

Geometry from quantum particles

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Abstract

We investigate the possibility that a background independent quantum theory of gravity is not a theory of quantum geometry. We provide a way for global spacetime symmetries to emerge from a background independent theory without geometry. In this, we use a quantum information theoretic formulation of quantum gravity and the method of noiseless subsystems in quantum error correction. This is also a method that can extract particles from a quantum geometric theory such as a spin foam model.

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1 Introduction

Is quantum gravity a theory of quantum geometry, or is spacetime only a classical concept?

Most developed background-independent approaches to quantum gravity (i.e., theories whose basic quantities do not refer to a fixed spacetime geometry), such as loop quantum gravity [1], causal sets [2], spin foams [3], quantum Regge calculus [4] or causal dynamical triangulations [5] provide a candidate for a fundamental microscopic structure of spacetime that is some kind of quantum geometry: the kinematical state space of loop quantum gravity is a quantum superposition of spatial geometry states, causal sets is a path-integral of discrete causal orders, etc. The goals of such theories are to: i) be a well-defined microscopic theory of quantum geometry, ii) show that general relativity (and possibly also quantum field theory or matter couplings) emerge as the low-energy limit of the theory, and iii) make predictions on the kind and magnitude of departure from the classical theory.

These are very ambitious goals and each theory has had various levels of success so far. Several of the difficulties these approaches face (with the notable exception of causal dynamical triangulations) can be traced to the apparently desirable feature of background-independence and resulting complications, especially for the dynamical part of the theory. At the same time, it has been suggested a number of times that the quantum theory of gravity may not be a quantization of general relativity (for example in [6]). While this suggestion has been supported by intriguing arguments, there are, so far, few suggestions on what the alternative may be. Furthermore, if this is so, does one need a whole new type of background-independent quantum gravity formalism, or can at least some of the known approaches be modified to fit an expanded use (presumably revising the meaning of background-independence) in which quantum geometry can only be classical?

What could possibly be a way for spacetime to be a classical concept only, yet emergent from a background-independent quantum theory? A possible answer is to start from a quantum, suitably background-independent theory and first look for long-range coherent degrees of freedom. These will characterize the low-energy limit. They can be thought of as particles even though, at this level, there is no spacetime and thus the usual notion of particles (as in Wigner) does not apply. Only then, if these behave as if they are in a spacetime, do we have a spacetime.

How can one make sense of this scenario? On the face of it, there is a problem with every single step in the above. First, what is a *long-range* propagating degree of freedom if there is no spacetime and thus no obvious notion of “long-range”? What can we mean by “low-energy limit”, when energy needs time-translation invariance? What can a particle be in this setup? And, if one can define it, what does it mean that “it behaves as if in a spacetime”?

The aim of this paper is to investigate how far one can go in this direction

by following [7] that suggested that unnecessary references to spacetime can be eliminated by using the language of quantum information processing. Indeed, an object such as a spin foam can be formulated as a quantum superposition of quantum information flows which then may be restricted to be geometric and thus reduced to the usual quantum geometry path integrals, or not, the case of interest for us.

In this setup, as suggested by [8], a suitable notion of a particle from quantum information processing can be used. This is the notion of a noiseless subsystem in quantum error correction, a subsystem protected from the noise, usually thanks to symmetries of the noise. In a quantum gravity context, this means a subsystem emergent (protected) from the microscopic Planckian evolution, and thus relevant for the effective theory. The extent to which this is background-independent will be discussed in detail in the next section and the concluding one. We will then suggest that these “behave as if they are in a spacetime” if they are invariant under Poincaré transformations, or deSitter in the case of a positive cosmological constant, and so do their interactions. That is, we turn around the usual order: a particle is not Poincaré invariant because it is in a Minkowski spacetime, rather, all we can mean by a Minkowski spacetime is that all coherent degrees of freedom and their interactions are Poincaré invariant at the relevant scale.

In a simplified setup (where all coherent degrees of freedom are free and the relevant transformations are not emergent, as one expects in the full theory, but already present in the microscopic dynamics) we provide the required conditions on the fundamental dynamics that realize this scheme.

A secondary goal of this paper is to address the low energy problem in background independent approaches to quantum gravity, namely the problem of extracting a semiclassical low energy geometry from a dynamical microscopic quantum geometry. That is, our results may also be useful to quantum theories of gravity *with* microscopic quantum geometry: the definition of a coherent degree of freedom we use can be applied, for example, to spin foams with a boundary to extract its effective particles (and, in fact, that is why it was originally considered in [8]). Our setup thus provides a new way to get to the much sought-after semiclassical limit. In future work, we hope to give an algorithmic construction of the class of microscopic dynamics that contains Poincaré-invariant particles.

