Đorđe Šuvaković Olgin and Goran Radosavljević*

MONOPSONY IN THE LABOR MARKET: PROFIT VS. WAGE MAXIMIZATION**

MONOPSON NA TRŽIŠTU RADA: MAKSIMIRANJE PROFITA PREMA MAKSIMIRANJU PLATA

ABSTRACT: *This paper compares the efficiency of profit- and wage-maximizing (PM and WM) monopsony in the labor market. We show that, both locally and globally, a PM monopsony may well be dominated by its WM twin, where the local and global dominance are defined with respect to a single (inverse) labor supply function and a single family of such functions. This family is always divided in the two disjoint (sub)families of the PM and WM dominance. We also analyze some major factors that explain the size of these (sub)families.*

KEY WORDS: *Monopsony; Labor Market; Profit Maximization; Wage Maximization*

APSTRAKT: *U ovom radu se poredi efikasnost monopsona na tržištu rada. Posmatraju se dva slučaja: monopson koji maksimizira profit (PM) i monopson koji maksimizira nadnice (WM). Pokazano je da WM monopson može da dominira, lokalno i globalno, nad PM monopsonom, pri čemu su lokalna i globalna dominacija definisane uzimajući u obzir jednu (inverznu) funkciju ponude rada i familiju takvih funkcija. Ta familija je uvek podeljena na dve disjunktne (pod)familije, tako da u okviru jedne (pod)familije dominira PM, a u okviru druge WM monopson. Analiziraju se i glavni faktori koji utiču na veličinu tih (pod)familija.*

KLJUČNE REČI: *monopson, tržište rada, maksimizacija profita, maksimizacija nadnice*

*   Foundation for the Advancement of Economics, Belgrade

Đorđe Šuvaković Olgin and Goran Radosavljević

1. Introduction

After a period of neglect, monopsony in the labor market seems to be regaining its place, both in the labor and industrial economics[1].

Thus, recent contributions focus either on the empirical relevance of monopsonistic behavior (Boal and Ransom, 1997; Manning, 2003a; 2003b; Staiger *et al*, 1999) or on theoretical explanations of the emergence of such behavior (Bhaskar *et al*, 2004; Boal and Ransom, 1997; Manning, 2003a; 2003b).

Our emphasis is different. We take for granted the existence of monopsonistic labor markets but concentrate on their comparative efficiency due to supposedly different objectives of firms that operate on such markets.

It is normally assumed that monopsonistic enterprises act as profit maximizers. The purpose of this paper is to examine the impact of wage-maximizing (WM) behavior - frequently analyzed in the corresponding literature[2] - on the efficiency of monopsonistic labor markets, as well as to compare this efficiency with that of labor markets populated with profit-maximizing firms (PMFs).

In this connection it should be emphasized that WM firms (WMFs) are most often identified with the Western type producer cooperatives and partnerships in the service sector (see, for example, Dreze, 1989; Pencavel and Craig, 1994).

Recently, such enterprises have also been linked with the many insider-controlled firms that have emerged during the privatization process in the post-socialist countries (Blanchard, 1997; IBRD, 1996; Roland, 2000).

In a sense, the present paper follows on from Domar's (1966) early analysis of a WM monopsony in the labor market. However, unlike Domar, we restrict ourselves to a systematic comparison of the efficiency of monopsonistic WMF and PMF, when they operate in the labor market.

---

[1] Valuable recent sources that assess the relevance of the non-wage-taking phenomenon are Boal and Ransom (1997; 2002) and Manning (2003a; 2003b).

[2] The comprehensive survey of the vast literature on WM firms is Bonin and Putterman (1987). For a concise review see Bonin, Jones and Putterman (1993).

The structure of the paper is as follows.

In Section 1 of Part II, we first define a typical family of increasing (inverse) labor supply or wage functions, which enable both PMF and WMF to earn nonnegative profit under convex technology.

To motivate the reader, we then give a graphic illustration of this family to focus, in Section 2 of Part II, on the member of the family of wage functions that yields exactly the Pareto optimal equilibrium of a no loss making WMF. Such a function always exists. This also means that in the case considered a WM monopsony Pareto dominates its PM twin, since the latter - apart from the case of perfect wage discrimination - can never reach the Paretian norm.

In Part III we represent the PM monopsony equilibrium in the form appropriate for straightforward efficiency comparisons. Then, we formally characterize the two types of WM monopsony equilibrium, evoked by Domar (1966).

In Part IV we show that the family of wage functions, defined in part II, is always divided, by some neutral member-function, into an upper and a lower subfamily, where the former ensures the dominance of a WMF over PMF, while within the latter the converse is true.

In Part V we discuss the results of systematic numerical simulations, performed to get an insight into the relative size of the WMF and PMF dominance regions as well as into the sensitivity of this size to the shape of labor's marginal product curve, the curvature of labor supply functions, and the level of the entry wage. In Appendix, we graphically present the three arguably most representative exercises, which test the sensitivity of the WMF/PMF dominance relation to the degree of curvature of the labor supply functions, in the "neutral" case of linear marginal product of labor.

Summary and concluding remarks, where the latter also address the issue of how to privatize a non-wage-taking firm, are left for Part VI.

2. A Typical Family of Wage Functions and the Case When a WM Monopsony Pareto Dominates its PM Twin

2.1. The S family of inverse labor supply or wage functions

In order to define the particular one-parameter family of inverse labor supply or wage functions that we consider and which yields nonnegative profit for a non-wage-taking firm, we first introduce the function of firm's (non-capital) income per unit of labor or a 'full' wage, y:

$$y = \frac{X(L) - C}{L} \tag{1}$$

where $X(L)$ and L are the short-run production function and the labor input respectively, and where, by suitably choosing the measure of X, its (constant) price, p, is normalized to unity. Finally, C stands for fixed (capital) costs.

The reader familiar with the theory of a WMF - see, for example, Dreze (1989), Bonin and Putterman (1987) - will recognize in (1) the objective function of such an enterprise. Here, the y function - depicted in Figures 1 and 2 below - will, inter alia, serve to define the steepest wage curve that yields zero profit both to a WM and conventional PM firm.

In the monopsonistic labor market a typical (inverse) labor supply or wage function faced by a firm may be represented as:

$$W_k = f(L, a_k) \equiv W_k(L) , L > 0 , \tag{2}$$

where W_k is a wage rate or a supply price of labor, L is a firm's demand for labor or potential labor supply and a_k is (nonnegative) parameter, which represents a measure of labor scarcity experienced by a firm.

