REPETITIVE CLUSTER-TILTED ALGEBRAS⋆

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ABSTRACT. Let H be a finite dimensional hereditary algebra over an algebraically closed field k and $\mathscr{C}_{F^m}$ be the repetitive cluster category of H with $m \geq 1$. We investigate the properties of cluster tilting objects in $\mathscr{C}_{F^m}$ and the structure of repetitive cluster-tilted algebras. Moreover, we generalized Theorem 4.2 in [4] (Buan A, Marsh R, Reiten I. Cluster-tilted algebra. Trans. Amer. Math. Soc., 359(1)(2007), 323-332.) to the situation of $\mathscr{C}_{F^m}$, and prove that the tilting graph $\mathscr{K}_{\mathscr{C}_{F^m}}$ of $\mathscr{C}_{F^m}$ is connected.

1. INTRODUCTION

Let H be a finite dimensional hereditary algebra over an algebraically closed field k. The endomorphism algebra of a tilting module over H is called tilted algebra. Cluster category of type H is the orbit category $\mathscr{C} = D^b(H)/(F)$ of the derived category $D^b(H)$ of H by an automorphism group generated by $F = \tau^{-1}[1]$, where τ is the Auslander-Reiten translation in $D^b(H)$ and [1] is the shift functor of $D^b(H)$. $\mathscr{C}$ is a triangulated category and is a Calabi-Yau categories of CY-dimension 2, see [4, 12]. It was shown that any cluster tilting object of $\mathscr{C}$ is induced by a tilting module of a hereditary algebra H' which is derived equivalent to H. The endomorphism algebra of a tilting object in $\mathscr{C}$ is called a cluster-tilted algebra.

Now cluster-tilted algebras and cluster categories provide an algebraic understanding of combinatorics of cluster algebras defined and studied by Fomin and Zelevinsky in [7]. In this connection, the indecomposable exceptional objects in cluster categories correspond to the cluster variables, and cluster tilting objects

⋆ Supported by the NSFC(11171183).

(maximal 1-orthogonal subcategories [10, 11]) correspond to clusters in the corresponding cluster algebras, see [5, 6]. Moreover by [12] or [14], cluster-tilted algebras provide a class of Gorenstein algebras of Gorenstein dimension 1, which is important in representation theory of algebras [15].

For any positive integer m, a repetitive cluster category $\mathscr{C}_{F^m} = D^b(H)/(F^m)$, which is defined by [17] as the orbit category of the derived category $D^b(H)$ by the group (F^m) generated by F^m, is a triangulated category by Keller [12], which is also a Calabi-Yau category of Calabi-Yau dimension 2m/m. The cluster tilting objects in this repetitive cluster category are shown to correspond one-to-one to those in the classical cluster categories. The endomorphism algebras of cluster tilting objects in $\mathscr{C}_{F^m}$ are called repetitive cluster-tilted algebras. They all have the same representation type and share a universal covering: the endomorphism algebra of corresponding cluster tilting subcategory in $D^b(H)$, see [17] for details.

In this paper, we investigate the properties of cluster tilting objects in $\mathscr{C}_{F^m}$ and the structure of repetitive cluster-tilted algebras. The article is organized as follows: In Section 2 we prove some basic facts about cluster tilting objects in $\mathscr{C}_{F^m}$. In Section 3, we investigate the structure of repetitive cluster-tilted algebras, and generalize Theorem 4.2 in [3] to the situation of $\mathscr{C}_{F^m}$, see Theorem 3.4. Furthermore, we also prove that the tilting graph $\mathscr{K}_{\mathscr{C}_{F^m}}$ of $\mathscr{C}_{F^m}$ is connected.

Throughout this paper, we fix an algebraically closed field k, and denote by H a finite dimensional hereditary k-algebra with n simple modules. We denote by $\mathscr{C} = D^b(H)/(F)$ the cluster category of H. A basic *tilting object* in $\mathscr{C}$ is an object T with n non-isomorphic indecomposable direct summands such that $\mathrm{Ext}^1_{\mathscr{C}}(T, T) = 0$. We follow the standard terminologies and notations used in the representation theory of algebras, see [1, 2, 8, 16].

2. Properties of cluster tilting objects in $\mathscr{C}_{F^m}$

In this section, we first recall some definitions and collect some known results which will be used later, then we prove some basic facts about cluster tilting objects in $\mathscr{C}_{F^m}$.

