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The standard model in the on-shell scheme*

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Abstract

We outline the renormalization of the standard model to all orders of perturbation theory in a way which does not make essential use of a specific subtraction scheme but is based on the Slavnov-Taylor identity. Physical fields and parameters are used throughout the paper. The Ward-identity for the global gauge invariance of the vertex functions is formulated. As an application the Callan-Symanzik equation is derived.

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The standard model in the on-shell scheme

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We outline the renormalization of the standard model to all orders of perturbation theory in a way which does not make essential use of a specific subtraction scheme but is based on the Slavnov-Taylor identity. Physical fields and parameters are used throughout the paper. The Ward-identity for the global gauge invariance of the vertex functions is formulated. As an application the Callan-Symanzik equation is derived.

1. Introduction

The renormalization of the electroweak standard model is a well studied subject (s.[1,2]). The one-loop approximation is almost complete, two-loop calculations have been started. For the all order treatment however only [3] is available, where the necessary reasoning has been performed in terms of the symmetric variables i.e. fields which would occur without the spontaneous symmetry breaking. But if one wants to support the explicit calculations and e.g. formulate rigid invariance as it is needed in practice then one has to renormalize in the on-shell scheme which means formulate the renormalization in terms of the physical fields. Due to the γ_5-problem dimensional regularization will not be invariant with respect to the symmetry hence a formulation of the model is desirable which does not make explicit use of a specific subtraction scheme, but relies only on general properties like locality and power counting. In the present note we describe a few results of the respective analysis. Specifically we write down the Slavnov-Taylor (ST) identity and discuss its general solution in the classical approximation. As important applications we derive then the rigid invariance and the Callan-Symanzik equation in the one-loop approximation. In order not to overload the paper we have never written down the fermionic sector (i.e. quarks, leptons and their couplings) but mentioned when we made simplifying assumptions (like CP-invariance) concerning them.

2. The classical Lagrangean

The standard model of electroweak interactions is a non-abelian gauge theory with gauge group $SU(2) \times U(1)$. After spontaneous symmetry breakdown one has to identify the physical fields $W^{\pm}_\mu, Z_\mu, A_\mu$ and H (for Higgs) in the bosonic sector, quarks and leptons in the fermionic sector by diagonalizing the mass matrices. At the same step one identifies the currents (charged, neutral, electromagnetic) by constructing the charge eigenstates. For the bosonic sector we have the following classical approximation

$$\Gamma_{YM} = -\frac{1}{4}\int G^{\mu\nu}_a \tilde{I}_{aa'} G_{\mu\nu a'} \tag{1}$$

$$\Gamma_{scalar} = \int (D^\mu\Phi)^\dagger D_\mu\Phi - V(\Phi) \tag{2}$$

$$V(\Phi) \equiv -\frac{1}{8}\frac{m_H^2}{M_W^2} g^2((\Phi^\dagger\Phi)^2 - v^2(\Phi^\dagger\Phi) - \frac{1}{4}v^4) \tag{3}$$

$$\Phi \equiv \begin{pmatrix} \phi^+(x) \\ 1/\sqrt{2}(v + H(x) + i\chi(x)) \end{pmatrix} \tag{4}$$

The field strength tensor and the covariant derivative have their usual form

$$G^{\mu\nu}_a = \partial^\mu V^\nu_a - \partial^\nu V^\mu_a + g\tilde{I}_{aa'}\hat{\varepsilon}_{a'bc}V^\mu_b V^\nu_c \tag{5}$$

$$D_\mu\Phi = \partial_\mu\Phi - gi\frac{\hat{\tau}^a}{2}\Phi V_{\mu a} \tag{6}$$

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We use the summation convention for the roman indices with values $+,-,Z,A$ and have introduced convenient notations. The tensor

$$\hat{\varepsilon}_{abc} = \begin{cases} \hat{\varepsilon}_{+-Z} & = & -i\cos\theta_W \\ \hat{\varepsilon}_{+-A} & = & i\sin\theta_W \end{cases} \qquad (7)$$

is completely antisymmetric and the matrices $\hat{\tau}_a \quad (a = +,-,Z,A)$ form a representation of $SU(2)\times U(1)$ according to

$$\left[\frac{\hat{\tau}_a}{2},\frac{\hat{\tau}_b}{2}\right] = i\hat{\varepsilon}_{abc}\tilde{I}_{cc'}\frac{\hat{\tau}_{c'}}{2} \qquad (8)$$