The outline of this paper is as follows. In the next section we clarify what we mean by background independence. In section 3, we review how a quantum gravity model can be described as a quantum information processing system. In section 4, we expand on the suggestion of [8] to use quantum error correction to identify long-range propagating degrees of freedom. We review the particular kind of quantum error correction that is relevant here: decoherence-free subspaces and noiseless subsystems. These coherent subsystems are required to be invariant under certain unitary transformations (Poincaré, Lorentz or deSitter would be an example) which implies further specific conditions on the fundamental dynamics.

We discuss the strengths, weaknesses and implications of this setup in the Conclusions.

2 Background-independent quantum gravity

General relativity tells us that only events and their relations are physical. Any coordinates we may use to describe them have no physical meaning and coordinate distances are not physical quantities. In general relativity the metric is dynamical: there is no background space and time. Possibly the most concise statement of background-independence is the one given by Stachel: “There is no kinematics independent of dynamics” [9]. Since the quantum theory of gravity is expected to contain general relativity, it is reasonable to ask that it maintains this important principle and is also background independent. Of course, we do not know what form background-independence will ultimately take, if it applies at all. There is a substantial literature discussing different possible forms of background independence (see [9, 10, 11] and references therein).

We are interested in a background independent quantum theory of gravity which is not based on quantum geometry. In that context, we note that in the literature and in quantum gravity research one finds two statements of background independence which lead to very different conclusions relevant for us. They can be summarized as follows [10]:

Background independence 1. A quantum theory of gravity is background independent if its basic quantities and concepts do not presuppose the existence of a given background metric.

Background independence 2. A quantum theory of gravity is background independent if there is no fixed theoretical structure. Any fixed structure will be regarded as a background.

The main background independent approaches to quantum gravity implement both 1 and 2 since they are given as theories of dynamical quantum geometry. That is, the two statements cannot be distinguished. Although there is no implementation of 2 outside the quantum geometry context, we feel it is important that it should be distinguished from 1.

It is clear that a quantum theory of gravity in which the basic quantities are not geometric in any sense ought to be automatically background-independent in the sense of 1, whether or not there is some fixed structure in it. This will be our position for the purposes of this article.

In a little more detail, since one of the main background independent quantum gravity candidates, loop quantum gravity, is a canonical theory, we would like to clarify the following. Often in the literature, a merging of 1 and 2 above,

appears to imply that the fundamental quantum theory of gravity must have a quantum Hamiltonian constraint and that a true Hamiltonian evolution in the microscopic theory is excluded. Our viewpoint is that this applies to (globally hyperbolic) quantum geometry and needs not be imposed on a candidate quantum theory of gravity in which geometry is only classical (in this we follow [12] and [13]). Such a theory, instead, only needs the classical Hamiltonian constraint to be present at the regime at which classical geometry arises¹.

In short, our position in this paper is that to impose the second form of background independence, whether or not the relevant quantum quantities are geometric, is to make an arbitrary extrapolation from the background-independence of general relativity. As for the first form, since its implementation in terms of quantum geometry creates a lot of problems, especially in the key issue of the low-energy limit of a microscopic quantum theory of gravity, we will not adopt it. Instead, we investigate the possible role of classical geometry in a quantum theory of gravity whose microscopic degrees of freedom are not geometric. This surely is background independent in the sense of 1.

3 Background-independent quantum gravity as a quantum information processing system

The type of quantum theory of gravity we will consider for the purposes of our work is a *quantum causal history* [7, 14, 15, 16, 17, 18, 19, 20]. Mathematically, a quantum causal history is a directed graph with a finite-dimensional Hilbert space associated with each vertex and a quantum channel associated with each edge (details follow). This formalism is of interest because it can be adapted to fit two different roles:

1. At the mathematical level, with no reference to any geometric properties of the graph and the edges, this is simply a quantum information processing system. This is (trivially) background-independent in the sense discussed in the previous section. In this form, it is suitable for our intended application of a quantum information theoretic method that introduces geometrical properties at the level of effective coherent degrees of freedom encoded in the system. The aim will be to find global symmetries of a classical geometry at the level of particles, without starting with a quantum geometry.

¹A common objection to a microscopic Hamiltonian that surely a true Hamiltonian must imply a preferred time, which is then ruled out by observations. This is not necessarily the case. First, there is no reason that the evolution of the fundamental quantum degrees of freedom has a direct correspondence to the geometric spacetime description. Second, even classically, one can have, for example, multifingered evolution with a fixed average lapse.

Also note that one can be perfectly relational without resorting to the extreme of the second form of background independence. All that is required is that any physically relevant observable refers to observers inside the system and their relations.

2. It is also possible to link this structure to quantum geometry via the interpretation of the directed graph as a causal set and the vertices as the events. Dynamics is introduced via a path-integral sum over all causal sets interpolating between given in and out states. This gives a spin foam model and thus a quantum geometry [7]. In fact, quantum causal histories were introduced in [7] and further developed in [14] as a formalism for a quantum cosmology which is locally finite, causal and background-independent. In this case, the same quantum information theoretic method can identify effective Poincaré particles in the spin foam, thus providing a new way to investigate the semiclassical limit of such models.