Let $\mathscr{A}$ be a Krull-Remark-Schmidt category and $\mathscr{B}$ a full subcategory of $\mathscr{A}$. We denote by ind $\mathscr{B}$ the set of all indecomposable objects in $\mathscr{B}$. For any object M in $\mathscr{A}$, we denote by add M the full subcategory of $\mathscr{A}$ consisting of finite direct sums of indecomposable summands of M and by $\delta(M)$ the number of non-isomorphic indecomposable summands of M.

Let m be an integer with $m \geq 1$ and $\mathscr{C}_{F^m} = D^b(H)/(F^m)$ be the repetitive cluster category defined in [11] with $F = \tau^{-1}[1]$. Note that $\mathscr{C}_{F^1} = \mathscr{C}$ is the classical cluster category. The following definition is defined in [10, 11, 17].

Definition 2.1. *An object T of $\mathscr{C}_{F^m}$ is a cluster tilting object provided $X \in$ add T if and only if $\mathrm{Ext}^1_{\mathscr{C}_{F^m}}(X, T) = 0$ and $X \in$ add T if and only if $\mathrm{Ext}^1_{\mathscr{C}_{F^m}}(T, X) = 0$.*

The triangle functor $\rho_m : \mathscr{C}_{F^m} \to \mathscr{C}$ is defined in [17] which is also a covering functor. The following lemma is taken from [17] which will be used in the sequel.

Lemma 2.2. (1). *$\mathscr{C}_{F^m}$ is a Krull-remark-Schmidt category.*

(2). $\mathrm{ind}\ \mathscr{C}_{F^m} = \bigcup_{i=0}^{m-1} (\mathrm{ind}\ F^i(\mathscr{C}))$.

(3). *T is a cluster tilting object in $\mathscr{C}$ if and only if $\rho_m^{-1}(T)$ is a cluster tilting object in $\mathscr{C}_{F^m}$.*

(4). *For any tilting H-module T, $\bigoplus_{i=0}^{m-1} F^i T$ is a cluster tilting object in $\mathscr{C}_{F^m}$, and any cluster tilting object M in $\mathscr{C}_{F^m}$ arises in this way, i.e., there is a hereditary algebra H', which is derived equivalent to H, and a tilting H'-module T' such that the cluster tilting object M is induced from T'.*

Remark. It is easy to see that $\delta(T) = mn$ provided T is a cluster tilting object in $\mathscr{C}_{F^m}$.

Let M be an object of $\mathscr{C}_{F^m}$. M is said to be 1-orthogonal if $\mathrm{Ext}^1_{\mathscr{C}_{F^m}}(M, M) = 0$. A basic 1-orthogonal object of $\mathscr{C}_{F^m}$ is said to be an almost tilting object if $\delta(M) = nm - 1$ and there exists an indecomposable object X of $\mathscr{C}_{F^m}$ such that $M \oplus X$ is a cluster tilting object of $\mathscr{C}_{F^m}$.

Definition 2.3. *An object M of $\mathscr{C}_{F^m}$ is said to be F-stable if M can be written as $M = X \oplus FX \oplus \cdots \oplus F^{m-1}X$ for some object X in $\mathcal{C}$. In this case, we say that the F-stable object M is determined by X.*

Lemma 2.4. *Let M be a F-stable object of $\mathscr{C}_{F^m}$ and $M = X \oplus FX \oplus \cdots \oplus F^{m-1}X$ for some object X in $\mathcal{C}$. Then $\mathrm{Ext}^1_{\mathscr{C}_{F^m}}(M, M) = 0$ if and only if $\mathrm{Ext}^1_{\mathscr{C}}(X, X) = 0$.*

Proof. It follows from that

$$\begin{aligned}
\mathrm{Ext}^1_{\mathscr{C}_{F^m}}(M, M) &= \mathrm{Hom}_{\mathscr{C}_{F^m}}(M, M[1]) \\
&= \bigoplus_{i \in \mathbb{Z}} \mathrm{Hom}_{\mathcal{D}^b(H)}(M, (F^m)^i M[1]) \\
&= \bigoplus_{i \in \mathbb{Z}} \mathrm{Hom}_{\mathcal{D}^b(H)}(\bigoplus_{j=0}^{m-1} F^j X, F^{mi}(\bigoplus_{l=0}^{m-1} F^l X)[1]) \\
&= \bigoplus_{m} \bigoplus_{i \in \mathbb{Z}} \mathrm{Hom}_{\mathcal{D}^b(H)}(X, F^{mi} X[1]) \\
&= \bigoplus_{m} \mathrm{Hom}_{\mathscr{C}}(X, X[1]) \\
&= \bigoplus_{m} \mathrm{Ext}^1_{\mathscr{C}}(X, X).
\end{aligned}$$

