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CP Violation in Neutrinoless Double Beta Decay and Neutrino Oscillation

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Taking possible CP violations into account, we discuss constraints on lepton mixing angles obtained from neutrinoless double beta decay and from neutrino oscillation for three flavour Majorana neutrinos. From the CHORUS oscillation experiment, combined with the data from neutrinoless double beta decay, we show that the large angle solution for θ_{23} is improbable if the neutrino mass m_3 of the third generation is a candidate for hot dark matter.

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I. INTRODUCTION

It is one of the most important problems in particle physics whether neutrinos have mass or not. From recent neutrino experiments [1] [2] [3] [4], it becomes very probable that neutrinos have mass. However, if neutrinos have mass, we must explain the reason why it would be so small relative to charged lepton masses. The seesaw mechanism is one of the most promising candidates for such an explanation. In this case, neutrinos become Majorana particles, and three CP violating phases remain in the lepton mixing matrix U for the three generation case [5]. This contrasts with the case of Dirac Neutrinos, in which only one CP-violating phase remains in U. Thus, for Majorana neutrinos, possible CP violations due to these phases in U complicates obtaining information about the lepton mixing angles from the corresponding experiments.

In this paper we consider neutrinos as massive Majorana particles with three generations and shaw how experiments, such as neutrinoless double beta decay ($(\beta\beta)_{0\nu}$) and neutrino oscillation, constrain lepton mixing angles and neutrino mass. In order to obtain these constraints we take possible CP-violating phases in U into account. In $(\beta\beta)_{0\nu}$ there are three CP-violating phases which contribute to possible CP violations. In the neutrino oscillation, on the other hand, there is only one such phase as in the case of Dirac neutrinos.

In sec.II we obtain constraints from $(\beta\beta)_{0\nu}$. The constraint which arises from the neutrino oscillation experiment CHORUS is described in sec.III. Combining these constraints, we find that the large mixing angle of θ_{23} becomes improbable if the third generation of neutrino is a candidate for hot dark matter and not degenerate.

II. NEUTRINOLESS DOUBLE BETA DECAY

Let us first consider the neutrinoless double beta decay ($(\beta\beta)_{0\nu}$) which occurs only in the case of Majorana neutrinos.

— FIG.1 —

The decay rate of $(\beta\beta)_{0\nu}$ is, in the absence of right-handed couplings, proportional to the "averaged" mass defined by [6]

$$\langle m_\nu \rangle \equiv |\sum_{j=1}^{3} U_{ej}^2 m_j|. \tag{2.1}$$

Here $U_{\alpha j}$ is the left-handed neutrino mixing matrix which combines the weak eigenstate ($\alpha = e, \mu$ and τ) to the mass eigenstate with mass m_j (j=1,2 and 3). $U_{\alpha j}$ takes the following form in the case of Majorana neutrinos,

$$U = \begin{pmatrix} c_1 c_3 & s_1 c_3 e^{i\beta} & s_3 e^{i(\rho-\phi)} \\ (-s_1 c_2 - c_1 s_2 s_3 e^{i\phi}) e^{-i\beta} & c_1 c_2 - s_1 s_2 s_3 e^{i\phi} & s_2 c_3 e^{i(\rho-\beta)} \\ (s_1 s_2 - c_1 c_2 s_3 e^{i\phi}) e^{-i\rho} & (-c_1 s_2 - s_1 c_2 s_3 e^{i\phi}) e^{-i(\rho-\beta)} & c_2 c_3 \end{pmatrix}. \tag{2.2}$$

Here $c_j = cos\theta_j$, $s_j = sin\theta_j$ ($\theta_1 = \theta_{12},\ \theta_2 = \theta_{23},\ \theta_3 = \theta_{31}$). Besides ϕ there appear two additional CP violating phases β and ρ for Majorana neutrinos. Hence $\langle m_\nu \rangle$ becomes

$$\langle m_\nu \rangle = |m_1 c_1^2 c_3^2 - m_2 s_1^2 c_3^2 e^{-2i\beta'} - m_3 s_3^2 e^{-2i\rho'}|, \tag{2.3}$$

where we have introduced

$$\beta' \equiv \frac{\pi}{2} - \beta, \quad \rho' \equiv \frac{\pi}{2} - (\rho - \phi). \tag{2.4}$$

The CP violating phases β' and ρ' complicate extracting the constraints on the mixing angles from $\langle m_\nu \rangle$. In the following discussion, we follow the method given in [7] in order to get the constraints which are independent on β' and ρ'.

From Eq.(2.3) it follows that

$$\begin{aligned} \langle m_\nu \rangle^2 = & (m_1 c_1^2 c_3^2 - m_2 s_1^2 c_3^2 cos2\beta' - m_3 s_3^2 cos2\rho')^2 \\ & + (m_2 s_1^2 c_3^2 sin2\beta' + m_3 s_3^2 sin2\rho')^2 \end{aligned} \tag{2.5}$$

Rewriting $cos2\rho'$ and $sin2\rho'$ as $tan\rho'$, we can consider Eq.(2.5) as an equation of $tan\rho'$:

$$a_{+\beta} tan^2\rho' + b_\beta tan\rho' + a_{-\beta} = 0. \tag{2.6}$$

Here $a_{\pm\beta}$ and b_β are defined by

$$\begin{aligned} a_{\pm\beta} \equiv & 4 sin^2\beta' m_2 s_1^2 c_3^2 (m_1 c_1^2 c_3^2 \pm m_3 s_3^2) \\ & + (m_1 c_1^2 c_3^2 - m_2 s_1^2 c_3^2 \pm m_3 s_3^2)^2 - \langle m_\nu \rangle^2 \\ b_\beta \equiv & 4 m_2 m_3 s_1^2 s_3^2 c_3^2 sin2\beta'. \end{aligned} \tag{2.7}$$

