contradictions: an absence of correct limiting transition from quantum theory to classical theory at $\hbar \to 0$. In fact, in quantum theory the intensity of spontaneous emission

$$W_{mn}^{sp} = A_{mn}\hbar\omega_{mn}$$

is equal to zero at the limit $\hbar \to 0$. At the same time, the zero-point fluctuations go to zero. But in traditional classical theory the spontaneous emission *is present,* in spite of the zero-point radiation absence. The second contradiction is also obvious. If the existence of the zero-point radiation stems naturally from the bases of quantum theory, why it is absent in the final density radiation expression? Thus, for the derivation of Planck's formula after quantization of electromagnetic field and obtaining the coefficients A_{mn} and B_{mn} one must refer to the Einstein method [3] of 1917. However, only the expression

$$\rho\left(\omega,T\right) = \frac{\omega^2}{\pi^2 c^3}\frac{\hbar\omega}{e^{\hbar\omega/kT}-1} \tag{1}$$

may be derived this way. The zero-point term $(\omega^2/\pi^2c^3)(\hbar\omega/2)$ is absent here, but it produces spontaneous transitions, according to quantum interpretation. Therefore, at the absolute zero of temperature, the spontaneous emission must be absent also.

In this paper we try to elucidate these questions and, at the same time, propose a new method of derivation of Planck's radiation law. Here we operate with classical ideas only and completely avoid quantum hypotheses. Such an opportunity is given to us by stochastic electrodynamics (SED). It is a classical theory with classical electromagnetic zero-point radiation which playing a role of the vacuum. SED has had almost 40 years history. It explains quantum phenomena by methods of classical physics. The most comprehensive recent surveys on the subject are [4] and [5] (see also now classical works [6] and [7]).

The basic postulate of SED is in the presence of classical stochastic electromagnetic zero-point (i.e. existing at the absolute zero of temperature) radiation in the universe. It is homogeneous, isotropic, Lorentz-invariant, and has the intensity $\frac{1}{2}\hbar\omega$ per normal mode. According to SED, all quantum effects, by their essence, are the interaction effects of matter with the zero-point radiation. Indeed, there have been made many calculations in SED which prove complete connections with the results of quantum mechanics (QM) on van der Waals forces [8-10], Casimir's effect [11,12], correct behavior of a simple harmonic oscillator [13,14]. The atomic stability [6,15], the electronic diamagnetism of metals [16], the thermal effects of acceleration [17,18] have been explained. There are works concerning classical hydrogen atom [19] which, as yet, have not reached the desirable goal. But in our work, for the first time, have obtained the exited states of harmonic oscillator, and the result, we expect, opens a new avenue of attack on the hydrogen atom problem.

The electromagnetic theory of gravitation [20] was proposed by H. E. Puthoff in 1989 on the ground of one old idea of A. D. Sakharov. It has initiated the investigations of B. Haisch and coworkers [21-24] on inertia. Actually, very interesting question arises: can one change the physical, inertial mass of an object situated in a container by changing the density of electromagnetic field enclosed in it? The calculations made in works [21] and [23], even though are too complicated and overweighed by details that hampers to see the physical essence of the subject, give very promising indications that it is true. This result is obtained in the present time only by methods of SED.

The derivation of Planck's radiation law within classical physics was put forward long ago, mostly in the works of T. H. Boyer [25-29]. However, only one method [29] was accepted as a correct one. We believe that our method is also correct and it has a close connection with the Einstein-Hopf model, regarded by T. H. Boyer in work [25]. In one of our future papers we are hoping to tie up our main equation (22) with the equation (35) of the T. H. Boyer work, proving by it that the atomic energy discreteness is not the necessary condition for derivation of Planck's radiation law.

Following the proponents of SED, we take seriously the presence of homogeneous isotropic Lorentz-invariant electromagnetic field with the density radiation

$$\rho_{zp} = \frac{\hbar\omega^3}{2\pi^2 c^3} \tag{2}$$

in the universe at the absolute zero of temperature. Therefore, we expect the following form of Planck's radiation law

$$\rho(\omega, T) = \frac{\omega^2}{\pi^2 c^3}\left(\frac{\hbar\omega}{e^{\hbar\omega/kT} - 1} + \frac{\hbar\omega}{2}\right). \tag{3}$$

In order to achieve this goal, our paper is arranged as follows. In the derivation of Eq. (3) we lean upon Einstein's paper [3] of 1917. We shortly remind its content in Sec. II. It is the irony of fate that in this paper Einstein tried to show a reality of quanta. However, such an interpretation of absorption and emission processes of the two-level system is far from being necessary. In Sections III and IV we show that this system may be classical, in fact, by supplying the Einstein reasoning by an idea of the classical zero-point radiation. The proof of our main result is made in Sec. V. Most part of calculations is placed in Appendix.