Where $\bigoplus_{m} X$ denotes the direct sum of m copies of X. □

Lemma 2.5. *Let X and Y be objects in $\mathscr{C}$, and $M = X \oplus FX \oplus \cdots \oplus F^{m-1}X$ be the object in $\mathscr{C}_{F^m}$ determined by X. Then $\mathrm{Hom}_{\mathscr{C}_{F^m}}(M, F^j Y) \simeq \mathrm{Hom}_{\mathscr{C}_{F^m}}(M, Y)$ for $0 \leq j \leq m - 1$.*

Proof. It follows from that $F^j M \simeq M$ in $\mathscr{C}_{F^m}$. □

Proposition 2.6. *Let M be a basic almost tilting object in $\mathscr{C}_{F^m}$ with $m \geq 2$. Then M has only one indecomposable complement X in $\mathscr{C}_{F^m}$.*

Proof. Let X be an indecomposable complement to M. Assume that Y is another indecomposable complement to M, we want to show that $Y \simeq X$. Since $M \oplus X$ is a basic tilting object in $\mathscr{C}_{F^m}$, we may assume that $M \oplus X$ is determined by a basic tilting object T in $\mathscr{C}$, that is, $M \oplus X = T \oplus FT \oplus \cdots \oplus F^{m-1}T$. Note that $m \geq 2$ and $\delta(M) = nm - 1$, we have that $T \oplus FT \oplus \cdots \oplus F^{m-1}T = M \oplus Y$. Then $Y \simeq X$ follows from Lemma 2.2 since $\mathscr{C}_{F^m}$ is a Krull-Remark-Schmidt category. □

Let M be the F-stable object in $\mathscr{C}_{F^m}$ determined by an object X in $\mathscr{C}$. We denote by $O(M)$ the number of F-orbit in M determined by the indecomposable summands of M. It is easy to see that $O(M) = \delta(M)$.

Lemma 2.7. *Let M be a F-stable object of $\mathscr{C}_{F^m}$ and $\mathrm{Ext}^1_{\mathscr{C}_{F^m}}(M, M) = 0$. Then M is a cluster tilting object of $\mathscr{C}_{F^m}$ if and only if $O(M) = n$.*

Proof. Assume that M is determined by X in $\mathscr{C}$, that is, $M = X \oplus FX \oplus \cdots \oplus F^{m-1}$. According to Lemma 2.4, we have that $\mathrm{Ext}^1_{\mathscr{C}}(X, X) = 0$. It is well known that X is a tilting object in $\mathscr{C}$ if and only if $\delta(X) = n$. The consequence follows from Lemma 2.2. □

Definition 2.8. *An object M of $\mathscr{C}_{F^m}$ is said to be rigid if M is F-stable and $\mathrm{Ext}^1_{\mathscr{C}_{F^m}}(M, M) = 0$. A rigid object M of $\mathscr{C}_{F^m}$ is said to be an almost near tilting object if $O(M) = n - 1$.*

Proposition 2.9. *Let M be an almost near tilting object of $\mathscr{C}_{F^m}$. Then M has exactly two kinds of complements. That is, there exist two F-stable objects M_1 and M_2, determined by non-isomorphic indecomposable objects X_1 and X_2 of $\mathscr{C}$ respectively, such that $M \oplus M_1$ and $M \oplus M_2$ are cluster tilting objects in $\mathscr{C}_{F^m}$.*

Proof. Assume that M is determined by an object X in $\mathscr{C}$ and $M = X \oplus FX \oplus \cdots \oplus F^{m-1}X$. Then $\delta(X) = n - 1$, and by using Lemma 2.4 we have that $\mathrm{Ext}^1_{\mathscr{C}}(X, X) = 0$. Therefore, X is an almost tilting object of $\mathscr{C}$. According to [4],

X has exactly two non-isomorphic complements X_1 and X_2 in $\mathscr{C}$. Assume that $M_i = X_i \oplus FX_i \oplus \cdots \oplus F^{m-1}X_i$ for $i = 1, 2$, then it is easy to see that $M \oplus M_1$ and $M \oplus M_2$ are cluster tilting objects in $\mathscr{C}_{F^m}$. □

3. Repetitive cluster-tilted algebras

Let M be a cluster tilting object in $\mathscr{C}_{F^m}$. Then the endomorphism algebra $\mathrm{End}_{\mathscr{C}_{F^m}}(M)$ is called a repetitive cluster-tilted algebra.

