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Mathematical Modeling of the Synchronous Phase Modifier

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Abstract: The mathematical model of the three-phase, synchronous phase modifier has been developed. This model has been represented as a system of six (salient-pole machine) and eight (nonsalient-pole machine) differential equations for flux-linkages and angular velocity determination. The differential equations for the flux-linkages of the stator, amortisseur and excitation windings have been expressed in synchronously rotational rectangular direct and quadrature coordinates. Both the rotational and transformation electromotive forces have been taken into consideration in the stator differential equations. The additional resistance for excitation current control has been taken into consideration in the differential equation of the excitation winding. The currents in both axes have been presented in matrix form. The curves of the synchronous phase modifier angular velocity; field and stator currents during induction starting have been calculated and illustrated.

Key Words: Synchronous Phase Modifier, Mathematical Model, Differential Equations, Direct and Quadrature Axes

Introduction

When an industrial plant has a lagging power factor, the value of the power factor should be maintained between 0.9 and 1.0, if possible (Smith, R.L. and S. Herman, 1999). This condition is desirable because of a number of factors including the need to reduce the reactive current to achieve more capacity for useful current on the mains, feeders and subfeeders; the need for better voltage regulation and stability; and the desirability of obtaining lower power rates from the power company.

The ability to adjust the power factor of one or more loads in a power system can significantly affect the operating efficiency of the power system. The lower the power factor of a system, the greater the losses in the power line feeding it.

Most loads on a typical power system are induction motors (Gordon, R.S., 1992), so power systems are almost invariably lagging in power factor. Having one or more leading loads on the system can be useful for the following reasons:

- a leading load can supply some reactive power for nearby lagging loads, instead it coming from the generator. Since the reactive power does not have to travel over the long and fairly high-resistance transmission lines, the transmission line current is reduced and the power system losses are much lower;
- since the transmission lines carry less current, they can be smaller for a given rated power flow. A lower equipment current rating reduces the cost of a power system.

When a synchronous phase modifier is connected across the line, it supplies the leading component of the current needed to offset or counteract the existing lagging component of the current. When used with a voltage regulator the phase modifier can automatically run overexcited at times of high load and under excited at light load (Weedy B.M. and B. J. Cory, 1998).

A great advantage is the flexibility of operation for all load conditions. Although the cost of such installations is high, in some circumstances it is justified, e.g. at the receiving-end busbar of a long high voltage line where transmission at power factors less than unity cannot be tolerated. By adjusting the field current of the synchronous phase modifier, causing the phase modifier to draw a lagging current from the line at no-load and a leading current at full load, the receiving-end voltage of the line can be kept constant. Being a rotating machine, its stored energy is useful for riding through transient disturbances, including voltage sags (Grainger J. and W. Stevenson. 1994).

Prediction of synchronous phase modifier performance depends on having an adequate analytical model. The models of synchronous motor and generator are subjected to intensive and detailed research (Al-Jufout, S.A., 1999), (Al-Jufout S. A., 2000), (Sivokobylenko, V.F., 1984) and (Rogozin, G.G., 1992) whereas the synchronous phase modifier is neglected, which should be highlighted in order to predict and analyze its performance in various modes of operation. Recently much attention is devoted to the development of models that can be used for both transient as well as steady-state conditions.

The Numerical Model of the Synchronous Phase Modifier: Since the synchronous phase modifier is similar to a synchronous motor or generator, its numerical model can be derived from the numerical model of the synchronous motor or generator (Sivokobylenko, V.F. and V. Kostenko, 1979). Analysis of synchronous machines is greatly facilitated by transforming armature quantities to a reference frame rotating at rotor speed. In such a reference frame, the armature quantities can be resolved into two components, one along the field winding axis, also known as the direct axis and the second along an orthogonal axis, known as the quadrature axis. The numerical model of the salient-pole synchronous phase modifier in synchronously rotational rectangular direct and quadrature axes can be written as follows:

$$\frac{d\lambda_{Sd}}{dt} = v_{Sd} - R_S i_{Sd} + \omega \lambda_{Sq}, \quad (1)$$

$$\frac{d\lambda_{Sq}}{dt} = v_{Sq} - R_S i_{Sq} - \omega \lambda_{Sd}, \quad (2)$$

$$\frac{d\lambda_f}{dt} = v_f - (R_f + R_{adj}) i_f, \quad (3)$$

$$\frac{d\lambda_{Rd}}{dt} = -R_{Rd} i_{Rd}, \quad (4)$$

$$\frac{d\lambda_{Rq}}{dt} = -R_{Rq} i_{Rq}, \quad (5)$$

$$\frac{d\omega}{dt} = \frac{1}{J}\left(i_{Sq}\lambda_{Sd} - i_{Sd}\lambda_{Sq}\right), \quad (6)$$

where

$\lambda_{Sd}, \lambda_{Sq}, \lambda_{Rd}, \lambda_{Rq}, \lambda_f$: the flux linkages of the stator, the amortisseur windings in direct and quadrature coordinates respectively and the flux linkage of the field winding;