In the remainder of this section we review the basics of quantum causal histories, both in its “bare bones” version and the quantum geometric one.

3.1 Quantum causal histories on directed graphs

Let Γ be a directed graph with vertices $x \in V(\Gamma)$ and directed edges $e \in E(\Gamma)$. The *source* $s(e)$ and *range* $r(e)$ of an edge e are, respectively, the initial and final vertices of e . A (finite) path $w = e_k \cdots e_1$ in Γ is a word in the edges of Γ such that $r(e_i) = s(e_{i+1})$ for $1 \leq i < k$. If $s(w) = r(w)$ then we say w is a *cycle*. We require that Γ has no cycles². If there exists a path w such that $s(w) = x$ and $r(w) = y$ let us write $x \leq y$ for the associated partial ordering. We call such vertices *related*. Otherwise, they are *unrelated* and we use $x \sim y$ to denote this. Given any $x, y \in V(\Gamma)$ there are finitely many $z \in V(\Gamma)$ such that $x \leq z \leq y$.

A *quantum causal history* is a directed graph Γ endowed with the following structure. For every vertex $x \in V(\Gamma)$ there is a (finite-dimensional) Hilbert space $\mathcal{H}(x)$. If x and y are unrelated, the joint Hilbert space is $\mathcal{H}(x) \otimes \mathcal{H}(y)$.

For every edge $e \in E(\Gamma)$ there is a quantum channel

$$\Phi_e : \mathcal{A}(s(e)) \rightarrow \mathcal{A}(r(e)), \quad (1)$$

where $\mathcal{A}(x)$ is the full matrix algebra on $\mathcal{H}(x)$. (Basic properties of quantum channels are discussed below.) Given $x \neq y$, without loss of generality we can assume there is at most one directed edge from x to y . If there are multiple edges then the corresponding channels can be combined into one channel which contains the pertinent information in the quantum causal history.

A *parallel set* $\xi \subseteq E(\Gamma)$ is defined by the property that $x \sim y$ whenever $x, y \in \xi$. The algebra $\mathcal{A}(\xi) = \otimes_{x \in \xi} \mathcal{A}(x)$ acts on the composite system Hilbert space $\mathcal{H}(\xi) = \otimes_{x \in \xi} \mathcal{H}(x)$. If ξ and ζ are two parallel sets such that all forward directed paths from ξ intersect ζ and all paths that arrive at ζ pass through ξ , then we have an evolution of a closed quantum system and a unitary operator

$$U(\xi, \zeta) : \mathcal{H}(\xi) \rightarrow \mathcal{H}(\zeta). \quad (2)$$

²Note that, while this condition was initially motivated by Γ being a causal set (see 3.3), the same condition is also natural if the quantum causal history is a quantum computer with Γ the circuit.

This determines an isomorphism $\Phi(\xi, \zeta) : \mathcal{A}(\xi) \rightarrow \mathcal{A}(\zeta)$ via

$$\Phi(\xi, \zeta)(\rho) = U(\xi, \zeta)\rho U(\xi, \zeta)^\dagger \quad \text{for all } \rho \in \mathcal{A}(\xi). \quad (3)$$

Eq. (1) is the restriction of (3) to $\mathcal{A}(x) \subseteq \mathcal{A}(\xi)$ for $x \in \xi$. In turn, (3) can be reconstructed from the local maps (1) using the appropriate precise mathematical definition of a quantum causal history (see [14] for more details).

At this level, the quantum causal history is simply a quantum information processing structure. Note that it is possible to mirror the original quantum geometric construction by introducing a “path-integral” superposition of all possible graphs interpolating between two given sets of commuting algebras. However, there is no motivation for doing so if Γ is an information flow circuit and not a discrete quantum geometry. In any case, our results apply to both cases.

3.2 Quantum Channels

Quantum channels are central both in quantum causal histories and in the method we will use to extract particles from them. Here we give a brief account of their basic features.

Let \mathcal{H}_S be the state space of a quantum system in contact with an environment \mathcal{H}_E . The standard characterization of evolution in open quantum systems starts with an initial state in the system space that, together with the state of the environment, undergoes a unitary evolution determined by a Hamiltonian on the composite Hilbert space $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_E$, and this is followed by tracing out the environment to obtain the final state of the system. The associated evolution map, or “superoperator”, $\Phi : \mathcal{B}(\mathcal{H}_S) \rightarrow \mathcal{B}(\mathcal{H}_S)$ is necessarily completely positive (see below) and trace preserving. More generally, the map could have different domain and range Hilbert spaces. Hence the operational definition of a *quantum channel* (or quantum evolution, or quantum operation) from a Hilbert space \mathcal{H}_1 to \mathcal{H}_2 , is a completely positive, trace preserving map $\Phi : \mathcal{B}(\mathcal{H}_1) \rightarrow \mathcal{B}(\mathcal{H}_2)$.