They are explicity given by ($\tau_i,\ i=1,2,3$ the Pauli matrices)

$$\hat{\tau}_+ = \tfrac{1}{\sqrt{2}}(\tau_1 + i\tau_2) \quad \hat{\tau}_Z = \tau_3\cos\theta_W - \mathbf{1}\frac{\sin^2\theta_W}{\cos\theta_W}$$
$$\hat{\tau}_- = \tfrac{1}{\sqrt{2}}(\tau_1 - i\tau_2) \quad \hat{\tau}_A = -\tau_3\sin\theta_W - \mathbf{1}\sin\theta_W \qquad (9)$$

The matrix $\tilde{I}_{aa'}$ guarantees the charge neutrality of the classical action

$$\tilde{I}_{+-} = \tilde{I}_{-+} = \tilde{I}_{ZZ} = \tilde{I}_{AA} = 1 \qquad (10)$$
$$\tilde{I}_{ab} = 0 \text{ else} \qquad (11)$$

The value $v = 2M_W/g$ for the shift parameter yields the desired masses for the physical fields. We may now turn to the global and local symmetries of the action. By construction we conserved the electric charge and can obviously express this conservation law by

$$\int \quad i\big(W_\mu^+\frac{\delta}{\delta W_\mu^+} - W_\mu^-\frac{\delta}{\delta W_\mu^-} + \Phi^+\frac{\delta}{\delta\Phi^+} - \Phi^-\frac{\delta}{\delta\Phi^-}\big)\Gamma_{GSW} = 0 \qquad (12)$$

$$\Gamma_{GSW} = \Gamma_{YM} + \Gamma_{scalar} \qquad (13)$$

According to Noethers theorem there exists an associated conserved current which can be found via

$$w_{em}\Gamma_{GSW} = -\partial_\mu j^\mu_{em} \qquad \int w_{em} = \mathcal{W}_{em} \qquad (14)$$

with w_{em} being the local charge operator. The explicit calculation yields

$$\partial_\mu j^\mu_{em} = -i\partial^\mu\Big(V_+^\nu G_{\mu\nu -} - V_-^\nu G_{\mu\nu +} + \phi^-(D_\mu\Phi)^+ - \phi^+(D_\mu\Phi)^-\Big) \qquad (15)$$

which according to (14) definitely *is* the electromagnetic current but nevertheless does not have a simple QED-like form because it contains non-abelian contributions in the field strenght $G_{\mu\nu}$ and the covariant derivative $D_\mu\Phi$. The final form of these relations is now obtained by verifying that

$$\partial_\mu j^\mu_{em} = \frac{1}{g\sin\theta_W}\partial^\mu\frac{\delta}{\delta A^\mu}\Gamma_{GSW} \qquad (16)$$

holds true and thus (14) can be brought in the form of the classical electromagnetic current identity

$$\left(w_{em} + \frac{1}{g\sin\theta_W}\partial^\mu\frac{\delta}{\delta A^\mu}\right)\Gamma_{GSW} = 0 \qquad (17)$$

This functional form of Noethers theorem and local gauge invariance permits to identify the coupling of the electromagnetic current to the photon as the elementary charge e

$$g\sin\theta_W = e \qquad (18)$$

It is noteworthy that the coupling g of the abelian factor in $SU(2)\times U(1)$ was not constrained by the algebra but fixed by the assignment of the weak hypercharge and is now identified by (18).

