

PROBABILISTIC MODELING OF IEEE 802.11 DISTRIBUTED COORDINATION FUNCTIONS

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Abstract. We introduce and analyze a new Markov model of the IEEE 802.11 Distributed Coordination Function (DCF) for wireless networks. The new model is derived from a detailed DCF description where transition probabilities are determined by precise estimates of collision probabilities based on network topology and node states. For steady state calculations, we approximate joint probabilities from marginal probabilities using product approximations. To assess the quality of the model, we compare detailed equilibrium node states with results from realistic simulations of wireless networks. We find very close correspondence between the model and the simulations in a variety of representative network topologies.

Key words. Wireless networks. Carrier sense multiple access. Hidden terminal problem. Stochastic modeling. Markov process.

1. Introduction. Wireless local area networks (WLANs) play a critical role in modern society. Efficient wireless communications in WLANs require independent nodes to coordinate transmissions and receptions of data packets over shared spectrum so as to mitigate collision. The coordination of such communications is accomplished through a Media Access Control (MAC) protocol, a set of rules that defines when and how to transmit data from one node to another. A number of MAC protocols, such as Aloha, CSMA/CD, CSMA/CA, etc., have proven to be effective both generally and in special circumstances. Most studies of MAC protocols are experimental, using either simulated or real network traffic to directly compare performance. MAC protocols themselves are complex and have resisted efforts to create consistent mathematical models that can reproduce detailed network performance timelines. The purpose of this paper is to derive and validate a predictive mathematical model for the protocol of Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) with binary exponential backoff, which forms the IEEE 802.11 Distributed Coordination Function (DCF). This detailed model is valuable in and of itself to understand how protocol parameters affect performance, and it is a natural building block for studying the performance of upper layer protocols that operate on top of IEEE 802.11 DCF.

All modeling efforts require that we make assumptions, but the complexity of IEEE 802.11 DCF under general network topologies requires that investigators make strong assumptions about potential collisions between nodes. The seminal work of Bianchi* [2] on fully connected single-hop saturated networks begins by assuming that the collision probability on each node is constant and independent of network topology and node states. As we shall see, this is clearly not the case in general and a Markov model based on this assumption cannot hope to model DCF. Numerous works have extended this approach to try to capture missing elements of DCF in a way that is both simpler than a full simulation and valuable as a predictive instrument for studying protocols.

There have been many extensions of Bianchi's work to model single hop transmissions where there are no hidden terminals. For instance, the basic model in [2] is adapted to the assumption of freezing backoff counter due to busy medium in [20],

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which is further polished and strengthened in [5] by introducing the dependence of consecutive slots, and also in [15] by redefining the discrete time scale given in [2]. Wu et al. [17] augment Bianchi's model by assuming finite retransmission attempts, which is also adopted in [11]. In [9], the authors propose another model extension for saturation throughput analysis by considering the effect of non-ideal channel conditions, while [4] presents a similar model for unsaturated cases. In addition to throughput analysis, a comprehensive analysis of delay performance is conducted by [19], where the authors modify node state transitions in [2] with signal transfer functions to characterize the probability distribution of MAC layer service time for WLANs in both saturated and non-saturated traffic situations. Others, for instance, [3], model the statistical behaviors of the Head-of-Line packet instead of nodes and perform unified study on both throughput and delay. A great deal of effort has also been made to model and analyze IEEE 802.11 DCF in the presence of *hidden terminals*, where some prospective senders are not within the sensing range of others. For instance, to model the existence of hidden terminals, [18] employs fix-sized time slots and details the state transition to formalize the channel status considering the interaction between physical and virtual carrier sensing in a discrete time Markov system. However, the authors follow the same assumption that collision probability is constant regardless of retransmission history. In contrast, [12] uses the joint backoff stage of the two stations that are hidden from each other as state in order to account for the interactions between them. Unfortunately, these models are limited to infrastructure scenarios using access points and depend on the network topology.

There has also been some effort to model and analyze multihop transmissions. Guillemin et al. propose a model for CSMA in multi-hop settings based on a random walk on lattice [8]. The underlying assumption in this model is that node behavior is synchronized so that the problem can be parametrized by the queue size on each node. However, nodes in a network undergo random exponential backoffs when there is channel contention so these assumptions are not valid. Efficiency requires that network protocols operate asynchronously with each node acting opportunistically to empty its queue or respond to other node's requests for it to accept data. Other investigators rely upon statistical descriptions of transmission nodes combining with channel behaviors to develop a model. Garetto et al. [7] model CSMA for various two contenting flow topologies to study the unfairness problem and further supplement it to predict throughput in arbitrary topology [6]. The authors implement a decoupling model for each individual node with an embedded discrete time renewal process based on the basic assumption that the current channel state is independent of previous state. However, [16] points out that the above assumption is unrealistic with the presence of hidden terminals and the consequent de-synchronization of the nodes. Instead, Tsertou and Laurenson describe the channel by modeling a first-order dependence between consecutive channel state and adjusted Bianchi's original model using fixed-sized time slot and contention window [16]. Mustapha et al. [13] apply a discrete-time modeling approach that combines a topology model, a channel model and a simplified node state model with only three states for analyzing throughput of multi-hop ad hoc networks. In the similar vein but different methodology, Shi et al. [14] extend Bianchi's assumptions on backoff-stage dependence of collision probabilities, non-saturate queues, *etc.*, and develop a detailed continuous-time model of CSMA networks where the correlations of nodes are described through a companion channel model of joint backoff states. Unfortunately, the true statistical description that they are attempting to capture depends upon network topology and queue sizes.

A more useful model will generate the statistical description given network parameters and topology. This is precisely what we set out to do.

The remainder of the paper is organized as follows. In Section 2 we review the IEEE 802.11 DCF and introduce assumptions used in this paper. In Section 3 we formulate and discuss the model in details. In Section 4 we apply the model in three basic network configurations and examine the results. Section 5 concludes the paper.

2. Review of IEEE 802.11 Distributed Coordination Function (DCF).

In computer networks, a channel access method allows multiple network devices or nodes to transmit data packets over the same physical transmission medium (*i.e.*, copper wire, air) and share its capacity. The simplest design is called random access. With this scheme, all network devices may transmit whenever they want without considering others' conditions. However, random access leads to packet collisions when two or more devices transmit at the same time. The resulting mingling of signals will corrupt all data packets involved, and they have to be retransmitted at a later time. Hence packet collisions cause lost of information and waste channel bandwidth.

To avoid packet collisions, MAC (Medium Access Control) layer is introduced in the OSI (Open Systems Interconnection) model of computer networks. For Wireless Local Area Networks (WLANs), IEEE 802.11, an international standard, provides a detailed MAC layer specification, in which the fundamental mechanism for network devices to access the medium without any centralized control is called Distributed Coordination Function (DCF).

IEEE 802.11 DCF is a contention based random access scheme, implementing the Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) protocols. Carrier sense is the ability of a network device to determine if the transmission medium is idle. In general, wireless carrier sense is composed of two distinct techniques: 1) CCA (Clear Channel Assessment), which is performed through physical evaluation of the signal energy on the station's radio interface, and 2) NAV (Network Allocation Vector), a virtual carrier sense mechanism, which is a data segment that indicates the amount of time required for the transmission immediately following the current packet that contains the NAV.

The collision avoidance feature of CSMA/CA requires that a station transmits only when the channel is sensed to be idle. Unfortunately, collisions may still occur when two stations sense an idle channel at the same instant and subsequently transmit. To reduce the chance of repeated collisions of retransmitted packets, CSMA/CA protocols apply a binary exponential back-off (BEB) algorithm, by which every station selects a random back-off time before each retransmission. The name binary exponential originates from the fact that at each retransmission attempt, the longest possible back-off time doubles. Hence it is less likely for two stations to retransmit at the same moment.

DCF specifies two approaches for packet transmission. The default scheme is called Basic Access mechanism. Provided the channel is sensed idle, a sender transmits the data packet after a random back-off time interval. However, the data transmission is still vulnerable to packet collision due to the well-known 'hidden terminal problem', or 'hidden node problem', in wireless networking. A node x is called hidden node of node y if x is outside the sensing range of y . A collision may still occur at the receiver node in the presence of other concurrent transmitters who are hidden from the sender. To address this issue, DCF provides an optional technique, known as a Request-to-Send/Clear-to-Send (RTS/CTS) mechanism. Instead of broadcasting a long and

valuable data packet directly, a sender/receiver pair operated in RTS/CTS mode reserves the channel by handshaking via RTS and CTS short packets. In particular, since NAV is transmitted along both RTS and CTS packets, a neighboring node (two nodes are neighbors if they can sense each other) overhearing either RTS or CTS packets will defer its own transmission long enough for the addressed communication to finish. Although collisions may still occur between RTS packets, RTS/CTS scheme can reduce the chance of collisions between data packets as long as RTS packets are significantly shorter than the data packets. A more comprehensive description of 802.11 DCF can be found in the standard [10].

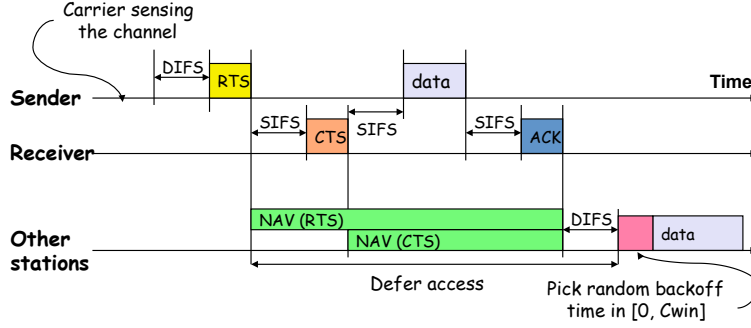
2.1. Preliminaries. In a wireless ad-hoc networks, not all nodes are necessarily within the sensing range of each other, creating hidden terminals. To address this, the 802.11 DCF adopts an RTS/CTS/DATA/ACK four way handshaking scheme, shown in Figure 2.1 and described as follows:

A sender, x , will constantly monitor the channel activity by carrier sensing. x will not attempt to transmit RTS unless the channel is sensed idle for a period of time called the Distributed InterFrame Space (DIFS). On the other hand, x accesses the channel following the BEB algorithm: at each transmission of RTS packet, the back-off counter is uniformly chosen between 0 and the current Contention Window size. Here the contention window determine the longest possible back-off time a node can choose. The back-off counter is decremented to zero unless x senses a busy channel. This will suspend the counter until the channel is sensed idle again after a DIFS. Broadcasting of RTS starts when the timer reaches zero. If the receiver y successfully captures the RTS packet, it will reply to x by broadcasting a CTS packet after a short period of time interval called the Short InterFrame Space (SIFS). The contention window will be reset to an initial value only when x correctly receives the CTS from y . However, CTS reception can be disrupted by a transmission from another node anywhere within range of x . If the CTS is not received, the contention window doubles, and x retransmits RTS according to the new contention window after waiting a specified time period of T_{out} , called CTS timeout. Thus, at each failed RTS/CTS handshaking attempt, w is doubled up to a maximum value. Then the window size remains at that threshold until it is reset. If the maximum transmission failure limit (Retry Limit) is reached, x will discard the data packet and the window size returns to an initial value. The RTS/CTS exchange improves the chances that two nodes will be able to reserve the channel and exchange data after another SIFS in a complex environment. At the end of the successful reception of the data packet from x , y immediately responds with a positive acknowledgement (ACK) after a SIFS. The RTS/CTS/DATA/ACK four way handshaking is complete whenever an ACK is correctly received by x . If not, x will reschedule the data packet transmission.

2.2. Assumptions. To systematically develop a predictive model of 802.11 DCF, we introduce the following notation and assumptions.

Network : We assume ideal channel conditions. This means there will be no noise and the propagation delay is ignored. Each node operates under homogeneous configurations. All nodes have the same sensing range R_s and transmission range R , where $R < R_s$.

Timescale : There exists a constant timescale of least duration, σ , which is equal to the time needed at any node to detect the transmission of a packet from any other node [2]. Because σ is very small, we shall assume that any node can immediately detect the transmission of a packet from any other node inside its sensing range R_s . All the time parameters in the model, i.e, T_{NAV} , T_{busy} ,

FIG. 2.1. *RTS/CTS Access mechanism*

etc, are assumed to be multiples σ .

MAC protocol : For simplification of modeling, we use a modified version of IEEE 802.11 DCF implementing the RTS/CTS mechanism: DIFS is set to be one time unit and SIFS is assumed to be negligible. RTS and CTS packets have the same size, hence their transmission delays, denoted as T_{RTS} and T_{CTS} , are equal. The protocol still adopt the BEB algorithm and the back-off counter is chosen uniformly between 1 and the contention window size. Furthermore, we set the retry limit of the RTS is the number of times that a contention window is allowed to double. Hence if the contention window achieves its threshold, we assume the data packet being sent is dropped.

Data : There is no retransmission of data packets. A data frame is dropped either because there is a collision at the receiver or retry limit of RTS reached. Also, we assume the acknowledgment packet (ACK) following a successful data packet transmission has fixed size (2 slots) and always succeeds. Hence the transmission time T_{DATA} includes the sending/receiving period of data plus ACK.

Carrier sense :

1. **CAA - Clear Channel Assessment**: Since the signals from different neighboring nodes can overlap, the busy period a node physically senses in general will not be constant and will most likely depend on the number of active neighbors.

2. **Network Allocation Vector (NAV)**: It is included in both RTS and CTS packets indicating how long the channel will be occupied. In the standard, the value of NAV is $T_{NAV_r} = T_{CTS} + T_{DATA} + T_{ACK}$ if contained in RTS, or $T_{NAV_c} = T_{DATA} + T_{ACK}$ if contained in CTS. When a node freezes through NAV, it will ignore arriving packets until the NAV period ends. On the other hand, a node will update the freezing period of NAV with the information overheard from either a CTS or RTS packet if a new NAV value is greater than the current NAV value. For simplicity, we employ fixed-size NAV period, and assume a node freezes at the end of NAV if the channel is busy.

CTS Timeout : Within the period of CTS timeout, $T_{out} = T_{CTS} + \sigma$, any incoming packets arrived from the physical medium, valid or not, will be ignored. At the end of CTS timeout, we assume a node freezes if the channel is occupied,

and resumes back-off/idle if otherwise.

3. Modeling the Distributed Coordination Function. In a single hop network (i.e. a fully connected graph), every node can sense each other and consequently experiences the same level of contention. However, in a WLAN, the competition among stations for channel access can be biased: a station with more nodes hidden from it may back off longer or encounter more packet collisions than the others which have fewer undetectable contenders. As a result, the performance of the DCF will vary for each node in the network.

3.1. Modeling of node states. We model each node x in the network as a multi-dimensional stochastic process, denoted by

$$\mathcal{H}_x(t) := (s_x(t), b_x(t), a_x(t), v_x(t), \vec{Q}_x(t))$$

with the discrete-time Markov chain, in which the uniform integer time scale, σ , is adopted: t_n and t_{n+1} correspond to the beginning of two consecutive slots. ($t_n := n\sigma$.)

$s_x(t)$: **Back-off stage** $(0, 1, 2, \dots, m)$ of node x at time t , where m is the maximum back-off stage. By the exponential back-off scheme described in section 2, $s_x(t) = i$ implies that the contention window size at time $t = w_i = 2^i w$. w is the initial window size.

$b_x(t)$: **Back-off counter** of node x at time t . At the beginning of any back-off stage i , the counter will randomly choose a value among $(1, \dots, w_i)$ based on the assumptions of protocol. Then for each following time step t_n , the back-off counter either decrements or freezes.

$a_x(t)$: **Action/Status** of node x at time t :

$$\left\{ \begin{array}{ll} I, & x \text{ is idle} \\ B, & x \text{ is back-off counting} \\ U, & x \text{ is waiting due to unidentified signals sensed} \\ R_{\vec{z}}, & x \text{ is sending RTS to } z \\ R_{\overleftarrow{z}}, & x \text{ is receiving an uncorrupted RTS from } z \\ R_{\overline{z}}, & x \text{ is overhearing an uncorrupted RTS from } z \\ C_{\vec{z}}, & x \text{ is sending a CTS to } z \\ C_{\overleftarrow{z}}, & x \text{ is receiving an uncorrupted CTS from } z \\ C_{\overline{z}}, & x \text{ is overhearing an uncorrupted CTS from } z \\ A_{\vec{z}}, & x \text{ is sending DATA to } z \\ A_{\overleftarrow{z}}, & x \text{ is receiving an uncorrupted DATA from } z \\ D_z, & x \text{ is waiting due to NAV triggered by RTS/CTS from } z \\ W, & x \text{ is waiting for a responding CTS} \end{array} \right.$$

Here $z \in N_x$ where N_x denotes the set of neighboring nodes of x . Remark on W : $a_x(t) = W$ implies that either the previous RTS packet has been dropped at the receiver so there will be no responding CTS, or the CTS has become unidentified due to collisions at x .

The following table characterizes the actions of x by the behaviors of x 's antenna, the channel conditions, and the status of x 's queue. For instance, if $a_x(t) = I$, x has nothing to send in the buffer and there is no signal in the medium. Hence its antenna keeps quiet, the channel is sensed free, and its queue is empty. If $a_x(t) = D_z$, x will be frozen because of NAV, which means the antenna is quiet, the channel can be either busy or free depending on the other nodes' actions, and x 's queue can be either empty or occupied. The other actions can be described similarly as above.

TABLE 3.1

$a_x(t)$	Antenna (Quiet/Sending)	Channel (Busy/Free)	Queue (Empty/Occupied)
I	Quiet	Free	Empty
B	Quiet	Free	Occupied
D_z	Quiet	Busy/Free	Empty/Occupied
W	Quiet	Busy/Free	Occupied
$U/R_{\bar{z}}/A_{\bar{z}}/R_{\bar{z}}/C_{\bar{z}}$	Quiet	Busy	Empty/Occupied
$C_{\bar{z}}$	Quiet	Busy	Occupied
$C_{\bar{z}}$	Sending	Busy	Empty/Occupied
$R_{\bar{z}}/A_{\bar{z}}$	Sending	Busy	Occupied

$v_x(t)$: **Virtual timer** associated with $a_x(t)$. It will start ($t = t_0$) at one of the following values and decrement to 0 at the beginning of each time slot. Otherwise the timer stays at 0.

$$v_x(t_0) = \begin{cases} t_{RTS}, & \text{if } a_x(t) \in \{R_{\bar{z}}, R_{\bar{z}}, R_{\bar{z}}\} \\ t_{out}, & \text{if } a_x(t) = W \\ t_{CTS}, & \text{if } a_x(t) \in \{C_{\bar{z}}, C_{\bar{z}}, C_{\bar{z}}\} \\ t_{DATA}, & \text{if } a_x(t) \in \{A_{\bar{z}}, A_{\bar{z}}\} \\ t_{NAV_r}/t_{NAV_c}, & \text{if } a_x(t) = D_z \end{cases}$$

where t_0 is the initial start time. Here, $t_{RTS} := \lceil T_{RTS}/\sigma \rceil - 1$ (similarly defined for other time parameters).