$i_{Sd}, i_{Sq}, i_{Rd}, i_{Rq}, i_f$: the currents of the stator, the amortisseur windings in direct and quadrature coordinates respectively and the current of the field winding;

R_S, R_f, R_{Rd}, R_{Rq}: the resistances of the stator, the field winding and the amortisseur winding in direct and quadrature coordinates respectively;

R_{adj}: adjustable resistance connected in series with the field circuit;

v_{Sd}, v_{Sq}, v_f: the voltages applied to the synchronous phase modifier in direct and quadrature coordinates respectively and the field winding voltage;

ω : the angular velocity of the rotor;

J : the rotor moment of inertia.

The currents of the stator, amortisseur and field windings can be computed by solving two systems of three and two algebraic equations respectively. These systems in matrix form are as follows:

$$\begin{bmatrix} i_{Sd} \\ i_{Rd} \\ i_f \end{bmatrix} = \begin{bmatrix} L_{\sigma S}+L_{\mu d} & L_{\mu d} & L_{\mu d} \\ L_{\mu d} & L_{\sigma Rd}+L_{\mu d} & L_{\mu d} \\ L_{\mu d} & L_{\mu d} & L_{\sigma f}+L_{\mu d} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \lambda_{Sd} \\ \lambda_{Rd} \\ \lambda_f \end{bmatrix}$$

$$\begin{bmatrix} i_{Sq} \\ i_{Rq} \end{bmatrix} = \begin{bmatrix} L_{\sigma S}+L_{\mu q} & L_{\mu q} \\ L_{\mu q} & L_{\sigma Rq}+L_{\mu q} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \lambda_{Sq} \\ \lambda_{Rq} \end{bmatrix},$$

where

$L_{\sigma S}, L_{\sigma f}, L_{\sigma Rd}, L_{\sigma Rq}$: the leakage inductances of the stator, the field winding and the amortisseur in direct and quadrature coordinates respectively;

$L_{\mu d}, L_{\mu q}$: the inductances of the magnetization branch in direct and quadrature coordinates respectively.

For the nonsalient-pole synchronous phase modifier, the current paths in the cylindrical-rotor are not as well defined as in the damper circuit of the salient-pole phase modifier. The rotor can be then represented as a set of direct-axis windings consisting of the field winding and additional two windings representing the rotor-body circuits (Fig.1), which produce flux along the direct axis and a set of two quadrature-axis windings, which corresponds to rotor circuits, which produce flux along the quadrature axis.

A mathematical model of the cylindrical-rotor synchronous phase modifier can be obtained by replacing equation (4) with equations (7) and (8):

$$\frac{d\lambda_{Rd}^{(1)}}{dt} = -R_{Rd}^{(1)} i_{Rd}^{(1)}, \quad (7)$$

$$\frac{d\lambda_{Rd}^{(2)}}{dt} = -R_{Rd}^{(2)} i_{Rd}^{(2)}, \quad (8)$$

and equation (5) with equations (9) and (10):

$$\frac{d\lambda_{Rq}^{(1)}}{dt} = -R_{Rq}^{(1)} i_{Rq}^{(1)}, \quad (9)$$

$$\frac{d\lambda_{Rq}^{(2)}}{dt} = -R_{Rq}^{(2)} i_{Rq}^{(2)}, \quad (10)$$

where

$\lambda_{Rd}^{(1)}, \lambda_{Rd}^{(2)}, \lambda_{Rq}^{(1)}, \lambda_{Rq}^{(2)}$: the flux linkages of the rotor circuits in direct and quadrature coordinates respectively;

$i_{Rd}^{(1)}, i_{Rd}^{(2)}, i_{Rq}^{(1)}, i_{Rq}^{(2)}$: the currents of the rotor circuits in direct and quadrature coordinates respectively;

$R_{Rd}^{(1)}, R_{Rd}^{(2)}, R_{Rq}^{(1)}, R_{Rq}^{(2)}$: the resistances of the rotor circuits in direct and quadrature coordinates respectively.