Proposition 3.1. *Let T be a basic tilting module of H and M be the F-stable object in $\mathscr{C}_{F^m}$ determined by T. Then M is a cluster tilting object in $\mathscr{C}_{F^m}$ and the endomorphism algebra $\mathrm{End}_{\mathscr{C}_{F^m}}(M)$ is isomorphic to*

$$\begin{pmatrix} C_0 & & & \\ E_1 & C_1 & & \\ & \ddots & \ddots & \\ & & E_{m-1} & C_{m-1} \end{pmatrix}.$$

Where $C_i = C = \mathrm{End}_H\ T$ and $E_i = \mathrm{Ext}^2_C(DC, C)$ for $0 \leq i \leq m-1$, all the remaining coefficients are zero and multiplication is induced from the canonical isomorphisms $C \otimes_C E \cong {}_CE \cong E \otimes_C C$ and the zero morphism $E \otimes_C E \longrightarrow 0$.

Proof. By the assumption, $M = T \oplus FT \oplus \cdots \oplus F^{m-1}T$. As a vector space, we have

$$\mathrm{End}_{\mathscr{C}_{F^m}}(M) = \bigoplus_{i\in\mathbb{Z}} \mathrm{Hom}_{\mathcal{D}^b(H)}(\bigoplus_{j=0}^{m-1} F^jT, F^{mi}(\bigoplus_{l=0}^{m-1} F^lT)).$$

Since T is a H-module, we have that $\mathrm{Hom}_{\mathcal{D}^b(H)}(F^iT, F^jT) = 0$ unless $i = j$ or $i = j-1$. Moreover, $\mathrm{Hom}_{\mathcal{D}^b(H)}(F^iT, F^iT) = \mathrm{Hom}_H(T,T) = C$ and $\mathrm{Hom}_{\mathcal{D}^b(H)}(F^iT, F^{i+1}T) = \mathrm{Hom}_{\mathcal{D}^b(H)}(T, FT) = \mathrm{Ext}^2_C(DC, C)$. □

Remark. According to Lemma 2.2.(4), every repetitive cluster-tilted algebra can be described as in Proposition 3.1.

Lemma 3.2. *Let M be an indecomposable object in $\mathscr{C}_{F^m}$ with $m \geq 2$. Assume that $M = F^jX$ with X, an indecomposable object in $\mathscr{C}$, and that $0 \leq j \leq m-1$.*

If $\widehat{M} = F^j X \oplus \cdots \oplus F^{m-1}X \oplus F^{j-1}X \oplus \cdots \oplus X$ *is a rigid object in* $\mathscr{C}_{F^m}$*, then* $\mathrm{End}_{\mathscr{C}_{F^m}}(M)$ *is a field.*

Proof. By assumption, we have that $\mathrm{Ext}^1_{\mathscr{C}_{F^m}}(\widehat{M}, \widehat{M}) = 0$. According to Lemma 2.4, we have that $\mathrm{Ext}^1_{\mathscr{C}}(X, X) = 0$. Hence $\mathrm{End}_{\mathcal{D}^b(H)}(X, X) \simeq k$, since $m \geq 2$. We have that

$$\begin{aligned}
\mathrm{End}_{\mathscr{C}_{F^m}}(M) &= \mathrm{End}_{\mathscr{C}_{F^m}}(X) \\
&= \bigoplus_{i \in \mathbb{Z}} \mathrm{Hom}_{\mathcal{D}^b(H)}(X, F^{mi}X) \\
&= \mathrm{Hom}_{\mathcal{D}^b(H)}(X, X) = k.
\end{aligned}$$