For the weak currents a similar reasoning is possible up to the point where the spontaneous breakdown of the chiral $SU(2)_L$ symmetry intervenes. So the global operator

$$\mathcal{W}^\pm_{cc} \equiv \sqrt{2}\int \hat{\varepsilon}_{ab\mp}V_a^\mu\tilde{I}_{bb'}\frac{\delta}{\delta V^\mu_{b'}} + i\Phi^\dagger\frac{\tau_\mp}{2}\begin{pmatrix}\frac{\overrightarrow{\delta}}{\delta\phi^-} \\ \frac{1}{\sqrt{2}}\frac{\overrightarrow{\delta}}{\delta(H-i\chi)}\end{pmatrix} - i\left(\frac{\overleftarrow{\delta}}{\delta\phi^+},\frac{1}{\sqrt{2}}\frac{\overleftarrow{\delta}}{\delta(H+i\chi)}\right)\frac{\tau_\mp}{2}\Phi \qquad (19)$$

when acting on Γ_{GSW} will not produce zero but soft breaking terms which correspond to the action of an inhomogeneous operator

$$\mathcal{W}^\pm_{cc}\Gamma_{GSW} = -\frac{i}{g}\int\sqrt{2}M_W\frac{\delta\Gamma_{GSW}}{\delta\phi^\mp} \qquad (20)$$

The respective local equation reads

$$w^\pm_{cc}\Gamma_{GSW} = -i\sqrt{2}\frac{M_W}{g}\frac{\delta\Gamma_{GSW}}{\delta\phi^\mp} - \partial_\mu j^{\mu\pm}_{cc}$$

$$= \quad -\sqrt{2}i\frac{M_W}{g}\frac{\delta\Gamma_{GSW}}{\delta\phi^{\mp}} - \frac{\sqrt{2}}{g}\partial^\mu\frac{\delta\Gamma_{GSW}}{\delta W^\mu_\mp} \qquad (21)$$

and the bosonic contributions to the charged weak current comprise also lower dimensional terms whose origin is the spontaneous breaking. Analogously one proceeds for the neutral current. The virtue of (17) and (21) is that they exhibit the currents and their (non-)conservation depending on their type. For the purposes of renormalization and compact writing one incorporates of course the inhomogeneous differential operators into the l.h.s. and defines

$$\mathcal{W}_a \equiv \int \tilde{I}_{aa'}\Big(\hat{\varepsilon}_{bc'a'}V^\mu_b\tilde{I}_{cc'}\frac{\delta}{\delta V^\mu_{c'}} + \Phi^\dagger\frac{i\hat{\tau}_{a'}}{2}\frac{\overrightarrow{\delta}}{\delta\Phi^\dagger} - \frac{\overleftarrow{\delta}}{\delta\Phi}\frac{i\hat{\tau}_{a'}}{2}\Phi\Big) \qquad (22)$$

as Ward identity operators satisfying the algebra

$$[\mathcal{W}_a, \mathcal{W}_b] = \hat{\varepsilon}_{abc}\tilde{I}_{cc'}\mathcal{W}_{c'} \quad a,b,c = +,-,Z,A \qquad (23)$$

The (broken) symmetry of Γ_{GSW} under global transformations is then expressed by the Ward identity

$$\mathcal{W}_a\Gamma_{GSW} = 0 \qquad (24)$$

whereas the current identities assume the form

$$\left(w_a + \frac{1}{g}\tilde{I}_{aa'}\partial^\mu\frac{\delta}{\delta V^\mu_{a'}}\right)\Gamma_{GSW} = 0 \qquad (25)$$

3. Gauge fixing, quantization and the Slavnor-Taylor identity

The importance of the standard model originates from the fact that it is a renormalizable field theory and thus permits consistent higher order calculations. A nessecary prerequisite for this is gauge fixing which very often is done in the terms of the so-called R_ξ-gauges:

$$\begin{aligned} \Gamma_{g.f.} &= \int -\frac{1}{2\xi}F_a\tilde{I}_{aa'}F_{a'} \\ F_+ &\equiv \partial_\mu W^\mu_+ - iM_W\xi_W\phi_+ \\ F_Z &\equiv \partial_\mu Z^\mu - M_Z\xi_Z\chi \\ F_A &\equiv \partial_\mu A^\mu \end{aligned} \qquad (26)$$