$\vec{Q}_x(t)$: **Queue status vector** of node x at time t . Here, $\vec{Q}_x(t) = \langle Y, L \rangle$, where Y is the receiver of the Head of Line (HoL) packet that being sent by node x . The second entry, L , represents the length of the queue (including the HoL packet) at node x . If there is no packet in the queue, we say $\vec{Q}_x(t) = \vec{0} = \langle \emptyset, 0 \rangle$. Furthermore, we say node x is on l -th layer at time t if $L = l$. Whenever the node x successfully receives a packet during the back-off counting, L is increased by 1. If node x finishes transmitting a packet (either success or failure), L is dropped by 1, and Y will be updated based on the receiver of the next packet in the queue.

3.2. Modeling of States Transitions.

3.2.1. x as a listener/receiver. A node x is a listener when it is in back-off counting (with occupied queue) or idle (with empty queue). It consistently monitors the channel by both physical and virtual carrier sense. Upon the successful reception of a RTS packet, x becomes a receiver by completing the RTS/CTS/DATA/ACK handshake. Diagram 3.1 and 3.2 represent the states' transitions for x based on the description of 802.11 DCF and the assumptions in section 2. Both diagrams share a similar structure, called **Carrier Sense Block (CSB)**, which repeatedly appears in our model for every pair of back-off stage and back-off counter.

For Figure 3.1, suppose that at time step t_n where $n = 0, 1, 2, \dots$, node x is at the k th step of the i th backoff stage for receiver y with l packets in the queue. At the next time step there are five possible state transitions on node x , associated with the

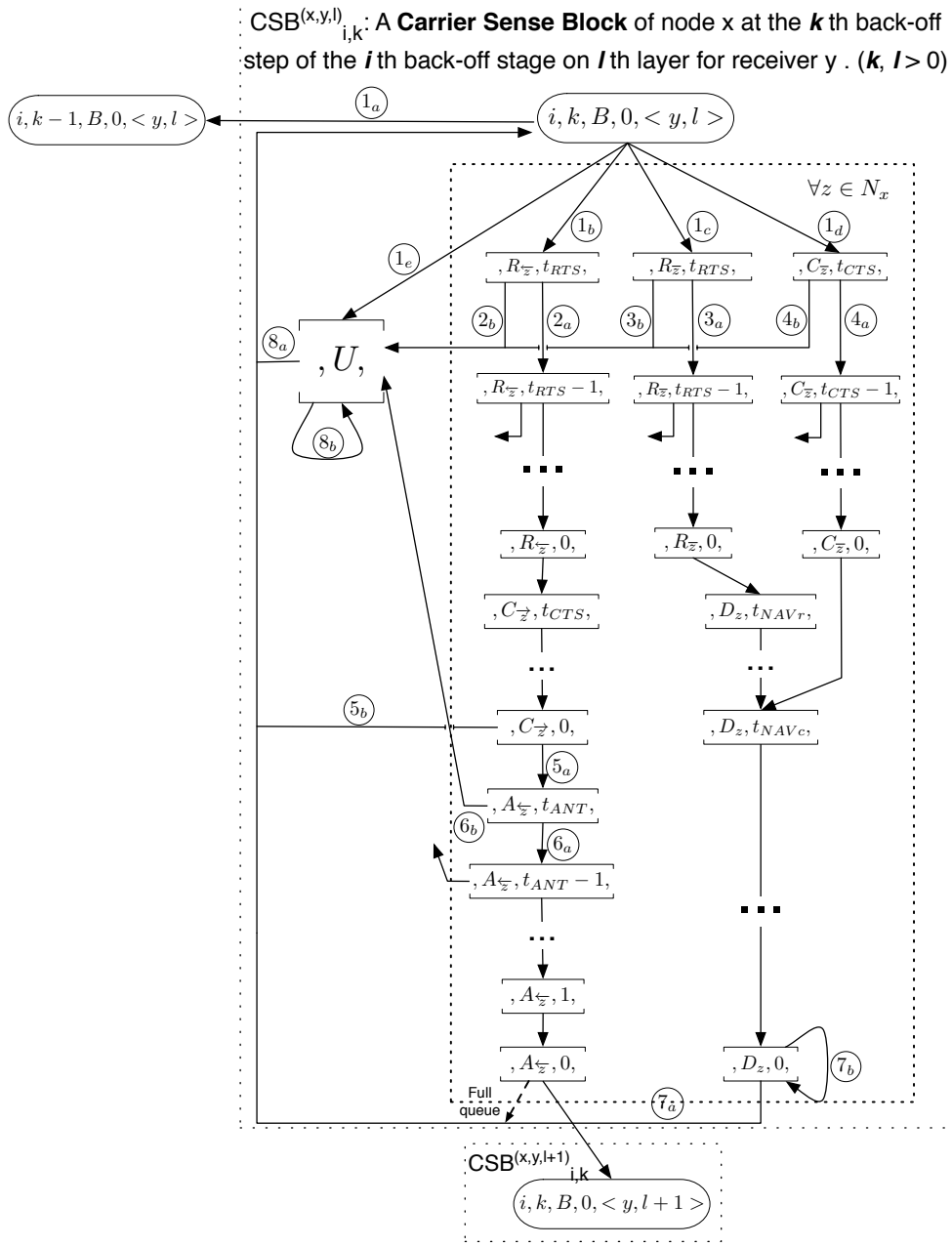


FIG. 3.1. Carrier Sense Block at upper layer

following probabilities respectively:

$$\textcircled{1}_a = \text{Prob}\{(i, k-1, B, 0, \langle y, l \rangle)_{n+1} | (i, k, B, 0, \langle y, l \rangle)_n\} \quad (3.2.1)$$

$$\textcircled{1}_b = \text{Prob}\{(i, k, R_{\bar{z}}, t_{RTS}, \langle y, l \rangle)_{n+1} | (i, k, B, 0, \langle y, l \rangle)_n\} \quad (3.2.2)$$

$$\textcircled{1}_c = \text{Prob}\{(i, k, R_{\bar{z}}, t_{RTS}, \langle y, l \rangle)_{n+1} | (i, k, B, 0, \langle y, l \rangle)_n\} \quad (3.2.3)$$

$$\textcircled{1}_d = \text{Prob}\{(i, k, C_{\bar{z}}, t_{CTS}, \langle y, l \rangle)_{n+1} | (i, k, B, 0, \langle y, l \rangle)_n\} \quad (3.2.4)$$

$$\textcircled{1}_e = \text{Prob}\{(i, k, U, 0, \langle y, l \rangle)_{n+1} | (i, k, B, 0, \langle y, l \rangle)_n\} \quad (3.2.5)$$

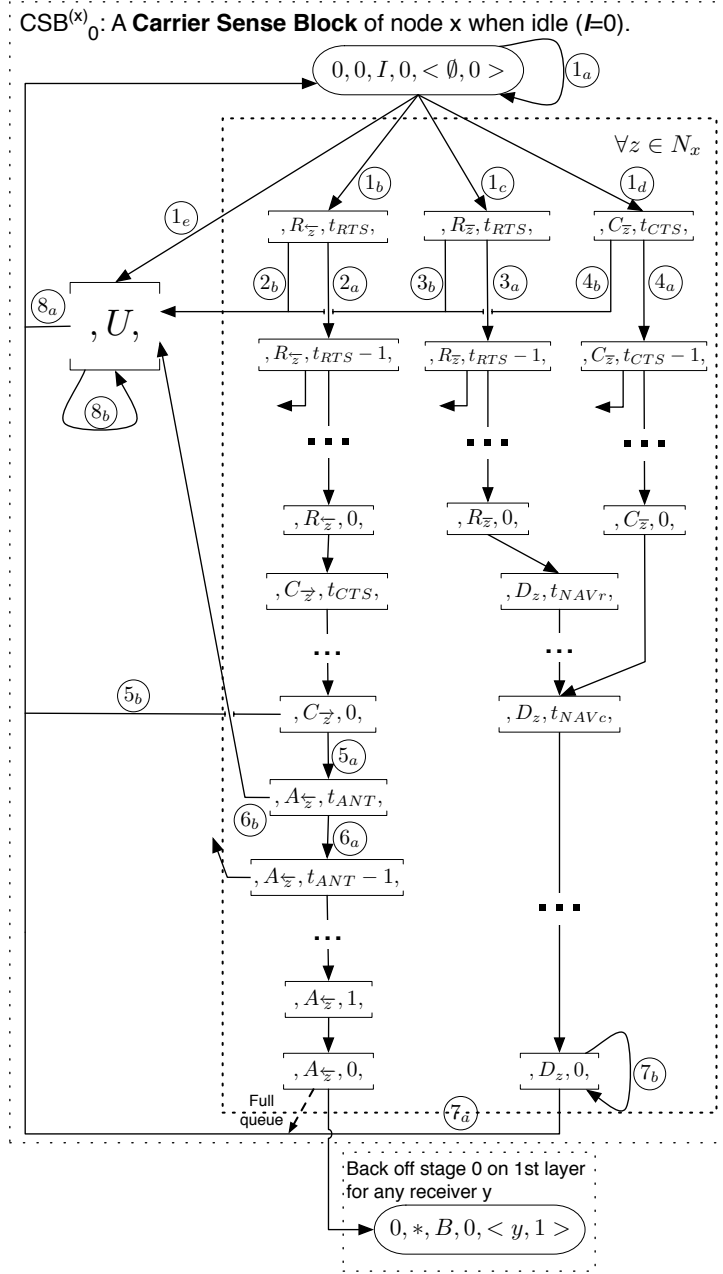


FIG. 3.2. Carrier Sense Block at base layer

Here we adopt the short notion:

$$\begin{aligned}
 & P\{(z_1, z_2, z_3, z_4, z_5)_{n+1} | (z'_1, z'_2, z'_3, z'_4, z'_5)_n\} \\
 & = P\{\mathcal{H}_x(t_{n+1}) = (z_1, z_2, z_3, z_4, z_5) | \mathcal{H}_x(t_n) = (z'_1, z'_2, z'_3, z'_4, z'_5)\}
 \end{aligned}$$

Transition (1_a) occurs when x detects a quiet channel, that is, currently no neighbors

of x are broadcasting or beginning to transmit any signals. As a result, the back off counter decrements by 1. Transition $\textcircled{1}_b$ accounts for the fact that, one of x 's neighbor, z , begins to send a RTS packet for x while others neighboring nodes stay quiet. In this case, node x takes the first step of receiving the RTS packet, so that $a_x(t_{n+1}) = R_{\overleftarrow{z}}, v_x(t_{n+1}) = t_{RTS}$. Transition $\textcircled{1}_c$ or $\textcircled{1}_d$ takes place provided that only z starts to broadcast a RTS packet or a CTS packet not for x . In those scenarios, $a_x(t_{n+1}) = R_{\overleftarrow{z}}, v_x(t_{n+1}) = t_{RTS}$ or $a_x(t_{n+1}) = C_{\overrightarrow{z}}, v_x(t_{n+1}) = t_{CTS}$. The transition $\textcircled{1}_e$, $a_x(t_{n+1}) = U$, happens when x detects disordered signals in the channel, caused by either corrupted or partial packets from x 's neighbors.

During the receiving (overhearing) of RTS or CTS from a neighbor z , node x may observe packet collisions when the hidden nodes of z initiate transmissions to x . Thus, given the j -th step of receiving ($v_x(t_n) = j$), we have the following probabilities associated with the transitions $\textcircled{2}_b$, $\textcircled{3}_b$ and $\textcircled{4}_b$:

$$\textcircled{2}_b = \text{Prob}\{(i, k, U, 0, \langle y, l \rangle)_{n+1} | (i, k, R_{\overleftarrow{z}}, j, \langle y, l \rangle)_n\} \quad (3.2.6)$$

$$\textcircled{3}_b = \text{Prob}\{(i, k, U, 0, \langle y, l \rangle)_{n+1} | (i, k, R_{\overrightarrow{z}}, j, \langle y, l \rangle)_n\} \quad (3.2.7)$$

$$\textcircled{4}_b = \text{Prob}\{(i, k, U, 0, \langle y, l \rangle)_{n+1} | (i, k, C_{\overrightarrow{z}}, j, \langle y, l \rangle)_n\} \quad (3.2.8)$$

Otherwise, x keeps receiving and the virtual counter $v_x(t)$ decreases by 1 at each time step with the probabilities:

$$\textcircled{2}_a = \text{Prob}\{(i, k, R_{\overleftarrow{z}}, j-1, 0, \langle y, l \rangle)_{n+1} | (i, k, R_{\overleftarrow{z}}, j, \langle y, l \rangle)_n\} \quad (3.2.9)$$

$$\textcircled{3}_a = \text{Prob}\{(i, k, R_{\overrightarrow{z}}, j-1, 0, \langle y, l \rangle)_{n+1} | (i, k, R_{\overrightarrow{z}}, j, \langle y, l \rangle)_n\} \quad (3.2.10)$$

$$\textcircled{4}_a = \text{Prob}\{(i, k, C_{\overrightarrow{z}}, j-1, 0, \langle y, l \rangle)_{n+1} | (i, k, C_{\overrightarrow{z}}, j, \langle y, l \rangle)_n\} \quad (3.2.11)$$

If a RTS is successfully received, that is, $a_x(t_n) = R_{\overleftarrow{z}}, v_x(t_n) = 0$, x will start to respond with a CTS to z , shown by $a_x(t_{n+1}) = C_{\overrightarrow{z}}, v_x(t_{n+1}) = t_{CTS}$. The transmission of the CTS takes t_{CTS} steps and if successful, x should begin to receive a data packet from z . Otherwise, no data will be sent, and x resumes carrier sensing. Thus we have the following transition probabilities:

$$\textcircled{5}_a = \text{Prob}\{(i, k, A_{\overleftarrow{z}}, t_{DATA}, \langle y, l \rangle)_{n+1} | (i, k, C_{\overrightarrow{z}}, 0, \langle y, l \rangle)_n\} \quad (3.2.12)$$

$$\textcircled{5}_b = \text{Prob}\{(i, k, B, 0, \langle y, l \rangle)_{n+1} | (i, k, C_{\overrightarrow{z}}, 0, \langle y, l \rangle)_n\} \quad (3.2.13)$$

At each step of receiving DATA, there are two possible transitions:

$$\textcircled{6}_a = \text{Prob}\{(i, k, A_{\overleftarrow{z}}, j-1, \langle y, l \rangle)_{n+1} | (i, k, A_{\overleftarrow{z}}, j, \langle y, l \rangle)_n\} \quad (3.2.14)$$

$$\textcircled{6}_b = \text{Prob}\{(i, k, U, 0, \langle y, l \rangle)_{n+1} | (i, k, A_{\overleftarrow{z}}, j, \langle y, l \rangle)_n\} \quad (3.2.15)$$

For the first transition, x correctly receives the next piece of data so $v_x(t)$ decrease by 1. Otherwise, x detects a collision, which implies the signal is corrupted, shown by $a_x(t_{n+1}) = U$. When $v_x(t) = 1$, the receiving of data is complete and x shall reply with an ACK packet. When $v_x(t)$ decreases to 0, that is, the DATA/ACK handshake is successful, x will resume back off counting on the next layer and the queue size increases by 1. If the queue is full, as shown by the dashed arrow in diagram 3.1, the data received will be dropped and x will resume back-off counting on the same layer.

If x successfully overhears a RTS, then with probability 1 it will go to silent mode D_z and update $v_x(t)$ to t_{NAV_r} . Similarly, if a CTS is overheard, $v_x(t)$ changes to

t_{NAVc} . Upon $v_x(t)$ reaches 0, the behavior of x at the next time step depends on the channel status. With probability $\overline{7}_a$, x resumes back-off counting because it senses a quiet channel, or with probability $\overline{7}_b$, x detects a busy channel and waits.

$$\overline{7}_a = \text{Prob}\{(i, k, B, 0, \langle y, l \rangle)_{n+1} | (i, k, D_z, 0, \langle y, l \rangle)_n\} \quad (3.2.16)$$

$$\overline{7}_b = \text{Prob}\{(i, k, D_z, 0, \langle y, l \rangle)_{n+1} | (i, k, D_z, 0, \langle y, l \rangle)_n\} \quad (3.2.17)$$

Finally, if x senses jumbled signals in the channel at time step t_n ($a_x(t_n) = U$), then after one discrete time step x either senses the channel is clear and resumes back-off counting ($a_x(t_{n+1}) = B$), or detects a busy channel ($a_x(t_{n+1}) = U$) and waits, with the following probabilities:

$$\overline{8}_a = \text{Prob}\{(i, k, B, 0, \langle y, l \rangle)_{n+1} | (i, k, U, 0, \langle y, l \rangle)_n\} \quad (3.2.18)$$

$$\overline{8}_b = \text{Prob}\{(i, k, U, 0, \langle y, l \rangle)_{n+1} | (i, k, U, 0, \langle y, l \rangle)_n\} \quad (3.2.19)$$

For Figure 3.2 where x has empty queue, the state transitions are similar except with probability $\overline{1}_a$ x stays idle and keeps monitoring the channel. After a data packet is received, if x is a relay node, it will randomly or deterministically choose a receiver in N_x and set a back-off counter between 1 and the initial contention window size w .

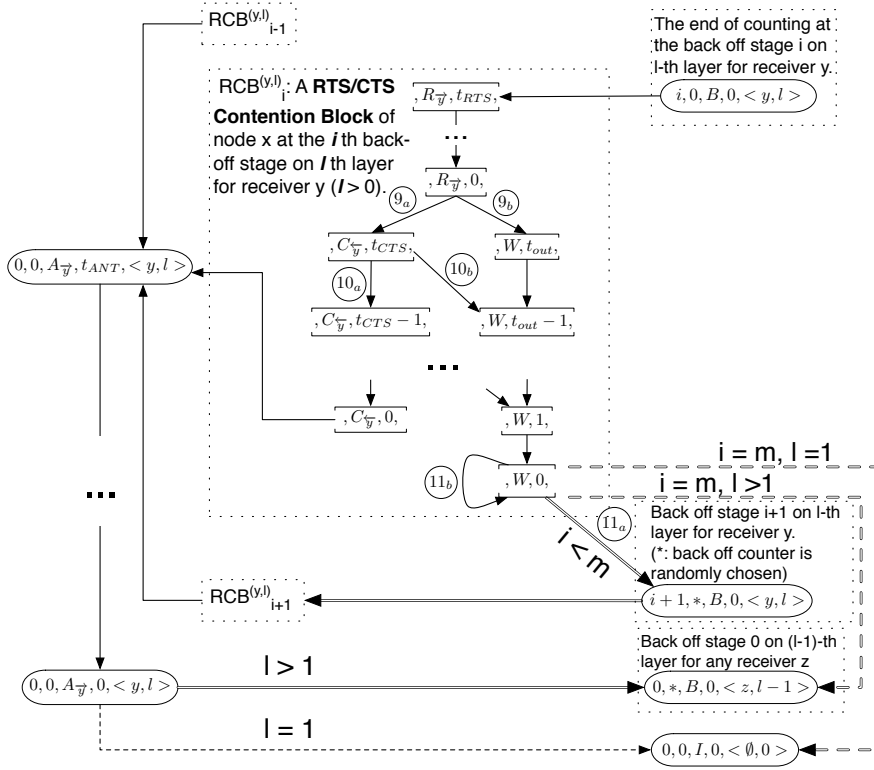


FIG. 3.3. RTS/CTS Contention Block

3.2.2. x as a sender. At the end of counting ($b_x(t) = 0$) at any back-off stage, x becomes a sender by immediately initiating a RTS transmission. The state transitions

of x as a sender are shown in Figure 3.3. A structure, called a **RTS/CTS Contention Block** (RCB) emerges in the model whenever x attempts a RTS/CTS handshake.