A *completely positive* (CP) map Φ is a linear map $\Phi : \mathcal{B}(\mathcal{H}_1) \rightarrow \mathcal{B}(\mathcal{H}_2)$ such that the maps

$$id_k \otimes \Phi : \mathcal{M}_k \otimes \mathcal{B}(\mathcal{H}_1) \rightarrow \mathcal{M}_k \otimes \mathcal{B}(\mathcal{H}_2)$$

are positive for all $k \geq 1$. Here we have written \mathcal{M}_k for the algebra $\mathcal{B}(\mathbb{C}^k)$ represented as the $k \times k$ matrices with respect to a given orthonormal basis. (The CP condition is independent of the basis that is used.)

A fundamental technical device in the study of CP maps is the *operator-sum representation* given by the theorem of Choi [21] and Kraus [22]. For every CP map Φ there is a set of operators $\{E_a\} \subseteq \mathcal{B}(\mathcal{H}_1, \mathcal{H}_2)$ such that

$$\Phi(\rho) = \sum_a E_a \rho E_a^\dagger \quad \text{for all } \rho \in \mathcal{B}(\mathcal{H}_1). \quad (4)$$

We shall write $\Phi = \{E_a\}$ when the E_a satisfy Eq. (4) for Φ . The family $\{E_a\}$ may be chosen with cardinality $|\{E_a\}| \leq \dim(\mathcal{H}_1) \dim(\mathcal{H}_2)$, and is easily seen to be non-unique³.

The class of CP maps that are quantum channels satisfy an extra constraint. Specifically, note that when Φ is represented as in (4), trace preservation is equivalent to the identity

$$\sum_a E_a^\dagger E_a = \mathbb{1}_{\mathcal{H}_1}. \quad (5)$$

Thus, a quantum channel Φ is a map which satisfies (4) and (5) for some set of operators $\{E_a\}$.

3.3 Quantum causal histories with geometrical information

The quantum causal history can also double as a formalism of a microscopic quantum geometry theory. We start by interpreting the directed graph Γ as a causal set (a discrete, locally finite analogue of the set of events of a Minkowski spacetime) and the vertices as events. These are the smallest Planck scale systems in a quantum spacetime. In a locally finite theory (i.e., with a finite number of relevant degrees of freedom in a finite volume) these quantum systems are assigned a simple matrix algebra $\mathcal{A}(x)$ for each event x . Two unrelated events are acausal, thus the operators on the corresponding algebras commute. Every causal relation $x \leq y$ and the corresponding edge e in the causal set is the evolution of an open quantum system and hence a quantum channel $\phi_e : \mathcal{A}(x) \rightarrow \mathcal{A}(y)$.

A path integral model of a quantum spacetime is then given as a quantum sum over all possible causal sets that interpolate between two given “parallel” sets of events S_i and S_f (corresponding to the universe at a given initial and final times), formally:

$$A_{S_i \rightarrow S_f} = \sum_{\Gamma: S_i \rightarrow S_f} \prod_{e \in \Gamma} \phi_e. \quad (6)$$

One can import further geometric information, for example by requiring that the local state spaces are the spin network intertwiner spaces of loop quantum gravity, i.e., $\mathcal{H}(x)$ is the vector space of so-called intertwiners. These are maps from the tensor product of representations of $SU(2)$ to the identity representation. This, and other assignments of different groups and intertwiners, are examples of *spin foam models* [3].

A microscopic model of spacetime is successful if it has a good low-energy limit in which it reproduces the known theories, namely general relativity with

³However, if $\{E_a\}$ and $\{F_b\}$ are two families of operators that implement the same channel Φ , then there is a scalar matrix $U = (u_{ab})$ such that $E_a = \sum_b u_{ab} F_b$ for all a .

quantum matter coupled to it. In the case of causal dynamical triangulations, impressive results show strong indications that this model has the desired features [5]. This hinges on specific features of the model that allow a Wick rotation to a statistical sum and thus technical control of the sum via both analytic and numerical methods. In the general spin foam case, this is a formidable problem, involving a quantum sum over group representations.

Our proposal regarding this problem is that, instead of considering the sum (6) directly, one can first look for long-range propagating degrees of freedom (particles) and reconstruct the geometry from these (if they exist). The specific method we adopt is promising because it deals directly with quantum systems and coarse-grains a quantum system to its effective particles. Our discussion applies to such models with a boundary.

4 Particle: Group-invariant noiseless subsystems

We now suggest that a suitable definition of a coherent degree of freedom in a quantum causal history is a *noiseless subsystem*.

Quantum channels depict the most general form of evolution in open quantum systems, and hence they play a central role in quantum computing. In this setting, the operators $\Phi = \{E_a\}$ in the operator-sum representation Eq. (4) for a channel are called the *error* or *noise* operators associated with Φ . It is precisely the effects of such operators that must be mitigated for in the context of quantum error correction.