Applying the Ward identity operators $\mathcal{W}_a$ to $\Gamma_{g.f.}$ one finds that only $\mathcal{W}_A$ yields zero this being due to charge neutrality. All other Ward identities are broken by the mass terms in (26). This is a fortiori true for the local version of which we give but one example:

$$(ew_{em} + \partial\frac{\delta}{\delta A})\Gamma_{g.f.} = \frac{1}{\xi}\Box\partial A - \frac{ie}{2\xi}\partial^\mu[(\partial^\nu W_{\nu+} - iM_W\xi_W\phi_+)W_{\mu-} - \text{h.c.}] \qquad (27)$$

The terms in $W_\pm$ indicate that ∂A is not a free field hence unitarity cannot be deduced from the local Ward identity. In order to remedy this situation one has to add the Faddeev-Popov ($\phi\pi$) fields c_a, $\bar{c}_a$ ($a = +,-,Z,A$) and to enlarge the gauge sector by a ghost action in such a way that BRS invariance is achieved. The BRS transformations of the physical fields read:

$$\begin{aligned} \mathrm{s}V_{\mu a} &= \partial_\mu c_a + g\hat{\varepsilon}_{abc}V_{\mu b}c_c \\ \mathrm{s}c_a &= -\frac{g}{2}\hat{\varepsilon}_{abc}c_b c_c \\ \mathrm{s}\bar{c} &= -\frac{1}{\xi}F_a \end{aligned} \qquad (28)$$

The complete action is invariant

$$\mathrm{s}\Gamma_{\mathrm{cl}} = 0 \qquad (29)$$

with

$$\Gamma_{\mathrm{cl}} \equiv \Gamma_{\mathrm{YM}} + \Gamma_{\mathrm{scalar}} + \Gamma_{\mathrm{g.f.}} + \Gamma_{\mathrm{ghost}} \qquad (30)$$

BRS invariance turns out to be the relevant invariance for quantization and renormalization because it fixes the interactions amongst the unphysical fields in such a way that the complete action is renormalizable and eventually the physical S-matrix unitary. The proof of renormalizability and unitarity is based on a specific functional form expressing the BRS invariance to all orders: the ST identity. The main reason for the formulation given below is the questionable existence of an invariant regularization for the standard model. One thus prefers not to rely on a very specific regularization scheme and to control the non-linear BRS transformations i.e. their non-trivial renormalization by a suitable technical tool: external fields coupled to the field variations.

$$\Gamma_{\mathrm{ext}} = \int \rho^\mu_a\tilde{I}_{aa'}\mathrm{s}V^\mu_{a'} + \sigma_a\tilde{I}_{aa'}\mathrm{s}c_{a'} + Y_a\tilde{I}_{aa'}\mathrm{s}\phi_{a'} \qquad (31)$$

Since for the scalar fields (like for the vector fields) one cannot stick to naive multiplet structure we label them also by the indices $+, -, Z, A$:

$$(\phi_+, \phi_-, \phi_Z, \phi_A) \equiv (\phi_+, \phi_-, H, \chi) \qquad (32)$$

The other assignments of quantum numbers are such that $\Gamma_{\rm ext}$ has dimension four and is neutral with respect to electric and $\phi\pi$ charges. The ST identity takes then the form

$$s(\Gamma) \equiv \int \frac{\delta\Gamma}{\delta\rho^\mu_a}\tilde{I}_{aa'}\frac{\delta\Gamma}{\delta V_{\mu a'}} + \frac{\delta\Gamma}{\delta\sigma_a}\tilde{I}_{aa'}\frac{\delta\Gamma}{\delta c_{a'}} + B_a\frac{\delta\Gamma}{\delta\bar{c}_a} + \frac{\delta\Gamma}{\delta Y_a}\tilde{I}_{aa'}\frac{\delta\Gamma}{\delta\phi_{a'}} = 0. \qquad (33)$$

and we have extended the classical action to

$$\Gamma = \Gamma_{cl} + \Gamma^{(1)} + \Gamma^{(2)} + \ldots, \qquad (34)$$

the vertex functional ordered according to the number of closed loops. In writing down (33) we have slightly changed the gauge fixing by introducing auxiliary fields B_a of dimension 2 and the BRS transformations

$$\mathrm{s}\bar{c}_a = B_a \qquad \mathrm{s}B_a = 0 \qquad (35)$$

The respective gauge fixing reads

$$\Gamma_{\rm g.f.} \;=\; \int \frac{1}{2}\xi B_a\tilde{I}_{aa'}B_{a'} + B_a\tilde{I}_{aa'}F_{a'} \qquad (36)$$