In what follows a_k will be (discretely) varied so as to cover all relevant degrees of labor scarcity - see relation (3) and the ensuing relations (4) - (5a).

The f function is further characterized as follows:

$$\frac{\partial f}{\partial L} \equiv f' > 0 \tag{2a}$$

$$\frac{\partial^2 f}{\partial L^2} \equiv f'' \geq 0 \tag{2b}$$

$$\frac{\partial f}{\partial a_k} \equiv f_{a_k} > 0 \tag{2c}$$

$$\frac{\partial f'}{\partial a_k} \equiv f'_{a_k} > 0 \tag{2d}$$

While the interpretation of (2a)-(2c) in straightforward, (2d) assumes that an increase in labor scarcity leads to a greater increase in the wage rate, given any (infinitesimal) increase in the demand for labor.

By varying the a_k parameter within the interval defined in (3) below, we obtain the one-parameter family of W_k functions, denoted by S:

$$S = \{W_k = f(L, a_k) \equiv W_k(L) , a_k \in (a_e, a_z)\} , \tag{3}$$

where the S family is bounded from below by the horizontal entry-wage schedule W_e, depicted in Figure 1[3],

$$W_e = f(L, a_e) = const > 0 \tag{4}$$

In (4) the a_e value of a_k generates the equilibrium labor use, L_e, by a wage taking PMF:

$$L_e = \arg [X'(L) - f(L, a_e) = 0], \quad \forall a_e \mid f(L, a_e) = const > 0 \tag{4a}$$

---

[3]   Note that in Figure 1, as in most of the numerical simulations of part V, we assume, for simplicity, that the entry-wage is insensitive to the value of the a_k parameter.

On the other hand, the upper boundary of S reduces to the function $W_z = f(L, a_z)$, which simultaneously satisfies the following two conditions, i.e., defines the function that has the tangency point with $y=y(L)$:

$$W_z(L) \equiv f(L, a_z) = y(L) , \quad W_z'(L) \equiv \frac{\partial f(L, a_z)}{\partial L} = y'(L) \tag{5}$$

In (5), $L = L_z$ represents the corresponding equilibrium employment, implied by a_z, which, following eq. (5), satisfies:

$$L_z = \arg[W_z'(L) - y'(L) = 0] , \quad L_z = \arg[W_z(L) - y(L) = 0] \tag{5a}$$

Thus, as already stated, eq. (4) defines the hypothetical case of a wage-taking enterprise - i.e., of zero labor scarcity faced by a single firm - while relation (5) defines the steepest relevant wage function which, by definition, yields zero profit both to a PMF and WMF - see the W_z curve of Figure 1.

Figure 1. The S family of wage functions of (3), represented by the shaded area bordered by the horizontal line W_e of (4) and the steepest relevant wage curve, W_z, of (5).

- $M_m = W_m + LW'_m$ is marginal cost of monopsonistic labor, obtained from the wage function W_m, displayed below, and depicted in Figure 2;
- the functions y and X' are those of (1) and (7);
- L_z and L_e are respectively from (5a) and (4a), while the PMF labor use may be written as $L_m^{\Pi} = \arg \max [\Pi(L) \equiv X(L) - LW_k(L) - C]$, *cf.* eq. (7a);

Figure 1 is obtained using the following parameter forms:

- $X' = 3.5 - 0.6L^2$, $C = 2.85$, $y = 3.5 - 0.2L^2 - C/L$, $W_e = 0.4$, $W_n = 0.4 + 0.0803L^2$, $W_m = 0.4 + 0.237L^2$, $W_z = 0.4 + 0.344L^2$, $M_m = 0.4 + 0.711L^2$

Note finally that, when coupled with the inequality $L > 0$ of (2), eqs. (2a)-(2d), which describe one well-behaved (inverse) labor supply or wage function, also ensure that any two member-functions of the S family do not have any common point. Among other things, this means that any member-function divides S in the two disjoint subfamilies.

2.2 The Pareto dominance of a WM monopsony over its PM twin: The graphical example

As we said, in this section we provide a particular numerically generated example in which a WM monopsony achieves exactly the Pareto optimum and thus, by definition, Pareto dominates its PM twin, since the latter, except in the case of perfect wage discrimination, can never reach the Paretian norm.

When labor is the only variable input, the WMF maximand of income-per-worker, y, is defined as in eq. (1) above.

At the same time, the unconstrained maximum of y in L is defined by the well-known WMF equilibrium condition, $X'(L) = y(L)$, displayed in relation (14) below, where $X'(L)$ and $y(L)$ are, respectively, labor's marginal product and income-per-labor-unit functions, and where the product price, p, is taken to be the numeraire, $p = 1$.

Suppose now that one family-member wage function - denoted by $W_m(L)$, defined in eq. (12), and depicted in Figure 2 – intersects the $y(L)$ function of eq. (1) just at its maximum, L_m^y, of eq. (12a).

In this case, we will have:

$$W_m(L) = X' \tag{5b}$$

Thus, for the $W_m(L)$ wage function, the WM monopsony equilibrium, L_y^m, coincides with the Pareto optimum, L_y^P - see also Figure 2. Hence, by definition, a WMF Pareto dominates PMF, and, in the case considered, a WMF produces (about) 12% more output than its PMF twin.

3. The Equilibrium of a PM and WM Monopsony: A general comparison

3.1. The two forms of the PM monopsony equilibrium

Starting from (2), the economic profit of the PM monopsony, Π may be represented as:

$$\Pi = X(L) - LW_k(L) - C , \tag{6}$$

where $X(L)$ and C appear in (1).

The standard first order condition for the maximum of Π in L reads:

$$X'(L) - [W_k(L) + LW'_k(L)] = 0 , \tag{7}$$

where X' and $W'_k(\equiv f')$ respectively denote the first derivatives of X and W_k with respect to L, and where, from (6), the value of L which achieves the maximum of Π may be written as:

$$L^\Pi = \arg\max [\Pi(L) \equiv X(L) - LW_k(L) - C] \tag{7a}$$

The standard PM monopsony equilibrium condition, obtained from (7), reads:

$$X'(L) = W_k(L) + LW'_k(L) \equiv M_k(L) , \tag{8}$$

where the R.H.S. of eq. (8) is the marginal labor cost of a PM monopsony, denoted by $M_k(L)$.