So, the discriminant D for Eq.(2.6) must satisfy the following inequality:

$$
\begin{aligned}
D &\equiv b_\beta^2 - 4a_{+\beta}a_{-\beta} \\
&= 4^3(m_1c_1^2c_3^2)^2(m_2s_1^2c_3^2)^2(f_+ - sin^2\beta')(sin^2\beta' - f_-) \\
&\geq 0, \qquad (2.8)
\end{aligned}
$$

where

$$
f_\pm \equiv \frac{(\langle m_\nu\rangle \pm m_3s_3^2)^2 - (m_1c_1^2c_3^2 - m_2s_1^2c_3^2)^2}{4m_1m_2c_1^2s_1^2c_3^4}. \qquad (2.9)
$$

So we obtain

$$
f_- \leq sin^2\beta' \leq f_+. \qquad (2.10)
$$

It follows from Eq.(2.10) that

$$
f_- \leq 1, \quad f_+ \geq 0. \qquad (2.11)
$$

Analogously, rewriting $cos2\beta'$ and $sin2\beta'$ as $tan\beta'$, and considering Eq.(2.5) as an equation of $tan\beta'$, we obtain the inequalities:

$$
g_- \leq sin^2\rho' \leq g_+. \qquad (2.12)
$$

Here

$$
g_\pm \equiv \frac{(\langle m_\nu\rangle \pm m_2s_1^2c_3^2)^2 - (m_1c_1^2c_3^2 - m_3s_3^2)^2}{4m_1m_3c_1^2s_3^2c_3^2}. \qquad (2.13)
$$

So we get

$$
g_- \leq 1, \quad g_+ \geq 0. \qquad (2.14)
$$

Conditions (2.11) and (2.14) are consistency conditions. The CP violation area is given by more stringent conditions

$$
0 \leq f_- \leq sin^2\beta' \leq f_+ \leq 1 \ or \ 0 \leq g_- \leq sin^2\rho' \leq g_+ \leq 1. \qquad (2.15)
$$

Using the inequalities (2.11) and (2.14), we can describe the allowed region for mixing angles in the s_1^2 versus s_3^2 plane once the neutrino masses m_i and the "averaged " neutrino mass $\langle m_\nu\rangle$ are known. Only an upper bound of O(1eV) for the magnitude of $\langle m_\nu\rangle$ has been determined experimentally [8].

The neutrino masses may be safely ordered as $m_1 \leq m_2 \leq m_3$. So in the following discussions we will consider three cases:

a) $\langle m_\nu\rangle \leq m_1$,
b) $m_1 \leq \langle m_\nu\rangle \leq m_2$,
and
c) $m_2 \leq \langle m_\nu\rangle \leq m_3$.

Note that the definition of $\langle m_\nu\rangle$ in Eq.(2.1) and the Schwartz inequality jointly imply that

$$
\langle m_\nu\rangle \leq \sum_{j=1}^{3} m_j|U_{ej}^2| \leq m_3\sum_{j=1}^{3}|U_{ej}^2| = m_3, \qquad (2.16)
$$

that is, $\langle m_\nu\rangle$ can not be larger than m_3.

The allowed regions in the s_1^2 versus s_3^2 plane for cases (a), (b) and (c) are obtained from Eqs.(2.11) and (2.14), and are shown in FIG.2.

— FIG.2 —

From FIG.2, we obtain the following upper bound on s_3^2 :

$$s_3^2 \leq \frac{m_2 + \langle m_\nu \rangle}{m_3 + m_2} \tag{2.17}$$

in each of these cases. The CP violation areas in the s_1^2 versus s_3^2 plane given by Eq.(2.15) are indicated by oblique lines in FIG.2. Case (a) has been considered also in [7] and [9]. In [7], the representation for the mixing matrix adopted by Cabibbo-Kobayashi-Maskawa was used. In [9], only the limiting case where all the neutrino masses are degenerate ($m_1 = m_2 = m_3$) was discussed. It should be noted that we consider the additional cases (b) and (c) and also that no conditions on the neutrino masses have been imposed so far.

FIG.2 describes the allowed regions for neutrinoless double beta decay in the most general case. In the following discussion we restrict cases (b) and (c) as follows: (b$'$) $m_1 \ll \langle m_\nu \rangle < m_2 \sim m_3$ and (c$'$) $m_1 \sim m_2 \ll \langle m_\nu \rangle < m_3$. We will see how the neutrinoless double beta decay constrains the mixing angles θ_2 and θ_3.

In case (b$'$) we obtain $\frac{m_1+\langle m_\nu \rangle}{m_3+m_1} \approx \frac{\langle m_\nu \rangle - m_1}{m_3-m_1} \approx \frac{\langle m_\nu \rangle}{m_3}$, $\frac{\langle m_\nu \rangle - m_1}{m_2-m_1} \approx \frac{m_1+\langle m_\nu \rangle}{m_2+m_1} \approx \frac{\langle m_\nu \rangle}{m_3}$ and $\frac{m_2 \pm \langle m_\nu \rangle}{m_3+m_2} \approx \frac{1}{2} \pm \frac{\langle m_\nu \rangle}{2m_3}$. So the allowed regions depend solely on the value of $\frac{\langle m_\nu \rangle}{m_3}$. In FIG.3, we depict the allowed region s_1^2 versus s_3^2 plane for $\frac{\langle m_\nu \rangle}{m_3} = 0.16, 0.33, 0.49, 0.65, 0.82$ and 0.98.