II. EINSTEIN'S DERIVATION

Let us recall, at first, the Einstein fundamental work [3]. More contracted exposition of its ideas was presented in his with Ehrenfest work [30]. Allow us to repeat their considerations.

An atom can be found in two states Z and Z^* with energies ε and ε^*, respectively, $(\varepsilon^* > \varepsilon)$. Assume their difference be equal $\varepsilon^* - \varepsilon = \hbar\omega$. The atom is immersed in radiation of density ρ and is in thermal equilibrium with it. Therefore, the level populations obey the canonical distribution law

$$n \sim e^{-\varepsilon/kT}, n^* \sim e^{-\varepsilon^*/kT}, \frac{n^*}{n} \sim e^{-\hbar\omega/kT}. \tag{4}$$

For transitions of the atom from Z^*-state to Z-state during the time interval dt, we have the probability

$$dw^* = (a^* + b^*\rho)dt, \tag{5}$$

where a^* is the spontaneous emission coefficient, b^* is the induced emission coefficient. For $Z \to Z^*$ atom transitions, we have the probability

$$dw = b\rho dt. \tag{6}$$

Hence, in equilibrium, the following equation is true

$$n^*(a^* + b^*\rho) = nb\rho. \tag{7}$$

Using the Eq. (4) and the fact that at $T \to \infty$, $\rho \to \infty$ and, therefore, $b = b^*$, we go to

$$\rho = \frac{a^*/b^*}{e^{\hbar\omega/kT} - 1}. \tag{8}$$

The relation a^*/b^* can be obtained by comparison of Eqs. (8) and (1). Obviously, if Einstein had had the correct microscopic theory, he could have obtained the a^* and b^* coefficients immediately from it. In Sec. III we will propose such a microscopic theory which is alternative to Dirac's quantum theory of radiation. More specifically, it already exists under the name of "stochastic electrodynamics", and we extend it by using the Einstein work, that has been exposed in this section.

III. QUALITATIVE INTRODUCTION TO STOCHASTIC ELECTRODYNAMICS

Of course, the standpoint of SED is extremely attractive for those who have strong inner protest against the absence of space-time description in the atomic world, against the duality of physics. Let us consider qualitatively some phenomena regarded earlier as particular quantum.

Photoeffect. Conductivity electrons in metal interact with the zero-point radiation. Hence, they have the kinetic energy $\frac{1}{2}\hbar\omega$ on a frequency ω. Incoming radiation is involved in a resonance process with the zero-point vibrations on a frequency ω, and the emitted electrons have the energy proportional to $\hbar\omega$. Thus, the energy of photoelectrons depends only on the frequency, not on the intensity of incoming radiation.

Stability of an atom in the ground state. Why doesn't the radiating electron fall on the nucleus? Because it absorbs energy continuously from the zero-point radiation. The balance of energy between emitting energy of the electron rotating around the nucleus and its absorbing energy dictates the radius of the principal orbit.

The existence of stationary atomic states. Let an electron move around the nucleus on the circular orbit with some frequency ω_0. Such a motion may be replaced by harmonic vibrations of two, perpendicular to each other, linear oscillators of natural frequency ω_0 and phases shifted on $\pi/2$. The oscillators interact with the zero-point radiation within a short frequency interval near ω_0. It would be natural to suppose that the stationary states (the ground state, in any case) are formed under an energetic balance of the electron and the field. Louis de Broglie assigned the wavelength to any material particle by extending relations $E = \hbar\omega$ and $\vec{p} = \hbar\vec{k}$ that had been applicable to the light only before. From the standpoint of SED the physical meaning of this procedure becomes clear enough. The energy and momentum of a particle (two oscillators) are equalled to the mean energy and to the mean momentum of fluctuations of the interacting field. An atom emits narrow lines of a spectrum, because an electron may rotate stationary only on the orbits which include an integer number of de Broglie waves. On the other orbits the de Broglie wave would cease because of phases differences.