The BRS transformations are then nilpotent on all elementary fields and $\Gamma_{\rm g.f.} + \Gamma_{\rm ghost}$ is a variation; this simplifies the algebraic analysis to be performed later on.

Quantization and consistent renormalization of the standard model means to establish (33) to all orders of perturbation theory for the 1PI Green functions collected in the functional Γ.

The use of (33) proceeds in two steps: First one has to show that the classical action (which is local) is the general solution of (33) on which one has imposed a set of normalization conditions i.e. one has disposed of a (finite) specific set of parameters. This set must not be changed in higher orders otherwise one has no operator interpretation in a fixed Fock space. Second one has to show that all possible breakings of the ST identity arising in higher orders can be removed by adjusting finitely many local counterterms.

For the purpose of the present paper we restrict ourselves to step one and refer for step two to the literature [3]. There it has been shown (in terms of the "symmetric" variables) that the only true obstruction to (33) is the Adler-Bardeen anomaly which is absent in the standard model family by family due to the doublet structure and colour multiplicity.

In order to simplify for a first trial our task we assume that CP is conserved. This amounts to forbid mixing in the fermion families. We choose the Higgs field to be even, the field χ to be odd under CP. Then we insert into (33) the most general $\Gamma^{\rm gen}_{\rm cl}$ which is neutral with respect to electric and $\phi\pi$ charge and has dimension less than or equal to four. The outcome of the rather lengthy calculation can be summarized as follows: The gauge fixing and ghost part of $\Gamma^{\rm gen}_{\rm cl}$ is completely separated from the rest. The vector and scalar field contributions are self-consistently determined from the external field part, which fixes the transformation properties of the fields. Input parameters are the masses M^2_W, M^2_Z, m^2_H of the vectors and Higgs respectively and one coupling g. If one requires that these parameters do not vanish, the action is uniquely determined up to (non-diagonal) field redefinitions. I.e. $\Gamma^{\rm gen}_{\rm cl}$ is obtained from the standard form by performing the field transformations

$$\begin{array}{ccccccc} V^a_\mu & \Rightarrow & \tilde{z}^V_{aa'}V^\mu_{a'} & \qquad & \rho^a_\mu & \Rightarrow & \rho^\mu_{a'}(\tilde{z}^V)^{-1}_{a'a} \\ \phi^a_\mu & \Rightarrow & \tilde{z}^H_{aa'}\phi_{a'} & & Y_a & \Rightarrow & Y_{a'}(\tilde{z}^H)^{-1}_{a'a} \\ c_a & \Rightarrow & \tilde{z}^G_{aa'}c_{a'} & & \sigma_a & \Rightarrow & \sigma_{a'}(\tilde{z}^G)^{-1}_{a'a} \end{array} \qquad (37)$$

The general form of $\tilde{z}$ is the following

$$\tilde{Z} = \begin{pmatrix} \tilde{z}_{++} & 0 & 0 & 0 \\ 0 & \tilde{z}_{--} & 0 & 0 \\ 0 & 0 & \tilde{z}_{ZZ} & \tilde{z}_{ZA} \\ 0 & 0 & \tilde{z}_{AZ} & \tilde{z}_{AA} \end{pmatrix} \qquad (38)$$

The ST identity is invariant under these field redefinitions. CP-invariance yields $\tilde{z}^H_{ZA} = \tilde{z}^H_{AZ} = 0$, $\tilde{z}^V_{++} = \tilde{z}^V_{--}$, $\tilde{z}^H_{++} = \tilde{z}^H_{--}$, all real.