— FIG.3 —

In case (c$'$) we have $\frac{m_1+\langle m_\nu \rangle}{m_3+m_1} \approx \frac{m_2+\langle m_\nu \rangle}{m_3+m_2} \approx \frac{\langle m_\nu \rangle - m_2}{m_3-m_2} \approx \frac{\langle m_\nu \rangle}{m_3}$. Therefore, the allowed region is $s_3^2 \approx \frac{\langle m_\nu \rangle}{m_3}$, $0 \leq s_1^2 \leq 1$. It is interesting that the value of s_3^2 is completely determined by $\frac{\langle m_\nu \rangle}{m_3}$ in this case.

III. NEUTRINO OSCILLATION

In this section, we consider the constraints obtained from the neutrino oscillation experiment CHORUS [10] and see how the method developed in the previous section can also be used to describe the allowed region in s_1^2 versus s_3^2 plane.

The CHORUS experiment investigates $\nu_\mu \to \nu_\tau$ oscillation through the observation of the τ leptons. The 90% C.L. upper limit of the probability of $P(\nu_\mu \to \nu_\tau)$ is 2.5×10^{-3} [11]. In order to obtain the constraints from this experiment we consider two cases:

(i) $m_3 \sim m_2 \gg m_1$

(ii) $m_3 \gg m_2 \sim m_1$

Firstly we consider case (i). In this case we have $\delta m_{31}^2 \equiv m_3^2 - m_1^2 \sim \delta m_{21}^2 \equiv m_2^2 - m_1^2 \gg \delta m_{32}^2 \equiv m_3^2 - m_2^2$. Substituting the experimental setting, $E_\nu = 27$GeV and

$L \approx 0.6$km, we have $\frac{\delta m^2 L}{4E_\nu} = 2.8 \times 10^{-2} \delta m^2\,\mathrm{eV}^2$. Hence we may set $sin\left(\frac{\delta m^2_{32} L}{4E_\nu}\right) \sim 0$. So the approximate oscillation probability is given by [12]

$$P(\nu_\mu \to \nu_\tau) = 4|U_{\mu 1}|^2 |U_{\tau 1}|^2 sin^2(\frac{\delta m^2_{31} L}{4E_\nu}). \tag{3.1}$$

Substituting the expression of Eq.(2.2) into Eq.(3.1), we obtain the following equation w.r.t $\cos\phi$,

$$\begin{aligned} \frac{P(\nu_\mu \to \nu_\tau)}{4\sin^2(\frac{\delta m^2_{31}}{4E_\nu} L)} &= a_+ \cos^2\phi - 2b\cos\phi + a_- \\ &\equiv f(\cos\phi). \end{aligned} \tag{3.2}$$

Here

$$\begin{aligned} a_+ &\equiv -(2 s_1 s_2 s_3 c_1 c_2)^2, \\ a_- &\equiv (s_1^2 c_2^2 + c_1^2 s_2^2 s_3^2)(s_1^2 s_2^2 + c_1^2 c_2^2 s_3^2), \\ b &\equiv s_1 s_2 s_3 c_1 c_2 (s_1^2 - c_1^2 s_3^2)(c_2^2 - s_2^2). \end{aligned} \tag{3.3}$$

The oscillation process does not distinguish Majorana neutrino from Dirac neutrino and only the ϕ phase occurs.

The constraints are obtained from Eq.(3.2). a_+ is negative definite and $f(\pm 1)$ is positive definite. Therefore the condition $-1 \leq cos\phi \leq 1$ implies the following inequalities:

Case a-1 : $0 \leq \frac{(s_2^2 - c_2^2)(s_1^2 - c_1^2 s_3^2)}{4 s_1 s_2 s_3 c_1 c_2} \leq 1$

$$(s_1 c_2 - c_1 s_2 s_3)^2 (s_1 s_2 + c_1 c_2 s_3)^2 \leq \frac{P(\nu_\mu \to \nu_\tau)}{4\sin^2(\frac{\delta m^2_{31}}{4E_\nu} L)} \leq \frac{1}{4}(s_1^2 + c_1^2 s_3^2)^2 \tag{3.4}$$

Case a-2 : $1 < \frac{(s_2^2 - c_2^2)(s_1^2 - c_1^2 s_3^2)}{4 s_1 s_2 s_3 c_1 c_2}$

$$(s_1 c_2 - c_1 s_2 s_3)^2 (s_1 s_2 + c_1 c_2 s_3)^2 \leq \frac{P(\nu_\mu \to \nu_\tau)}{4\sin^2(\frac{\delta m^2_{31}}{4E_\nu} L)} \leq (s_1 c_2 + c_1 s_2 s_3)^2 (s_1 s_2 - c_1 c_2 s_3)^2 \tag{3.5}$$

Case b-1 : $-1 \leq \frac{(s_2^2 - c_2^2)(s_1^2 - c_1^2 s_3^2)}{4 s_1 s_2 s_3 c_1 c_2} \leq 0$

$$(s_1 c_2 + c_1 s_2 s_3)^2 (s_1 s_2 - c_1 c_2 s_3)^2 \leq \frac{P(\nu_\mu \to \nu_\tau)}{4\sin^2(\frac{\delta m^2_{31}}{4E_\nu} L)} \leq \frac{1}{4}(s_1^2 + c_1^2 s_3^2)^2 \tag{3.6}$$