The noiseless subsystem method (also called decoherence-free subspaces and subsystems) is the fundamental passive technique for error correction in quantum computing. Recently a framework for studying noiseless subsystems that applies to arbitrary quantum channels was presented [23, 24, 25]. This framework is built upon earlier work in passive quantum error correction [26, 27, 28, 29, 30, 31, 32], and is a centrepiece of the unified approach to quantum error correction, called “operator quantum error correction”, introduced in [23, 24]. The basic idea in this setting is to (when possible) encode initial states in sectors that will remain immune to the degrading effects of errors $\Phi = \{E_a\}$ associated with a channel.

The mathematical description is given as follows. Let Φ be a channel on \mathcal{H} and suppose that \mathcal{H} decomposes as $\mathcal{H} = (\mathcal{H}^A \otimes \mathcal{H}^B) \oplus \mathcal{K}$, where A and B are subsystems and $\mathcal{K} = (\mathcal{H}^A \otimes \mathcal{H}^B)^\perp$. We say that B is *noiseless* for Φ if

$$\forall \sigma^A \forall \sigma^B, \exists \tau^A : \Phi(\sigma^A \otimes \sigma^B) = \tau^A \otimes \sigma^B. \quad (7)$$

Here we have written σ^A (resp. σ^B) for operators on \mathcal{H}^A (resp. \mathcal{H}^B), and we regard $\sigma = \sigma^A \otimes \sigma^B$ as an operator that acts on \mathcal{H} by defining it to be zero on \mathcal{K} . Let P^{AB} be the projection of \mathcal{H} onto $\mathcal{H}^A \otimes \mathcal{H}^B$ and define a “compression superoperator” $\mathcal{P}^{AB}(\cdot) = P^{AB}(\cdot)P^{AB}$ on \mathcal{H} . That is, \mathcal{P}^{AB} is the map on $\mathcal{B}(\mathcal{H})$ defined by $\mathcal{P}^{AB}(\sigma) = P^{AB}\sigma P^{AB}$, $\forall \sigma \in \mathcal{B}(\mathcal{H})$. Then in terms of the partial trace operation on A , Eq. (7) is

equivalent to the statement

$$\mathrm{Tr}_A \circ \mathcal{P}^{AB} \circ \Phi \circ \mathcal{P}^{AB} = \mathrm{Tr}_A \circ \mathcal{P}^{AB}. \quad (8)$$

See the Appendix for an example of a noiseless subsystem. One would like to eventually generalize this method to a suitable notion of approximate noiseless subsystems.

We now attempt to identify what it should mean for a quantum causal history to (i) contain a subsystem that evolves in a well defined unitary manner and (ii) for this notion to be invariant in a group-theoretic sense.

Let G be a group and suppose that $\pi : G \rightarrow \mathcal{B}(\mathcal{H}^{\mathrm{rep}})$ is a (unitary) representation of G on a Hilbert space $\mathcal{H}^{\mathrm{rep}}$. We identify G with the unitary group $\pi(G)$. For each U in G denote the corresponding superoperator on $\mathcal{B}(\mathcal{H})$ by $\mathcal{U}(\cdot) = U(\cdot)U^\dagger$. To further simplify the notation below, we shall denote the representation Hilbert space as $\mathcal{H}^B \equiv \mathcal{H}^{\mathrm{rep}}$. We are interested in scenarios for which \mathcal{H}^B is a subsystem of a larger Hilbert space \mathcal{H} within a causal history. Specifically, \mathcal{H} decomposes as $\mathcal{H} = (\mathcal{H}^A \otimes \mathcal{H}^B) \oplus \mathcal{K}$, where $\mathcal{K} = (\mathcal{H}^A \otimes \mathcal{H}^B)^\perp$. Suppose now that we have a quantum channel Φ on \mathcal{H} .

Definition 1. We say that B is G -invariant under Φ if there is a U in G such that

$$\forall \sigma^A \forall \sigma^B, \exists \tau^A : \Phi(\sigma^A \otimes \sigma^B) = \tau^A \otimes \mathcal{U}(\sigma^B). \quad (9)$$

This terminology is justified in the following sense. As a consequence of the theorem below, observe that Eq. (9) may be restated as

$$(\mathrm{Tr}_A \circ \Phi \circ \iota_B)(\sigma^B) = \mathcal{U}(\sigma^B) \quad \forall \sigma^B, \quad (10)$$

where $\iota_B : \mathcal{B}(\mathcal{H}^B) \rightarrow \mathcal{B}(\mathcal{H})$ is the map given by $\iota_B(\sigma^B) = \alpha(\mathbb{1}^A \otimes \sigma^B)$ with $\alpha = (\dim \mathcal{H}^A)^{-1}$. Notice in particular that this implies

$$\mathcal{V} \circ \mathrm{Tr}_A \circ \Phi \circ \iota_B = \mathcal{V} \circ \mathcal{U} \quad \forall V \in G. \quad (11)$$

The map $\mathcal{V} \circ \mathcal{U}$ is implemented by the unitary $VU \in G$. It follows that evolution of the B subsystem under Φ is invariant for the natural group action of G .