To simplify the efficiency comparison - which will be performed in Part IV below - we will write the PMF equilibrium of (8) in the form

$$X'(L) - LW'_k(L) \equiv g_k(L) = W_k(L) \tag{8a}$$

Where, obviously, we have:

$$g_k(L) < X'(L) , L>0 , \tag{8b}$$

$$\frac{\partial g_k}{\partial L} \equiv g'_k(L) < 0 , \; L \geq 0$$

Finally, the corresponding second order condition, derived from (7), is

$$X'' - 2W'_k - LW''_k < 0 \tag{9}$$

where X'' and $W''_k (\equiv f'')$ respectively denote the second derivatives of X and W_k.

3.2. The WM monopsony constrained equilibrium

Depending on the degree of labor scarcity, the monopsonistic WMF is characterized by the two types of equilibrium, initially considered, though for different reasons, by Domar (1966).

The first type of equilibrium is the constrained one. Here the wage function $W_k(L)$ is binding on the maximum of $y(L)$ - see point C in Figure 2 below - where this maximum reduces to

$$L_C^y = \sup \arg[y(L) - W_k(L) = 0], \tag{10}$$

where:

$$W_k(L) \equiv f(L, a_k) , a_k \in (a_m, a_z) \tag{10a}$$

Thus a WMF attains the constrained maximum L_c^y of (10) within the open interval

$$L_z < L_c^y < L_m^y \, , \tag{11}$$

where L_z and L_m^y are given in (5a) and (12a), while the a_m value of the labor scarcity parameter of (10a) – which generates the maximum of y in L - is defined as follows:

$$a_m \mid f(L, a_m) \equiv W_m(L) = y(L_m^y), \tag{12}$$

where in (11) the maximum of y in L has been denoted by L_m^y, i.e.:

$$L_m^y = \arg \max y(L) \tag{12a}$$

Finally, the entire subfamily of $W_k(L)$ functions of (10a) - which is a subfamily of S - may be written as:

$$S_y^c = \{W_k(L) \mid W_m(L) < W_k(L) < W_z(L)\} \, , \tag{13}$$

where $W_m(L)$ is of (12), and where the $W_z(L)$ function is defined in (5).

As will be demonstrated in Part IV, within the S_y^c subfamily, a WM monopsony exhibits greater efficiency than a PM monopsony - compare also the typical WMF and PMF equilibrium points generated within this family, and respectively denoted by C and G in Figure 2 below.

3.3. The WM monopsony unconstrained equilibrium

The second type of WM monopsony equilibrium is obtained when the wage function $W_k(L)$ is not binding on the maximum of $y(L)$ of (1):

$$X' = \frac{X(L) - C}{L} \equiv y \tag{14}$$

The subfamily of wage functions, that yield the WMF unconstrained equilibrium of (14), is generated by varying the a_k parameter between its values a_e and a_m, of (12) and (4a), and will be denoted by S^u:

$$S^u = \{W_k(L) \mid W_e < W_k(L) < W_m(L)\}, \tag{14a}$$

As will be shown in Part IV, within the part of the S^u subfamily - denoted by S^u_y - a WM monopsony efficiency dominates its PM twin, just as within the WMF constrained equilibrium region, S^c_y, defined in the previous section.

The 'wage-maximizing' S^u_y subfamily just introduced can be presented via the typical wage function W_k,

$$W_k = W_k(L) \equiv f(L, a_k), \ a_k \in (a_n, a_m], \tag{15}$$

$$S^u_y = \{W_k(L) \mid W_n(L) < W_k(L) < W_m(L)\}, \tag{16}$$

where $W_m(L)$ is from (12), and where $W_n(L)$ - which will be labeled the neutral wage function - may be written as:

$$W_n = f(L, a_n) \equiv W_n(L) \tag{17}$$

Figure 2. The PM, the WM constrained, and the Paretian PM equilibrium, respectively represented by the points G, C, and P. The point M also defines both the WM unconstrained and the (identical) Paretian equilibrium
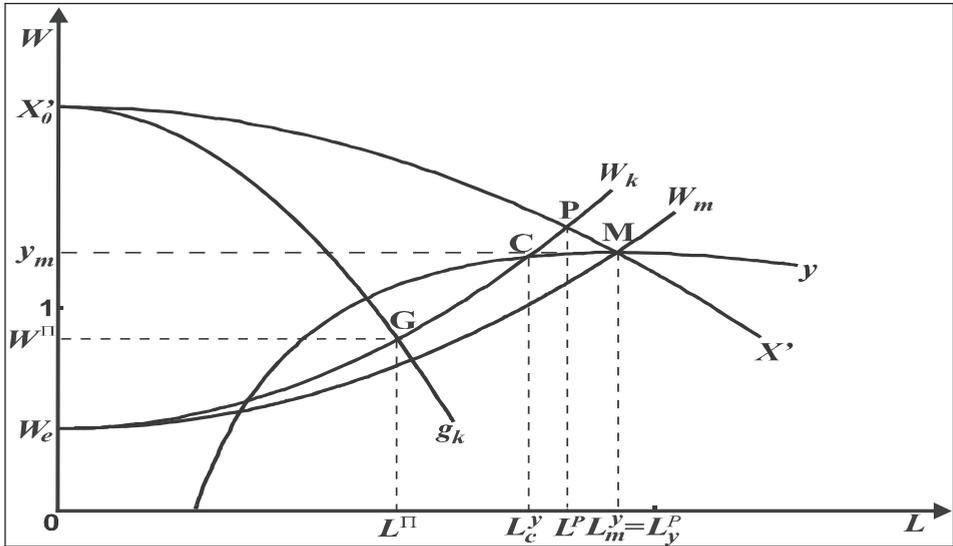
- the PMF equilibrium employment, L^{Π}, is from (7a);

- the WMF constrained, L_c^y, employment equilibrium is from (10);

- the Pareto optimal labor use, L^P, is from (21) below;

- the M point is defined by the unconstrained maximum of y, L_m^y, given in (14a), and the corresponding (maximal) value of $y = y_m$, M=(L_m^y, y_m);

- the Pareto optimal labor use L_y^P, is identical in the case considered with the optimal labor use by a WM monopsony, L_m^y, defined by (12a) - see also eq. (5b) of Part II;

- the functions W_k, W_m and g_k are respectively from (2), (12) and (8a); other functions are already depicted in Figure 1.

The required feature of the neutral $W_n(L)$ function, where this feature is demonstrated in subsection 1.2 of Part IV[4], is that it generates the maximum of $\Pi(L)$, L_n^Π, equal to the *un*constrained maximum of y, given in (12a):

$$
\begin{aligned}
L_n^\Pi &= \arg\ \max\big[X(L) - LW_n(L) - C\big] \\
&= L_m^y
\end{aligned}
\tag{18}
$$

Thus, a_n of (17) represents the degree of labor scarcity which yields the same equilibria for a PMF and WMF. The linear version of $W_n(L)$ is depicted in Figure A1 of Appendix.