Case b-2 : $\frac{(s_2^2 - c_2^2)(s_1^2 - c_1^2 s_3^2)}{4 s_1 s_2 s_3 c_1 c_2} < -1$

$$(s_1 c_2 + c_1 s_2 s_3)^2 (s_1 s_2 - c_1 c_2 s_3)^2 \leq \frac{P(\nu_\mu \to \nu_\tau)}{4\sin^2(\frac{\delta m^2_{31}}{4E_\nu} L)} \leq (s_1 c_2 - c_1 s_2 s_3)^2 (s_1 s_2 + c_1 c_2 s_3)^2 \tag{3.7}$$

As we have mentioned before, only an upper bound of $P(\nu_\mu \to \nu_\tau)$ has been deteermined experimentally. So the meaningful inequalities arise from the lower bounds

of Eqs.(3.4) $\sim$ (3.7). Namely

Case a

$$(s_1c_2 - c_1s_2s_3)^2(s_1s_2 + c_1c_2s_3)^2 \leq \frac{P(\nu_\mu \to \nu_\tau)}{4\sin^2(\frac{\delta m^2_{31}}{4E_\nu}L)} \quad for \quad (s_2^2 - c_2^2)(s_1^2 - c_1^2s_3^2) \geq 0 \tag{3.8}$$

Case b

$$(s_1c_2 + c_1s_2s_3)^2(s_1s_2 - c_1c_2s_3)^2 \leq \frac{P(\nu_\mu \to \nu_\tau)}{4\sin^2(\frac{\delta m^2_{31}}{4E_\nu}L)} \quad for \quad (s_2^2 - c_2^2)(s_1^2 - c_1^2s_3^2) \leq 0 \tag{3.9}$$

From these inequalities (3.8) and (3.9), we obtain the allowed region in the s_1^2 versus s_3^2 plane for a fixed value of θ_2. Using $\delta m^2_{31} \sim \delta m^2_{21} = 6\text{eV}^2 \gg \delta m^2_{32}$ and $\frac{P(\nu_\mu \to \nu_\tau)}{\sin^2 \frac{\delta m^2_{31}L}{4E_\nu}} < 0.088$, we show the allowed regions for $\theta_2 = 0, \frac{\pi}{24}, \frac{\pi}{12}, \cdots, \frac{\pi}{2}$ in FIG.4.

— FIG.4 —

Next we consider case (ii). Here $\delta m^2_{31} \sim \delta m^2_{32} \gg \delta m^2_{21}$, and we have

$$P(\nu_\mu \to \nu_\tau) = 4|U_{\mu 3}|^2|U_{\tau 3}|^2 sin^2(\frac{\delta m^2_{31}L}{4E_\nu}). \tag{3.10}$$

Here, analogously to case (i), $\sin^2\left(\frac{\delta m^2_{21}L}{4E_\nu}\right) \sim 0$ has been assumed. Substituting the expression of Eq.(2.2) into Eq.(3.10), we obtain

$$P(\nu_\mu \to \nu_\tau) = 4\sin^2(2.8 \times 10^{-2}m_3^2)s_2^2c_2^2c_3^4. \tag{3.11}$$

Combining Eq.(3.11) with the constraints from the neutrinoless double beta decay Eq.(2.17), we also obtain

$$\begin{aligned} &P(\nu_\mu \to \nu_\tau) \\ &\quad \geq 4\sin^2(2.8 \times 10^{-2}m_3^2)s_2^2c_2^2\left(\frac{m_3 - \langle m_\nu \rangle}{m_3}\right)^2. \end{aligned} \tag{3.12}$$

Using the experimental upper bound of $P(\nu_\mu \to \nu_\tau) < 2.5 \times 10^{-3}$ and Eq.(3.11), we obtain the allowed regions in the s_2^2 versus s_3^2 plane for the possible values of m_3 (FIG.5).

— FIG.5 —

Eq.(3.12) with $P(\nu_\mu \to \nu_\tau) < 2.5 \times 10^{-3}$ gives more interesting constraints in the s_2^2 versus $\langle m_\nu \rangle$ plane for the possible values of m_3, which is depicted in FIG.6.

— FIG.6 —

It should be noted that $\langle m_\nu \rangle$ can not be larger than m_3 as seen from Eq.(2.16). If we input the experimental upper bound of $\langle m_\nu \rangle \lesssim O(1\text{eV})$ in FIG.6, we obtain more stringent constraints on θ_2. It is interesting that the allowed region becomes more restrictive as the experimental results become more precise. That is, the smaller the upper bounds of $P(\nu_\mu \to \nu_\tau)$ and $\langle m_\nu \rangle$ become, the more restricted the allowed region is. Especially, the large mixing angle solution of θ_2 becomes improbable if m_3 is a candidate for hot dark matter.

IV. CONCLUSION

We have obtained constraints from $(\beta\beta)_{0\nu}$ and the CHORUS neutrino oscillation experiment, taking into account possible CP violation phases. From $(\beta\beta)_{0\nu}$, the allowed regions on s_2^2 versus s_3^2 plane are obtained for the respective cases (case(a): $\langle m_\nu \rangle \leq m_1$, case(b): $m_1 \leq \langle m_\nu \rangle \leq m_2$, and case(c): $m_2 \leq \langle m_\nu \rangle \leq m_3$). From the CHORUS neutrino oscillation experiment, we have obtained the allowed regions in the s_1^2 versus s_3^2 plane for given values of θ_2 for case (i); $\delta m_{31}^2 \sim \delta m_{21}^2 = 6\mathrm{eV}^2 \gg \delta m_{32}^2$. For case(ii), $\delta m_{31}^2 \sim \delta m_{32}^2 \gg \delta m_{21}^2$, we have obtained the allowed regions in the s_2^2 versus s_3^2 plane and those in the $\langle m_\nu \rangle$ versus s_2^2 plane for given values of m_3. Combining the constraint from the CHORUS neutrino oscillation experiment with those obtained from $(\beta\beta)_{0\nu}$, we have found that the large mixing angle solution of θ_2 becomes improbable if the neutrino mass m_3 of the third generation is a candidate for hot dark matter and not degenerate.