The parameters $\tilde{z}_{ab}$ have to be fixed by appropriate normalization conditions. As an example we treat the bilinear vector terms of the general action

$$\int -\frac{1}{4}(\partial^\mu V^\nu_a - \partial^\nu V^\mu_a)Z^V_{ab}\,(\partial_\mu V_{\nu b} - \partial_\nu V_{\mu b})$$

$$-\frac{1}{2}V_a^\mu M_{ab}V_{\mu b} \quad (39)$$

where Z^V_{ab} is a symmetric matrix defined by

$$Z^V_{ab} = \tilde{z}^T_{aa'}\tilde{I}_{a'b'}\tilde{z}_{b'b} \quad (40)$$

This equation determines $\tilde{z}_{ab}$ only up to an orthogonal matrix $O(\vartheta)$

$$O(\vartheta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\vartheta & \sin\vartheta \\ 0 & 0 & -\sin\vartheta & \cos\vartheta \end{pmatrix} \quad (41)$$

hence it is convenient to write $\tilde{z}$ as

$$\tilde{z} = O(\vartheta)z \quad (42)$$

with z a symmetric matrix. After these preparations the mass matrix reads

$$(M_{ab}) = M_Z^2 z(m_{ab})z \quad (43)$$

where (m_{ab}) is symmetric with $m_{+-} = \cos^2\theta_W$, $m_{ZZ} = \cos^2\theta_V$, $m_{ZA} = \sin\theta_V\cos\theta_V$, $m_{AA} = \sin^2\theta_V$ and $\cos^2\theta_W = M_W^2/M_Z^2$. Continuing the example in the neutral sector we impose now the normalization conditions [4]

$$\begin{array}{rcccl} \partial_{p^2}\Gamma^T_{ZZ}(p^2=M_Z^2) & = & 1 & \Rightarrow & z_{ZZ} \\ \partial_{p^2}\Gamma^T_{AA}(p^2=0) & = & 1 & \Rightarrow & z_{AA} \\ \Gamma^T_{AZ}(p^2=0) & = & 0 & \Rightarrow & \theta_V \\ \Gamma^T_{AZ}(p^2=M_Z^2) & = & 0 & \Rightarrow & z_{AZ} \\ \Gamma^T_{ZZ}(p^2=M_Z^2) & = & 0 & & \end{array} \quad (44)$$

They separate the Z-boson and the photon on-shell and identify the photon as the massless particle. As the explicit one-loop calculations have shown one needs precisely these parameter redefinitions found here in the general solution of the ST identity if one wants to adjust the non-local mixing terms between Z_μ and A_μ.

4. Ward-identities of global symmetries

In section 2 we have indicated the global symmetries valid for the classical action of the standard model, which are however violated in the course of quantization by the mass terms of the gauge fixing. In higher orders the splitting up into gauge fixing and physical part becomes nontrivial, because the breaking can be inserted in any loop diagram with internal vector and scalar lines, but still is soft, i.e. it vanishes for momenta larger than all masses. In order to formulate rigid symmetries in higher orders systematically and independently of the scheme we couple the soft breaking to external non-propagating fields, so that the symmetry is formally restored. Explicitly: First we enlarge the gauge fixing by an external scalar doublet

$$\hat{\Phi} = \begin{pmatrix} \hat{\phi}^+ \\ \frac{1}{\sqrt{2}}(\hat{v} + \hat{H} + i\hat{\chi}) \end{pmatrix} \quad (45)$$

with the same quantum numbers as the scalar doublet Φ and a shift in its CP-even neutral component $\hat{H}$. Then we write

$$\Gamma_{\rm g.f.} = \int (\tfrac{1}{2}\xi B_a\tilde{I}_{ab}B_b + B_a\tilde{I}_{ab}\partial V_b - \frac{ie}{\sin\theta_W}(\hat{\Phi}^\dagger\frac{\hat{\tau}_a}{2}B_a\Phi - \Phi^\dagger\frac{\hat{\tau}_a}{2}B_a\hat{\Phi}) \quad (46)$$

with $\hat{\tau}_a$ given in (8) and (9).