In what follows, we will denote by S_y the family that consists of disjoint subfamilies S_y^c and S_y^u, where, by analogy with the set notation[5], we can write

$$
S_y = S_y^c \cup S_y^u,\ S_y^c \cap S_y^u \equiv \emptyset
\tag{19}
$$

Given the definitions of S_y^c and S_y^u of (13) and (16), the S_y subfamily may also be written as:

$$
S_y = \{W_k = f(L, a_k) \equiv W_k(L)\, ,\ a_k \in (a_n, a_z)\}\, ,
\tag{19a}
$$

where a_z is that given by (5).

Finally, we introduce the remaining subfamily of S, denoted by S_Π, and making again the analogy with the set notation, we may write

$$
S_\Pi = S \setminus S_y
\tag{20}
$$

or, via the typical wage function W_k,

$$S_{\Pi} = \{W_k = f(L, a_k) \equiv W_k(L) , a_k \in (a_e, a_n)\}, \tag{20a}$$

where a_e and a_n respectively appear in (4) and (17).

4. The Alternating Dominance of a WM and PM Monopsony

4.1. The dominance of a WMF over PMF within the S_y subfamily of S

The dominance of a WMF over PMF within the S_y^c subfamily of S

In the model, for the typical wage function of *S*, the Pareto optimal employment, L^P, depicted in Figure 2, is defined by the standard equation of Pareto optimum in the labor market:

$$L^P = \arg[X'(L) - W_k(L) = 0] , \quad W_e \leq W_k(L) \leq W_z(L) \tag{21}$$

At the same time, the local dominance of a WMF over PMF, or *vice versa*, is defined as follows:

Definition 1 - The Local Efficiency Dominance - Given the *S* family of wage functions, which all yield nonnegative profit both to a WMF and PMF, a WMF (PMF) is defined to locally efficiency dominate a PMF (WMF) if, for some function in *S*, a WMF (PMF) employs more labor, and thus produces more output and makes a greater total surplus than a PMF (WMF).

Now, starting from (1), we may write the first derivative of $y(L)$ as:

$$y'(L) \equiv \frac{X'(L) - y(L)}{L} \tag{22}$$

Also, solving (22) for $X'(L)$ and substituting the latter into (8a), we can write the PMF equilibrium of (8a) in the form appropriate for a direct comparison of firms' employment and output, and thus of efficiency:

$$W_k(L) = y(L) - L[W'_k(L) - y'(L)] \equiv g_k(L) , \tag{23}$$

where in (23) the $g_k(L)$ function appears in a slightly different form than in (8a).

On the other hand, within the S_y^c subfamily of (13), the WMF constrained equilibrium from (10) always satisfies the condition - see also point C in Figure 2:

$$W_k(L) = y(L) \tag{24}$$

Now, within the relevant interval, given in (11), we have:

$$W_k'(L) > y'(L) > 0 \quad , \quad L \in \left(L_z, L_m^y \right) \tag{25}$$

Hence, due to (25), it follows that in (23) the $g_k(L)$ function satisfies the following inequality:

$$g_k(L) < y(L) \, , \ L \in \left(L_z, L_m^y \right) \tag{26}$$

The monopsonistic PMF equilibrium, L^Π, obtained via the $g_k(L)$ function, is depicted in Figure 2 above, where the G point of this Figure may be written as G $= (L^\Pi, W^\Pi)$, where:

$$L^\Pi = \arg \left[g_k(L) - W_k(L) = 0 \right] , \ W^\Pi = W_k(L^\Pi) \tag{27}$$

Now, since W_k is increasing in L, and due to (24), (25), (26) and (21), for any wage function of S_y^c, a WMF has a higher employment, L_c^y (and output) than a PMF, L^Π, though less than the Pareto optimal one (L^P):

$$L^\Pi < L_c^y < L^P , \tag{28}$$

where L_c^y is given in (10).

Finally, we summarize relation (28) by the following lemma:

Lemma 1. Within the S_y^c subfamily of wage functions a WMF efficiency dominates a PMF.

The dominance of the WMF over PMF equilibrium within the S_y^c subfamily of wage functions is depicted in Figure 2 above.

The dominance of a WMF over PMF within the S_y^u subfamily of S

Now we focus on the S_y^u subfamily of wage functions - defined in (16) - which allow a WM monopsony to reach its unconstrained equilibrium but, as will be easily seen, still ensure the dominance of such a firm over its conventional PM twin.

A curious feature of the upper boundary function of S_y^u which, as already pointed out, is $W_m(L)$, is that it generates the identity of the Pareto optimal and WM equilibrium - due to (12a), (14) and (12) :

$$X'_y\left(L_m^y\right) = y\left(L_m^y\right) = W_m\left(L_m^y\right) , \tag{29}$$

where X'_y denotes the WMF labor's equilibrium marginal product.

At the same time, for the $W_m(L)$ wage function, a PMF is still inferior to a WMF since, due to (8) and $k = m$, we have:

$$X'_\Pi(L) = W_m(L) + LW'_m(L) > W_m(L) = X'_y(L), \tag{30}$$

that is,

$$X'_\Pi(L) > X'_y(L) \implies L_m^\Pi < L_m^y = L^P , \tag{30a}$$

where X'_Π denotes the PMF labor's equilibrium marginal productivity and L^P is the Pareto optimal labor use of (21).

Thus, it appears that there always exists some wage function - the $W_m(L)$ function in our case - for which a non-wage-taking WMF reaches the Pareto optimum, and thus Pareto dominates the non-wage-taking PMF.

Furthermore, a decrease in the a_k parameter from its a_m level does not affect the WMF equilibrium, $L^y = L^y_m$.