Suppose node x transmits a RTS packet to y during i th backoff stage with l packets in the queue. After a time period of t_{RTS} , the RTS transmission either succeeds and begins to receive a CTS from y with probability

$$\textcircled{9_a} = \text{Prob}\{(i, 0, C_{\overleftarrow{y}}, t_{CTS}, \langle y, l \rangle)_{n+1} | (i, 0, R_{\overrightarrow{y}}, 0, \langle y, l \rangle)_n\} \quad (3.2.20)$$

or fails with probability

$$\textcircled{9_b} = \text{Prob}\{(i, 0, W, t_{out}, \langle y, l \rangle)_{n+1} | (i, 0, R_{\overrightarrow{y}}, 0, \langle y, l \rangle)_n\} \quad (3.2.21)$$

In this case, there will be no reply so that x waits until the virtual counter $v_x(t)$ reaches 0.

At each step of receiving a CTS, depending on whether there is a collision at x , we have the following transition probabilities:

$$\textcircled{10_a} = \text{Prob}\{(i, 0, C_{\overleftarrow{y}}, j-1, \langle y, l \rangle)_{n+1} | (i, 0, C_{\overleftarrow{y}}, j, \langle y, l \rangle)_n\}$$

$$\textcircled{10_b} = \text{Prob}\{(i, 0, W, t_{out} - j, \langle y, l \rangle)_{n+1} | (i, 0, C_{\overleftarrow{y}}, j, \langle y, l \rangle)_n\}$$

When the receiving of the CTS is complete, x will initiate an DATA/ACK handshake, which lasts t_{DATA} time steps. In the end, if $l = 1$, i.e. the queue is empty, x becomes idle, otherwise x restarts the back-off procedure for the next HoL packet and the queue size decreases by 1.

Finally, suppose the RTS/CTS handshake fails, x senses the channel at the end of the CTS timeout. Given no transmitting neighbors, if the current back-off stage is less than the maximum stage allowed ($i < m$), x will reset the back off counter between 1 and the doubled contention window size, then resume counting procedure at the back off stage $i + 1$. The associated probability function is:

$$\textcircled{11_a} = \sum_k \text{Prob}\{(i+1, k, B, 0, \langle y, l \rangle)_{n+1} | (i, 0, W, 0, \langle y, l \rangle)_n\}$$

However, if the maximum stage is reached, then the data packet will be dropped. Based on the current queue size, x can either restart back-off procedure ($l > 1$) or become idle ($l = 1$):

$$\textcircled{11_a} = \sum_k \text{Prob}\{(0, k, B, 0, \langle y, l-1 \rangle)_{n+1} | (m, 0, W, 0, \langle y, l \rangle)_n\}$$

$$\textcircled{11_b} = \text{Prob}\{(0, 0, I, 0, \langle \emptyset, 0 \rangle)_{n+1} | (m, 0, W, 0, \langle y, 1 \rangle)_n\}$$

For the case that a busy channel is sensed, x will freeze, as shown by,

$$\textcircled{11_b} = \text{Prob}\{(i, 0, W, 0, \langle y, l \rangle)_{n+1} | (i, 0, W, 0, \langle y, l \rangle)_n\}$$

3.3. Representation of Transition Probabilities. In this section, we address the formulations of transition probability functions in detail. For simplicity, we first denote the probability density function for any node x in the network at time step t_n by

$$P^{(n)}([\chi_{i,k,j}^{\langle y,l \rangle}]_x) := \text{Prob}\{\mathcal{H}_x(t_n) = \mathbf{h}_x\} = \text{Prob}\{\mathcal{H}_x(t_n) = (i, k, \chi, j, \langle y, l \rangle)\}$$

Here $i \in [0, m]$, $k \in [0, 2^i w]$, $\chi \in \{I, B, U, R_{\bar{z}/\bar{z}/\bar{z}}, C_{\bar{z}/\bar{z}/\bar{z}}, A_{\bar{z}/\bar{z}}, D_z, W\}$, $j \in [0, t_{NAV_r}]$, $y, z \in N_x$ and $l \in [0, L_x]$ where N_x is the set that contains x 's neighbors (and \emptyset), and L_x represents the maximum queue size of x . The joint probability density functions are similarly defined and symmetric:

$$P^{(n)}([\chi_{i,k,j}^{\langle y,l \rangle}]_x, [\bar{\chi}_{i',k',j'}^{\langle y',l' \rangle}]_{x'}, \dots) = P^{(n)}([\chi_{i,k,j}^{\langle y,l \rangle}]_{x'}, [\bar{\chi}_{i',k',j'}^{\langle y',l' \rangle}]_x, \dots)$$

The probability density function of node x can be obtained by marginalizing out other nodes in the joint state probability density function, i.e.

$$\begin{aligned} \text{Prob}(\mathcal{H}_x(t_n) = \mathbf{h}_x) &= \text{Prob}(\mathcal{H}_x(t_n) = \mathbf{h}_x, \bullet) \\ &= \sum_{(\mathbf{h}'_x, \dots) \in \Omega(\mathbf{h}_x; x', \dots)} \text{Prob}(\mathcal{H}_x(t_n) = \mathbf{h}_x, \mathcal{H}_x(t_n) = \mathbf{h}'_x, \dots) \end{aligned}$$

where $\Omega(\mathbf{h}_x; x', \dots)$ represents the sub state space of nodes $\{x', \dots\}$ such that,

$$\text{Prob}(\mathcal{H}_x(t_n) = \mathbf{h}_x, \mathcal{H}_{x'}(t_n) = \mathbf{h}'_x, \dots) \neq 0$$

$$\forall (\mathbf{h}'_x, \dots) \in \Omega(\mathbf{h}_x; x', \dots)$$

On the other hand, given a set of marginal densities, the joint distribution in general cannot be uniquely determined unless the random variables are independent. This brings forward the main challenge in our modeling framework since for each node, all the critical state transitions mentioned in the last section are dependent on the concurrent states of its neighboring nodes. To be precise, suppose $N_x = \{x_1, x_2, \dots, x_r\}$ and expanding the marginal probability density function of x on $\Omega(\mathbf{h}_x; x_1, \dots, x_r)$, we have

$$\begin{aligned} &\text{Prob}\{\mathcal{H}_x(t_{n+1}) = \mathbf{h}'_x | \mathcal{H}_x(t_n) = \mathbf{h}_x\} \\ &= \sum_{(\mathbf{h}_{x_1}, \dots, \mathbf{h}_{x_r}) \in \Omega_{\mathcal{A}}(\mathbf{h}_x; x_1, \dots, x_r)} \frac{\text{Prob}\{\mathcal{H}_x(t_n) = \mathbf{h}_x, \mathcal{H}_{x_1}(t_n) = \mathbf{h}_{x_1}, \dots, \mathcal{H}_{x_r}(t_n) = \mathbf{h}_{x_r}\}}{\text{Prob}\{\mathcal{H}_x(t_n) = \mathbf{h}_x\}} \\ &= \frac{\sum_{\Omega_{\mathcal{A}}([\chi_{i,k,j}^{\langle y,l \rangle}]_x; x_1, \dots, x_r)} P^{(n)}([\chi_{i,k,j}^{\langle y,l \rangle}]_x, [\chi_{i,k,j}^{\langle y,l \rangle}]_{x_1}, \dots, [\chi_{i,k,j}^{\langle y,l \rangle}]_{x_r})}{\sum_{\Omega([\chi_{i,k,j}^{\langle y,l \rangle}]_x; x_1, \dots, x_r)} P^{(n)}([\chi_{i,k,j}^{\langle y,l \rangle}]_x, [\chi_{i,k,j}^{\langle y,l \rangle}]_{x_1}, \dots, [\chi_{i,k,j}^{\langle y,l \rangle}]_{x_r})} := \frac{\mathcal{F}_{\Omega_{\mathcal{A}}}([\chi_{i,k,j}^{\langle y,l \rangle}]_x)}{\mathcal{F}_{\Omega}([\chi_{i,k,j}^{\langle y,l \rangle}]_x)} \end{aligned}$$

where $\Omega_{\mathcal{A}}(\mathbf{h}_x; x_1, \dots, x_r) \subseteq \Omega(\mathbf{h}_x; x_1, \dots, x_r)$ and

$$\begin{aligned} &\text{Prob}\{\mathcal{H}_x(t_{n+1}) = \mathbf{h}'_x | \mathcal{H}_x(t_n) = \mathbf{h}_x, \mathcal{H}_{x_1}(t_n) = \mathbf{h}_{x_1}, \dots, \mathcal{H}_{x_r}(t_n) = \mathbf{h}_{x_r}\} \\ &= \begin{cases} 1, & \text{if } (\mathbf{h}_{x_1}, \dots, \mathbf{h}_{x_r}) \in \Omega_{\mathcal{A}}(\mathbf{h}_x; x_1, \dots, x_r) \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

For the purpose of evaluating the transition probability functions introduced in Section 3.2, we shall establish their connections (shown by functions \mathcal{F}_{Ω} and $\mathcal{F}_{\Omega_{\mathcal{A}}}$) to the probability density functions of joint states with the neighboring nodes. The joint state spaces Ω and $\Omega_{\mathcal{A}}$ will be discussed based on four categories of actions \mathcal{A} that x takes.

3.3.1. Carrier sensing while in the idle or back-off states. Let us suppose at the current time step t_n x is sensing a free channel and not freezing or waiting, that is, x is in back off state B (or equivalently, idle state I , if its queue is empty),

and the parameters i', k', y', l' are fixed: $\mathcal{H}_x(t_n) = (i', k', B, 0, \langle y', l' \rangle)$. Referring to Table 3.1 the channel must be quiet, hence all the neighboring nodes of x are not sending and not receiving from x or common neighbors with x (as x is known to be in the back-off state). Using the notations of cartesian product, we then have

$$\begin{aligned} \Omega([B_{i',k',0}^{\langle y',l' \rangle}]_x; x_1, \dots, x_r) &= \Omega([B_{i',k',0}^{\langle y',l' \rangle}]_x; x_1) \times \dots \times \Omega([B_{i',k',0}^{\langle y',l' \rangle}]_x; x_r) \\ &= \times_{x_\alpha \in N_x} \Omega([B_{i',k',0}^{\langle y',l' \rangle}]_x; x_\alpha) \\ &= \times_{x_\alpha \in N_x} \{ \mathcal{H}_{x_\alpha}(t_n) \mid \underbrace{\chi_\alpha \notin \{R_{\bar{z}}, C_{\bar{z}}, A_{\bar{z}}\}}_{\text{not transmitting}} \& \underbrace{\chi_\alpha \notin \{R_{\bar{z}'/\bar{z}'}, C_{\bar{z}'/\bar{z}'}, A_{\bar{z}'}, D_{z'}\}}_{\text{not interacting with } x \text{ and } N_x}, z' \notin N_x \} \end{aligned}$$

such that $P^{(n)}([B_{i',k',0}^{\langle y',l' \rangle}]_x) = \mathcal{F}_\Omega([B_{i',k',0}^{\langle y',l' \rangle}]_x)$. At the next time step t_{n+1} , if no neighbors of x are ready to send any signals, the channel will remain quiet. Hence we conclude that

$$\textcircled{1a} = \frac{\mathcal{F}_{\Omega_{1a}}([B_{i',k',0}^{\langle y',l' \rangle}]_x)}{\mathcal{F}_\Omega([B_{i',k',0}^{\langle y',l' \rangle}]_x)} = \frac{\sum_{\Omega_{1a}([B_{i',k',0}^{\langle y',l' \rangle}]_x; x_1, \dots, x_r)} P^{(n)}([B_{i',k',0}^{\langle y',l' \rangle}]_x, [\chi_{i,k,j}^{\langle y,l \rangle}]_{x_1}, \dots, [\chi_{i,k,j}^{\langle y,l \rangle}]_{x_r})}{\sum_{\Omega([B_{i',k',0}^{\langle y',l' \rangle}]_x; x_1, \dots, x_r)} P^{(n)}([B_{i',k',0}^{\langle y',l' \rangle}]_x, [\chi_{i,k,j}^{\langle y,l \rangle}]_{x_1}, \dots, [\chi_{i,k,j}^{\langle y,l \rangle}]_{x_r})}$$

Here, $\Omega_{1a}([B_{i',k',0}^{\langle y',l' \rangle}]_x; x_1, \dots, x_r) \subseteq \Omega([B_{i',k',0}^{\langle y',l' \rangle}]_x; x_1, \dots, x_r)$ and includes an extra restriction:

$$\begin{aligned} \Omega_{1a}([B_{i',k',0}^{\langle y',l' \rangle}]_x; x_1, \dots, x_r) &= \times_{x_\alpha \in N_x} \Omega_{1a}([B_{i',k',0}^{\langle y',l' \rangle}]_x; x_\alpha) \\ &= \times_{x_\alpha \in N_x} \{ \mathcal{H}_{x_\alpha}(t_n) \in \Omega([B_{i',k',0}^{\langle y',l' \rangle}]_x; x_\alpha) \mid \underbrace{(\chi, k, j)_{x_\alpha} \notin \{(B, 0, 0), (R_{\bar{z}}, k, 0), (C_{\bar{z}}, 0, 0)\}}_{\text{not begin to send RTS/CTS/DATA}} \}. \end{aligned}$$

For transition $\textcircled{1b}$, it accounts for the fact that one neighbor of x , for example, x' , begins to send a RTS packet to x , while the rest neighbors do not begin to send. We thus have

$$\begin{aligned} \Omega_{1b}([B_{i',k',0}^{\langle y',l' \rangle}]_x; x', \dots, x_r) &= \Omega_{1b}([B_{i',k',0}^{\langle y',l' \rangle}]_x; x') \times_{x_\alpha \in N_x \setminus x'} \Omega_{1a}([B_{i',k',0}^{\langle y',l' \rangle}]_x; x_\alpha) \\ &= \{ \mathcal{H}_{x'}(t_n) \mid \underbrace{(\chi, k, y)_{x'} = (B, 0, x)}_{\text{begins to sent a RTS to } x} \} \times_{x_\alpha \in N_x \setminus x'} \Omega_{1a}([B_{i',k',0}^{\langle y',l' \rangle}]_x; x_\alpha) \end{aligned}$$

such that

$$\textcircled{1b} = \frac{\mathcal{F}_{\Omega_{1b,x'}}([B_{i',k',0}^{\langle y',l' \rangle}]_x)}{\mathcal{F}_\Omega([B_{i',k',0}^{\langle y',l' \rangle}]_x)}.$$

On the other hand, if x' begins to send a RTS not to x while all other neighbors remain quiet and do not start to transmit any packet, x will start to overhear the RTS. The probability $\textcircled{1c}$ is given by

$$\textcircled{1c} = \frac{\mathcal{F}_{\Omega_{1c,x'}}([B_{i',k',0}^{\langle y',l' \rangle}]_x, x')}{\mathcal{F}_\Omega([B_{i',k',0}^{\langle y',l' \rangle}]_x)}$$

where $\Omega_{1c}([B_{i',k',0}^{(y',l')}]_x; x', \dots, x_r)$ is similarly defined by

$$\begin{aligned} & \Omega_{1c}([B_{i',k',0}^{(y',l')}]_x; x', \dots, x_r) \\ &= \underbrace{\{\mathcal{H}_{x'}(t_n) \mid (\chi, k)_{x'} = (B, 0) \ \& \ y_{x'} \neq x\}}_{\text{begins to sent a RTS not to } x} \times_{x_\alpha \in N_x \setminus x'} \Omega_{1a}([B_{i',k',0}^{(y',l')}]_x; x_\alpha) \end{aligned}$$

Likewise, if x' starts to sent a CTS not to x while the remaining neighbors stay quiet and do not initiate a transmission, we get

$$\begin{aligned} & \Omega_{1d}([B_{i',k',0}^{(y',l')}]_x; x', \dots, x_r) \\ &= \underbrace{\{\mathcal{H}_{x'}(t_n) \mid (\chi, k)_{x'} = (R_{\bar{z}}, 0), z \notin N_x\}}_{\text{begins to sent a CTS not to } x \text{ (or } N_x)} \times_{x_\alpha \in N_x \setminus x'} \Omega_{1a}([B_{i',k',0}^{(y',l')}]_x; x_\alpha) \end{aligned}$$

so that

$$\textcircled{1d} = \frac{\mathcal{F}_{\Omega_{1d}, x'}([B_{i',k',0}^{(y',l')}]_x)}{\mathcal{F}_\Omega([B_{i',k',0}^{(y',l')}]_x)}$$

Otherwise, x detects an unidentified busy channel. The corresponding transition has probability computed by

$$\textcircled{1e} = 1 - \frac{\mathcal{F}_{\Omega_{1a}}([B_{i',k',0}^{(y',l')}]_x)}{\mathcal{F}_\Omega([B_{i',k',0}^{(y',l')}]_x)} - \sum_{x' \in N_x} \left(\frac{\mathcal{F}_{\Omega_{1b}, x'}([B_{i',k',0}^{(y',l')}]_x)}{\mathcal{F}_\Omega([B_{i',k',0}^{(y',l')}]_x)} + \frac{\mathcal{F}_{\Omega_{1c}, x'}([B_{i',k',0}^{(y',l')}]_x)}{\mathcal{F}_\Omega([B_{i',k',0}^{(y',l')}]_x)} + \frac{\mathcal{F}_{\Omega_{1d}, x'}([B_{i',k',0}^{(y',l')}]_x)}{\mathcal{F}_\Omega([B_{i',k',0}^{(y',l')}]_x)} \right)$$

3.3.2. Receiving/overhearing packets. Next suppose at t_n x is receiving or overhearing a packet from a neighbor x' without interference by the others that are hidden from x' : $\mathcal{H}_x(t_n) = (i', k', \tilde{\chi}, j', \langle y', l' \rangle)$, $\tilde{\chi} \in \{R_{x'}^{\leftarrow}, R_{x'}^{\rightarrow}, C_{x'}^{\leftarrow}, C_{x'}^{\rightarrow}, A_{x'}^{\leftarrow}\}$, $j' \neq 0$. We observe that x' is at the j' -th step of transmitting the same packet, and all the other neighbors of x that are hidden from x' are quiet and do not interact with x . The common neighbors of x and x' are ignored because they share the same channel and will not intervene. Now assume $N_{xx'} := \{x_1, x_2, \dots, x_h\}$ are hidden from x' , we can write

$$\Omega([\tilde{\chi}_{i',k',j'}^{(y',l')}]_x; x', \dots, x_r) = \Omega([\tilde{\chi}_{i',k',j'}^{(y',l')}]_x; x') \times_{x_\alpha \in N_{xx'}} \Omega([\tilde{\chi}_{i',k',j'}^{(y',l')}]_x; x'; x_\alpha),$$

such that

$$\left\{ \begin{array}{l} \Omega([R_{x'/i',k',j'}^{(y',l')}]_x; x') = \{\mathcal{H}_{x'}(t_n) \mid \underbrace{(\chi, j)_{x'} = (R_{\bar{x}}, j')}_{j'\text{-th step of sending RTS to } x}\} \\ \Omega([R_{x'/i',k',j'}^{(y',l')}]_x; x') = \{\mathcal{H}_{x'}(t_n) \mid \underbrace{(\chi, j)_{x'} = (R_{\bar{z}}, j'), z \neq x}_{j'\text{-th step of sending RTS not to } x}\} \\ \Omega([C_{x'/i',k',j'}^{(y',l')}]_x; x') = \{\mathcal{H}_{x'}(t_n) \mid \underbrace{(\chi, j)_{x'} = (C_{\bar{z}}, j'), z \notin N_x}_{j'\text{-th step of sending CTS not to } x \text{ and } N_x}\} \\ \Omega([C_{x'/i',k',j'}^{(y',l')}]_x; x') = \{\mathcal{H}_{x'}(t_n) \mid \underbrace{(\chi, j)_{x'} = (C_{\bar{x}}, j')}_{j'\text{-th step of sending CTS to } x}\} \\ \Omega([A_{x'/i',k',j'}^{(y',l')}]_x; x') = \{\mathcal{H}_{x'}(t_n) \mid \underbrace{(\chi, j)_{x'} = (A_{\bar{x}}, j')}_{j'\text{-th step of sending DATA to } x}\} \end{array} \right. ,$$

where $\Omega([\tilde{\chi}_{i',k',j'}^{(y',l')}]_x; x'; x_\alpha)$ contains all the possible states of neighboring node x_α in $N_{x'}$ given ongoing communication between x and x' . If x is receiving a RTS or overhearing a RTS/CTS from x' ($\tilde{\chi} \in \{R_{x'}, R_{x'}, C_{x'}\}$), then for the hidden nodes x_α , x should appear to be in a back-off or idle state since the conversation between x and x' are concealed:

$$\Omega([\tilde{\chi}_{i',k',j'}^{(y',l')}]_x; x'; x_\alpha) = \Omega([B_{i',k',0}^{(y',l')}]_x; x_\alpha), \quad \tilde{\chi} \in \{R_{x'}, R_{x'}, C_{x'}\}.$$