Recently the Super Kamiokande group announced that they obtained the large angle solution $\sin^2 2\theta_{23} \geq 0.8$ with $\delta m_{23}^2 = 10^{-4} \sim 10^{-2}\mathrm{eV}^2$ from the atmospheric neutrino deficit [13]. This conclusion does not contradict our result since their δm_{23}^2 is of order less than $1\mathrm{eV}^2$.

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FIG. 1. Feynman diagram of neutrinoless double beta decay.

FIG. 2. The allowed region in the $sin^2\theta_{12}$ versus $\sin^2\theta_{31}$ plane obtained from neutrinoless double beta decay is given by the shaded areas in the cases:
(a) $\langle m_\nu \rangle \leq m_1$
(b) $m_1 \leq \langle m_\nu \rangle \leq m_2$
(c) $m_2 \leq \langle m_\nu \rangle \leq m_3$
In the allowed region, CP-violation area is specially indicated by the oblique lines.

FIG. 3. Each allowed region obtained from neutrinoless double beta decay for the cases:
$\frac{\langle m_\nu \rangle}{m_3} = 0.16,\ 0.33,\ 0.49,\ 0.65,\ 0.82,\ 0.98$
under the assumption that $m_1 \ll \langle m_\nu \rangle < m_2 \sim m_3$

FIG. 4. The allowed regions (shaded regions) by the inequalities of Eqs.(3.8) and (3.9) under the condition that $P_{CHORUS} < 2.5 \times 10^{-3}$, $\delta m^2_{31} \sim \delta m^2_{21} = 6\mathrm{eV}^2 \gg \delta m^2_{32}$ with given θ_2:
(a)$\theta_2 = 0, \frac{\pi}{2}$ (b)$\theta_2 = \frac{\pi}{24}, \frac{11\pi}{24}$ (c)$\theta_2 = \frac{\pi}{12}, \frac{5\pi}{12}$ (d)$\theta_2 = \frac{\pi}{8}, \frac{3\pi}{8}$
(e)$\theta_2 = \frac{\pi}{6}, \frac{\pi}{3}$ (f)$\theta_2 = \frac{5\pi}{24}, \frac{7\pi}{24}$ (g)$\theta_2 = \frac{\pi}{4}$

FIG. 5. The allowed regions (shaded regions) in the s_2^2 versus s_3^2 plane for the possible values of m_3 obtained from the CHORUS experiment.

FIG. 6. The allowed regions (shaded regions) in the $\langle m_\nu \rangle$ versus s_2^2 plane for the possible values of m_3 obtained from the CHORUS experiment and $(\beta\beta)_{0\nu}$.