On the other hand, a decrease in a_k (expectedly) increases the PMF labor use, L^Π. To verify this, we write the PMF equilibrium of (8) as:

$$X'(L) = f(L, a_k) + Lf'(L, a_k) \tag{31}$$

Then, we differentiate (31) with respect to a_k and use the envelope theorem to obtain, due to (2c), (2a) and (9a):

$$\frac{dL}{da_k} = \frac{f_{a_k} + Lf'_{a_k}}{X'' - 2f' - Lf''} < 0 \tag{31a}$$

Thus, with a_k decreasing from a_m of (12) to a_e of (4a), the PMF equilibrium labor use, L^Π, is strictly monotonically increasing, until it (hypothetically) reaches its wage-taking level L^Π_e of (4a), where, due to $X'' < 0$, we have:

$$L^\Pi_e > L^y_m \tag{32}$$

But, this further implies that there always exists some value a_n of the a_k parameter, where $a_e < a_n < a_m$, and the corresponding neutral wage function $W_n(L) \equiv f(L,a_n)$, already introduced in (17), which defines the identity between the PM and WM employment - see equation (18), which we write again here:

$$L^\Pi_n = \arg\ \max\left(X(L) - LW_n(L) - C\right) = L^y_m \tag{33}$$

In other words, within the S^u family of wage functions there always exists a single wage function, $W_n(L)$, which equalizes the PMF and WMF equilibrium and thus implies identical efficiency of the two types of firm. At the same time, note that $W_n(L)$ divides the S_y^u family in the disjoint subfamilies S_y^u and S_Π, respectively defined in (16) and (20a).

We now collect the results of this subsection to obtain the following lemma:

Lemma 2. Within the S_y^u subfamily of wage functions a WMF efficiency dominates PMF.

Finally, we integrate Lemma 1 and Lemma 2, to obtain:

Lemma 3. Within the S_y family of wage functions a WMF efficiency dominates PMF, where, using again the set notation, $S_y = S_y^c \cup S_y^u$, $S_y^c \cap S_y^u \equiv \emptyset$.

4.2. The dominance of a PMF over WMF within the S_Π subfamily of S and The Alternating Dominance Theorem

Due to the previous results, for the remaining S_Π subfamily of wage functions, defined in (20) and (20a), we immediately get the following lemma:

Lemma 4. Within the S_Π subfamily of wage functions a PMF efficiency dominates a WMF, where $S_\Pi = S \setminus S_y$, i.e., where S_y and S_Π are disjoint subfamilies.

Finally, we integrate Lemma 3 and Lemma 4 to get the general proposition on the alternating (efficiency) dominance of a WMF and PMF:

Proposition 1 - The Alternating Dominance Theorem. Given the income-per-worker function $y = y(L)$, the continuous S family of all relevant wage functions - that make no losses both to a WMF and PMF - is divided by one, neutral member-function, $W_n(L)$, into two disjoint subfamilies, S_y and S_Π , where for any function of S_y a WMF dominates PMF, while for any function of S_Π the converse is true.

In the next part we present systematic numerical simulations that provide an insight into the relative size of S_y and S_Π , and single out the major factors that explain this size.

5. The WMF/PMF Dominance Ratio, δ, and the size of the WMF and PMF Dominance Regions

We now temporarily assume that S is not a continuous but *discrete* family, characterized with a small uniform step in the a_k scarcity parameter, $\Delta a_k = \Delta a$, $k = 1,\ldots,n$, where n is big. Thus we will have the (big number of) wage functions, evenly spread across S.

In principle, to measure the (approximate) size of relevant subfamilies one can count the wage curves that belong to these subfamilies[6].

In order to calculate the WMF/PMF dominance relation, we first introduce what seems to be its natural definition:

Definition 2. The WMF/PMF dominance relation is identified with the δ ratio, where the numerator and denominator of δ respectively reduce to the shares of S_y and S_Π in S, and where these shares, denoted by $N(S_y)$ and $N(S_\Pi)$, represent the size of the WMF and PMF dominance regions,

$$\delta = \frac{N(S_y)}{N(S_\Pi)} \; , \tag{34}$$

where:

$$N(S_y) = \frac{\delta}{1+\delta} \; , \; N(S_\Pi) = \frac{1}{1+\delta} \tag{35}$$

---

6   Given the total of 30 simulations, see Tables 1-3 and Footnote 11 below, we had to omit the counting procedure. Instead, as approximation, we have applied the Borell measure, which is error-free in the case of continuous families, while in the (present) discrete case it makes the computational error arbitrarily small, provided the number of family-member functions is arbitrarily big.

The δ dominance ratio also suggests introducing the concept of global efficiency dominance, as distinct from local dominance:

Definition 3. Given the *S* family of wage functions, a WMF (PMF) is defined to globally dominate PMF (WMF) if the δ dominance ratio is greater (smaller) than unity.

To get an idea about the magnitude of the δ ratio and about the major factors that explain this magnitude, we present in Tables 1-3, displayed at the end of this section, the corresponding systematically organized numerical simulations. A few of them are also graphically presented in Appendix.

First, the simulations examine the sensitivity of δ to switches from technologies T1, in which labor's marginal productivity $X'(L)$, is a strictly concave function of to T2 technologies, typical of linear $X'(L)$, and, finally, to technologies with a strictly convex $X'(L)$. Of course, the technologies are selected so as to be commensurable, in the sense that - given the price of output $p(=1)$, fixed capital costs, C, and the entry-wage, W_e - they yield income per worker functions, characterized by (approximately) the same maximum, $L_y^m(\approx 1.98)$, and the same (maximal) value of income per worker, computed at this maximum, $y_m(\approx 1.26)$.

The changes in δ due to switches of technologies can be seen by looking at any row of Tables 1-3.

Second, the simulations focus on the sensitivity of δ to changes in the degree of curvature of the family-member (inverse) labor supply or wage functions. These functions are first assumed to be linear, S_1, then quadratic, S_2, and, finally, cubic, S_3. Here, the resultant changes in δ can be seen by looking at any column of Tables 1-3.

Third, simulations examine the sensitivity of δ to changes in the entry wage, W_e, that assumes values of 0.4 (Table 1), 0.63 (Table 2) and 1.0 (Table 3), which respectively comprise around 30%, 50% and 80% of the maximal income per worker, $y_m \approx 1.26$. Here, the underlying changes in δ can be seen by looking at its particular values in each field of Table 1, and then in *the corresponding* field in Tables 2 and 3.

What can first be concluded from the performed simulations is that the changes in δ assume the following regularities:

i)   δ increases by switching from T_1 to T_2, and, then, from T_2 to T_3 technologies;
ii)   δ increases with the curvature of the wage functions, i.e., by switching from linear to quadratic and, then, to cubic wage functions;
iii) A fall in the entry wage, W_e leads to an increase in the δ ratio.

As for the direction and magnitude of the δ ratio, it appears that a WM monopsony strongly dominates its profit-maximizing twin[7].

Thus, the WMF dominance region is (significantly) greater than that of a PMF in 26 of 27 simulations performed.

Finally, the average magnitude of the δ dominance ratio, denoted by $\overline{\delta}$ and based on all 27 simulations, is very high and implies that, on average, around 94% of *all* the wage functions considered belong to the WMF dominance regions[8].