On the other hand, if x is receiving a CTS or DATA from x' , then x_α should be informed because of the network allocation vector (NAV) incorporated inside the previous RTS/CTS packets sent from x . As a result, x_α should be in corresponding step NAV delay. If not, it is also impossible for x_α to receive any CTS/DATA packets because its own RTS/CTS handshakes should have failed. To summarize, we have the following:

$$\Omega([C_{x'/i',k',j'}^{(y',l')}]_x; x'; x_\alpha) = \Omega([B_{i',k',0}^{(y',l')}]_x; x_\alpha) \bigcup \{\mathcal{H}_{x'}(t_n) \mid \underbrace{(\chi, j)_{x_\alpha} = (D_x, t_{NAVc} + 1 + j')}_{t_{NAVc} + 1 + j'\text{-th step of NAV delay}} \& \underbrace{\chi_{x_\alpha} \notin \{C_{\bar{z}}, A_{\bar{z}}\}}_{\text{not receiving CTS/DATA}} \& \underbrace{(\chi, j)_{x_\alpha} \neq (R_{\bar{z}}, 0)}_{\text{not begin to send CTS}}\}$$

$$\Omega([A_{x'/i',k',j'}^{(y',l')}]_x; x'; x_\alpha) = \Omega([B_{i',k',0}^{(y',l')}]_x; x_\alpha) \bigcup \{\mathcal{H}_{x'}(t_n) \mid \underbrace{(\chi, j)_{x_\alpha} = (D_x, j')}_{j'\text{-th step of NAV delay}} \& \underbrace{\chi_{x_\alpha} \notin \{C_{\bar{z}}, A_{\bar{z}}\}}_{\text{not receiving CTS/DATA}}\}.$$

Note that in general $t_{DATA} \gg t_{RTS}(t_{CTS})$, thus it is possible that x_α finishes receiving a RTS and starts to broadcast a CTS during the period of DATA reception at x .

At the next time step t_{n+1} , x will continue to receive from x' unless some neighbors starts to broadcast, thus

$$\Omega_2([R_{x'/i',k',j'}^{(y',l')}]_x; x', \dots, x_r) = \Omega([R_{x'/i',k',j'}^{(y',l')}]_x; x') \times_{x_\alpha \in N_{x'}} \{\mathcal{H}_{x_\alpha}(t_n) \in \Omega([R_{x'/i',k',j'}^{(y',l')}]_x; x'; x_\alpha) \mid \underbrace{(\chi, k, j)_{x_\alpha} \notin \{(B, 0, 0), (R_{\bar{z}}, k, 0), (C_{\bar{z}}, 0, 0)\}}_{\text{not begin to send RTS/CTS/DATA}}\}$$

$$\Omega_3([R_{x'/i',k',j'}^{(y',l')}]_x; x', \dots, x_r) = \Omega([R_{x'/i',k',j'}^{(y',l')}]_x; x') \times_{x_\alpha \in N_{x'}} \\ \{\mathcal{H}_{x_\alpha}(t_n) \in \Omega([R_{x'/i',k',j'}^{(y',l')}]_x; x'; x_\alpha) \mid \underbrace{(\chi, k, j)_{x_\alpha} \notin \{(B, 0, 0), (R_{\bar{x}}, k, 0), (C_{\bar{x}}, 0, 0)\}}_{\text{not begin to send RTS/CTS/DATA}}\}$$

$$\Omega_4([C_{x'/i',k',j'}^{(y',l')}]_x; x', \dots, x_r) = \Omega([C_{x'/i',k',j'}^{(y',l')}]_x; x') \times_{x_\alpha \in N_{x'}} \\ \{\mathcal{H}_{x_\alpha}(t_n) \in \Omega([C_{x'/i',k',j'}^{(y',l')}]_x; x'; x_\alpha) \mid \underbrace{(\chi, k, j)_{x_\alpha} \notin \{(B, 0, 0), (R_{\bar{x}}, k, 0), (C_{\bar{x}}, 0, 0)\}}_{\text{not begin to send RTS/CTS/DATA}}\}$$

$$\Omega_6([A_{x'/i',k',j'}^{(y',l')}]_x; x', \dots, x_r) = \Omega([A_{x'/i',k',j'}^{(y',l')}]_x; x') \times_{x_\alpha \in N_{x'}} \\ \{\mathcal{H}_{x_\alpha}(t_n) \in \Omega([A_{x'/i',k',j'}^{(y',l')}]_x; x'; x_\alpha) \mid \underbrace{(\chi, k, j)_{x_\alpha} \notin \{(B, 0, 0), (R_{\bar{x}}, k, 0)\}}_{\text{not begin to send RTS/CTS}}\}$$

$$\Omega_{10}([C_{x'/i',k',j'}^{(y',l')}]_x; x', \dots, x_r) = \Omega([C_{x'/i',k',j'}^{(y',l')}]_x; x') \times_{x_\alpha \in N_{x'}} \\ \{\mathcal{H}_{x_\alpha}(t_n) \in \Omega([C_{x'/i',k',j'}^{(y',l')}]_x; x'; x_\alpha) \mid \underbrace{(\chi, k, j)_{x_\alpha} \neq (B, 0, 0)}_{\text{not begin to send RTS}}\}$$

and the transition probability functions during receiving are given by

$$\textcircled{2}_a = \frac{\mathcal{F}_{\Omega_2}([R_{x'/i',k',j'}^{(y',l')}]_x)}{\mathcal{F}_\Omega([R_{x'/i',k',j'}^{(y',l')}]_x)}, \quad \textcircled{3}_a = \frac{\mathcal{F}_{\Omega_3}([R_{x'/i',k',j'}^{(y',l')}]_x)}{\mathcal{F}_\Omega([R_{x'/i',k',j'}^{(y',l')}]_x)}, \quad \textcircled{4}_a = \frac{\mathcal{F}_{\Omega_4}([C_{x'/i',k',j'}^{(y',l')}]_x)}{\mathcal{F}_\Omega([C_{x'/i',k',j'}^{(y',l')}]_x)}, \\ \textcircled{6}_a = \frac{\mathcal{F}_{\Omega_6}([A_{x'/i',k',j'}^{(y',l')}]_x)}{\mathcal{F}_\Omega([A_{x'/i',k',j'}^{(y',l')}]_x)}, \quad \textcircled{10}_a = \frac{\mathcal{F}_{\Omega_{10}}([C_{x'/i',k',j'}^{(y',l')}]_x)}{\mathcal{F}_\Omega([C_{x'/i',k',j'}^{(y',l')}]_x)}.$$

3.3.3. End of sending. At the last step of transmissions from x to x' , $\mathcal{H}_x(t_n) = (i', k', \tilde{\chi}, 0, \langle y', l' \rangle)$, $\tilde{\chi} \in \{R_{x'}, C_{x'}\}$, we know the communications are successful only if x' also reaches the last step of receiving. Thus we only consider the joint state probability functions between x and x' in this case:

$$\textcircled{9}_a = \sum_{\Omega_9([R_{x'/i',0,0}^{(y',l')}]_x; x')} P^{(n)}([R_{x'}^{(y',l')}]_x, [\chi_{i,k,j}^{(y,l)}]_{x'}) \frac{1}{P^{(n)}([R_{x'}^{(y',l')}]_x)}, \\ \textcircled{5}_a = \sum_{\Omega_5([C_{x'/i',k',0}^{(y',l')}]_x; x')} P^{(n)}([C_{x'}^{(y',l')}]_x, [\chi_{i,k,j}^{(y,l)}]_{x'}) \frac{1}{P^{(n)}([C_{x'}^{(y',l')}]_x)},$$

where

$$\Omega_9([R_{x'/i',0,0}^{(y',l')}]_x; x') = \{\mathcal{H}_{x'}(t_n) \mid \underbrace{(\chi, j)_{x'} = (R_{\bar{x}}, 0)}_{\text{last step of receiving RTS from } x}\}, \\ \Omega_5([C_{x'/i',k',0}^{(y',l')}]_x; x') = \{\mathcal{H}_{x'}(t_n) \mid \underbrace{(\chi, j)_{x'} = (C_{\bar{x}}, 0)}_{\text{last step of receiving CTS from } x}\}.$$

Furthermore, since the RTS/CTS x' received must be sent from x , we have

$$P^{(n)}([R_{\bar{x}/i,k,0}^{(y,l)}]_{x'}) = \mathcal{F}_\Omega([R_{\bar{x}/i,k,0}^{(y,l)}]_{x'}) = \sum_{\Omega([R_{\bar{x}/i,k,0}^{(y,l)}]_{x'};x)} P^{(n)}([R_{\bar{x}/i,k,0}^{(y,l)}]_{x'}, [\chi_{i,k,j}^{(y,l)}]_x),$$

$$P^{(n)}([C_{\bar{x}/i,0,0}^{(x,l)}]_{x'}) = \mathcal{F}_\Omega([C_{\bar{x}/i,0,0}^{(x,l)}]_{x'}) = \sum_{\Omega([C_{\bar{x}/i,0,0}^{(x,l)}]_{x'};x)} P^{(n)}([C_{\bar{x}/i,0,0}^{(x,l)}]_{x'}, [\chi_{i,k,j}^{(y,l)}]_x),$$

where i and l are fixed for x' and

$$\Omega([R_{\bar{x}/i,k,0}^{(y,l)}]_{x'}; x) = \{ \mathcal{H}_x(t_n) \mid \underbrace{(\chi, j)_x = (R_{\bar{x}}^{\rightarrow}, 0)}_{\text{last step of sending RTS to } x'} \} ,$$

$$\Omega([C_{\bar{x}/i,0,0}^{(x,l)}]_{x'}; x) = \{ \mathcal{H}_x(t_n) \mid \underbrace{(\chi, j)_x = (C_{\bar{x}}^{\rightarrow}, 0)}_{\text{last step of sending CTS to } x'} \} .$$

We can now rewrite the transition probability functions as

$$\textcircled{9a} = \sum_{\Omega_9([R_{\bar{x}/i,0,0}^{x',l'}]_{x'};x')} \left(\frac{P^{(n)}([R_{\bar{x}/i,0,0}^{x',l'}]_{x'}, [\chi_{i,k,j}^{(y,l)}]_{x'})}{\mathcal{F}_\Omega([R_{\bar{x}/i,k,0}^{(y,l)}]_{x'})} \frac{P^{(n)}([R_{\bar{x}/i,k,0}^{(y,l)}]_{x'})}{P^{(n)}([R_{\bar{x}/i,0,0}^{x',l'}]_{x'})} \right),$$

$$\textcircled{5a} = \sum_{\Omega_5([C_{\bar{x}/i',k',0}^{y',l'}]_{x'};x')} \left(\frac{P^{(n)}([C_{\bar{x}/i',k',0}^{y',l'}]_{x'}, [\chi_{i,k,j}^{(y,l)}]_{x'})}{\mathcal{F}_\Omega([C_{\bar{x}/i,0,0}^{(x,l)}]_{x'})} \frac{P^{(n)}([C_{\bar{x}/i,0,0}^{(x,l)}]_{x'})}{P^{(n)}([C_{\bar{x}/i',k',0}^{y',l'}]_{x'})} \right).$$

3.3.4. End of waiting. Finally, suppose at time t_n node x is waiting (busy channel/NAV/CTS timeout) as well as sensing the channel: $\mathcal{H}_x(t_n) = (i', k', \tilde{\chi}, 0, \langle y', l' \rangle)$, $\tilde{\chi} \in \{U, D_z, W\}$. If a busy channel is sensed by x during backing-off or idle, then there must be at least one active neighbor accessing the channel at the same time. Moreover, it is impossible for x to send any packet, or receive a responding CTS (as x will be a sender in that case). Thus

$$\begin{aligned} \Omega([U_{x'/i',k',0}^{(y',l')}]_{x'}; x_1, \dots, x_r) &= \times_{x_\alpha \in N_x} \Omega([U_{x'/i',k',0}^{(y',l')}]_{x'}; x_\alpha; x_1, \dots, x_{\alpha-1}, x_{\alpha+1}, \dots, x_r) \\ &= \bigcup_{x_\alpha \in N_x} \Omega([U_{x'/i',k',0}^{(y',l')}]_{x'}; x_\alpha) \times_{x_\beta \in N_x \setminus x_\alpha} \Omega([U_{x'/i',k',0}^{(y',l')}]_{x'}; x_\alpha; x_\beta) \\ &= \bigcup_{x_\alpha \in N_x} \{ \mathcal{H}_{x_\alpha}(t_n) \mid \underbrace{\chi_{x_\alpha} \in \{R_{\bar{z}}^{\rightarrow}, C_{\bar{z}}^{\rightarrow}, A_{\bar{z}}^{\rightarrow}\}, z' \neq x}_{\text{transmitting (except CTS to } x)} \} \\ &\quad \times_{x_\beta \in N_x \setminus x_\alpha} \{ \mathcal{H}_{x_\beta}(t_n) \mid \underbrace{\chi_{x_\beta} \notin \{R_{\bar{x}/\bar{x}}, C_{\bar{x}/\bar{x}}, A_{\bar{x}/\bar{x}}\}}_{\text{not receiving from } x} \ \& \ \underbrace{\chi_{x_\beta} \neq C_{\bar{x}}^{\rightarrow}}_{\text{not sending CTS to } x} \} \end{aligned}$$

At the next time step if at least one neighbor is at the end of transmitting while no neighbors are in the middle of or ready to initiate a broadcasting, x will sense a free channel again and resume back-off counting (or become idle if the queue is empty) at

the next time step. Otherwise, x will continue waiting. Therefore,

$$\begin{aligned}
& \Omega_8([U_{x'/i',k',0}^{(y',l')}]_x; x_1, \dots, x_r) \\
&= \bigcup_{x_\alpha \in N_x} \underbrace{\{\mathcal{H}_{x_\alpha}(t_n) \mid (\chi, j)_{x_\beta} \in \{(R_{\vec{z}}, 0), (C_{\vec{z}}, 0), (A_{\vec{z}}, 0)\}, z' \neq x\}}_{\text{end of transmitting (except CTS to } x)} \\
&\quad \times_{x_\beta \in N_x \setminus x_\alpha} \underbrace{\{\mathcal{H}_{x_\beta}(t_n) \in \Omega([U_{x'/i',k',0}^{(y',l')}]_x; x_\alpha; x_\beta) \mid \\
&\quad \quad (\chi, j)_{x_\beta} \notin \{(R_{\vec{z}}, j'), (C_{\vec{z}}, j'), (A_{\vec{z}}, j')\}, j' \neq 0\}}_{\text{not in the middle of sending}} \\
&\quad \& \underbrace{\{(\chi, k, j)_{x_\beta} \notin \{(B, 0, 0), (R_{\vec{z}}, k, 0), (C_{\vec{z}}, 0, 0)\}\}}_{\text{not begin to send RTS/CTS/DATA}}
\end{aligned}$$

so that

$$\textcircled{8a} = \frac{\mathcal{F}_{\Omega_8}([U_{x'/i',k',0}^{(y',l')}]_x)}{\mathcal{F}_\Omega([U_{x'/i',k',0}^{(y',l')}]_x)}$$

If x is at the end of NAV waiting period due to RTS or CTS from x' . Using the facts that x can not interact with other nodes during waiting and if x' is sending or receiving DATA, it must be at the last step, we conclude that

$$\begin{aligned}
& \Omega([D_{x'/i',k',0}^{(y',l')}]_x; x', \dots, x_r) \\
&= \underbrace{\{\mathcal{H}_{x'}(t_n) \mid (\chi, j)_{x'} \notin \{(A_{\vec{z}/\vec{z}'}, j')\}, j' \neq 0\}}_{\text{not in the middle of DATA}} \& \underbrace{\{\chi_{x'} \notin \{R_{\vec{x}/\vec{x}}, C_{\vec{x}/\vec{x}/\vec{x}}, A_{\vec{x}/\vec{x}}, D_x\}\}}_{\text{not interacting with } x} \\
&\quad \times_{x_\alpha \in N_x \setminus x'} \underbrace{\{\mathcal{H}_{x_\alpha}(t_n) \mid \chi_{x_\alpha} \notin \{R_{\vec{x}/\vec{x}}, C_{\vec{x}/\vec{x}/\vec{x}}, A_{\vec{x}/\vec{x}}, D_x\}\}}_{\text{not interacting with } x}
\end{aligned}$$

At the next time step if all neighbors of x are not transmitting or begin to send, x will detect a free channel and consequentially resume back-off counting (or become idle). Otherwise we assume x will wait until the channel is clear. The corresponding conditions are

$$\begin{aligned}
& \Omega_7([D_{x'/i',k',0}^{(y',l')}]_x; x', \dots, x_r) = \Omega([D_{x'/i',k',0}^{(y',l')}]_x; x', \dots, x_r) \bigcap \times_{x_\alpha \in N_x} \{\mathcal{H}_{x_\alpha}(t_n) \mid \\
&\quad \underbrace{\chi_{x_\alpha} \notin \{R_{\vec{z}}, C_{\vec{z}}, A_{\vec{z}}\}}_{\text{not transmitting}} \& \underbrace{(\chi, k, j)_{x_\alpha} \notin \{(B, 0, 0), (R_{\vec{z}}, k, 0), (C_{\vec{z}}, 0, 0)\}}_{\text{not begin to send RTS/CTS/DATA}}\}
\end{aligned}$$

such that

$$\textcircled{7a} = \frac{\mathcal{F}_{\Omega_7}([D_{x'/i',k',0}^{(y',l')}]_x)}{\mathcal{F}_\Omega([D_{x'/i',k',0}^{(y',l')}]_x)}$$

If x is at the last step of CTS timeout for x' , which implies the previous RTS/CTS between x and x' fails, the possible concurrent states of x' are

$$\Omega([W_{i',0,0}^{(x',l')}]_x; x') = \{\mathcal{H}_{x'}(t_n) \mid \chi_{x'} \in \{I, B, U, R_{\vec{z}}, R_{\vec{z}'/\vec{z}'}\}, z' \neq x\}$$