In the context of a quantum causal history, this means that the microscopic evolution in the history: i) leaves \mathcal{H}^B to evolve unitarily, i.e., it is an effective coherent degree of freedom and ii) \mathcal{H}^B is invariant under G -transformations, where the interesting implementations are when G is the Poincaré, Lorentz or deSitter group. Also note that the formulation of Eq. (9) can be widely applied within the causal history framework. In particular, it could be applied to a single edge map ϕ_e , to a map $\phi_w = \phi_{e_k} \circ \dots \circ \phi_{e_1}$ associated with a path $w = e_k \dots e_1$, or, indeed, to any evolution map associated with the structure of the history, such as partial traces over any given subsystem, etc.

There is some notational clarification required for Eq. (9). The channels $\phi_e : \mathcal{B}(\mathcal{H}(s(e))) \rightarrow \mathcal{B}(\mathcal{H}(r(e)))$ associated with a quantum causal history map between different Hilbert spaces. This would appear to be problematic in connection with Eq. (9), as the operator $U\sigma^B U^\dagger$ acts on the subsystem \mathcal{H}^B of, in this case, $\mathcal{H}(s(e))$, and not of $\mathcal{H}(r(e))$. However, our formulation of Eq. (9) is simply a notational convenience. We could identify \mathcal{H}^B with a subsystem \mathcal{H}^D of $\mathcal{H}(r(e))$ via a unitary $V^{BD} : \mathcal{H}^B \rightarrow \mathcal{H}^D$ (or more generally an isometry), and under this map σ^B is identified with $\sigma^D = V^{BD}\sigma^B(V^{BD})^\dagger$ and the group element U is identified with $U' = V^{BD}U(V^{BD})^\dagger$. In the case of a quantum computer, for instance, this could simply be the identification of the standard basis for n -qubit space with itself, after a certain time step. Thus, for brevity, we have suppressed this notational issue, effectively assuming the map V^{BD} is the identity map. With this identification in mind, we can write Eq. (9) unambiguously.

We note that Eq. (9) first appeared in [24] in the context of operator quantum error correction as a natural generalization of the notion of noiseless subsystems for quantum operations introduced in [23, 24].

The following theorem gives a number of testable conditions that are equivalent to G -invariance. Namely, the result shows how this notion may be phrased in terms of the partial trace operation on A ; that it is enough to satisfy this equation for the maximally mixed state on A ; how to test if a given subsystem satisfies this equation if a choice of operator elements for the evolution map is known; and the corresponding statement in terms of operator algebras.

Theorem 1. *Let G be a group represented on a Hilbert space \mathcal{H}^B . Suppose that \mathcal{H} is a Hilbert space that decomposes as $\mathcal{H} = (\mathcal{H}^A \otimes \mathcal{H}^B) \oplus \mathcal{K}$, and that $\Phi : \mathcal{B}(\mathcal{H}) \rightarrow \mathcal{B}(\mathcal{H})$ is a quantum channel. Then the following five conditions are equivalent:*

1. B is G -invariant under Φ .
2. $\exists U \in G : \forall \sigma^B, \exists \tau^A : \Phi(\mathbb{1}^A \otimes \sigma^B) = \tau^A \otimes \mathcal{U}(\sigma^B)$
3. $\exists U \in G : \text{Tr}_A \circ \mathcal{P}^{AB} \circ \Phi \circ \mathcal{P}^{AB} = \mathcal{U} \circ \text{Tr}_A \circ \mathcal{P}^{AB}$.
4. Let $\{|\alpha_k\rangle\}$ be an orthonormal basis for \mathcal{H}^A and let $\{P_{kl} = |\alpha_k\rangle\langle\alpha_l| \otimes \mathbb{1}^B\}$ be the corresponding family of matrix units in $\mathcal{B}(\mathcal{H}^A) \otimes \mathbb{1}^B$. Let $\Phi = \{E_a\}$ be a choice of operator elements for Φ . Then there is a $U \in G$ such that

$$P_{kk}(\mathbb{1}^A \otimes U^\dagger)E_a P_{ll} = \lambda_{akl} P_{kl} \quad \forall a, k, l \quad (12)$$

for some set of scalars $\{\lambda_{akl}\}$ and

$$(\mathbb{1}^A \otimes U^\dagger)E_a P^{AB} = P^{AB}(\mathbb{1}^A \otimes U^\dagger)E_a P^{AB} \quad \forall a. \quad (13)$$

5. There is a $U \in G$ such that the subspace $\mathcal{H}^A \otimes \mathcal{H}^B$ is invariant for the operators $\{(\mathbb{1}^A \otimes U^\dagger)E_a\}$, and the restricted operators $\{(\mathbb{1}^A \otimes U^\dagger)E_a P^{AB}\}$ belong to the operator algebra $\mathcal{B}(\mathcal{H}^A) \otimes \mathbb{1}^B$.