In Appendix we graphically present the WMF and PMF dominance regions, obtained for We=0.4, linear $X'(L)$ and, respectively, for the families of linear, quadratic and cubic wage functions.

---

[7]   The values of $\delta=\delta_{ij}$ (i,j=1,2,3), and thus of $\overline{N}$ (S_y), are approximate - see Footnotes 6 above and 11 below.

[8]   To test the robustness of the performed simulations, we have also done the three modified exercises, where the (previously parametric) entry-wage has been modeled as an increasing function of the labor scarcity parameter. The simulations have been designed to be comparable with the three arguably most representative simulations - of column 2, Table 2. However, this exercise has not altered the tenor of the previous results - the WMF average dominance region has become 91%, comparing to 93% of the corresponding fixed entry wage simulations.

Table 1. The values of the **δ** dominance ratio for the entry-wage $W_e = 0.4$; $\delta = \delta_{ij}$ $(i,j=1,2,3)$ is of Definition 2 of Part V. The technologies T1, T2 and T3, and the wage functions S1, S2 and S3, are also of Part V.

$We = 0.4$	T1 $X' = 3.5 - 0.6L^2$ $y = 3.5 - 0.2L^2 - C/L$ $C = 2.85$	T2 $X' = 2 - 0.4L$ $y = 2 - 0.2L - C/L$ $C = 0.680$	T3 $X' = 2.08/(1.45L^{0.2})$ $y = (2.6/1.45L^{0.2}) - C/L$ $C = 0.600$
S1 $W_n = W_e + a_n L$ $W_z = W_e + a_z L$	$a_n = 0.231$ $a_z = 0.505$ $\delta_{11} = 1.19$	$a_n = 0.160$ $a_z = 0.743$ $\delta_{12} = 3.64$	$a_n = 0.227$ $a_z = 0.980$ $\delta_{13} = 3.32$
S2 $W_n = W_e + a_n L^2$ $W_z = W_e + a_z L^2$	$a_n = 0.0803$ $a_z = 0.344$ $\delta_{21} = 3.28$	$a_n = 0.0850$ $a_z = 1.01$ $\delta_{22} = 10.9$	$a_n = 0.0790$ $a_z = 1.86$ $\delta_{23} = 22.5$
S3 $W_n = W_e + a_n L^3$ $W_z = W_e + a_z L^3$	$a_n = 0.0314$ $a_z = 0.261$ $\delta_{31} = 7.31$	$a_n = 0.0350$ $a_z = 1.61$ $\delta_{32} = 45.0$	$a_n = 0.0314$ $a_z = 4.15$ $\delta_{33} = 131$

Table 2. The values of the **δ** dominance ratio for the entry-wage $W_e = 0.63$; $\delta = \delta_{ij}$ $(i,j=1,2,3)$ is of Definition 2 of Part V. The technologies T1, T2 and T3, and the wage functions S1, S2 and S3, are also of Part V.

$We = 0.63$	T1 $X' = 3.5 - 0.6L^2$ $y = 3.5 - 0.2L^2 - C/L$ $C = 2.85$	T2 $X' = 2 - 0.4L$ $y = 2 - 0.2L - C/L$ $C = 0.6800$	T3 $X' = 2.08/(1.45L^{0.2})$ $y = (2.6/1.45L^{0.2}) - C/L$ $C = 0.600$
S1 $W_n = W_e + a_n L$ $W_z = W_e + a_z L$	$a_n = 0.171$ $a_z = 0.363$ $\delta_{11} = 1.12$	$a_n = 0.172$ $a_z = 0.490$ $\delta_{12} = 1.85$	$a_n = 0.166$ $a_z = 0.628$ $\delta_{13} = 2.78$
S2 $W_n = W_e + a_n L^2$ $W_z = W_e + a_z L^2$	$a_n = 0.0595$ $a_z = 0.231$ $\delta_{21} = 2.88$	$a_n = 0.0620$ $a_z = 0.564$ $\delta_{22} = 8.10$	$a_n = 0.0580$ $a_z = 1.00$ $\delta_{23} = 16.2$
S3 $W_n = W_e + a_n L^3$ $W_z = W_e + a_z L^3$	$a_n = 0.0232$ $a_z = 0.161$ $\delta_{31} = 5.94$	$a_n = 0.0250$ $a_z = 0.756$ $\delta_{32} = 29.2$	$a_n = 0.0230$ $a_z = 1.86$ $\delta_{33} = 79.9$

Table 3. The values of the δ dominance ratio for the entry-wage $W_e = 1$; $\delta = \delta_{ij}$ (i,j=1,2,3) is of Definition 2 of Part V. The technologies T1, T2 and T3, and the wage functions S1, S2 and S3, are also of Part V.

$We = 1$	T1 $X' = 3.5 - 0.6L^2$ $y = 3.5 - 0.2L^2 - C/L$ $C = 2.85$	T2 $X' = 2 - 0.4L$ $y = 2 - 0.2L - C/L$ $C = 0.680$	T3 $X' = 2.08/(1.45L^{0.2})$ $y = (2.6/1.45L^{0.2}) - C/L$ $C = 0.600$
S1 $W_n = W_e + a_n L$ $W_z = W_e + a_z L$	$a_n = 0.0750$ $a_z = 0.149$ $\delta_{11} = 0.987$	$a_n = 0.0720$ $a_z = 0.168$ $\delta_{12} = 1.33$	$a_n = 0.0690$ $a_z = 0.195$ $\delta_{13} = 1.83$
S2 $W_n = W_e + a_n L^2$ $W_z = W_e + a_z L^2$	$a_n = 0.0260$ $a_z = 0.085$ $\delta_{21} = 2.27$	$a_n = 0.0260$ $a_z = 0.136$ $\delta_{22} = 4.23$	$a_n = 0.0240$ $a_z = 0.204$ $\delta_{23} = 7.50$
S3 $W_n = W_e + a_n L^3$ $W_z = W_e + a_z L^3$	$a_n = 0.0102$ $a_z = 0.0510$ $\delta_{31} = 4.00$	$a_n = 0.0110$ $a_z = 0.124$ $\delta_{32} = 10.3$	$a_n = 0.00900$ $a_z = 0.245$ $\delta_{33} = 26.2$

6. Summary and Concluding Remarks

In this paper we have compared the short-run efficiency of profit- and wage-maximizing firms (PMFs and WMFs) when they operate at the monopsonistic labor market. To perform the comparison, we have first ruled out any type of wage discrimination as well as the (possible) existence of a market for WMF membership rights. We have then defined local efficiency dominance, according to which one no loss making firm dominates the other when, for a single inverse labor supply or wage function, the former produces more output - and thus creates a greater total surplus - than the latter.