For any other neighbor $x_\alpha \in N_x \setminus x'$, the previous RTS from x maybe overheard. Then

$$\Omega([W_{i',0,0}^{(x',l')}]_x; x'; x_\alpha) = \Omega([W_{i',0,0}^{(x',l')}]_x; x_\alpha) \bigcup \{ \mathcal{H}_{x_\alpha}(t_n) \mid \underbrace{(\chi, j)_{x_\alpha} = (D_x, t_{NAV_c})}_{t_{NAV_c}\text{-th step of NAV for } x} \}$$

Thus for all neighboring nodes of x we have

$$\Omega([W_{i',0,0}^{(x',l')}]_x; x', \dots, x_r) = \Omega([W_{i',0,0}^{(x',l')}]_x; x') \times_{x_\alpha \in N_x \setminus x'} \Omega([W_{i',0,0}^{(x',l')}]_x; x'; x_\alpha)$$

At the next time step, if no neighbors are sending or begin to send RTS:

$$\begin{aligned} \Omega_{11}([W_{i',0,0}^{(x',l')}]_x; x', \dots, x_r) &= \Omega([W_{i',0,0}^{(x',l')}]_x; x', \dots, x_r) \bigcap \\ &\quad \times_{x_\alpha \in N_x} \{ \mathcal{H}_{x_\alpha}(t_n) \mid \underbrace{(\chi, k)_{x_\alpha} \neq (B, 0) \ \& \ \chi_{x_\alpha} \neq R_{\bar{z}}}_{\text{not sending or begin to send RTS}} \} \end{aligned}$$

x resumes idle or back-off with probability evaluated by

$$\textcircled{11}_a = \frac{\mathcal{F}_{\Omega_{11}}([W_{i',0,0}^{(x',l')}]_x)}{\mathcal{F}_\Omega([W_{i',0,0}^{(x',l')}]_x)}$$

3.4. Equilibrium Distribution. In this section we will set up the balance equations for solving the stationary distribution, $\pi[\chi_{i,k,j}^{(y,l)}]_x$, of the discrete time Markov chain as $n \rightarrow \infty$. i.e. $\pi[\chi_{i,k,j}^{(y,l)}]_x = \lim_{n \rightarrow \infty} P^{(n)}([\chi_{i,k,j}^{(y,l)}]_x)$. For the transition probabilities such as $\textcircled{1a}$, we adopt the following notation:

$$p_{1a}^x = \lim_{n \rightarrow \infty} P_{1a}^x(t_n) := \lim_{n \rightarrow \infty} \textcircled{1}_a$$

If the transition such as $\textcircled{2a}$ involves a specific neighboring node x' , we use

$$p_{2a}^{xx'} = \lim_{n \rightarrow \infty} P_{2a}^{xx'}(t_n) := \lim_{n \rightarrow \infty} \textcircled{2}_a$$

3.4.1. System formulation. We start building the system from the base layer where $l = 0$:

$$\begin{aligned} \pi[I_{0,0,0}^{(\emptyset,0)}]_x &= p_{1a}^x \pi[I_{0,0,0}^{(\emptyset,0)}]_x + p_{8a}^x \pi[U_{0,0,0}^{(\emptyset,0)}]_x + \sum_{z \in N_x} (p_{5b}^{xz} \pi[C_{\bar{z}/0,0,0}^{(\emptyset,0)}]_x + p_{7a}^{xz} \pi[D_{z/0,0,0}^{(\emptyset,0)}]_x) \\ &\quad + \begin{cases} \sum_{z \in N_x} (\pi[W_{m,0,0}^{(z,1)}]_x + \pi[A_{\bar{z}/0,0,0}^{(z,1)}]_x), & L_x > 0 \\ \sum_{z \in N_x} \pi[A_{\bar{z}/0,0,0}^{(\emptyset,0)}]_x, & L_x = 0 \end{cases} \end{aligned} \quad (3.4.1)$$

$L_x = 0$ implies that node x has an empty queue. Next, suppose the queue is non-empty ($l > 0$) and node x is backing off for node y where $y \in N_x$. If $k = 0$, x transmits immediately, thus

$$\pi[B_{i,0,0}^{(y,l)}]_x = p_{1a}^x \pi[B_{i,1,0}^{(y,l)}]_x \quad (3.4.2)$$

Otherwise, depending on the value of i (back-off stage), k (back-off counter) and l (size of queue), we have

$$\begin{aligned} \pi[B_{i,k,0}^{(y,l)}]_x &= \begin{cases} p_{1a}^x \pi[B_{i,k+1,0}^{(y,l)}]_x + p_{8a}^x \pi[U_{i,k,0}^{(y,l)}]_x + \sum_{z \in N_x} (p_{5b}^{xz} \pi[C_{\bar{z}/i,k,0}^{(y,l)}]_x + p_{7a}^{xz} \pi[D_{z/i,k,0}^{(y,l)}]_x), & 0 < k < 2^i w \\ p_{8a}^x \pi[U_{i,k,0}^{(y,l)}]_x + \sum_{z \in N_x} (p_{5b}^{xz} \pi[C_{\bar{z}/i,k,0}^{(y,l)}]_x + p_{7a}^{xz} \pi[D_{z/i,k,0}^{(y,l)}]_x), & k = 2^i w \end{cases} \\ &+ \begin{cases} \sum_{z \in N_x} \pi[A_{\bar{z}/i,k,0}^{(y,l-1)}]_x, & 1 < l < L_x \\ \sum_{z \in N_x} (\pi[A_{\bar{z}/i,k,0}^{(y,L_x-1)}]_x + \pi[A_{\bar{z}/i,k,0}^{(y,L_x)}]_x), & l = L_x \\ 0, & l = 1 \end{cases} \\ &+ \begin{cases} \frac{1}{2^i w} \pi[W_{i-1,0,0}^{(y,l)}]_x, & i > 0 \\ \sum_{z \in N_x} \frac{P_{xy}}{w} (\pi[W_{m,0,0}^{(z,l+1)}]_x + \pi[A_{\bar{z}/0,0,0}^{(z,l+1)}]_x), & i = 0 \end{cases} \\ &+ \begin{cases} \sum_{z \in N_x} \frac{P_{xy}}{w} \pi[A_{\bar{z}/0,0,0}^{(\emptyset,0)}]_x, & i = 0, l = 1 \\ 0, & \text{otherwise} \end{cases} \end{aligned} \quad (3.4.3)$$

where P_{xy} denotes the probability of x sending a data packet to its neighbor y . Within each CSB and RCB, the steady state distribution of x should satisfy:

$$\begin{aligned} \pi[A_{\bar{z}/i,k,0}^{(y,l)}]_x &= \dots = (p_{6a}^{xz})^{t_{DATA}} \pi[A_{\bar{z}/i,k,t_{DATA}}^{(y,l)}]_{zx} = (p_{6a}^x)^{t_{DATA}} \cdot p_{5a}^{xz} \pi[C_{\bar{z}/i,k,0}^{(y,l)}]_x \\ &= \dots = (p_{6a}^{xz})^{t_{DATA}} \cdot p_{5a}^{xz} \cdot (p_{2a}^{xz})^{t_{RTS}} \pi[R_{\bar{z}/i,k,t_{RTS}}^{(y,l)}]_x \\ &= ((p_{6a}^{xz})^{t_{DATA}} \cdot p_{5a}^{xz} \cdot (p_{2a}^{xz})^{t_{RTS}} \cdot p_{1b}^{xz}) \pi[B_{i,k,0}^{(y,l)}]_x \end{aligned} \quad (3.4.4)$$

$$\begin{aligned} p_{7a}^{xz} \pi[D_{z/i,k,0}^{(y,l)}]_x &= \dots = \pi[D_{z/i,k,t_{NAVc}+1}^{(y,l)}]_x + \pi[C_{\bar{z}/i,k,0}^{(y,l)}]_x \\ &= \dots = (p_{3a}^{xz})^{t_{RTS}} \pi[R_{\bar{z}/i,k,t_{RTS}}^{(y,l)}]_x + (p_{4a}^{xz})^{t_{CTS}} \pi[C_{\bar{z}/i,k,t_{CTS}}^{(y,l)}]_x \\ &= ((p_{3a}^{xz})^{t_{RTS}} \cdot p_{1c}^{xz} + (p_{4a}^{xz})^{t_{CTS}} \cdot p_{1d}^{xz}) \pi[B_{i,k,0}^{(y,l)}]_x \end{aligned} \quad (3.4.5)$$

$$\begin{aligned} p_{8a}^x \pi[U_{i,k,0}^{(y,l)}]_x &= \sum_{z \in N_x} \left(\sum_{j=1}^{t_{RTS}} p_{2b}^{xz} \pi[R_{\bar{z}/i,k,j}^{(y,l)}]_x + \sum_{j=1}^{t_{DATA}} p_{6b}^{xz} \pi[A_{\bar{z}/i,k,j}^{(y,l)}]_x + \sum_{j=1}^{t_{RTS}} p_{3b}^{xz} \pi[R_{\bar{z}/i,k,j}^{(y,l)}]_x \right) \\ &+ \sum_{j=1}^{t_{CTS}} p_{4b}^{xz} \pi[C_{\bar{z}/i,k,j}^{(y,l)}]_x + p_{1e}^x \pi[B_{i,k,0}^{(y,l)}]_x \end{aligned} \quad (3.4.6)$$

$$\begin{aligned} \pi[A_{\bar{y}/0,0,0}^{(y,l)}]_x &= \dots = \sum_{i=0}^m (p_{10a}^{xy})^{t_{CTS}} \pi[C_{\bar{y}/0,0,t_{CTS}}^{(y,l)}]_x = \sum_{i=0}^m (p_{10a}^{xy})^{t_{CTS}} \cdot p_{9a}^{xy} \pi[R_{\bar{y}/i,0,0}^{(y,l)}]_x \\ &= \dots = \sum_{i=0}^m (p_{10a}^{xy})^{t_{CTS}} \cdot p_{9a}^{xy} \pi[B_{i,0,0}^{(y,l)}]_x \end{aligned} \quad (3.4.7)$$

$$\begin{aligned} p_{11a}^{xy} \pi[W_{i,0,0}^{(y,l)}]_x &= \dots = \sum_{j=1}^{t_{CTS}} p_{10b}^{xy} \pi[C_{i,0,j}^{(y,l)}]_x + \pi[W_{i,0,t_{out}}^{(y,l)}]_x \\ &= \dots = (1 - (p_{10a}^{xy})^{t_{CTS}} \cdot p_{9a}^{xy}) \pi[B_{i,0,0}^{(y,l)}]_x \end{aligned} \quad (3.4.8)$$

Now using equations (3.4.4)–(3.4.8), we can rewrite (3.4.1) and (3.4.3) as:

$$\begin{aligned} \pi[I_{0,0,0}^{(\emptyset,0)}]_x &= (p_{1a}^x + p_{1e}^x + \sum_{z \in N_x} (p_{1b}^{xz}(1 - P_R^{xz}) + p_{1c}^{xz} + p_{1d}^{xz})) \pi[I_{0,0,0}^{(\emptyset,0)}]_x \\ &\quad + \sum_{z \in N_x} (P_S^{xz} \sum_{i=0}^{m-1} \pi[B_{i,0,0}^{(z,1)}]_x + \pi[B_{m,0,0}^{(z,1)}]_x) \end{aligned} \quad (3.4.1a)$$

$$\begin{aligned} \pi[B_{i,k,0}^{(y,l)}]_x &= \begin{cases} p_{1a}^x \pi[B_{i,k+1,0}^{(y,l)}]_x + (p_{1e}^x + \sum_{z \in N_x} (p_{1b}^{xz}(1 - P_R^{xz}) + p_{1c}^{xz} + p_{1d}^{xz})) \pi[B_{i,k,0}^{(y,l)}]_x, & 1 < k < 2^i w \\ (p_{1e}^x + \sum_{z \in N_x} (p_{1b}^{xz}(1 - P_R^{xz}) + p_{1c}^{xz} + p_{1d}^{xz})) \pi[B_{i,k,0}^{(y,l)}]_x, & k = 2^i w \end{cases} \\ &\quad + \begin{cases} \sum_{z \in N_x} p_{1b}^{xz} P_R^{xz} \pi[B_{i,k,0}^{(y,l-1)}]_x, & 1 < l < L_x \\ \sum_{z \in N_x} p_{1b}^{xz} P_R^{xz} (\pi[B_{i,k,0}^{(y,L_x-1)}]_x + \pi[B_{i,k,0}^{(y,L_x)}]_x), & l = L_x \\ 0 & \text{otherwise} \end{cases} \\ &\quad + \begin{cases} \frac{1 - P_S^{xy}}{2^{i w}} \pi[B_{i-1,0,0}^{(y,l)}]_x, & i > 0 \\ \frac{P_{xy}}{w} \sum_{z \in N_x} (P_S^{xz} \sum_{i=0}^{m-1} \pi[B_{i,0,0}^{(z,l+1)}]_x + \pi[B_{m,0,0}^{(z,l+1)}]_x), & i = 0 \end{cases} \\ &\quad + \begin{cases} \sum_{z \in N_x} \frac{P_{xy}}{w} p_{1b}^{xz} P_R^{xz} \pi[I_{0,0,0}^{(\emptyset,0)}]_x, & i = 0, l = 1 \\ 0, & \text{otherwise} \end{cases} \end{aligned} \quad (3.4.3a)$$

where $P_S^{xz} := (P_{10a}^{xz})^{t_{CTS}} \cdot P_{9a}^{xz}$ represents the probability of a successful sending of a data packet at x to z , while $P_R^{xz} := (p_{6a}^{xz})^{t_{DATA}} \cdot p_{5a}^{xz} \cdot (p_{2a}^{xz})^{t_{RTS}}$ denotes the probability of a successful receiving of a data packet at x from z .

3.4.2. System Closure. Notice that the transition probability functions are still related to unknown joints probability functions. As a first step to conclude a solution, we complete the nonlinear system by applying naive product approximations:

$$P^{(n)}([\chi_{i,k,j}^{(y,l)}]_{x_1}, [\chi_{i,k,j}^{(y,l)}]_{x_2}, \dots, [\chi_{i,k,j}^{(y,l)}]_{x_r}) \approx \prod_{\alpha=1}^{\alpha=r} P^{(n)}([\chi_{i,k,j}^{(y,l)}]_{x_\alpha})$$

For simplicity we shall write

$$\frac{\sum_{\Omega_\alpha} \prod_{\gamma=1}^{\gamma=r} P^{(n)}([\chi_{i,k,j}^{(y,l)}]_{x_\gamma})}{\sum_{\Omega_\beta} \prod_{\gamma=1}^{\gamma=r} P^{(n)}([\chi_{i,k,j}^{(y,l)}]_{x_\gamma})} = \mathcal{P}_{\frac{x_1 \times \dots \times x_r}{x_1 \times \dots \times x_r}} \left(\frac{\Omega_\alpha}{\Omega_\beta} \right)$$

for any summation conditions Ω_α and Ω_β , and nodes x_1, \dots, x_r . Furthermore, if Ω_α and Ω_β can be decomposed as $\prod_{\gamma=1}^{\gamma=r} \Omega_\alpha(x_\gamma)$ and $\prod_{\gamma=1}^{\gamma=r} \Omega_\beta(x_\gamma)$, we can interchange the summation and product and denote:

$$\prod_{\gamma=1}^{\gamma=r} \frac{\sum_{\Omega_\alpha} P^{(n)}([\chi_{i,k,j}^{(y,l)}]_{x_\gamma})}{\sum_{\Omega_\beta} P^{(n)}([\chi_{i,k,j}^{(y,l)}]_{x_\gamma})} = \prod_{\gamma=1}^{\gamma=r} \mathcal{P}_{\frac{x_\gamma}{x_\gamma}} \left(\frac{\Omega_\alpha(x_\gamma)}{\Omega_\beta(x_\gamma)} \right)$$

In terms of each types of actions in section 3.3, we summarize the approximations using tables 3.2 to 3.5. Note that if $\forall x'' \in N_x$, $N_{x''} = \{x\}$, all messages from x will be successfully admitted by its neighbors. Thus we should have $\textcircled{6}_a \approx 1$ and $\textcircled{10}_a \approx 1$ since the channel has already been reserved by x through NAV.

4. Examples.

TABLE 3.2
Transition probability function approximations - case 1

Transition Probability	Approximations
P_{1a}^x	$\prod_{x_\alpha \in N_x} \mathcal{P}_{x_\alpha}^{\frac{x_\alpha}{x}} \left(\frac{\Omega_{1a}([B_{i',k',0}^{(y',l')}]_x; x_\alpha)}{\Omega([B_{i',k',0}^{(y',l')}]_x; x_\alpha)} \right)$
$P_{1b}^{xx'}$	$\mathcal{P}_{\frac{x'}{x}} \left(\frac{\Omega_{1b}([B_{i',k',0}^{(y',l')}]_x; x')}{\Omega([B_{i',k',0}^{(y',l')}]_x; x')} \right) \prod_{x_\alpha \in N_x \setminus x'} \mathcal{P}_{x_\alpha}^{\frac{x_\alpha}{x}} \left(\frac{\Omega_{1a}([B_{i',k',0}^{(y',l')}]_x; x_\alpha)}{\Omega([B_{i',k',0}^{(y',l')}]_x; x_\alpha)} \right)$
$P_{1c}^{xx'}$	$\mathcal{P}_{\frac{x'}{x}} \left(\frac{\Omega_{1c}([B_{i',k',0}^{(y',l')}]_x; x')}{\Omega([B_{i',k',0}^{(y',l')}]_x; x')} \right) \prod_{x_\alpha \in N_x \setminus x'} \mathcal{P}_{x_\alpha}^{\frac{x_\alpha}{x}} \left(\frac{\Omega_{1a}([B_{i',k',0}^{(y',l')}]_x; x_\alpha)}{\Omega([B_{i',k',0}^{(y',l')}]_x; x_\alpha)} \right)$
$P_{1d}^{xx'}$	$\mathcal{P}_{\frac{x'}{x}} \left(\frac{\Omega_{1d}([B_{i',k',0}^{(y',l')}]_x; x')}{\Omega([B_{i',k',0}^{(y',l')}]_x; x')} \right) \prod_{x_\alpha \in N_x \setminus x'} \mathcal{P}_{x_\alpha}^{\frac{x_\alpha}{x}} \left(\frac{\Omega_{1a}([B_{i',k',0}^{(y',l')}]_x; x_\alpha)}{\Omega([B_{i',k',0}^{(y',l')}]_x; x_\alpha)} \right)$
P_{1e}^x	$1 - P_{1a}^x - \sum_{x' \in N_x} (P_{1b}^{xx'} + P_{1c}^{xx'} + P_{1d}^{xx'})$