One and two-slit experiments. Let us imagine a wall with a slit and the zero-point radiation around it. The zero-point radiation can be essentially homogeneous and isotropic only far from matter. Near matter, the pattern of electromagnetic zero-point radiation is modified to reflect presence of the slit. If an electron or other particle goes through the slit, it will be influenced by the field fluctuations. The probability of its deflection, when it achieves the far screen, will be accounted by the de Broglie wave diffraction.

If a wall has two slits, the zero-point radiation pattern will reflect this fact. It is evident that every particular electron goes through only one of the two slits, and the diffraction picture arises as a consequence of the wavelike properties of the zero-point radiation.

IV. DERIVATION OF ENERGY SPECTRUM FOR HARMONIC OSCILLATOR

Let us look at the two-level model of Einstein from the new point of view. Now, a discreteness of stationary states can be understood qualitatively. In this Section we will show quantitatively, how the existence of the ground and exited states are explained in SED for Hertz's resonator, extending the properties of which, Einstein came to his model.

In SED, just as in quantum electrodynamics (QED), a harmonic oscillator is the most favorite object. At a time unit interval it emits the energy (see Appendix Eq. (A4)):

$$W_{em} = \frac{1}{3}\frac{e^2\omega_0^4 r_0^2}{c^3}. \tag{9}$$

At the same time it absorbs the energy (see Appendix Eq. (A19)):

$$W_{abs} = \frac{1}{3}\frac{e^2\hbar\omega_0^3}{mc^3} \tag{10}$$

from the zero-point radiation. Equating (9) and (10), we obtain

$$m\omega_0 r_0^2 = \hbar \tag{11}$$

which is the Bohr condition for atomic ground state. In the same manner formula (11) was first obtained in the paper [15], where H.E.Puthoff has corrected the arithmetic error 3/4 made by T.H.Boyer in the work [6]. His result, made for two oscillators, is valid also for one. In this case the oscillator full energy

$$\mathrm{E} = \frac{m\omega_0^2 r_0^2}{2} \tag{12}$$

is equal to

$$\varepsilon = \frac{1}{2}\hbar\omega_0. \tag{13}$$

The result can be derived by another way. It is more common and was already used in the early quantum theory. This is the introduction of the de Broglie wave. If we put one de Broglie wave on the principal orbit, then the relation

$$2\pi r_0 = \lambda_0 = \frac{2\pi\hbar}{mv_0} \tag{14}$$

will be true, or

$$r_0^2 = \frac{\hbar}{m\omega_0}. \tag{15}$$

By substituting Eq. (15) into (12), we obtain again the expression (13). It is important to remember that the appearance of the de Broglie wave (14), in our interpretation, connected with the idea that the particle moving with a constant velocity extracts from the zero-point radiation *a single frequency* corresponding to the equality of momenta for the particle (constituting of two oscillators) and the fluctuations of radiation.

The further extension on the exited states, by such simple way, however, does not give the required results. We would obtain the states with energies $\hbar\omega$, $\frac{3}{2}\hbar\omega$, $2\hbar\omega$ and so forth, that is inconsistent with the quantum formula. Thus, let us consider the atom-field interaction more closely. On this way, in the future, we can refuse from the idea of the de Broglie wave completely, especially, from the idea of moving on the orbit the de Broglie wave.

To understand what pattern the radiation has in vicinity of the atom, let us forget, for a while, the model of rotating around the nucleus electron and look at the equivalent (in a sense of the wave processes) picture of two perpendicular to each other oscillators. One is on the x axis, another on the y axis. Let the monochromatic wave $\xi_1 = A_0\cos\omega_0(t - \frac{x}{c})$ propogate to positive direction of x axis; c is a velocity of the wave. It extracts a single wave with the same frequency and phase from all vibrations of the zero-point radiation, propogating into the opposite direction, $\xi_2 = A_0\cos\omega_0(t + \frac{x}{c})$. As a result the standing wave forms

$$\xi = \xi_1 + \xi_2 = 2A_0\cos 2\pi\frac{x}{\lambda}\cos 2\pi\frac{t}{T}, \tag{16}$$

here $\lambda = cT$ and $T = 2\pi/\omega_0$. Its amplitude is equal to zero in knots, when

$$\cos 2\pi\frac{x}{\lambda} = 0$$

or

$$2\pi\frac{x}{\lambda} = \pm(2n+1)\frac{\pi}{2}, \qquad n = 1, 2, 3, \ldots .$$

The knots are located on the distances

$$x = \pm(2n+1)\frac{\lambda}{4} \tag{17}$$

from the origin.