□

Let M' be a basic almost near tilting object of $\mathscr{C}_{F^m}$. We may assume that $M' = X \oplus FX \oplus \cdots \oplus F^{m-1}X$ with X be a basic almost tilting object in $\mathscr{C}$. Let N_1 and N_2 be non-isomorphic F-stable complements of M'. Then $M_1 = M' \oplus N_1$ and $M_2 = M' \oplus N_2$ are cluster tilting objects in $\mathscr{C}_{F^m}$. We denote by $\Lambda_i = \mathrm{End}_{\mathscr{C}_{F^m}}(M_i)$ for $i = 1, 2$. If N_1 and N_2 are determined by non-isomorphic indecomposable objects X_1 and X_2 of $\mathscr{C}$ respectively, then $N_1 = X_1 \oplus FX_1 \oplus \cdots \oplus F^{m-1}X_1$ and $N_2 = X_2 \oplus FX_2 \oplus \cdots \oplus F^{m-1}X_2$.

Lemma 3.3. *Take the notation as above. Then* $S_{12} = \mathrm{Hom}_{\mathscr{C}_{F^m}}(M_1, N_2[1])$ *is a semisimple* Λ_1*-module and* $S_{21} = \mathrm{Hom}_{\mathscr{C}_{F^m}}(M_2, N_1[1])$ *is a semisimple* Λ_2*-module.*

Proof. By duality, we only need to prove that S_{12} is a semisimple Λ_1-module.

For an integer j with $0 \leq j \leq m-1$, by Lemma 2.5 we have the following isomorphism of k-spaces: $\mathrm{Hom}_{\mathscr{C}_{F^m}}(M_1, F^j X_2[1]) \simeq \mathrm{Hom}_{\mathscr{C}_{F^m}}(M_1, X_2[1])$.

We claim that $\mathrm{Hom}_{\mathscr{C}_{F^m}}(M_1, X_2[1])$ is a simple Λ_1-module.

In fact, if $m = 1$, our claim is proved in [3].

If $m \geq 2$, we consider the following triangle $X_2 \to B \to X_1 \to X_2[1]$, where $B \to X_1$ is the minimal right add X-approximation. Applying $\mathrm{Hom}_{\mathscr{C}_{F^m}}(M_1, -)$ we obtain the following exact sequence $\mathrm{Hom}_{\mathscr{C}_{F^m}}(M_1, B) \to \mathrm{Hom}_{\mathscr{C}_{F^m}}(M_1, X_1) \to \mathrm{Hom}_{\mathscr{C}_{F^m}}(M_1, X_2[1]) \to 0$

$$\begin{aligned}
\mathrm{Hom}_{\mathscr{C}_{F^m}}(M_1, X_2[1]) &= \mathrm{Hom}_{\mathscr{C}_{F^m}}(M' \oplus N_1, X_2[1]) \\
&= \mathrm{Hom}_{\mathscr{C}_{F^m}}(N_1, X_2[1]) \\
&= \mathrm{Hom}_{\mathscr{C}_{F^m}}(X_1 \oplus FX_1 \oplus \cdots \oplus F^{m-1}X_1, X_2[1]) \\
&= \bigoplus_{j\in\mathbb{Z}} \mathrm{Hom}_{\mathcal{D}^b(H)}(X_1 \oplus FX_1 \oplus \cdots \oplus F^{m-1}X_1, F^{jm}X_2[1]) \\
&= \mathrm{Hom}_{\mathcal{D}^b(H)}(X_1, X_2[1]) \oplus \mathrm{Hom}_{\mathcal{D}^b(H)}(X_1, FX_2[1]) \\
&= \mathrm{Hom}_{\mathscr{C}}(X_1, X_2[1]).
\end{aligned}$$

Since k is an algebraically closed field, by [4], $\mathrm{Hom}_{\mathscr{C}}(X_1, X_2[1])$ is one-dimensional k-space. Thus $\mathrm{Hom}_{\mathscr{C}_{F^m}}(M_1, X_2[1])$ is a simple Λ_1-module, our claim is proved.

By Lemma 2.5 again, we have that $S_{12} = \mathrm{Hom}_{\mathscr{C}_{F^m}}(M_1, N_2[1])$ is a semisimple Λ_1-module. This completes the proof. □

According to Corollary 4.4 in [14], we know that mod $\Lambda_1 \simeq \mathscr{C}_{F^m}/\mathrm{add}\ M_1[1]$ and mod $\Lambda_2 \simeq \mathscr{C}_{F^m}/\mathrm{add}\ M_2[1]$. Let $\widetilde{M} = M' \oplus N_1 \oplus N_2$. We denote by S_{12} the semisimple Λ_1-module $\mathrm{Hom}_{\mathscr{C}_{F^m}}(M_1, N_2[1])$ and by S_{21} the semisimple Λ_2-module $\mathrm{Hom}_{\mathscr{C}_{F^m}}(M_2, N_1[1])$, then we get equivalences mod $\Lambda_1/\mathrm{add}\ S_{12} \simeq \mathscr{C}_{F^m}/\mathrm{add}\ \widetilde{M}[1]$ and mod $\Lambda_2/\mathrm{add}\ S_{21} \simeq \mathscr{C}_{F^m}/\mathrm{add}\ \widetilde{M}[1]$.