Equ. (46) is written in a manifestly invariant form [5]. It reduces to the original gauge fixing for vanishing external fields, because it is the shift of the external Higgs, which produces the gauge fixing of the quantum fields

$$\hat{v} = \frac{2}{e}\zeta\cos\theta_W\sin\theta_W \quad \text{and} \quad \zeta_W = \zeta_Z = \zeta \quad (47)$$

Enlarging $\mathcal{W}_a$ by the transformation of $\hat{\Phi}$ and B_a, the latter transforming in the same way as the vectors V_a^μ, $\Gamma_{\rm g.f.}$ is seen to be invariant. Finally we read off from the BRS-transformations that the $\phi\pi$-ghosts transform also like the vectors, and from the ST identity we read off the transformation of the respective external fields. We thus arrive at

$$\mathcal{W}_a\Gamma_{\rm cl} = 0 \quad (48)$$

$$\mathcal{W}_a = \tilde{I}_{aa'}\int \hat{\epsilon}_{bca'}V_b^\mu\tilde{I}_{cc'}\frac{\delta}{\delta V^\mu_{c'}} + \{\rho^\mu,\sigma,c,B,\bar{c}\} + i\Phi^\dagger\frac{\hat{\tau}_{a'}}{2}\frac{\overrightarrow{\delta}}{\delta\Phi^\dagger} - i\frac{\overleftarrow{\delta}}{\delta\Phi}\frac{\hat{\tau}_{a'}}{2}\Phi + \{Y,\hat{\Phi},q\} \quad (49)$$

and having formulated the symmetry as a Ward identity one again can interpret $\Gamma_{\rm cl}$ as the lowest order of the functional of 1PI Green functions and deduce consequences for the 1-loop order by using the action principle. It tells us that the WI is at most broken by local contributions, hence all the non-local contributions sum up to symmetry. It is clear that such identities constitute important consistency checks in concrete calculations.

How to proceed for higher orders? Neglecting for the moment the fermion sector with the parity violating interactions we remain with the Higgs-vector model and know that dimensional regularization is an invariant scheme. Then from the general classical action one reads off that the WI in its classical form is valid for the bare fields. The renormalized Green functions satisfy therefore deformed WI which have 1-loop corrections due to the field renormalizations (37). For the vector part this reads:

$$\mathcal{W}_a = \tilde{I}_{aa'} \int V^\mu_b z_{bb'} \hat{\epsilon}_{b'c'a'}(\theta_W + \theta_V)\tilde{I}_{c'c} z_{cd} \frac{\delta}{\delta V^\mu_d} \qquad (50)$$

with $\theta_V = O(\hbar)$, $z \equiv z^v = \tilde{I} + \delta z^{(1)}$ and $\delta z^{(1)}$ symmetric according to the definition (42).

For the complete theory this simple argument does no longer hold and one has to carry out the proof of rigid symmetry constructively by algebraic consistency. The main point is the observation that the deformed Ward operators (50) are derived from an equivalence transformation and are the unique CP-odd solution of the $SU(2) \times U(1)$ algebra (23). Then a deformed WI can be established including the fermion sector to all orders of perturbation theory.

5. The Callan-Symanzik equation

Finally as an application of rigid symmetry we derive the Callan-Symanzik (CS) equation of the standard model in the spontaneously broken phase. The CS equation describes the behaviour of the Green functions under dilatations. The hard breakings of dilatations which appear due to the existence of asymptotic logarithms are the β-functions and anomalous dimensions. These functions are constrained in one loop by the global symmetry of the classical action.

The soft breaking of dilatations is in the classical approximation given by the mass terms, in higher orders it can be constructed along the lines of [6].