Sketch of proof. Suppose that B is G -invariant under Φ , and so Eq. (9) is satisfied for some $U \in G$. This is equivalent to the statement that $\forall \sigma^A \forall \sigma^B, \exists \tau^A$ such that

$$((id_A \otimes \mathcal{U}^\dagger) \circ \Phi)(\sigma^A \otimes \sigma^B) = \tau^A \otimes \sigma^B, \quad (14)$$

where $\mathcal{U}^\dagger(\cdot) = U^\dagger(\cdot)U$. Thus, in the terminology of [23, 24], this shows that \mathcal{H}^B is a noiseless subsystem for the map $(id_A \otimes \mathcal{U}^\dagger) \circ \Phi$. The result now follows from the characterization of noiseless subsystems from [23, 24]. \square

Let us discuss a mathematical problem motivated by this discussion. In the quantum causal history setting, we wish to regard the group G and a particular representation of G as given (namely, we know what the classical spacetime is and that particles are representations of the Poincaré group). Thus, it is of interest to find the evolution maps Φ such that Eq. (9) holds for some element U of G . To be precise, given G and a representation of G on a Hilbert space $\mathcal{H}^{\text{rep}} = \mathcal{H}^B$, find the set of all Φ such that B is G -invariant under Φ . This is of interest as it would give a class of microscopic quantum evolutions that contain particles with the desired classical geometric properties.

This problem is also of interest in the context of error correction in quantum computing. Indeed, in the case that G is the trivial group, this problem may be interpreted as follows: Given a subsystem \mathcal{H}^B of a system \mathcal{H} , find the quantum operations Φ on \mathcal{H} for which \mathcal{H}^B is a noiseless subsystem. The recent work [25] solves a different, but related problem. Specifically, an explicit method is presented to compute all noiseless subsystems when Φ is given. We expect that the techniques used in that work could lead to progress on the problem described here.

5 Conclusions: Are the Poincaré transformations the chicken or the egg?

Our task in this paper was to investigate how a classical spacetime may be emergent from a background independent quantum theory of gravity whose basic quantities are not quantum geometric. We proposed that the classical geometry can be present as symmetries of the emergent coherent degrees of freedom. That is, we wish to understand how global spacetime symmetries can emerge in a background independent theory with no spacetime. In parallel, we were interested in addressing the low energy issue of spin foam-like models via the use of emergent particles in the model.

We used a quantum information theoretic formulation and the specific method of noiseless subsystems used in the quantum error correction literature to characterize these coherent degrees of freedom, encoded in the microscopic dynamics. We generalized this to a suitable notion of group-invariant noiseless subsystems, thus giving a condition for the theory to have the required global symmetries. This

opens up the exciting possibility of having an algorithmic construction of the class of microscopic dynamics that contains the desired encoded particles.

Let us note some of the most interesting features of noiseless subsystems in a quantum gravity context. First, they are not localized, thus their symmetry is global (see the example in the Appendix). This is also relevant to the discussion of microscopic versus emergent locality in quantum gravity. They illustrate the fact that the emergent degrees of freedom can bear little relation in their interactions to the underlying microscopic theory, known of course from condensed matter physics, but now in a manifestly background independent form. Second, the construction employs quantum channels, rather than a partition function of the usual spin foam type, which applies both to a single underlying circuit (or history) or to a path integral sum. Finally, it is very important that the existence and properties of the noiseless subsystems depends entirely on the properties of the dynamics. As can be seen in the quantum information literature [26, 27, 28, 29, 30, 31, 32, 23, 24, 25] and in the application of this method to quantum black holes [33], as well as the example in the Appendix, in concrete examples of noiseless subsystems their existence depends on having symmetries in the dynamics.

It is also of interest that our results can be applied to spin foams with a boundary to extract the particles they contain and thus address the outstanding low energy issue of these models. (The importance of the boundary is also emphasized in [34]).

The following are shortcomings in the current application of noiseless subsystems to quantum gravity and will need to be addressed in future work: The G -invariant noiseless subsystems are not truly emergent but encoded in the microscopic dynamics, in the sense that both the symmetries of the dynamics that guarantee the existence of the noiseless subsystems and the group G are present in the microscopic dynamics. One would like to extend the relevant notions to an appropriate definition of an approximate G -invariant noiseless subsystem. Also, in order to claim that there is a flat spacetime in the microscopic theory, we need to have G -invariant interactions between the noiseless subsystems.