For well-behaved wage functions we have then varied a suitably defined labor scarcity parameter, from zero to its zero-profit level. Given a turned U-shaped income-per-worker schedule, the latter level defines the steepest wage curve that yields zero profit both to a WMF and PMF, and therefore has a tangency point with the income-per-worker schedule. We have thus generated the continuous family of wage functions, all of which ensure nonnegative profit for a PMF and WMF where, by definition, the number of such functions is infinite.

Under quite general assumptions on the shape of production and wage functions, we finally demonstrate that this family is always divided, by some neutral member-function, into the upper and lower subfamily, where for any function of the former a WMF (locally) dominates a PMF, while for any function of the latter the converse is true. Hence, we also show that, in general, for a given single wage function, a WMF can efficiency dominate a PMF, and vice versa.

After detecting this alternating WMF/PMF dominance, we have focused on the size of the PMF and WMF dominance regions, identified with the ratio of shares of the corresponding subfamilies in the above defined family of all relevant wage functions.

To achieve this, we have temporarily assumed that this family is discrete, where its member-functions, the number of which is big, are evenly spread across the family. This leads to the introduction of the concept of global dominance, where one firm is defined to (globally) dominate the other when the former locally dominates the latter for more than a half of wage functions that constitute the entire family.

After this, we have performed 27 systematically organized numerical simulations, which combine three types of technology, three types of wage functions, and three levels of the entry-wage. It turns out that these simulations present some remarkable regularities.

First, the simulations indicate that the WMF dominance region (relatively) increases by switching from technologies with a concave marginal product of labor to those characterized by a convex labor's marginal productivity. Second, this region also increases by switching from families of linear wage functions to families of (strictly) convex functions with smaller curvatures and, finally, to families that consist of more convex (wage) functions. Third, the WMF dominance is stronger for lower levels of the entry wage.

Here, the basic result is that, on average, a WMF (strongly) globally dominates PMF, where the average size of the WMF dominance region amounts to 94% of all considered wage functions, and where just one of 27 simulations yields (a modest) PMF's dominance.

Thus - in the broader context, and on the theoretical level - we have shown that for monopsonized labor market there exists a type of maximizing behavior (often discussed in the corresponding literature), where this behavior may well be superior to conventional profit-maximization.

Finally, two notes on the novel concept of global efficiency dominance are in order.

First, the concept does not seem to be restricted to the present case of monopsonistic labor markets and could also be used under some other market structures, e.g. under monopoly and monopolistic competition.

Second, on the empirical level, the concept would require each family-member function to be weighted by the probability of its occurrence at the market in question. Still, on the theoretical level, here considered, the (implicitly) assumed equal probability of all member-functions seems legitimate. This is due to the fact that all considered wage functions enable firms to make no losses, and from this perspective it seems reasonable not to discriminate between them, at least not on this stage of analysis.

The third note may be of relevance for the theory and policy of privatizing a non wage-taking firm, which is supposed to be the wage maximizer. If in such a case one reveals the local dominance of a WMF over PMF, a higher local efficiency of the former - due to its objective of wage maximization - ought to be weighed against the possibly higher technical productivity of the latter, observed in many outsider-privatized PM firms (see, for example, Frydman, Gray, Hessel and Rapaczynski (1999)).

In any case, and irrespective of these remarks, the key result of the paper clearly points to the fact that in non-wage-taking environments, and with equal technical and market opportunities, a wage-maximizing firm may be more efficient, both locally and globally, than a conventional, profit-maximizing enterprise.

Đorđe Šuvaković Olgin and Goran Radosavljević

REFERENCES

Bhaskar, V., Manning, A. and To T. (2004), 'Oligopsony and the Monopsonistic Competition in Labor Markets', mimeo.

Blanchard, O. (1997), *The Economics of Post-Communist Transition,* Clarendon Press

Boal, W. and Ransom, M. (1997), "Monopsony in the Labor Market", *Journal of Economic Literature*, 35, 86-112.

Boal, W. and Ransom, M. (2002), "Monopsony in American Labor Markets", EH.NET's *Online Encyclopedia of Economic History*

Bonin, J. and Putterman, V. (1987), *Economics of Cooperation and the Labor-Managed Economy*, Harwood Academic Publishers

Bonin, J., Jones, D. and Putterman, L. (1993), 'Theoretical and Empirical Studies of Producer Co-operatives: Will Ever the Twain Meet?' *Journal of Economic Literature*, 33, 1290-320.

Domar, E. (1966), "The Soviet Collective Farm as a Producer Cooperative", *American Economic Review*, 56, 734-57.

Dreze, J. (1989), *Labor-Management, Contracts and Capital Markets: A General Equilibrium Approach*, Basil Blackwell

Frydman, R., Gray, C., Hessel, M. and Rapaczynski, A. (1999), "When does Privatization Work? The Impact of Private Ownership on Corporate Performance in the Transition Economies", *Quarterly Journal of Economics*, 114, 1153-92.

IBRD (1996), *World Development Report 1996: From Plan to Market*, Oxford University Press

Manning, A. (2003a), *Monopsony in Motion: Imperfect Competition in Labor Markets*, Princeton University Press.

Manning, A. (2003b), "The Real Thin Theory: Monopsony in the Modern Labour Markets", *Labour Economics*, 10, 105-131.

Pencavel, J. and Craig, B. (1994), "The Empirical Performance of Orthodox Models of the Firm: Conventional Firms and Worker Cooperatives", *Journal of Political Economy*, 102, 718-44.

Roland, G. (2000), *Transition and Economics: Politics, Markets and Firms*, MIT Press

Staiger, D., Spetz, J. and Phibbs, C. (1999), 'Is There Monopsony in the Labor Market? Evidence from a Natural Experiment', NBER Working Paper 7258.