According to the action principle one has in 1-loop order

$$\underline{m}\partial_{\underline{m}}\Gamma = [\Delta_m]^3_3 \cdot \Gamma + \alpha_i \Delta_i \qquad (51)$$

where $\underline{m}\partial_{\underline{m}} = M_Z\partial_{M_Z} + M_W\partial_{M_W} + m_H\partial_{m_H} + \kappa\partial_\kappa$ sums all massive parameters of the theory. Δ_i is a basis of all 4-dimensional polynomials in the fields of the standard model. From BRS invariance one can deduce that these local polymials can be written as differential operators acting on the classical action. From algebraic consistency with rigid symmetry one finds that the hard breakings are moreover globally symmetric:

$$\mathcal{W}_a\Delta_i \longrightarrow 0 \quad \text{for asymptotic momenta} \qquad (52)$$

This entails relations for the coefficient functions. The final form of the 1-loop CS equation is given by:

$$\begin{aligned}
&\Big(\underline{m}\partial_{\underline{m}} + \beta_e e\partial_e + \beta_{m_H}\partial_{m_H} - 2\hat{\gamma}_{\hat{\xi}}\partial_{\hat{\xi}} \\
&+\beta_{M_W}\big(\sin\theta_W \partial_{\frac{M_W}{M_Z}} \\
&\qquad\qquad + \int (Z^\mu\delta_{A^\mu} - A^\mu\delta_{Z^\mu} + \{B, \bar{c}, c\})\big) \\
&-\gamma^V \int \Big(V^\mu_a\delta_{V^\mu_a} - B_a\delta_{B_a} - \bar{c}_a\delta_{\bar{c}_a} + 2\xi\partial_\xi\Big) \\
&-\hat{\gamma}^V \int \big((A^\mu + \tfrac{\sin\theta_W}{\cos\theta_W}Z^\mu)(\delta_{A^\mu} + \tfrac{\sin\theta_W}{\cos\theta_W}\delta_{Z^\mu}) - \{B, \bar{c}\}\big) \\
&-\hat{\gamma}^g \int \big(c_A + \tfrac{\sin\theta_W}{\cos\theta_W}c_Z\big)\big(\delta_{c_A} + \tfrac{\sin\theta_W}{\cos\theta_W}\delta_{c_Z}\big) \\
&-\gamma^g \int c_a\delta_{c_a} - \gamma^s \int \phi_a\delta_{\phi_a}\Big)\Gamma = [\Delta_m]^3_3 \cdot \Gamma \qquad (53)
\end{aligned}$$

This form is valid for all tests with respect to propagating fields. To simplify the writing the external fields have been put to zero.

There are two remarks concerning the CS equation: In the spontaneously broken phase the CS equation contains the β-functions with respect to the on-shell masses of the theory. A similar feature we have already discussed in the $U(1)$-axial model with fermions [7].

The anomalous dimensions $\hat{\gamma}$ appear in connection with the abelian subgroup of the standard model. There we have to introduce also the differentiation with respect to the abelian gauge parameter $\hat{\xi}$;

$$\int \hat{\xi}\big(\frac{\sin\theta_W}{\cos\theta_W}B_Z + B_A\big)^2 \tag{54}$$

This gauge fixing contains non-diagonal contributions and is generally taken to be zero in lowest order. The fact, that it has to be introduced into the CS equation with a nonvanishing coefficient $\hat{\gamma}_{\hat{\xi}} = (\beta_e - 2\beta_{M_W}\frac{\sin\theta_W}{\cos\theta_W})\hat{\xi} + \hat{\gamma}^V\xi$ shows that the choice $\hat{\xi} = 0$ is not stable under renormalization.

Finally we want to mention that there is a relation between the β-functions and anomalous dimensions due to the abelian subgroup:

$$\beta_e = \beta_{M_W}\frac{\sin\theta_W}{\cos\theta_W} + \gamma^V + \hat{\gamma}^V\frac{1}{\cos^2\theta_W} \tag{55}$$

6. Conclusions

The ST identity governs the construction of gauge theories. It defines the model in question once the symmetry group and the multiplets are given; it then determines the rigid invariance which embodies the remaining physical symmetry. Within the standard model we have sketched as an example how the general solution of the classical approximation exhibits the free parameters which have to be fixed by normalization conditions or can partly be constrained by the rigid invariance. The expression of the rigid symmetry in terms of a Ward identity turns out to be non-trivial. The effect of the classical symmetry to the one-loop approximation and the CS equation has been indicated.

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