If we return back to the model of the rotating electron and try to combine it with the resulting field pattern (16), we will go to the relation $x = r$. In this case, the electron occurs in the knots of the standing wave with every revolution, and, therefore, will be able to conserve the uniform motion on the orbit. Because the equality (13) is valid for the ground state (when there is only one standing halfwave inside the orbit), it would be natural to assume that the equation

$$\vec{p} = \frac{\hbar}{2}\vec{k} \tag{18}$$

is also valid for all states, since the momentum transferred from the zero-point radiation to the electron is the same in any state. Or, in a nonrelativistic approximation

$$mv\vec{e} = \frac{\hbar}{2}\frac{2\pi}{\lambda}\vec{n}.$$

Geometrical considerations require that $\vec{n}/\vec{e} = 2/\pi$, therefore

$$\lambda = \frac{2\hbar}{m\omega_0 r}. \tag{19}$$

Setting the formula into Eq. (17), at $x = r$, we go to

$$r^2 = (n + \frac{1}{2})\frac{\hbar}{m\omega_0}. \tag{20}$$

As we have two oscillators, then using relation (12) we arrive at

$$E_n = (n + \frac{1}{2})\hbar\omega_0 \tag{21}$$

which is in full accordance with the quantum result.

Thus, the idea of the de Broglie wave is appeared to be only a trick to account the interaction of multiply periodic atomic system with the zero-point radiation.

It is interesting now to make a historical remark. When, in 1926, Bohr and Schrödinger argued about the interpretation of wave mechanics, and the latter put the statement that quanta are unnecessary for understanding the quantum phenomena, Bohr referred to the derivation of Planck's radiation law in Einstein's paper [3] as to his basic counterargument (the details of the discussion were vividly described by Heisenberg in [31]). We will show here that such a reference has a fragile basis.

V. DERIVATION OF PLANCK'S RADIATION LAW WITHIN A FRAMEWORK OF SED

Our next step will be to obtain in a framework of SED such an equation that is a counterpart to Einstein's Eq. (7). For this purpose we must, first at all, realize, in a limiting clarity, what is the spontaneous emission. After the appearance of Einstein's work [3] and Dirac's theory [1], in a physical literature frantic and fruitless discussions began [2, 32-34] on the following questions. Have the spontaneous emission either quantum or classical origin? Does the zero-point radiation induce the spontaneous emission? Why does the zero-point radiation affect on matter twice as much as usual thermal radiation?

We believe that all those discussions occur because a mess of classical and quantum ideas. In stochastic electrodynamics the spontaneous emission has the unique and clear physical meaning: it is the emission of classical system in the absence of external fields. A classical Hertz's oscillator emits energy (9). This is the spontaneous emission. It presents both in the exited state and in the ground state. In the ground state it cancels out by absorption completely and the atom does not emit, whereas in the exited state the cancelation is in part only, and the atom emits. Naturally, the oscillator must absorb energy from the zero-point radiation in the both states.

It is convinient to exhibit all said in the tabulated form.

in Einstein's work		in SED
$(n^*a^*+n^*b^*\rho_T)\hbar\omega$	emitted energy	$n^*a^*+na+n^*b\rho_T$
$nb\rho_T\hbar\omega$	absorbed energy	$nb\rho_0+n^*b^*\rho_0+nb\rho_T$

In spite of the fact that the number of terms are doubled in SED, by comparison with Einstein's work, a striking symmetry is appeared now that was absent before. A physical meaning of the coefficients is altered in some details. Now a is the energy emitted by one atom and $b\rho_0$ is the energy absorbed from the zero-point radiation by one atom. The asterisk means, as usual, belonging to the exited state. In SED we do not speak anymore about a probability of transitions, since we have no quanta of energy $\hbar\omega$ now. Energy emits and absorbs continuously. However, taking into account the assumption that the transition times is much smaller than the living times in the ground and in the exited state, we can write down the energy balance equation as

$$n^*a^*+na+n^*b\rho_T=n^*b\rho_0+nb\rho_0+nb\rho_T. \tag{22}$$

Here, just like Einstein, we have used the limit $T\to\infty$, from which follows that $b^*=b$. It is not difficult to see from Eq. (22) that in SED, in contrast to Bohr's theory, the emission processes occur when an electron is in the exited state, not during a time of transitions between the states.