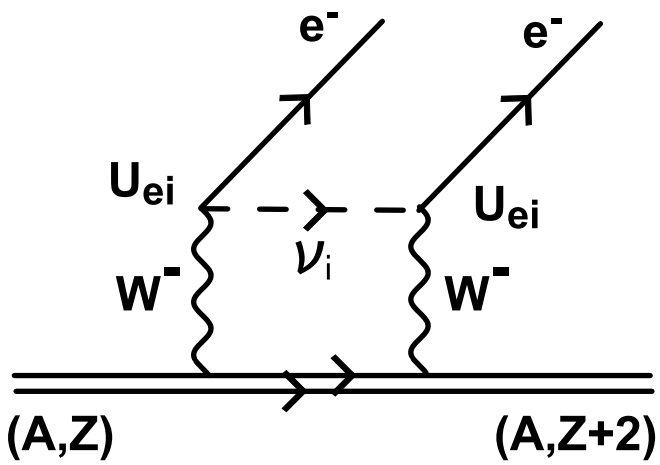

FIG.1

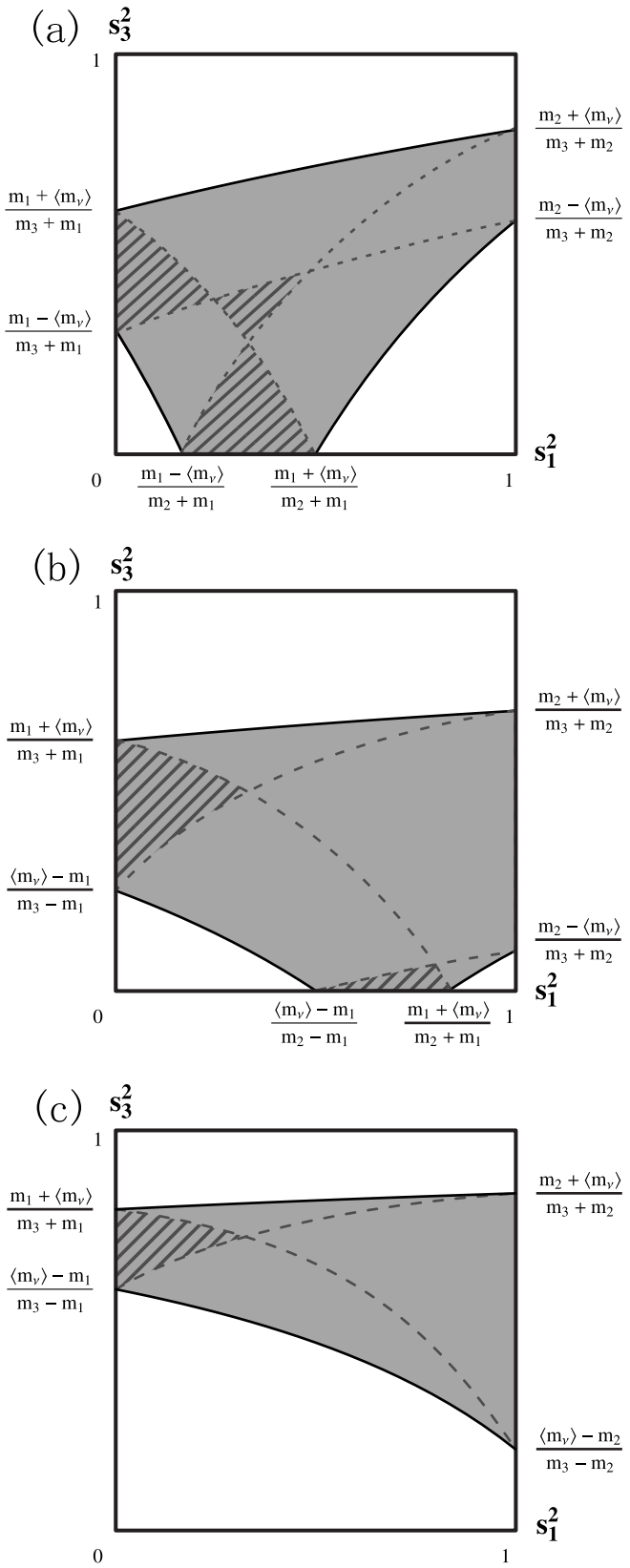
(a)
s_3^2
1
$\frac{m_2 + \langle m_\nu \rangle}{m_3 + m_2}$
$\frac{m_1 + \langle m_\nu \rangle}{m_3 + m_1}$
$\frac{m_2 - \langle m_\nu \rangle}{m_3 + m_2}$
$\frac{m_1 - \langle m_\nu \rangle}{m_3 + m_1}$
s_1^2
0
$\frac{m_1 - \langle m_\nu \rangle}{m_2 + m_1}$
$\frac{m_1 + \langle m_\nu \rangle}{m_2 + m_1}$
1
(b)
s_3^2
1
$\frac{m_2 + \langle m_\nu \rangle}{m_3 + m_2}$
$\frac{m_1 + \langle m_\nu \rangle}{m_3 + m_1}$
$\frac{\langle m_\nu \rangle - m_1}{m_3 - m_1}$
$\frac{m_2 - \langle m_\nu \rangle}{m_3 + m_2}$
s_1^2
0
$\frac{\langle m_\nu \rangle - m_1}{m_2 - m_1}$
$\frac{m_1 + \langle m_\nu \rangle}{m_2 + m_1}$
1
(c)
s_3^2
1
$\frac{m_2 + \langle m_\nu \rangle}{m_3 + m_2}$
$\frac{m_1 + \langle m_\nu \rangle}{m_3 + m_1}$
$\frac{\langle m_\nu \rangle - m_1}{m_3 - m_1}$
$\frac{\langle m_\nu \rangle - m_2}{m_3 - m_2}$
s_1^2
0
1

FIG.2

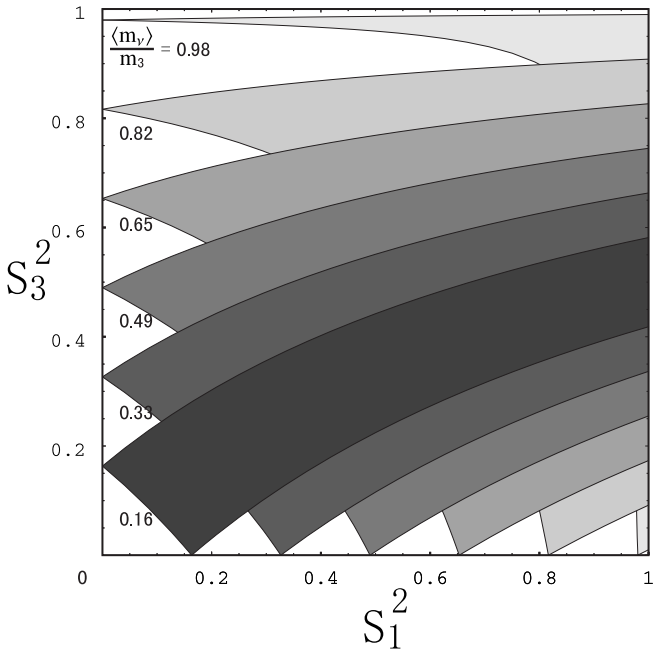
$\frac{\langle m_\nu \rangle}{m_3}$ = 0.98
0.82
0.65
0.49
0.33
0.16
0
0.2
0.4
0.6
0.8
1
S_3^2
S_1^2