TABLE 3.3
Transition probability function approximations - case 2

Transition Probability	Approximations
P_{2a}^x	$\prod_{x_\alpha \in N_{xx'}} \mathcal{P}_{x_\alpha}^{\frac{x_\alpha}{x}} \left(\frac{\Omega_2([R_{x'/i',k',j'}^{(y',l')}]_x; x')}{\Omega([B_{i',k',0}^{(y',l')}]_x; x_\alpha)} \right)$
$P_{3a}^{xx'}$	$\prod_{x_\alpha \in N_{xx'}} \mathcal{P}_{x_\alpha}^{\frac{x_\alpha}{x}} \left(\frac{\Omega_3([R_{x'/i',k',j'}^{(y',l')}]_x; x')}{\Omega([B_{i',k',0}^{(y',l')}]_x; x_\alpha)} \right)$
$P_{4a}^{xx'}$	$\prod_{x_\alpha \in N_{xx'}} \mathcal{P}_{x_\alpha}^{\frac{x_\alpha}{x}} \left(\frac{\Omega_4([C_{x'/i',k',j'}^{(y',l')}]_x; x')}{\Omega([B_{i',k',0}^{(y',l')}]_x; x_\alpha)} \right)$
$P_{6a}^{xx'}$	$\prod_{x_\alpha \in N_{xx'}} \mathcal{P}_{x_\alpha}^{\frac{x_\alpha}{x}} \left(\frac{\Omega_6([A_{x'/i',k',j'}^{(y',l')}]_x; x')}{\Omega([A_{x'/i',k',j'}^{(y',l')}]_x; x_\alpha)} \right)$
$P_{10a}^{xx'}$	$\prod_{x_\alpha \in N_{xx'}} \mathcal{P}_{x_\alpha}^{\frac{x_\alpha}{x}} \left(\frac{\Omega_{10}([C_{x'/i',k',j'}^{(y',l')}]_x; x')}{\Omega([C_{x'/i',k',j'}^{(y',l')}]_x; x_\alpha)} \right)$

TABLE 3.4
Transition probability function approximations - case 3

Transition Probability	Approximations
$P_{5a}^{xx'}$	$\mathcal{P}_{\frac{x'}{x}} \left(\frac{\Omega_5([C_{x'/i',k',0}^{(y',l')}]_x; x')}{\Omega([C_{x'/i,0,0}^{(x,l')}]_{x'}; x)} \right)$
$P_{9a}^{xx'}$	$\mathcal{P}_{\frac{x'}{x}} \left(\frac{\Omega_9([R_{x'/i',0,0}^{(x,l')}]_x; x')}{\Omega([R_{x'/i,k,0}^{(y,l')}]_{x'}; x)} \right)$

4.1. QualNet Simulation. To validate our model, we compare the detailed equilibrium node states of three representative networks with results from realistic simulations of wireless networks in the QualNet simulator [1]. QualNet employs high fidelity wireless channel models and model of IEEE 802.11 DCF. The parameters used in the QualNet 5.0 simulator are summarized in Table 4.1. Accordingly, the model parameters used in all examples are concluded as follows. (i) The RTS retry limit m varies from 0 to $\sqrt{\frac{CW_{\max}}{CW_{\min}}} = 2$, that is, we allow RTS to retransmit 0, 1 or 2 times

TABLE 3.5
Transition probability function approximations - case 4

Transition Probability	Approximations
$P_{7a}^{xx'}$	$\mathcal{P}_{\frac{x' \times \dots \times x_r}{x' \times \dots \times x_r}} \left(\frac{\Omega_7([D_{x'/i',k',j'}^{(y',l')}]_x; x', \dots, x_r)}{\Omega([D_{x'/i',k',j'}^{(y',l')}]_x; x', \dots, x_r)} \right)$
P_{8a}^x	$\mathcal{P}_{\frac{x_1 \times \dots \times x_r}{x_1 \times \dots \times x_r}} \left(\frac{\Omega_8([U_{i',k',j'}^{(y',l')}]_x; x_1, \dots, x_r)}{\Omega([U_{i',k',j'}^{(y',l')}]_x; x_1, \dots, x_r)} \right)$
$P_{11a}^{xx'}$	$\mathcal{P}_{\frac{x' \times \dots \times x_r}{x' \times \dots \times x_r}} \left(\frac{\Omega_{11}([W_{i',0,0}^{(x',l')}]_x; x', \dots, x_r)}{\Omega([D_{i',0,0}^{(x',l')}]_x; x', \dots, x_r)} \right)$

in each example. (ii) The initial window size w is set as $CW_{min} = 3$. (iii) The transmission time of RTS/CTS are discretized as $t_{RTS} = t_{CTS} = \lceil \frac{T_{RTS}}{\sigma} \rceil - 1 = 1$, similarly $t_{DATA} = 5$ and $t_{out} = 2$. Note that t_{DATA} combines the transmission time of both data payload and ACK frame. (iv) The NAV contained in RTS frame should include the remaining time of a complete RTS/CTS/DATA/ACK handshakes thus $t_{NAV_r} = \lceil \frac{T_{CTS} + T_{DATA} + T_{ACK}}{\sigma} \rceil - 1 = 7$, similarly $t_{NAV_c} = 5$.

TABLE 4.1
Parameters used in QualNet simulation

Terrain size	1500×1500 m ²
Mobility	0
Radio range	up to 500 m
PHY protocol	802.11b
Bandwidth	5 Mbps
MAC protocol	MAC 802.11b
Slot time	140 Microsecond
SIFS	0 Microsecond
DIFS	140 Microsecond
RTS/CTS/ACK Tx time	280 Microsecond
CTS Timeout time	420 Microsecond
Data Tx time	562 Microsecond
CWmin	3
CWmax	12

4.2. A 2-node network. We first consider the following scenario. The network contains two nodes, x_1 and x_2 . Both nodes have infinite number of packets in their queue and consecutively transmit to each other. Due to symmetry, we only focus on the behaviors of node x_1 . Since there is no hidden terminal problem in this simple network and channel conditions are assumed to be ideal, all transmissions are guaranteed against collision and interference unless both nodes reach out at the same moment.

In this case only four non-trivial transition probabilities exists at any time step t_n , which are described in Table 4.2.

Using product approximations, we evaluate the above transition probabilities as

TABLE 4.2
Non-Trivial Transition Probabilities

(1a)	x_1 detects a quiet channel while back-off
(1b)	x_1 detects a RTS while back-off
(9a)	RTS from x_1 succeeds
(9b)	RTS from x_1 fails

follows in terms of the marginal densities of x_2 only.

$$P_{1a}^{x_1}(t_n) = \mathcal{P}_{\frac{x_2}{x_2}} \left(\frac{\sum_{\chi=B, k \neq 0} + \sum_{\chi=W}}{\sum_{\chi \in \{B, W\}}} \right) \quad (4.2.1)$$

$$P_{1b}^{x_1 x_2}(t_n) = \mathcal{P}_{\frac{x_2}{x_2}} \left(\frac{\sum_{\chi=B, k=0}}{\sum_{\chi \in \{B, W\}}} \right) \quad (4.2.2)$$

$$P_{9a}^{x_1 x_2}(t_n) = \mathcal{P}_{\frac{x_2}{x_1}} \left(\frac{\sum_{\chi=R_{\bar{x}_1}, j=0}}{\sum_{\chi=R_{\bar{x}_2}, j=0}} \right) \quad (4.2.3)$$

$$P_{9b}^{x_1 x_2}(t_n) = 1 - P_{9a}^{x_1 x_2}(t_n) \quad (4.2.4)$$

For the next step, let $t_n \rightarrow \infty$ and employing the equilibrium equations (3.4.2), (3.4.3a), (3.4.4), (3.4.7) and (3.4.8), we have:

$$\pi[B_{i,0,0}^{(x_2, \infty)}]_{x_1} = p_{1a}^{x_1} \pi[B_{i,1,0}^{(x_2, \infty)}]_{x_1} \quad (4.2.5)$$

$$\pi[B_{i,k,0}^{(x_2, \infty)}]_{x_1} = \begin{cases} p_{1a}^{x_1} \pi[B_{i,k+1,0}^{(x_2, \infty)}]_{x_1} + p_{1b}^{x_1 x_2} \pi[B_{i,k,0}^{(x_2, \infty)}]_{x_1}, & 0 < k < 2^i * 3 \\ p_{1b}^{x_1 x_2} \pi[B_{i,2^i w, 0}^{(x_2, \infty)}]_{x_1}, & k = 2^i * 3 \end{cases} \quad (4.2.6)$$

$$+ \begin{cases} \frac{1}{2^{i+3}} p_{9b}^{x_1 x_2} \pi[B_{i-1,0,0}^{(x_2, \infty)}]_{x_1}, & i > 0, k \neq 0 \\ \frac{1}{3} (p_{9a}^{x_1 x_2} \sum_{i=0}^{m-1} \pi[B_{i,0,0}^{(x_2, \infty)}]_{x_1} + \pi[B_{m,0,0}^{(x_2, \infty)}]_{x_1}), & i = 0, k \neq 0 \end{cases} \quad (4.2.7)$$

$$\pi[A_{\frac{x_2}{x_2}/i,k,0}^{(x_2, \infty)}]_{x_1} = \dots = \pi[A_{\frac{x_2}{x_2}/i,k,5}^{(x_2, \infty)}]_{x_1} = \pi[C_{\frac{x_2}{x_2}/i,k,0}^{(x_2, \infty)}]_{x_1} = \dots = \pi[R_{\frac{x_2}{x_2}/i,k,1}^{(x_2, \infty)}]_{x_1} = p_{1b}^{x_1 x_2} \pi[B_{i,k,0}^{(x_2, \infty)}]_{x_1} \quad (4.2.8)$$

$$\pi[A_{\frac{x_2}{x_2}/0,0,0}^{(x_2, \infty)}]_{x_1} = \dots = \sum_{i=0}^m \pi[C_{\frac{x_2}{x_2}/i,0,1}^{(x_2, \infty)}]_{x_1} = \sum_{i=0}^m p_{9a}^{x_1 x_2} \pi[R_{\frac{x_2}{x_2}/i,0,0}^{(x_2, \infty)}]_{x_1} = \dots = \sum_{i=0}^m p_{9a}^{x_1 x_2} \pi[B_{i,0,0}^{(x_2, \infty)}]_{x_1} \quad (4.2.9)$$

$$\pi[W_{i,0,0}^{(x_2, \infty)}]_{x_1} = \dots = \pi[W_{i,0,2}^{(x_2, \infty)}]_{x_1} = p_{9b}^{x_1 x_2} \pi[R_{\frac{x_2}{x_2}/i,0,0}^{(x_2, \infty)}]_{x_1} = \dots = p_{9b}^{x_1 x_2} \pi[B_{i,0,0}^{(x_2, \infty)}]_{x_1} \quad (4.2.10)$$

Given the transition probability functions from (4.2.1) to (4.2.4), together with the symmetry conditions:

$$\pi[\chi_{i,k,j}^{(x_2, \infty)}]_{x_1} = \pi[\chi_{i,k,j}^{(x_1, \infty)}]_{x_2} \quad (4.2.11)$$

for any i, k, χ, j , and the normalization constraints

$$\sum_{i,k,\chi,j} \pi[\chi_{i,k,j}^{(x_2, \infty)}]_{x_1} = \sum_{i,k,\chi,j} \pi[\chi_{i,k,j}^{(x_1, \infty)}]_{x_2} = 1, \quad (4.2.12)$$

the resulting non-linear system can be solved for stationary distributions at x_1 using Matlab's `fsolve` subroutine. The initial conditions of the system are

$$P^{(0)}[\chi_{i,k,j}^{(x_2,\infty)}]_{x_1} = \begin{cases} \frac{1}{3}, & \chi = B, i = 0, j = 0, k > 0 \\ 0, & \text{otherwise} \end{cases}$$

and $P_{1a}^{x_1}(0) = 1, P_{1b}^{x_1}(0) = 0, P_{9a}^{x_1}(0) = 1, P_{9b}^{x_1}(0) = 0$.

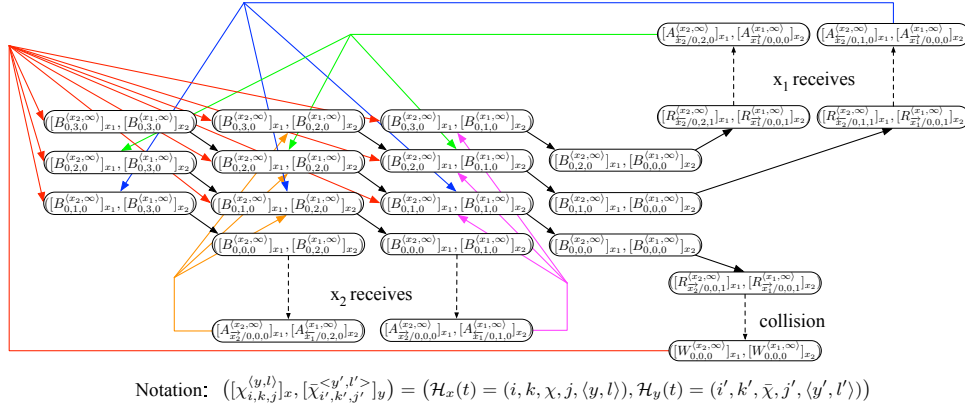


FIG. 4.1. 2-node joint state model ($m = 0$)

Figure 4.1 illustrates the dynamic among all the possible joint states of x_1 and x_2 when no RTS retry is allowed ($m = 0$). In the first plot in Figure 4.2, we compare the steady state distributions attained separately by solving the non-linear system using the product approximation (blue bars), exploring the joint state diagram (red bars) and implementing QualNet simulation (green bars). Overall, the joint state diagram accurately catch the nodes' behaviors and interactions under 802.11 DCF, and more importantly, so does the nonlinear system representing our model. The following two graphs in 4.2 show the results when we allow RTS retransmits 1 or 2 times. The joint state model between x_1 and x_2 contains highly irregular structure and therefore is difficult to solve directly. Nevertheless, our non-linear model closely reproduces the network behavior captured by the QualNet simulation.

4.3. A triangle network. As an interesting extension of the simple two-node system, we now consider an equilateral triangle network where three nodes x_1, x_2 and x_3 share the medium. Again, we assume each node has infinite packets in the queue and randomly chooses a receiver. Due to symmetry, only x_1 will be considered.

Since there are no hidden nodes, transmissions always succeed unless two or three nodes start to send simultaneously. As a consequence, only 8 transition probability functions at time t_n are non-trivial, summarized by the following table. Notice in this example, x_1 may detect a busy channel during back-off if x_2 and x_3 send a RTS at the same moment, resulting in a collision at x_1 . By applying product approximation, we can evaluate the above probabilities in terms of the marginal densities of x_2 and

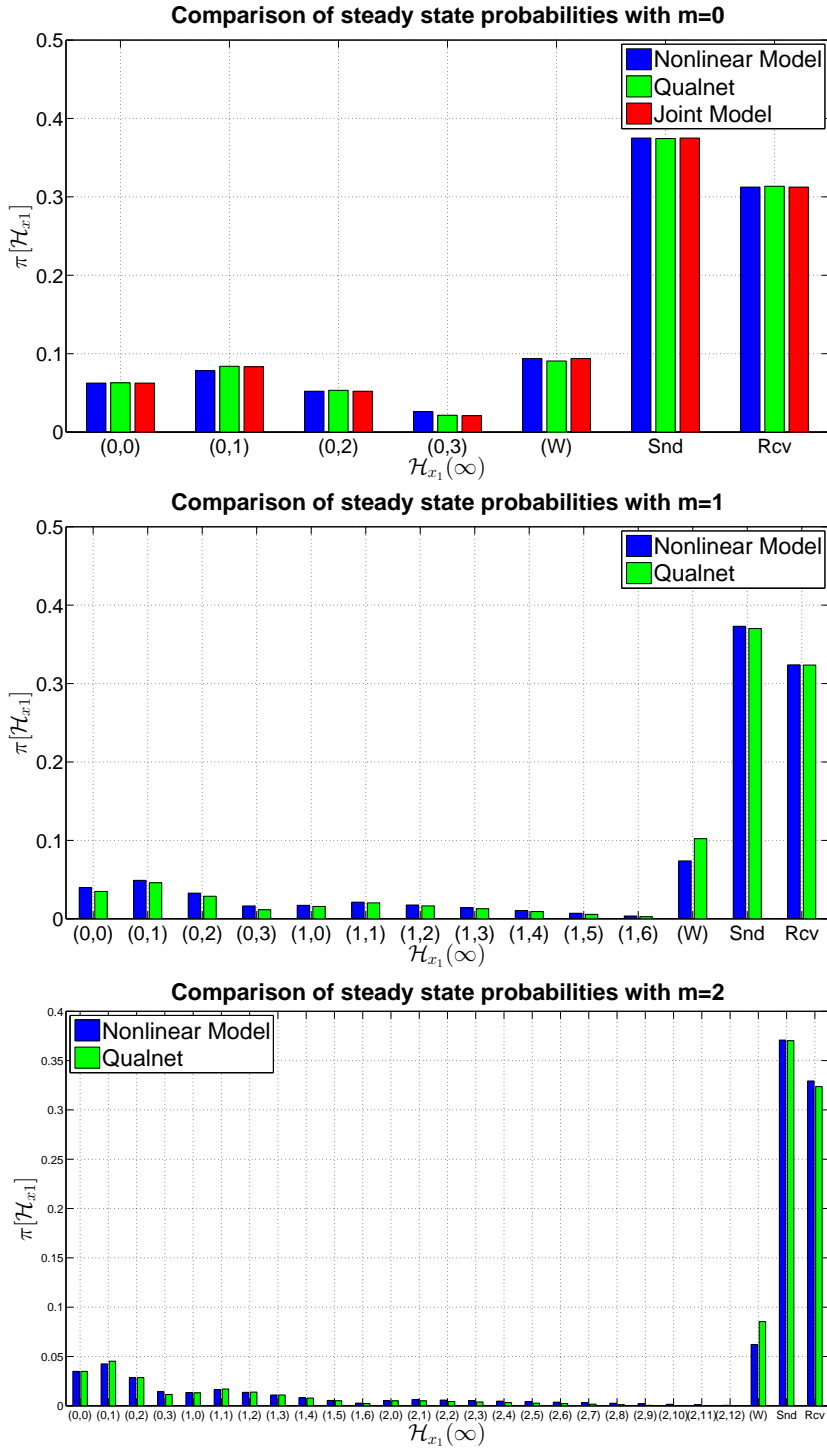


FIG. 4.2. Comparison at x_1 : the tuples represent (back-off stage, back-off counter); 'Snt' combines states of sending RTS/ receiving CTS/ sending DATA; 'Rcv' combines states of receiving RTS/ sending CTS/ receiving DATA

TABLE 4.3
Non-Trivial Transition Probabilities

(1 _a)	x_1 detects a quiet channel while back-off	(8 _a)	x_1 detects the channel is clear
(1 _b)	x_1 detects a RTS while back-off	(8 _b)	x_1 detects the channel is still busy
(1 _c)	x_1 overhears a RTS while back-off	(9 _a)	RTS sent from x_1 succeeds
(1 _e)	x_1 detects an busy channel	(9 _b)	RTS sent from x_1 fails

x_3 as the following.