We consider now an another limit, $T=0$,

$$n^*a^*+na=n^*b\rho_0+nb\rho_0,$$

then

$$n^*(a^*+b\rho_0)=n(-a+b\rho_0),$$

and we find that

$$a=b\rho_0. \tag{23}$$

The result is already known. It means the existence of the full energy balance in the ground state at $T=0$. The equation (22) reduces to

$$n^*a^*+n^*b\rho_T=n^*b\rho_0+nb\rho_T. \tag{24}$$

Furthermore, since $\varepsilon^*=\frac{3}{2}\hbar\omega_0$, we have $a^*=3a$ (see Eq. (6A)), therefore

$$n^*(2\rho_0+\rho_T)=n\rho_T. \tag{25}$$

From this equation it is easy to obtain the radiation law for the function

$$\rho=\rho_0+\rho_T \tag{26}$$

which is a reduced form of Eq. (3). Indeed, we have

$$n^*(\rho_0+\rho)=n(\rho-\rho_0) \tag{27}$$

or

$$\rho=\rho_0\frac{e^{\hbar\omega/kT}+1}{e^{\hbar\omega/kT}-1}. \tag{28}$$

Finally, substituting the explicit expression of ρ_0 into (28), we obtain

$$\rho=\frac{\hbar\omega^3}{\pi^2c^3}\left(\frac{1}{e^{\hbar\omega/kT}-1}+\frac{1}{2}\right).$$

VI. CONCLUSION

In the textbooks on quantum electrodynamics (QED) and quantum optics one can often find the statement that the spontaneous transitions appear as a consequence of atomic electrons interaction with the zero-point vibrations of electromagnetic field, or, the photon vacuum, because there can not be self-emergent transitions in Nature, going on without any interactions. Needless to say, the latter is true. However, the cause of the Hertz resonator emission in classical electrodynamics is not the interaction with a vacuum, but the interaction with a self-field, a self-action. In this paper we adhere strictly to the classical concept. In the case a mess of ideas does not arise.

According to quantum theory, an atom can have only a discrete set of states $Z_1, Z_2, \ldots, Z_n$ with eigenenergies $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$. Einstein, studying the problem of derivation of Planck's radiation law, has proposed the model of an atom which have only two states: Z — the ground state, from where the spontaneous transitions are absent, and Z^* — the exited state from where occur as spontaneous as stimulated transitions. From a view point of stochastic electrodynamics, classical theory with classical electromagnetic zero-point radiation, the existence of the ground state Z is explainable by an energy balance between self-emission and absorption from the zero-point radiation. If we assume that the exited state is formed in the similar way (i.e. there are both absorption and emission for it), then, as we had shown, it is easy to derive Planck's radiation law within a framework of classical physics. Einstein's work on spontaneous and induced radiation would present itself in another light before us. After making a proper interpretation to the spontaneous emission, we obtained the coefficients a and b without the postulates of quantum mechanics and the second quantization procedure. It proves that the zero-point fluctuations are not the reason of spontaneous transitions. The connection between SED and QED is in using the same properties of a simple harmonic oscillator. However, their interpretations of electromagnetic radiation are diametrically opposite. And today we could incline to prefer SED because of its simplicity and duality absence.

After completing the calculations, exhibited in this paper, the author became aware that he is hardly the first who had hit upon a connection of the de Broglie wave with the zero-point radiation [35].

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APPENDIX

The motion equation of a harmonic oscillator have a form

$$m\ddot{x} + \omega_0^2 x = 0. \tag{A1}$$

Its solution

$$x = r_0 \cos \omega t \tag{A2}$$

shows that the acceleration

$$\ddot{x} = -\omega^2 r_0 \cos \omega t \tag{A3}$$

yields to the charged particle emission with the intensity

$$W_{em} = \frac{2e^2}{3c^3} \left\langle \ddot{x}^2 \right\rangle = \frac{e^2 \omega_0^4 r_0^2}{3c^3}, \tag{A4}$$

because

$$\left\langle \cos^2 \omega t \right\rangle = \frac{1}{2}.$$

Full oscillator energy $E = T + V$ is equal to

$$E = \frac{m\omega_0^2 x^2}{2} + \frac{mx^2}{2} = \frac{m\omega_0^2 r_0^2}{2}. \tag{A5}$$