APPENDIX: A graphical presentation of the WM and PM dominance regions for the three types of wage functions[9]

Figure A1. The WMF and PMF dominance regions, identified with the S_y and S_Π subfamilies of the discrete S family, are approximated by the darker and lighter shaded areas, bordered by the W_z, W_n, and W_e functions: The case of linear labor's marginal product and linear wage functions, $\delta_{12} = 3.64$, see Table 1 of Part V.
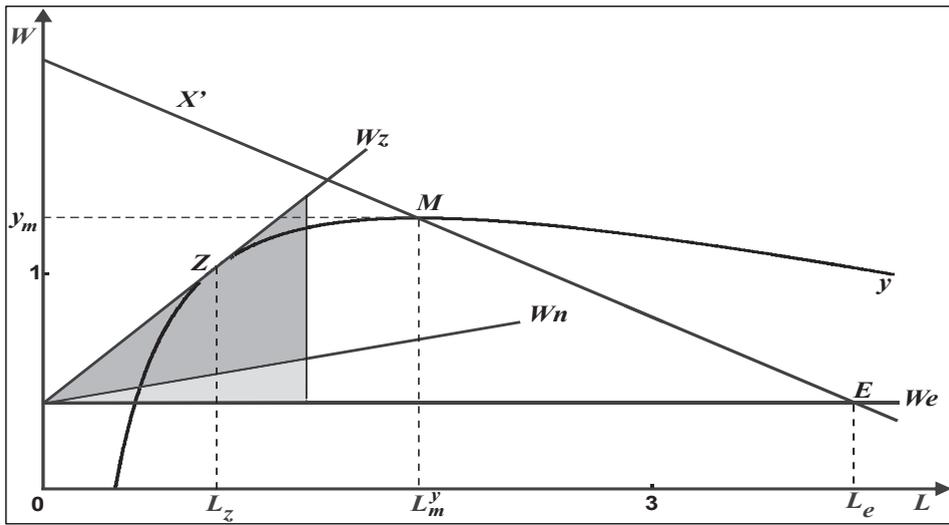
$X = 2L - 0.2L^2$ - production function
$X' = 2 - 0.4L$ - labor's marginal product
$y = 2 - 0.2L - C/L$ - income per worker
$C = 0.68$ - fixed costs
$W_e = 0.4$ - entry wage horizontal line, $a_k = 0$
$a_k = a$ - typical (varying) labor scarcity parameter
$W_k = W_e + a_k L^2$ - typical wage function
$W_n = W_e + a_n L$ -   neutral wage function that implies equal equilibrium
        of WMF and PMF, $a_n = 0.160$

$W_z = W_e + a_z L$ - wage function that implies zero-profit and identical equilibrium of a WMF and PMF, $a_z = 0.743$

Figure A2. The WMF and PMF dominance regions, identified with the S_y and S_Π subfamilies of the discrete S family, are approximated by the darker and lighter shaded areas, bordered by the W_z, W_n, and W_e functions: The case of linear labor's marginal product and quadratic wage functions, $\delta_{22} = 10.9$, see Table 1 of Part V.
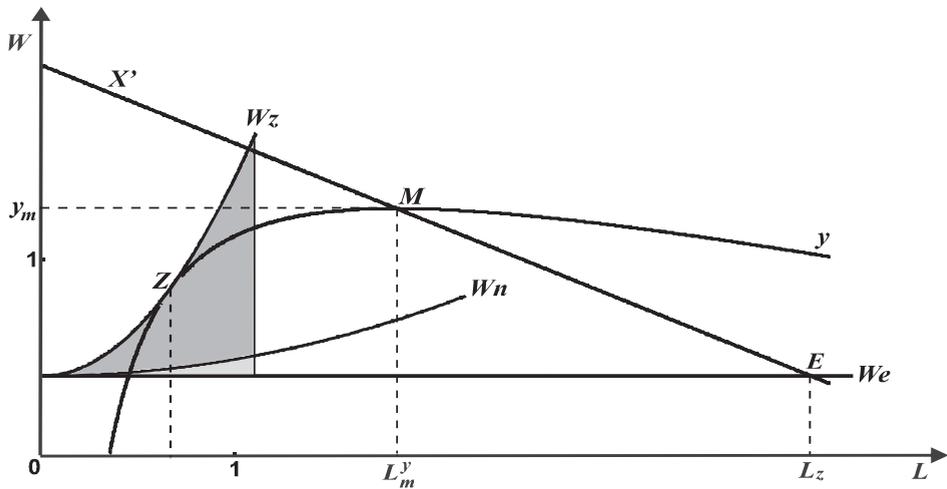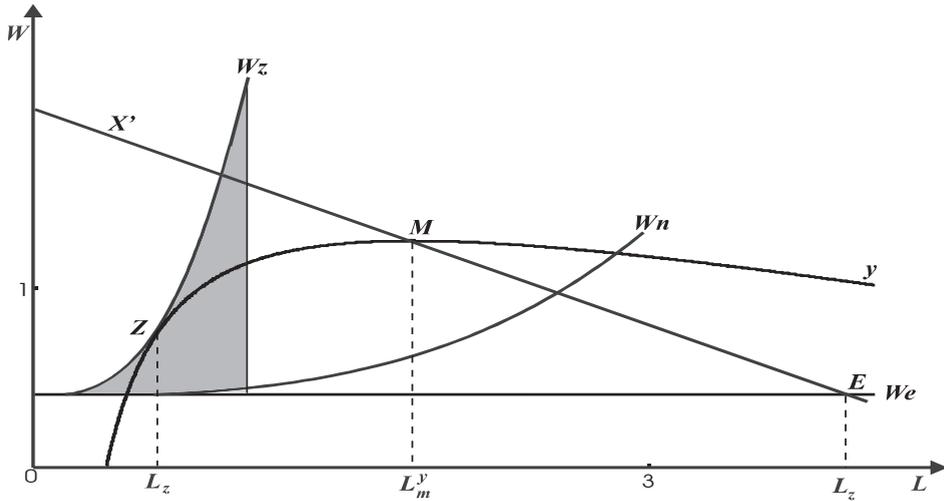

X, X', y , C, W_e, a_k, and W_k - as in legend of Figure A1
$W_n = W_e + a_n L^2$ - neutral wage function, defined as in Figure A1, $a_n = 0.085$
$W_z = W_e + a_z L^2$ - zero-profit wage function, defined as in Figure A1, $a_z = 1.01$

Figure A3. The WMF and PMF dominance regions, identified with the S_y and S_Π subfamilies of the discrete S family, and approximated by the darker and (very small) lighter shaded areas, bordered by the W_z, W_n, and W_e functions: The case of linear labor's marginal product and cubic wage functions, $\delta_{32} = 45.0$, see Table 1 of Part V.

X, X', y, C, W_e, a_k, and W_k - as in legend of Figure A1.
$W_n = W_e + a_n L^3$ - neutral wage function, defined as in Figure A1, $a_n = 0.0347$
$W_z = W_e + a_z L^3$ - zero-profit wage function, defined as in Figure A1, $a_z = 1.61$