FIG.3

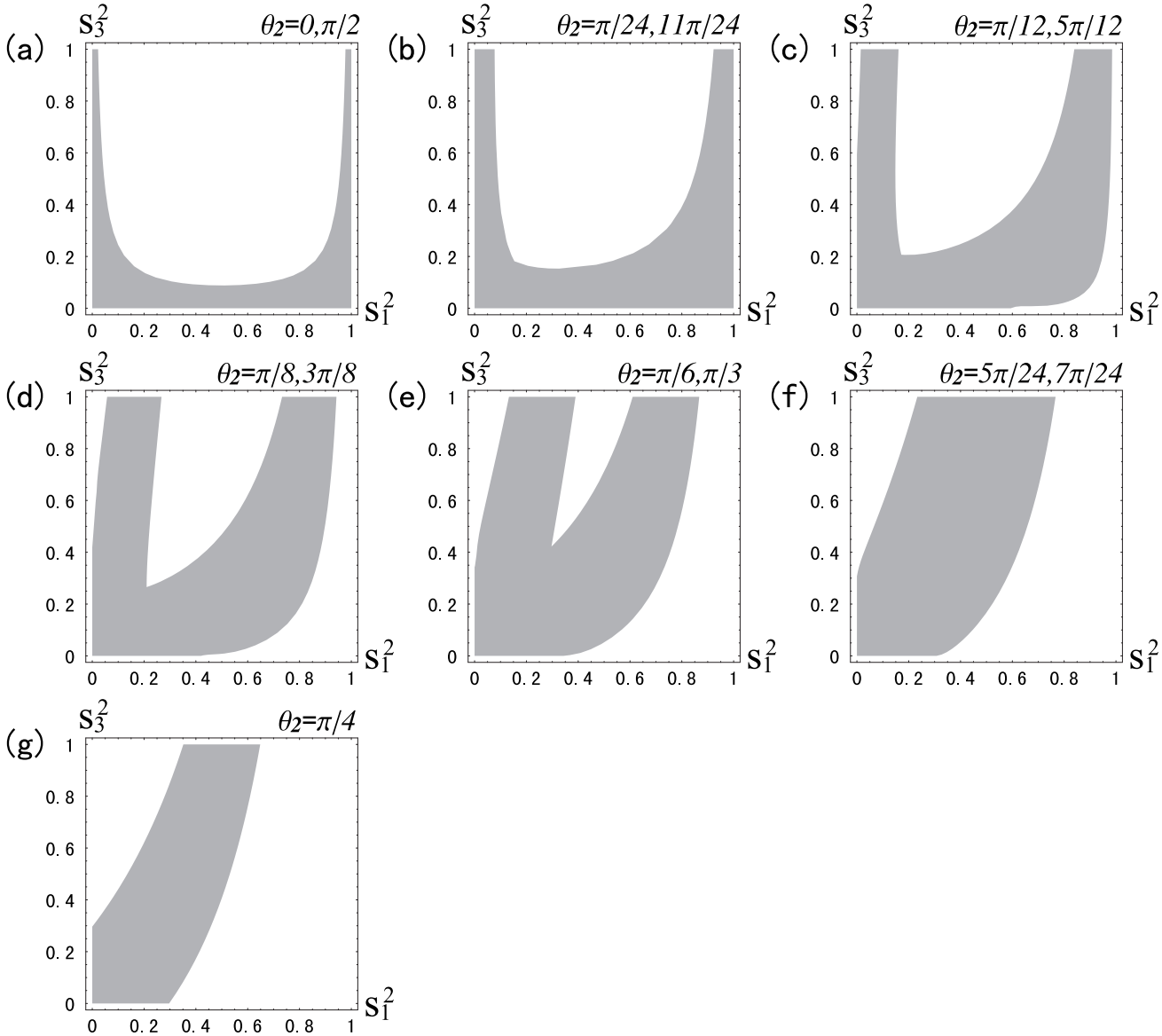
(a)
$\theta_2=0,\pi/2$
(b)
$\theta_2=\pi/24,11\pi/24$
(c)
$\theta_2=\pi/12,5\pi/12$
(d)
$\theta_2=\pi/8,3\pi/8$
(e)
$\theta_2=\pi/6,\pi/3$
(f)
$\theta_2=5\pi/24,7\pi/24$
(g)
$\theta_2=\pi/4$
s_3^2
s_1^2
0
0.2
0.4
0.6
0.8
1