Substituting r_0 of (5) into Eq. (4) we go to

$$W_{em} = \frac{2}{3} \frac{e^2 \omega_0^2 E}{mc^3}. \tag{A6}$$

The zero-point radiation can be written as a sum over plane waves

$$E_{zp}\left(r,t\right)=\sum_{\lambda=1}^{2}d^3k\hat{\varepsilon}\left(\vec{k},\lambda\right)h\left(\omega\right)\cos\left[\vec{k}\vec{r}-\omega t+\theta\left(\vec{k},\lambda\right)\right]$$

$$=\sum_{\lambda=1}^{2}\int d^3k\hat{\varepsilon}\left(\vec{k},\lambda\right)\frac{h\left(\omega\right)}{2}\left[a\left(\vec{k},\lambda\right)e^{i\left(\vec{k}\vec{r}-\omega t\right)}+a^*\left(\vec{k},\lambda\right)e^{-i\left(\vec{k}\vec{r}-\omega t\right)}\right], \tag{A7}$$

where $h(\omega)$ is the factor that puts a scale on the wave energy. For the zero-point radiation the relation

$$h_{zp}^2\left(\omega\right)=\frac{\hbar\omega}{2\pi^2} \tag{A8}$$

is valid,

$$a\left(\vec{k},\lambda\right)=e^{i\theta\left(\vec{k},\lambda\right)},\quad a^*\left(\vec{k},\lambda\right)=e^{-i\theta\left(\vec{k},\lambda\right)},$$

$\theta\left(\vec{k},\lambda\right)$ — a random phase distributed uniformely on the interval $[0,2\pi]$, independently distributed for each $\vec{k}$ and λ.

To obtain the energy absorbed by the linear harmonic oscillator from the field we must solve the linear stochastic equation

$$m\frac{d^2x}{dt^2}+m\omega_0^2x-\frac{2}{3}\frac{e^2}{mc^3}\frac{d^3x}{dt^3}=eE\left(0,t\right) \tag{A9}$$

Its solution can be found by Fourier method

$$x\left(t\right)=\frac{e}{m}\sum_{\lambda=1}^{2}\int d^3k\hat{\varepsilon}_x\left(\vec{k},\lambda\right)\frac{h\left(\omega\right)}{2}\left[\frac{a\left(\vec{k},\lambda\right)}{C\left(\omega\right)}e^{i\left(\vec{k}\vec{r}-\omega t\right)}+\frac{a^*\left(\vec{k},\lambda\right)}{C^*\left(\omega\right)}e^{-i\left(\vec{k}\vec{r}-\omega t\right)}\right], \tag{A10}$$

where

$$C\left(\omega\right)=\omega_0^2-\omega^2-i\Gamma\omega^3\quad\text{and}\quad\Gamma=\frac{2}{3}\frac{e^2}{mc^3},$$

$\hat{\varepsilon}\left(\vec{k},\lambda\right)$ in Eqs. (A7) and (A9) is a polarization vector. It obeys the relations

$$\hat{\varepsilon}\left(\vec{k}_1,\lambda\right)\hat{\varepsilon}\left(\vec{k}_2,\lambda\right)=\delta_{\lambda_1\lambda_2},\quad\vec{k}\hat{\varepsilon}\left(\vec{k},\lambda\right)=0, \tag{A11}$$

from those one can obtain the very useful identities for summing over the two possible polarizations:

$$\sum_{\lambda=1}^{2}\varepsilon_i\left(\vec{k},\lambda\right)\varepsilon_j\left(\vec{k},\lambda\right)=\delta_{ij}-\frac{k_ik_j}{k^2}. \tag{A12}$$

Therefore, the absorption energy is

$$W=\int_0^\tau dt\dot{x}\left(eE_{zp}\right)=\frac{e^2}{m}\sum_{\lambda_1=1}^{2}\sum_{\lambda_2=1}^{2}\int d^3k_1\int d^3k_2\hat{\varepsilon}_{x1}\hat{\varepsilon}_{x2}(-i\omega)\frac{h_1\left(\omega_1\right)}{2}\frac{h_2\left(\omega_2\right)}{2}$$

$$\times\left[\frac{a_1}{C_1}e^{i\left(\vec{k}_1\vec{r}_1-\omega_1t\right)}-\frac{a_1^*}{C_1^*}e^{-i\left(\vec{k}_1\vec{r}_1-\omega_1t\right)}\right]\left[a_2e^{i\left(\vec{k}_2\vec{r}_2-\omega_2t\right)}+a_2^*e^{-i\left(\vec{k}_2\vec{r}_2-\omega_2t\right)}\right]. \tag{A13}$$