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Appendix: An example of a noiseless subsystem in quantum computing

We will now present a widely used example of a noiseless subsystem in quantum computing. See [35, 36, 37] for a detailed analysis of related and more general noiseless subsystems. The system \mathcal{S} is composed of three spin- $\frac{1}{2}$ particles (labeled A , B , and C), so $\mathcal{H}_{\mathcal{S}} = (\mathbb{C}^2)^{\otimes 3}$. The CP map is a collective rotation: all three spins get rotated around a common axis and by a common angle, but these specifications of the rotation are chosen at random and are unknown. The rotation operator of a single spin- $\frac{1}{2}$ particle (an element of the Lie group $SU(2)$) can be written in terms of the Pauli operators (the generators of the Lie algebra $su(2)$): $\exp\{-i\theta\vec{n} \cdot \vec{\sigma}\}$ where \vec{n} is a real three dimensional unit norm vector defining the axis of rotation, $\theta \in [0, 2\pi]$ is the rotation angle, and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$. Defining $\vec{J} = \vec{\sigma}_A \otimes \mathbb{1}_B \otimes \mathbb{1}_C + \mathbb{1}_A \otimes \vec{\sigma}_B \otimes \mathbb{1}_C + \mathbb{1}_A \otimes \mathbb{1}_B \otimes \vec{\sigma}_C$ as the total spin operator, a collective rotation operator of all three spins can be written as $\exp\{-i\theta\vec{n} \cdot \vec{J}\}$. The map Φ is therefore the statistical average of all such collective rotations

$$\begin{aligned} \Phi[\rho] &= \frac{1}{4\pi} \int_{\mathcal{S}} \exp\{-i\theta\vec{n} \cdot \vec{J}\} \rho \exp\{i\theta\vec{n} \cdot \vec{J}\} d\vec{n} \\ &= \frac{1}{3} [E_x \rho E_x^\dagger + E_y \rho E_y^\dagger + E_z \rho E_z^\dagger] \end{aligned} \quad (15)$$

where $E_k = \exp\{-i\theta J_k\}$, $k = x, y, z$. The second line follows by the symmetry of the integration region.

Hence, the collective rotation channel is characterized by the three angular momentum operators J_x , J_y , and J_z . The noiseless subsystems for Φ are encoded in its “noise commutant”. This is the operator algebra $\mathcal{A}' = \{E_x, E_y, E_z\}' = \{J_x, J_y, J_z\}'$. Operators in the noise commutant are fixed points for Φ , and hence are immune to the noise of Φ . This algebra is unitarily equivalent to the algebra $\mathcal{A}' \cong \mathbb{C}\mathbb{1}_4 \oplus (\mathbb{1}_2 \otimes \mathcal{M}_2)$. Thus, the states encoded inside the subalgebra isomorphic to $\mathbb{1}_2 \otimes \mathcal{M}_2$ remain error-free under Φ , making use of symmetries in the noise. It is important to note that the tensor structure determined by this subalgebra is different than the initial system tensor presentation determined by the three particles A , B and C . Let us be more specific.

The operators J_x , J_y , and J_z form a representation of $su(2)$, whose irreducible sectors are given by the eigenspaces of the total angular momentum operator J^2 . An orthonormal basis for $\mathcal{H}_{\mathcal{S}}$ is $|j, m, \mu\rangle$, where $j = \frac{1}{2}, \frac{3}{2}$ is the total spin number, where $J^2|j, m, \mu\rangle = j(j+1)|j, m, \mu\rangle$, $m = -j, -j+1, \dots, j$, is the projection of the spin along the z axis $J_z|j, m, \mu\rangle = m|j, m, \mu\rangle$, and where μ labels the multiplicity of the representation.

From elementary composition of angular momentum, it can be found that there is a single copy of the spin- $\frac{3}{2}$ representation while the spin- $\frac{1}{2}$ representation appears in two copies. Hence, in the subspace associated to the eigenvalue $j = \frac{1}{2}$,

$\mathcal{H}_{\frac{1}{2}}$, the states can be represented with two quantum numbers $|m, \mu\rangle$, $m = \pm\frac{1}{2}$ labeling the eigenvalue of J_z and $\mu = 1, 2$ labeling the two copies of the irreducible representation. One can think of these two quantum numbers as resulting from the tensor product of the Hilbert space of two subsystems $\mathcal{H}_{\frac{1}{2}} = \mathcal{H}_m \otimes \mathcal{H}_\mu$. The system \mathcal{H}_m gets completely mixed by the map Φ while the system \mathcal{H}_μ is completely immune to noise. Thus, as discussed above, any state of the form $\mathbb{1}_m \otimes \rho_\mu$ is a fixed point of Φ . More generally, Eq. (7) is satisfied here since $\Phi(\rho_m \otimes \rho_\mu) = \mathbb{1}_m \otimes \rho_\mu$.

Let us emphasize again that in this example, the division of the Hilbert space $\mathcal{H}_S = \mathcal{H}_{\frac{3}{2}} \oplus (\mathcal{H}_m \otimes \mathcal{H}_\mu)$ has no relation with its natural division $\mathcal{H}_S = (\mathbb{C}^2)^{\otimes 3}$ into three subsystems A, B , and C . The noiseless subsystem \mathcal{H}_μ is an abstract construction which involves all three spin- $\frac{1}{2}$ particles in a non-trivial way. Furthermore, the subsystem \mathcal{H}_m is virtually absent from the dynamics as its state gets randomized by Φ . It is irrelevant to the physics of the system as it does not and cannot convey information.

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