Summarizing the above discussions, we get the following theorem which is a generalization of Theorem 4.2 in [3].

Theorem 3.4. *Let M' be a basic almost near tilting object of $\mathscr{C}_{F^m}$ determined by a basic almost tilting object X in $\mathscr{C}$ with non-isomorphic complements X_1 and X_2 be of in $\mathscr{C}$. Then $N_1 = X_1 \oplus FX_1 \oplus \cdots \oplus F^{m-1}X_1$ and $N_2 = X_2 \oplus FX_2 \oplus \cdots \oplus F^{m-1}X_2$ are non-isomorphic F-stable complements of M'. Moreover, $M_1 = M' \oplus N_1$ and $M_2 = M' \oplus N_2$ are cluster tilting objects in $\mathscr{C}_{F^m}$. Let $\Lambda_1 = \mathrm{End}_{\mathscr{C}_{F^m}}(M_1)$ and $\Lambda_2 = \mathrm{End}_{\mathscr{C}_{F^m}}(M_2)$. Then* mod $\Lambda_1/\mathrm{add}\ S_{12} \simeq$ mod $\Lambda_2/\mathrm{add}\ S_{21}$, *where $S_{12} = \mathrm{Hom}_{\mathscr{C}_{F^m}}(M_1, N_2[1])$ and $S_{21} = \mathrm{Hom}_{\mathscr{C}_{F^m}}(M_2, N_1[1])$ are semisimple modules.*

Let $\mathcal{T}_{\mathscr{C}_{F^m}}$ be the set of all basic cluster tilting objects in $\mathscr{C}_{F^m}$ up to isomorphism. According to [9], the tilting graph $\mathscr{K}_{\mathscr{C}_{F^m}}$ of $\mathscr{C}_{F^m}$ is defined as the following. The vertices of $\mathscr{K}_{\mathscr{C}_{F^m}}$ are the elements of $\mathcal{T}_{\mathscr{C}_{F^m}}$. There is an edge between M' and M if there exists an almost near tilting object B such that $M' = B \oplus N_1$ and $M = B \oplus N_2$ with N_1 and N_2 are determined by indecomposables X and Y in $\mathscr{C}$. That is, B is determined by an almost tilting object T in $\mathscr{C}$ and T has exactly two non-isomorphic indecomposable complements X and Y, such that N_1 and N_2 are determined by X and Y respectively.

Theorem 3.5. *The tilting graph $\mathscr{K}_{\mathscr{C}_{F^m}}$ of $\mathscr{C}_{F^m}$ is connected.*

Proof. Let M_1 and M_2 be two elements in $\mathcal{T}_{\mathscr{C}_{F^m}}$. We suppose that M_1 and M_2 are determined by basic tilting objects T_1 and T_2 of $\mathscr{C}$ respectively. According to [4] the tilting graph $\mathscr{K}_{\mathscr{C}}$ of $\mathscr{C}$ is connected, hence there exist basic tilting objects $X_1, \cdots, X_t$ of $\mathscr{C}$ such that there is a path $T_1 - X_1 - \cdots - X_t - T_2$ in tilting graph $\mathscr{K}_{\mathscr{C}}$ of $\mathscr{C}$. We denote by N_i the element of $\mathcal{T}_{\mathscr{C}_{F^m}}$ determined by X_i, i.e., $N_i = X_i \oplus FX_i \oplus \cdots \oplus F_{m-1}X_i$ for $1 \leq i \leq t$, according to Proposition 2.9, we obtain a path $M_1 - N_1 - \cdots - N_t - M_2$ in tilting graph $\mathscr{K}_{\mathscr{C}_{F^m}}$. The proof is completed. □

Acknowledgments. We would like to thank the referee for his or her valuable comments and suggestions, which improve the presentation of this paper.

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