FIG.4

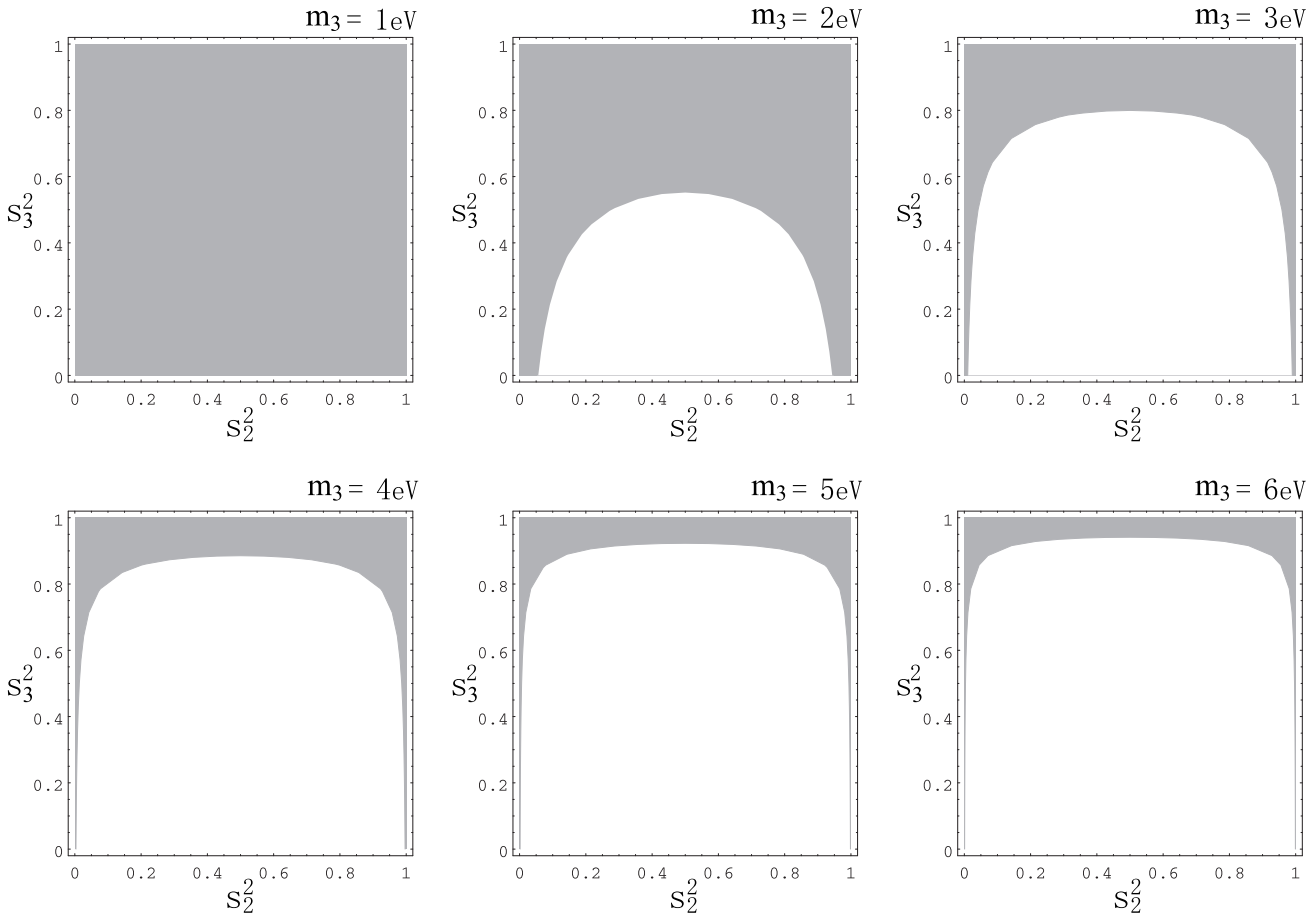

FIG.5

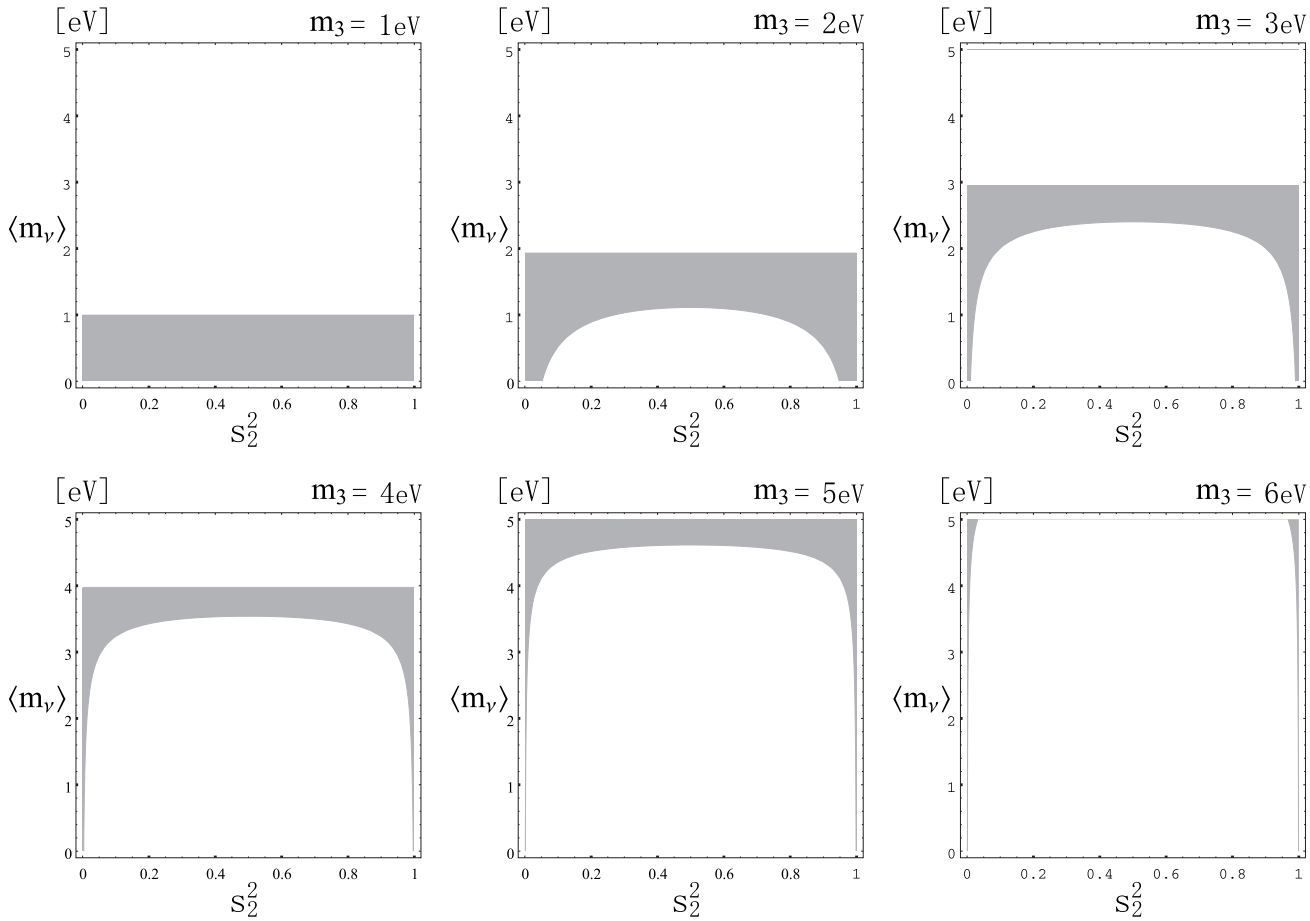

FIG.6