$$P_{1a}^{x_1}(t_n) = \prod_{x_\alpha \in \{x_2, x_3\}} \mathcal{P}_{x_\alpha}^{x_\alpha} \left(\frac{\sum_{\chi=B, k \neq 0} + \sum_{\chi \in \{W, U\}}}{\sum_{\chi \in \{B, W, U\}}} \right) \quad (4.3.1)$$

$$P_{1b}^{x_1 x_2}(t_n) = \mathcal{P}_{x_2}^{x_2} \left(\frac{\sum_{\chi=B, k=0, y=x_1}}{\sum_{\chi \in \{B, W, U\}}} \right) \mathcal{P}_{x_3}^{x_3} \left(\frac{\sum_{\chi=B, k \neq 0} + \sum_{\chi \in \{W, U\}}}{\sum_{\chi \in \{B, W, U\}}} \right) \quad (4.3.2)$$

$$P_{1b}^{x_1 x_3}(t_n) = \mathcal{P}_{x_3}^{x_3} \left(\frac{\sum_{\chi=B, k=0, y=x_1}}{\sum_{\chi \in \{B, W, U\}}} \right) \mathcal{P}_{x_2}^{x_2} \left(\frac{\sum_{\chi=B, k \neq 0} + \sum_{\chi \in \{W, U\}}}{\sum_{\chi \in \{B, W, U\}}} \right) \quad (4.3.3)$$

$$P_{1c}^{x_1 x_2}(t_n) = \mathcal{P}_{x_2}^{x_2} \left(\frac{\sum_{\chi=B, k=0, y=x_3}}{\sum_{\chi \in \{B, W, U\}}} \right) \mathcal{P}_{x_3}^{x_3} \left(\frac{\sum_{\chi=B, k \neq 0} + \sum_{\chi \in \{W, U\}}}{\sum_{\chi \in \{B, W, U\}}} \right) \quad (4.3.4)$$

$$P_{1c}^{x_1 x_3}(t_n) = \mathcal{P}_{x_3}^{x_3} \left(\frac{\sum_{\chi=B, k=0, y=x_2}}{\sum_{\chi \in \{B, W, U\}}} \right) \mathcal{P}_{x_2}^{x_2} \left(\frac{\sum_{\chi=B, k \neq 0} + \sum_{\chi \in \{W, U\}}}{\sum_{\chi \in \{B, W, U\}}} \right) \quad (4.3.5)$$

$$P_{1e}^{x_1}(t_n) = 1 - P_{1a}^{x_1}(t_n) - P_{1b}^{x_1 x_2}(t_n) - P_{1b}^{x_1 x_3}(t_n) - P_{1c}^{x_1 x_2}(t_n) - P_{1c}^{x_1 x_3}(t_n) \quad (4.3.6)$$

$$P_{8a}^{x_1}(t_n) = \mathcal{P}_{x_2 \times x_3}^{x_2 \times x_3} \left(\frac{\sum \Omega_{8a}([U_{i,k,0}^{(y,\infty)}]_{x_1; x_2, x_3})}{\sum \Omega([U_{i,k,0}^{(y,\infty)}]_{x_1; x_2, x_3})} \right) \quad (4.3.7)$$

$$P_{8b}^{x_1}(t_n) = 1 - P_{8a}^{x_1}(t_n) \quad (4.3.8)$$

$$P_{9a}^{x_1 x_2}(t_n) = \mathcal{P}_{x_1}^{x_2} \left(\frac{\sum_{\chi=R_{\hat{x}_1}, j=0}}{\sum_{\chi=R_{\hat{x}_2}, j=0}} \right) \quad (4.3.9)$$

$$P_{9b}^{x_1 x_2}(t_n) = 1 - P_{9a}^{x_1 x_2}(t_n) \quad (4.3.10)$$

$$P_{9a}^{x_1 x_3}(t_n) = \mathcal{P}_{x_1}^{x_3} \left(\frac{\sum_{\chi=R_{\hat{x}_1}, j=0}}{\sum_{\chi=R_{\hat{x}_3}, j=0}} \right) \quad (4.3.11)$$

$$P_{9b}^{x_1 x_3}(t_n) = 1 - P_{9a}^{x_1 x_3}(t_n) \quad (4.3.12)$$

where $\Omega([U_{i,k,0}^{(y,\infty)}]_{x_1; x_2, x_3})$ represents

$$\begin{aligned} & \{\mathcal{H}_{x_2}(t_n) | \chi_{x_2} \in \{(R/A)_{\hat{x}_1/\hat{x}_2}, C_{\hat{x}_2}\}\} \times \{\mathcal{H}_{x_3}(t_n) | \chi_{x_3} \notin \{R_{\hat{x}_1/\hat{x}_1}, C_{\hat{x}_1/\hat{x}_1/\hat{x}_1}, A_{\hat{x}_1}\}\} \\ & \cup \{\mathcal{H}_{x_3}(t_n) | \chi_{x_3} \in \{(R/A)_{\hat{x}_1/\hat{x}_2}, C_{\hat{x}_2}\}\} \times \{\mathcal{H}_{x_2}(t_n) | \chi_{x_2} \notin \{R_{\hat{x}_1/\hat{x}_1}, C_{\hat{x}_1/\hat{x}_1/\hat{x}_1}, A_{\hat{x}_1}\}\} \end{aligned}$$

and $\Omega_{8a}([U_{i,k,0}^{(y,\infty)}]_{x_1}; x_2, x_3)$ stands for

$$\begin{aligned} & \{\mathcal{H}_{x_2}(t_n) \in \Omega([U_{i,k,0}^{(y,\infty)}]_{x_1}; x_2) | j_{x_2} = 0\} \times \{\mathcal{H}_{x_3}(t_n) \in \Omega([U_{i,k,0}^{(y,\infty)}]_{x_1}; x_2; x_3) | \\ & \quad (\chi_{x_3}, j_{x_3}) \notin \{((R/C/A)_{\vec{x}_1/\vec{x}_2}, j), j \neq 0, (\chi_{x_3}, k_{x_3}) \neq (B, 0)\}\} \\ & \cup \{\mathcal{H}_{x_3}(t_n) \in \Omega([U_{i,k,0}^{(y,\infty)}]_{x_1}; x_3) | j_{x_3} = 0\} \times \{\mathcal{H}_{x_2}(t_n) \in \Omega([U_{i,k,0}^{(y,\infty)}]_{x_1}; x_3; x_2) | \\ & \quad (\chi_{x_2}, j_{x_2}) \notin \{((R/C/A)_{\vec{x}_1/\vec{x}_3}, j), j \neq 0, (\chi_{x_2}, k_{x_2}) \neq (B, 0)\}\} \end{aligned}$$

The stationary distribution of back-off states as $t_n \rightarrow \infty$ satisfy:

$$\begin{aligned} \pi[B_{i,0,0}^{(y,\infty)}]_{x_1} &= p_{1a}^{x_1} \pi[B_{i,1,0}^{(y,\infty)}]_{x_1} & (4.3.13) \\ \pi[B_{i,k,0}^{(y,\infty)}]_{x_1} &= \begin{cases} p_{1a}^{x_1} \pi[B_{i,k+1,0}^{(y,\infty)}]_{x_1} + (1 - p_{1a}^{x_1}) \pi[B_{i,k,0}^{(y,\infty)}]_{x_1}, & 0 < k < 2^i * 3 \\ (1 - p_{1a}^{x_1}) \pi[B_{i,2^i w, 0}^{(y,\infty)}]_{x_1}, & k = 2^i * 3 \end{cases} \\ &+ \begin{cases} \frac{P_{x_1 y}}{2^i * 3} (p_{9b}^{x_1 x_2} \pi[B_{i-1,0,0}^{(x_2,\infty)}]_{x_1} + p_{9b}^{x_1 x_3} \pi[B_{i-1,0,0}^{(x_3,\infty)}]_{x_1}), & i > 0, k \neq 0 \\ \frac{P_{x_1 y}}{3} (p_{9a}^{x_1 x_2} \sum_{i=0}^{m-1} \pi[B_{i,0,0}^{(x_2,\infty)}]_{x_1} + \pi[B_{m,0,0}^{(x_2,\infty)}]_{x_1} \\ \quad + p_{9a}^{x_1 x_3} \sum_{i=0}^{m-1} \pi[B_{i,0,0}^{(x_3,\infty)}]_{x_1} + \pi[B_{m,0,0}^{(x_3,\infty)}]_{x_1}), & i = 0, k \neq 0 \end{cases} & (4.3.14) \end{aligned}$$

for any $y \in \{x_2, x_3\}$. Note we assume at the beginning of each new DATA session, the sender chooses its receiver randomly, thus $P_{x_1 y} = \frac{1}{|N_{x_1}|} = \frac{1}{2}$. In general, this will be set as a parameter that is determined by the routing algorithm or experimental settings.

For the remaining part of the distribution, one can conclude that

$$\begin{aligned} \pi[A_{\frac{x_2}{x_2}/i,k,0}^{(y,\infty)}]_{x_1} &= \dots = \pi[A_{\frac{x_2}{x_2}/i,k,5}^{(y,\infty)}]_{x_1} = \pi[C_{\frac{x_2}{x_2}/i,k,0}^{(y,\infty)}]_{x_1} = \dots = \pi[R_{\frac{x_2}{x_2}/i,k,1}^{(y,\infty)}]_{x_1} = p_{1b}^{x_1 x_2} \pi[B_{i,k,0}^{(y,\infty)}]_{x_1} \\ \pi[A_{\frac{x_3}{x_3}/i,k,0}^{(y,\infty)}]_{x_1} &= \dots = \pi[A_{\frac{x_3}{x_3}/i,k,5}^{(y,\infty)}]_{x_1} = \pi[C_{\frac{x_3}{x_3}/i,k,0}^{(y,\infty)}]_{x_1} = \dots = \pi[R_{\frac{x_3}{x_3}/i,k,1}^{(y,\infty)}]_{x_1} = p_{1b}^{x_1 x_3} \pi[B_{i,k,0}^{(y,\infty)}]_{x_1} \end{aligned} \quad (4.3.15)$$

$$\begin{aligned} \pi[D_{\frac{x_2}{x_2}/i,k,0}^{(y,\infty)}]_{x_1} &= \dots = \pi[D_{\frac{x_2}{x_2}/i,k,7}^{(y,\infty)}]_{x_1} = \pi[R_{\frac{x_2}{x_2}/i,k,0}^{(y,\infty)}]_{x_1} = \pi[R_{\frac{x_2}{x_2}/i,k,1}^{(y,\infty)}]_{x_1} = p_{1c}^{x_1 x_2} \pi[B_{i,k,0}^{(y,\infty)}]_{x_1} \\ \pi[D_{\frac{x_3}{x_3}/i,k,0}^{(y,\infty)}]_{x_1} &= \dots = \pi[D_{\frac{x_3}{x_3}/i,k,7}^{(y,\infty)}]_{x_1} = \pi[R_{\frac{x_3}{x_3}/i,k,0}^{(y,\infty)}]_{x_1} = \pi[R_{\frac{x_3}{x_3}/i,k,1}^{(y,\infty)}]_{x_1} = p_{1c}^{x_1 x_3} \pi[B_{i,k,0}^{(y,\infty)}]_{x_1} \end{aligned} \quad (4.3.16)$$

$$\pi[U_{i,k,0}^{(y,\infty)}]_{x_1} = \frac{p_{1e}^{x_1}}{p_{8a}^{x_1}} \pi[B_{i,k,0}^{(y,\infty)}]_{x_1} \quad (4.3.17)$$

$$\begin{aligned} \pi[A_{\frac{x_2}{x_2}/0,0,0}^{(y,\infty)}]_{x_1} &= \dots = \sum_{i=0}^m \pi[C_{\frac{x_2}{x_2}/i,0,1}^{(y,\infty)}]_{x_1} = \sum_{i=0}^m p_{9a}^{x_1 x_2} \pi[R_{\frac{x_2}{x_2}/i,0,0}^{(y,\infty)}]_{x_1} = \dots = \sum_{i=0}^m p_{9a}^{x_1 x_2} \pi[B_{i,0,0}^{(y,\infty)}]_{x_1} \\ \pi[A_{\frac{x_3}{x_3}/0,0,0}^{(y,\infty)}]_{x_1} &= \dots = \sum_{i=0}^m \pi[C_{\frac{x_3}{x_3}/i,0,1}^{(y,\infty)}]_{x_1} = \sum_{i=0}^m p_{9a}^{x_1 x_3} \pi[R_{\frac{x_3}{x_3}/i,0,0}^{(y,\infty)}]_{x_1} = \dots = \sum_{i=0}^m p_{9a}^{x_1 x_3} \pi[B_{i,0,0}^{(y,\infty)}]_{x_1} \end{aligned} \quad (4.3.18)$$

$$\pi[W_{i,0,0}^{(y,\infty)}]_{x_1} = \dots = \pi[W_{i,0,2}^{(y,\infty)}]_{x_1} = p_{9b}^{x_1 y} \pi[R_{\frac{y}{y}/i,0,0}^{(y,\infty)}]_{x_1} = \dots = p_{9b}^{x_1 y} \pi[B_{i,0,0}^{(y,\infty)}]_{x_1} \quad (4.3.19)$$

Given the transition probability functions from (4.3.1) to (4.3.12), together with the condition of symmetric topology:

$$\pi[\chi_{i,k,j}^{(y,\infty)}]_{x_1} = \pi[\chi_{i,k,j}^{(y,\infty)}]_{x_2} = \pi[\chi_{i,k,j}^{(y,\infty)}]_{x_3} \quad (4.3.20)$$

for any i, k, χ, j, y , and the normalization conditions

$$\sum_{i,k,\chi,j,y} \pi[\chi_{i,k,j}^{(y,\infty)}]_{x_1} = \sum_{i,k,\chi,j,y} \pi[\chi_{i,k,j}^{(y,\infty)}]_{x_2} = \sum_{i,k,\chi,j,y} \pi[\chi_{i,k,j}^{(y,\infty)}]_{x_3} = 1 \quad (4.3.21)$$

we form a non-linear system which can be solved again by Matlab with initial conditions:

$$P^{(0)}[\chi_{i,k,j}^{(y,\infty)}]_{x_1} = \begin{cases} \frac{1}{6}, & \chi = B, i = 0, j = 0, k > 0 \\ 0, & \text{otherwise} \end{cases}$$

and $P_{1a}^{x_1}(0) = P_{8a}^{x_1}(0) = P_{9a}^{x_1 x_2}(0) = P_{9a}^{x_1 x_3}(0) = 1$. The remaining significant transition probabilities are initialized as 0.

The plots in Figure 4.3 compare the steady state distributions with analytical results from the above system and QualNet simulations where the RTS retransmission limit (m) is set to be 0, 1, 2 respectively.

4.4. 2 Senders and 1 receiver. Finally we examine the following topology where three nodes, x_1 , x_2 and x_3 , are presented in the network. Both x_1 and x_3 have infinite data packets destined to node x_2 in their queues. This case includes hidden terminals because node x_3 is not within range of x_1 and vice versa. Node x_2 has no queue and absorbs data packet from x_1 and x_3 . Due to symmetry, only x_1 and x_2 will be considered.

Unlike the previous example, x_1 and x_3 are hidden to each other thus the receiving procedure of RTS at x_2 may be interrupted by collision. On the other hand the receiving of CTS packets at x_1 always succeeds since x_1 has no other neighbors (beside x_2) to interfere. Notice this fact indicates all the subsequent data packets (in this case, from x_3 to x_2) will be protected by NAV hence probability $\textcircled{6}_a \equiv 1$.

TABLE 4.4
Non-Trivial Transition Probabilities for x_1

$\textcircled{1}_a$	x_1 detects a quiet channel while back-off
$\textcircled{1}_d$	x_1 overhears a CTS while back-off
$\textcircled{9}_a$	RTS sent from x_1 succeeds
$\textcircled{9}_b$	RTS sent from x_1 fails

TABLE 4.5
Non-Trivial Transition Probabilities for x_2

$\textcircled{1}_a$	x_2 detects a quiet channel while idle
$\textcircled{1}_b$	x_2 detects a RTS while idle
$\textcircled{1}_e$	x_2 detects a busy channel while idle
$\textcircled{2}_a$	x_2 receives RTS correctly
$\textcircled{2}_b$	x_2 detects a collision while receiving RTS
$\textcircled{8}_a$	x_2 detects the channel is clear
$\textcircled{8}_b$	x_2 detects the channel is still busy

The only possible non-trivial transition probability functions at time t_n are sum-

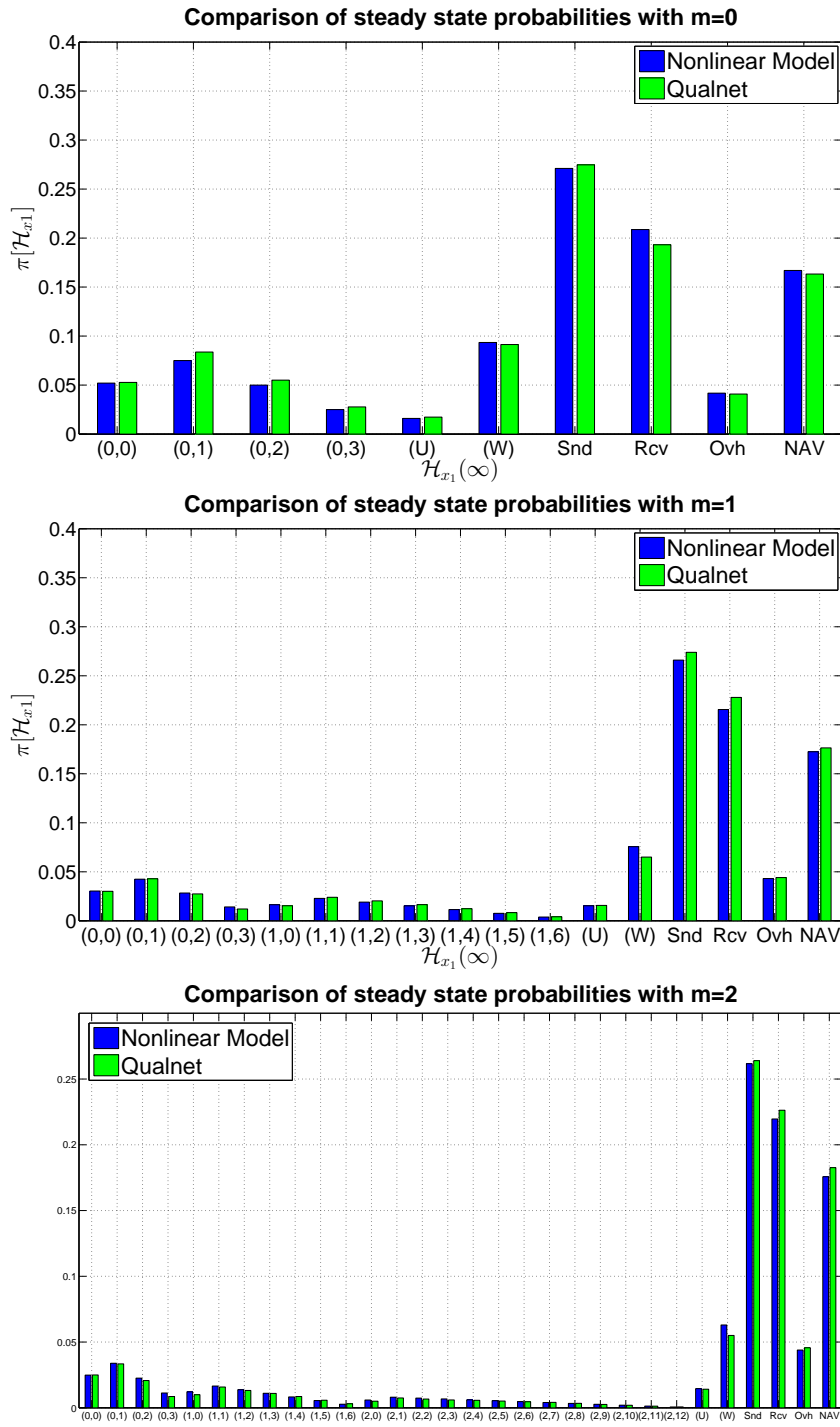


FIG. 4.3. Comparison at x_1 : the tuples represent (back-off stage, back-off counter); 'Snt' combines states of sending RTS/ receiving CTS/ sending DATA; 'Rcv' combines states of receiving RTS/ sending CTS/ receiving DATA; 'Ovh' denotes overhearing RTS