Thus, the currents can be computed by solving two systems of four and three algebraic equations respectively. These systems in matrix form are as follows:

$$\begin{bmatrix} i_{Sd} \\ i_{Rd}^{(1)} \\ i_{Rd}^{(2)} \\ i_f \end{bmatrix} = \begin{bmatrix} L_{\sigma S}+L_{\mu d} & L_{\mu d} & L_{\mu d} & L_{\mu d} \\ L_{\mu d} & L_{\sigma Rd}^{(1)}+L_{\mu d} & L_{\mu d} & L_{\mu d} \\ L_{\mu d} & L_{\mu d} & L_{\sigma Rd}^{(2)}+L_{\mu d} & L_{\mu d} \\ L_{\mu d} & L_{\mu d} & L_{\mu d} & L_{\sigma f}+L_{\mu d} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \lambda_{Sd} \\ \lambda_{Rd}^{(1)} \\ \lambda_{Rd}^{(2)} \\ \lambda_f \end{bmatrix}$$

$$\begin{bmatrix} i_{Sq} \\ i_{Rq}^{(1)} \\ i_{Rq}^{(2)} \end{bmatrix} = \begin{bmatrix} L_{\sigma S}+L_{\mu q} & L_{\mu q} & L_{\mu q} \\ L_{\mu q} & L_{\sigma Rq}^{(1)}+L_{\mu q} & L_{\mu q} \\ L_{\mu q} & L_{\mu q} & L_{\sigma Rq}^{(2)}+L_{\mu q} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \lambda_{Sq} \\ \lambda_{Rq}^{(1)} \\ \lambda_{Rq}^{(2)} \end{bmatrix}$$

where

$L_{\sigma Rd}^{(1)}, L_{\sigma Rd}^{(2)}, L_{\sigma Rq}^{(1)}, L_{\sigma Rq}^{(2)}$: the leakage inductances of the rotor circuits in direct and quadrature coordinates respectively.

The active and reactive powers can be computed by the following formulas:

$$P = v_{Sd} i_{Sd} + v_{Sq} i_{Sq},$$

$$Q = v_{Sq} i_{Sd} - v_{Sd} i_{Sq}.$$

In the developed model, the series adjustable resistance in the field circuit is used to control the field