We averaging now on the random phases. The phasis $\theta\left(\vec{k},x\right)$ have a normal distribution, i.e. its second moments are equal to

$$\left\langle\varepsilon^{i\theta\left(\vec{k}_1,\lambda_1\right)}e^{-i\theta\left(\vec{k}_2,\lambda_2\right)}\right\rangle=\left\langle a_1a_2^*\right\rangle=\delta_{\lambda_1\lambda_2}\delta^3\left(\vec{k}_1-\vec{k}_2\right)$$

$$\langle a_1 a_2 \rangle = 0, \quad \langle a_1^* a_2^* \rangle = 0. \tag{A14}$$

From it follows that

$$\left\langle \left[\frac{a_1}{C_1} e^{-i\omega_1 t} - \frac{a_1^*}{C_1^*} e^{i\omega_1 t} \right] \left[a_2 e^{-i\omega_2 t} + a_2^* e^{i\omega_2 t} \right] \right\rangle$$

$$= \left(\frac{1}{C_1} - \frac{1}{C_1^*} \right) \delta_{\lambda_1 \lambda_2} \delta^3 \left(\vec{k}_1 - \vec{k}_2 \right) = \frac{2i\Gamma\omega_1^3}{\left| C_1 \left(\omega_1 \right) \right|^2} \delta_{\lambda_1 \lambda_2} \delta^3 \left(\vec{k}_1 - \vec{k}_2 \right). \tag{A15}$$

Thus, averaging, we go to

$$W_{abs} = \frac{e^2}{m} \sum_{\lambda=1}^{2} \int d^3 k \hat{\varepsilon}_{1x} \hat{\varepsilon}_{2x} \frac{h^2 \left(\omega \right)}{2} \frac{\Gamma \omega^4}{\left| C \left(\omega \right) \right|^2}. \tag{A16}$$

Taking into account (A12), the angle integration yields to

$$\sum_{\lambda=1}^{2} \int d^3 k \hat{\varepsilon}_{1x} \hat{\varepsilon}_{2x} = \int d\Omega \left(1 - \frac{k_x^2}{k^2} \right) \int dk k^2 = \frac{8\pi}{3} \int d\omega \frac{\omega^2}{c^3}. \tag{A17}$$

The function $1/\left| C \left(\omega \right) \right|^2$ have a narrow maximum near $\omega = \omega_0$ in the limit $e^2/m \to 0$. Therefore, we can invoke the standard resonance approximation. We change ω on ω_0 for it everywhere except the difference $\omega - \omega_0$ and take the integration limits go to $-\infty$. Then we take the analogous inpact of $\omega + \omega_0$ into account. So, prove that

$$\lim_{e^2/m \to 0} \frac{e^2}{m} \frac{1}{\left| C \left(\omega \right) \right|^2} = \frac{3\pi c^3}{4\omega_0^4} \left[\delta \left(\omega - \omega_0 \right) + \delta \left(\omega + \omega_0 \right) \right]. \tag{A18}$$

Indeed

$$\frac{e^2}{m} \int_{-\infty}^{\infty} d\omega \frac{1}{\left(\omega_0^2 - \omega^2 \right)^2 + \Gamma^2 \omega^6} = \frac{3}{2} \Gamma c^3 \int_{-\infty}^{\infty} \left(\omega - \omega_0 \right) \frac{1}{\left(\omega - \omega_0 \right)^2 4\omega_0^2 + \Gamma^2 \omega^6}$$

$$= \frac{3}{4} \frac{c^3}{\omega_0^4} \int_{-\infty}^{\infty} d \left(\omega - \omega_0 \right) \frac{\Gamma \omega_0^2 / 2}{\left(\omega - \omega_0 \right)^2 + \left(\Gamma \omega_0^2 / 2 \right)^2} = \frac{3\pi c^3}{4\omega_0^4}.$$

Substituting (A18) and (A17) into Eq. (A16) we obtain

$$W_{abs} = \frac{1}{3} \frac{e^2 \hbar \omega_0^3}{m c^3}. \tag{A19}$$

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