marized by table 4.4 and 4.5. For node x_1 we have

$$P_{1a}^{x_1}(t_n) = \mathcal{P}_{\frac{x_2}{x_2}} \left(\frac{\sum_{\chi=R_{\bar{x}_3}, j \neq 0} + \sum_{\chi \in \{I, U\}}}{\sum_{\chi \in \{I, R_{\bar{x}_3}, U\}}} \right) \quad (4.4.1)$$

$$P_{1d}^{x_1 x_2}(t_n) = \mathcal{P}_{\frac{x_2}{x_2}} \left(\frac{\sum_{\chi=R_{\bar{x}_3}, j=0}}{\sum_{\chi \in \{I, R_{\bar{x}_3}, U\}}} \right) \quad (4.4.2)$$

$$P_{9a}^{x_1 x_2}(t_n) = \mathcal{P}_{\frac{x_2}{x_1}} \left(\frac{\sum_{\chi=R_{\bar{x}_1}, j=0}}{\sum_{\chi=R_{\bar{x}_2}, j=0}} \right) \quad (4.4.3)$$

$$P_{9b}^{x_1 x_2}(t_n) = 1 - P_{9a}^{x_1 x_2}(t_n) \quad (4.4.4)$$

For node x_2 , we have

$$P_{1a}^{x_2}(t_n) = \prod_{x_\alpha \in \{x_1, x_3\}} \mathcal{P}_{\frac{x_\alpha}{x_\alpha}} \left(\frac{\sum_{\chi=B, k \neq 0} + \sum_{\chi=W}}{\sum_{\chi \in \{B, W\}}} \right) \quad (4.4.5)$$

$$P_{1b}^{x_2 x_1}(t_n) = \mathcal{P}_{\frac{x_1}{x_1}} \left(\frac{\sum_{\chi=B, k=0, y=x_2}}{\sum_{\chi \in \{B, W\}}} \right) \mathcal{P}_{\frac{x_3}{x_3}} \left(\frac{\sum_{\chi=B, k \neq 0} + \sum_{\chi=W}}{\sum_{\chi \in \{B, W\}}} \right) \quad (4.4.6)$$

$$P_{1b}^{x_2 x_3}(t_n) = \mathcal{P}_{\frac{x_3}{x_3}} \left(\frac{\sum_{\chi=B, k=0, y=x_2}}{\sum_{\chi \in \{B, W\}}} \right) \mathcal{P}_{\frac{x_1}{x_1}} \left(\frac{\sum_{\chi=B, k \neq 0} + \sum_{\chi=W}}{\sum_{\chi \in \{B, W\}}} \right) \quad (4.4.7)$$

$$P_{1e}^{x_2}(t_n) = 1 - P_{1a}^{x_2}(t_n) - P_{1b}^{x_2 x_1}(t_n) - P_{1b}^{x_2 x_3}(t_n) \quad (4.4.8)$$

$$P_{2a}^{x_2 x_1}(t_n) = \mathcal{P}_{\frac{x_3}{x_3}} \left(\frac{\sum_{\chi=B, k \neq 0} + \sum_{\chi=W}}{\sum_{\chi \in \{B, W\}}} \right) \quad (4.4.9)$$

$$P_{2b}^{x_2 x_1}(t_n) = 1 - P_{2a}^{x_2 x_1}(t_n) \quad (4.4.10)$$

$$P_{2a}^{x_2 x_3}(t_n) = \mathcal{P}_{\frac{x_1}{x_1}} \left(\frac{\sum_{\chi=B, k \neq 0} + \sum_{\chi=W}}{\sum_{\chi \in \{B, W\}}} \right) \quad (4.4.11)$$

$$P_{2b}^{x_2 x_3}(t_n) = 1 - P_{2a}^{x_2 x_3}(t_n) \quad (4.4.12)$$

$$P_{8a}^{x_2}(t_n) = \mathcal{P}_{\frac{x_1 \times x_3}{x_1 \times x_3}} \left(\frac{\sum \Omega_{8a}([U_{0,0,0}^{(\emptyset,0)}]_{x_2}; x_1, x_3)}{\sum \Omega([U_{0,0,0}^{(\emptyset,0)}]_{x_2}; x_1, x_3)} \right) \quad (4.4.13)$$

$$P_{8b}^{x_2}(t_n) = 1 - P_{8a}^{x_2}(t_n) \quad (4.4.14)$$

where $\Omega([U_{0,0,0}^{(\emptyset,0)}]_{x_2}; x_1, x_3)$ represents

$$\begin{aligned} & \{\mathcal{H}_{x_1}(t_n) | \chi_{x_1} \in \{R_{\bar{x}_2}, A_{\bar{x}_2}\}\} \times \{\mathcal{H}_{x_3}(t_n) | \chi_{x_3} \notin \{C_{\bar{x}_2}, \bar{C}_{\bar{x}_2}\}\} \\ & \cup \{\mathcal{H}_{x_3}(t_n) | \chi_{x_3} \in \{R_{\bar{x}_2}, A_{\bar{x}_2}\}\} \times \{\mathcal{H}_{x_1}(t_n) | \chi_{x_1} \notin \{C_{\bar{x}_2}, \bar{C}_{\bar{x}_2}\}\} \end{aligned}$$

and $\Omega_{8a}([U_{0,0,0}^{(\emptyset,0)}]_{x_2}; x_1, x_3)$ stands for

$$\begin{aligned} & \{\mathcal{H}_{x_1}(t_n) \in \Omega([U_{0,0,0}^{(\emptyset,0)}]_{x_2}; x_1) | j_{x_1} = 0\} \times \{\mathcal{H}_{x_3}(t_n) \in \Omega([U_{0,0,0}^{(\emptyset,0)}]_{x_2}; x_1; x_3) | \\ & \quad (\chi_{x_3}, j_{x_3}) \notin \{(R_{\bar{x}_2}, 1), (A_{\bar{x}_2}, j), j \neq 0, (\chi_{x_3}, k_{x_3}) \neq (B, 0)\}\} \\ & \cup \{\mathcal{H}_{x_3}(t_n) \in \Omega([U_{0,0,0}^{(\emptyset,0)}]_{x_2}; x_3) | j_{x_3} = 0\} \times \{\mathcal{H}_{x_1}(t_n) \in \Omega([U_{0,0,0}^{(\emptyset,0)}]_{x_2}; x_3; x_1) | \\ & \quad (\chi_{x_1}, j_{x_1}) \notin \{(R_{\bar{x}_2}, 1), (A_{\bar{x}_2}, j), j \neq 0, (\chi_{x_1}, k_{x_1}) \neq (B, 0)\}\} \end{aligned}$$

The equilibrium equations for x_1 as $t_n \rightarrow \infty$ are

$$\pi[B_{i,0,0}^{\langle x_2, \infty \rangle}]_{x_1} = p_{1a}^{x_1} \pi[B_{i,1,0}^{\langle x_2, \infty \rangle}]_{x_1} \quad (4.4.15)$$

$$\pi[B_{i,k,0}^{\langle x_2, \infty \rangle}]_{x_1} = \begin{cases} p_{1a}^{x_1} \pi[B_{i,k+1,0}^{\langle x_2, \infty \rangle}]_{x_1} + (1 - p_{1a}^{x_1}) \pi[B_{i,k,0}^{\langle x_2, \infty \rangle}]_{x_1}, & 0 < k < 2^i * 3 \\ (1 - p_{1a}^{x_1}) \pi[B_{i,2^i w, 0}^{\langle x_2, \infty \rangle}]_{x_1}, & k = 2^i * 3 \\ \frac{1}{2^{i*3}} p_{9b}^{x_1 x_2} \pi[B_{i-1,0,0}^{\langle x_2, \infty \rangle}]_{x_1}, & i > 0, k \neq 0 \\ \frac{1}{3} (p_{9a}^{x_1 x_2} \sum_{i=0}^{m-1} \pi[B_{i,0,0}^{\langle x_2, \infty \rangle}]_{x_1} + \pi[B_{m,0,0}^{\langle x_2, \infty \rangle}]_{x_1}), & i = 0, k \neq 0 \end{cases} \quad (4.4.16)$$

$$\pi[D_{x_2/i,k,0}^{\langle x_2, \infty \rangle}]_{x_1} = \dots = \pi[D_{x_2/i,k,7}^{\langle x_2, \infty \rangle}]_{x_1} = \pi[C_{x_2/i,k,0}^{\langle x_2, \infty \rangle}]_{x_1} = \pi[C_{x_2/i,k,1}^{\langle x_2, \infty \rangle}]_{x_1} = p_{1d}^{x_1 x_2} \pi[B_{i,k,0}^{\langle x_2, \infty \rangle}]_{x_1} \quad (4.4.17)$$

$$\pi[A_{x_2^2/0,0,0}^{\langle x_2, \infty \rangle}]_{x_1} = \dots = \sum_{i=0}^m \pi[C_{x_2^2/i,0,1}^{\langle x_2, \infty \rangle}]_{x_1} = \sum_{i=0}^m p_{9a}^{x_1 x_2} \pi[R_{x_2^2/i,0,0}^{\langle x_2, \infty \rangle}]_{x_1} = \dots = \sum_{i=0}^m p_{9a}^{x_1 x_2} \pi[B_{i,0,0}^{\langle x_2, \infty \rangle}]_{x_1} \quad (4.4.18)$$

$$\pi[W_{i,0,0}^{\langle x_2, \infty \rangle}]_{x_1} = \dots = \pi[W_{i,0,2}^{\langle x_2, \infty \rangle}]_{x_1} = p_{9b}^{x_1 x_2} \pi[R_{x_2^2/i,0,0}^{\langle x_2, \infty \rangle}]_{x_1} = \dots = p_{9b}^{x_1 x_2} \pi[B_{i,0,0}^{\langle x_2, \infty \rangle}]_{x_1} \quad (4.4.19)$$

The equilibrium equations for x_2 follows (3.4.1), (3.4.6), (3.4.7):

$$\pi[I_{0,0,0}^{\langle \emptyset, 0 \rangle}]_{x_2} = \frac{1}{1 - p_{1a}^{x_2}} (p_{8a}^{x_2} \pi[U_{0,0,0}^{\langle \emptyset, 0 \rangle}]_{x_2} + \pi[A_{x_1/0,0,0}^{\langle \emptyset, 0 \rangle}]_{x_2} + \pi[A_{x_3/0,0,0}^{\langle \emptyset, 0 \rangle}]_{x_2}) \quad (4.4.20)$$

$$\begin{aligned} \pi[A_{x_1/0,0,0}^{\langle \emptyset, 0 \rangle}]_{x_2} &= \dots = \pi[R_{x_1/0,0,0}^{\langle \emptyset, 0 \rangle}]_{x_2} = p_{2a}^{x_2 x_1} \pi[R_{x_1/0,0,1}^{\langle \emptyset, 0 \rangle}]_{x_2} = p_{2a}^{x_2 x_1} p_{1b}^{x_2 x_1} \pi[I_{0,0,0}^{\langle \emptyset, 0 \rangle}]_{x_2} \\ \pi[A_{x_3/0,0,0}^{\langle \emptyset, 0 \rangle}]_{x_2} &= \dots = \pi[R_{x_3/0,0,0}^{\langle \emptyset, 0 \rangle}]_{x_2} = p_{2a}^{x_2 x_3} \pi[R_{x_3/0,0,1}^{\langle \emptyset, 0 \rangle}]_{x_2} = p_{2a}^{x_2 x_3} p_{1b}^{x_2 x_3} \pi[I_{0,0,0}^{\langle \emptyset, 0 \rangle}]_{x_2} \end{aligned} \quad (4.4.21)$$

$$\pi[U_{i,k,0}^{\langle y, \infty \rangle}]_{x_1} = \frac{1}{p_{8a}^{x_2}} (p_{1e}^{x_2} \pi[I_{0,0,0}^{\langle \emptyset, 0 \rangle}]_{x_2} + p_{8b}^{x_2 x_1} \pi[R_{x_1/0,0,1}^{\langle \emptyset, 0 \rangle}]_{x_2} + p_{8b}^{x_2 x_3} \pi[R_{x_3/0,0,1}^{\langle \emptyset, 0 \rangle}]_{x_2}) \quad (4.4.22)$$

Finally, using the symmetric conditions

$$\pi[\chi_{i,k,j}^{\langle x_2, \infty \rangle}]_{x_1} = \pi[\chi_{i,k,j}^{\langle x_2, \infty \rangle}]_{x_3} \quad (4.4.23)$$

for any i, k, χ, j , and the normalization conditions

$$\sum_{i,k,\chi,j} \pi[\chi_{i,k,j}^{\langle x_2, \infty \rangle}]_{x_1} = \sum_{i,k,\chi,j} \pi[\chi_{i,k,j}^{\langle \emptyset, 0 \rangle}]_{x_2} = \sum_{i,k,\chi,j} \pi[\chi_{i,k,j}^{\langle x_2, \infty \rangle}]_{x_3} = 1 \quad (4.4.24)$$

we form a non-linear system by combing equations (4.4.1) - (4.4.22). The initial conditions for x_1 are similar to the settings in section 4.2. For x_2 , we have

$$P^{(0)}[\chi_{0,0,j}^{\langle \emptyset, 0 \rangle}]_{x_1} = \begin{cases} 1, & \chi = I \\ 0, & \text{otherwise} \end{cases}$$

and $P_{1a}^{x_2}(0) = P_{2a}^{x_2 x_1}(0) = P_{2a}^{x_2 x_3}(0) = P_{8a}^{x_2}(0) = 1$ while the remaining non-trivial transition probabilities are initialized as 0.

The plots in Figure 4.4 show the difference between analytical results and QualNet simulations. When RTS retransmission limit (m) is 0, it is possible to construct

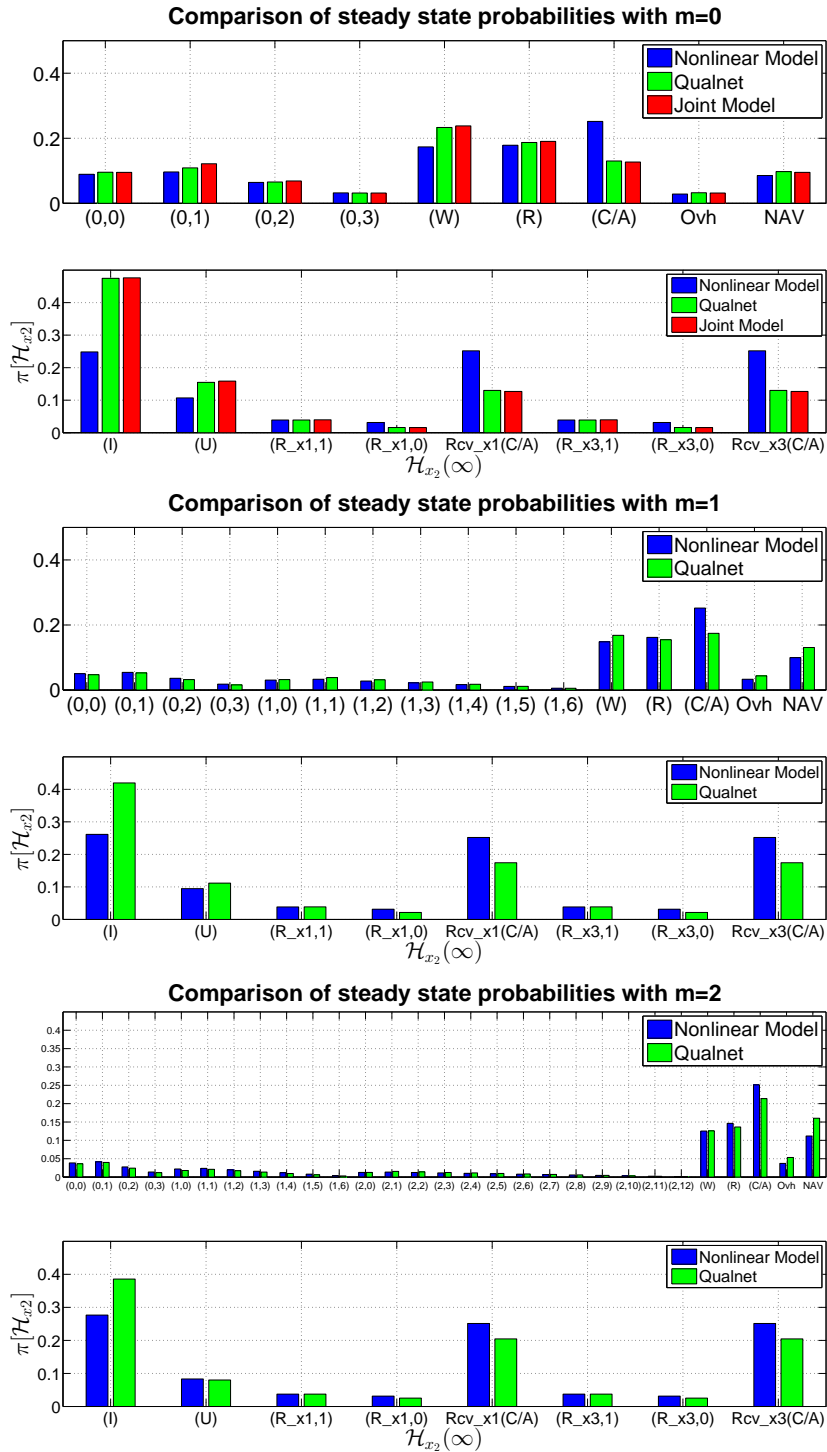


FIG. 4.4. Comparison at x_1 : the tuples denote (back-off stage, back-off counter); 'Rcv' combines states of receiving RTS/ sending CTS/ receiving DATA; 'Ovh' denotes overhearing CTS. Comparison at x_2 : the tuples represent the 1st step or last step of receiving RTS from x_1 or x_3

the stochastic joint state model of x_1 , x_2 and x_3 . The corresponding results are shown by the red bar. The joint model accurately predicts the behaviors of DCF. For the non-linear system, we observe some significant deviation mainly due to the product approximation approach as we bring closure to the system. However, as the complexity of the system increases, i.e. $m = 1, 2$, the results improve.

5. Conclusion. In this paper, we have introduced a new Markov model for the IEEE 802.11 Distributed Coordination Function (DCF), a central mechanism of our wireless infrastructure. Our Markov model does not rely upon the assumption that collision probabilities on each node are constant or independent of network topology. Instead, we have developed a detailed model of interconnected node states including multiple back-off stages and binary exponential back-off counters to capture the dominant first order effects of nodes' responses to contention. The model is complex, but it is necessarily so, and it is not so elaborate that it cannot be analyzed. Using the model, we have calculated steady-state node states for two and three node networks including a configuration that includes a hidden terminals with varying numbers of back-off stages. To determine the transition probabilities for steady-state calculations we approximate the joint probability densities with marginal probability densities using a product approximation. While this only uses a small subset of the information available in the network description, we find it sufficient to achieve excellent agreement with realistic simulations of network traffic.

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