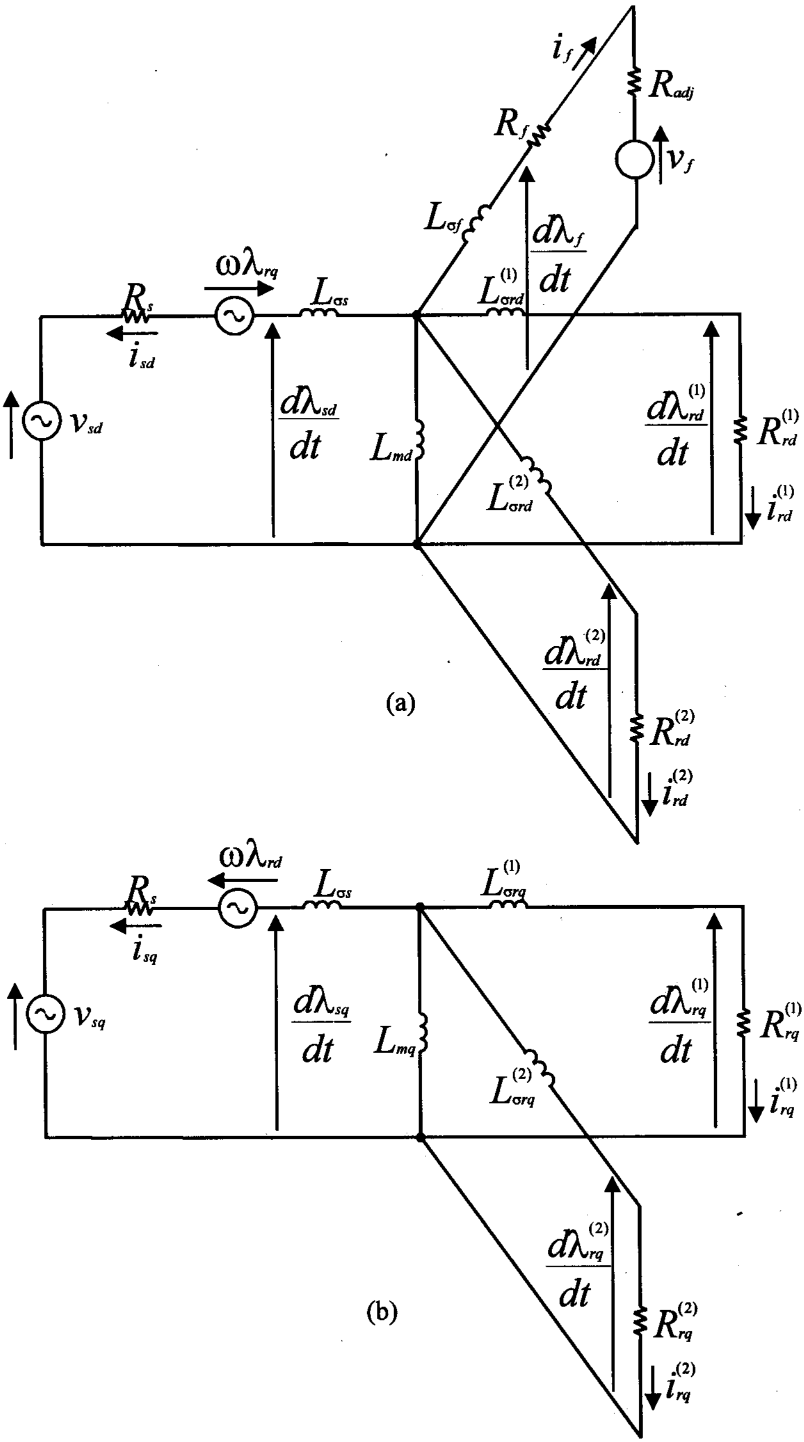

Fig. 1: The Equivalent Circuit of the Synchronous Phase Modifier with two *RL*-Parallel Circuits: (a) in the Direct Axis (b) in the Quadrature Axis

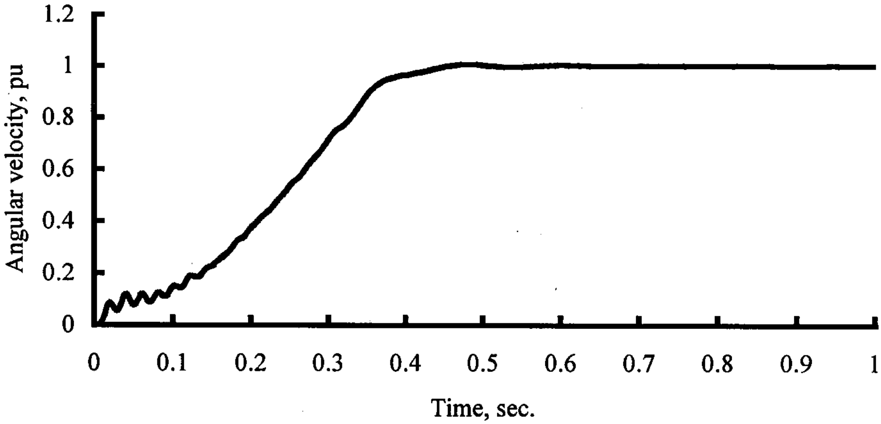

Fig. 2: The Angular Velocity of the Synchronous Phase Modifier During Induction Starting

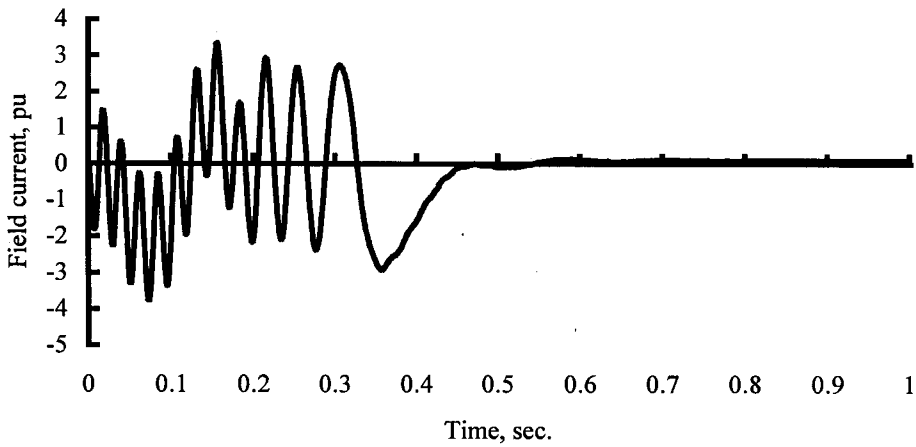

Fig. 3: The Field Current of the Synchronous Phase Modifier During Induction Starting

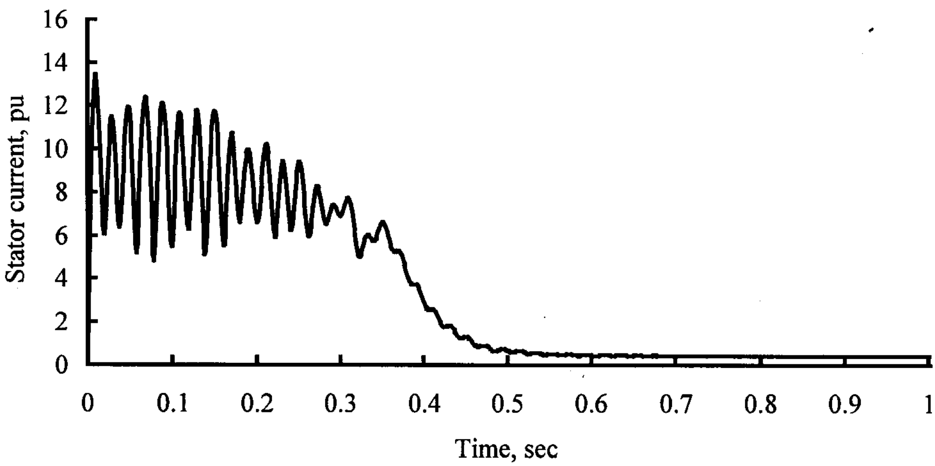

Fig. 4: The Magnitude of the Stator Current of the Synchronous Phase Modifier During Induction Starting

current. On large synchronous phase modifiers, Silicon Controlled Rectifiers (SCRs) are used to control the field current by changing the dc voltage applied to the field winding. By selecting different firing angles, the rectified dc voltage will be somewhere between the maximum value and zero. This voltage will have harmonic content and some form of filter on its output is important. Modeling of the control circuit with SCRs is investigated in (Lander C., 1993).

Model Application: The developed model can be used as a tool to predict the performance of the synchronous phase modifier in both transient and steady-state conditions. The systems of the differential and algebraic equations can be solved via analog or digital computer.

The voltages applied to the stator and field windings are given in per unit. The parameters of the equivalent model circuit (resistances and inductances) can be determined in per unit (pu) by engineering methods. The value of the adjustable resistance can be selected to obtain the desirable field current value. This model allows simulating the starting, running down, three-phase short-circuit at the terminals of the phase modifier and other symmetrical modes of operation.

To demonstrate the application of this model, the induction starting process followed by steady-state conditions is computed and analyzed (transient case followed by steady-state conditions). To simulate the induction starting process, the initial values of the variables (flux linkages and speed) are assumed equal to zero. The field winding is left unexcited and shunted by a resistance. If the phase modifier terminals are now connected to an ac supply, it will start as an induction motor because currents will be induced in the amortisseur winding to produce torque. The phase modifier will speed up and when the speed approaches 95% of the synchronous value, the rotor poles are excited by a field current from a dc source, the rotor poles, closely following the stator poles, will be locked to them. The rotor will then run at synchronous speed (Fig.2). Fig.3 shows the current flowing in the field winding during the starting process, where is the frequency of the induced ac component decreases while the motor speeds up. Fig.3 shows the magnitude of the stator current where the inrush starting current is equal to 13.4 pu.

Conclusion

The mathematical model of the three-phase synchronous phase modifier for transient as well as steady-state conditions analysis is developed. This model is represented as a system of six (salient-pole machine) and eight (nonsalient-pole machine) differential equations for flux-linkages and angular velocity determination. The differential equations for the flux-linkages of the stator, amortisseur and excitation windings are expressed in synchronously rotational rectangular direct and quadrature coordinates. Both the rotational and transformation electromotive forces are taken into consideration in the stator differential equations. The additional adjustable resistance for excitation current control is taken into consideration in the differential equation of the excitation winding. The currents in both axes are presented in matrix form. The skin effect in the cylindrical rotor is considered by representing the rotor by two parallel-connected resistive-inductive circuits. The curves of the synchronous phase modifier angular velocity; field and stator currents during induction starting have been calculated and illustrated. Such investigation increases the reliability of operation by accurate selection of the control and protective devices' settings particularly when microprocessors are used.

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