

Chaotic Arithmetic Coding for Secure Video Multicast

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Abstract—Arithmetic Coding (AC) is widely used for the entropy coding of text and video data. It involves recursive partitioning of the range [0,1) in accordance with the relative probabilities of occurrence of the input symbols. A data (image or video) encryption scheme based on arithmetic coding called as Chaotic Arithmetic Coding (CAC) has been presented in previous works. In CAC, a large number of chaotic maps can be used to perform coding, each achieving Shannon optimal compression performance. The exact choice of map is governed by a key. CAC has the effect of scrambling the intervals without making any changes to the width of interval in which the codeword must lie, thereby allowing encryption without sacrificing any coding efficiency. In this paper, we use a redundancy in CAC procedure for secure multicast of videos where multiple users are distributed with different keys to decode same encrypted file. By encrypting once, we can generate multiple keys, either of which can be used to decrypt the encoded file. This is very suitable for video distribution over Internet where a single video can be distributed to multiple clients in a privacy preserving manner.

Keywords—chaotic maps, arithmetic coding, secure multicast

I. INTRODUCTION (Heading 1)

Due to the emergence of the cloud computing paradigm, and the ubiquitousness of portable multimedia-capable devices, the issue of real-time multimedia delivery has gained a lot of importance. The technical challenges involved in such scenarios include providing a compression mechanism that is highly scalable, secure, easily search-able and index-able. All of this should be done that the compression properties are not lost. Providing security in a video communication context is especially challenging, as the security requirements tend to be application and platform-specific, and the input data is characterized by large storage requirements, real-time processing latencies, and the use of standardized video codecs.

Arithmetic coding is extremely efficient for compression efficiency in large data- sets and it achieves the Shannon compression efficiency for large chunks of data [2]. However, conventional implementations are not particularly secure. Thus, arithmetic coding has been interpreted in terms of chaotic maps [11] to be useful for joint compression and encryption of video data [1,12-16].

Many other techniques based on varying the statistical model of entropy coders have been proposed in the research literature [3-6], however these techniques suffer from losses in compression efficiency that result from changes in entropy model statistics and are weak against known attacks [7-10].

Encryption schemes like AES and DES can be used for video data also, but they require specialized hardware for encryption [17-19], whereas these schemes can achieve encryption while compression.

In this paper, we explain how CAC can be used for secure multicast. In typical secure multicast scenario, the content is encoded using different keys and then distributed to multiple users, or the content is not encrypted. With CAC, we can generate multiple keys to decode the same content. Hence, only one encode is required for the entire multicast group and different users can obtain different keys to decode their content.

II. ARITHMETIC CODING WITH PIECE-WISE LINEAR CHAOTIC MAPS

Before we go ahead, we review the basics of chaotic arithmetic coding. We consider a scenario where we have a string $S = x_1, x_2, \dots, x_N$ consisting of N symbols to be encoded. The probability of occurrence of a symbol s_i , $i \in 1, 2, \dots, n$ is given by p_i such that $p_i = N_i/N$ and N_i is the number of times the symbol s_i appears in the given string S . We next consider a piece-wise linear map (ρ) with the following properties:

1. It is defined on the interval $[0, 1)$ to $[0, 1)$ i.e.

$$P: [0, 1) \rightarrow [0, 1)$$

2. The map can be decomposed into N piece-wise linear parts, k i.e.

$$P = \bigcup_{k=1}^N q_k$$

3. Each part k maps the region on the x axis $[beg_k, end_k)$ to the interval $[0, 1)$ i.e.

$$q_k : [beg_k, end_k) \rightarrow [0, 1)$$

4. The last two propositions lead to:

$$\bigcup_{k=1}^N [beg_k, end_k) = [0, 1)$$

5. The map q_k is one-one and onto i.e.:

$$\forall x \in [beg_k, end_k)$$

$$\exists y \in [0, 1): y = q_k(x), \text{ and}$$

$$\forall y \in [0, 1)$$

$$\exists x \in [beg_k, end_k): q_k(x) = y$$

6. ρ is a many-one mapping from $[0, 1)$ to $[0, 1)$. This implies that the decomposed linear maps q_k don't intersect each other i.e.

$$\forall (k \neq j): [begk, endk] \cap [begj, endj] = \emptyset$$

7. Each linear map q_k is associated uniquely with one symbol s_i . The mapping $q_k \rightarrow s_i$ is define arbitrarily but the one-one relationship must hold.
8. The valid-input width of each map (q_k), given by $(endk - begk)$ is proportional to a probability of occurrence of symbol s_i .

$$endk - begk \propto p_i$$

$$\Rightarrow endk - begk = C \times p_i$$

We recall that $\sum_{k=1}^N (endk - begk)$ is same as the input width of $\cup_{k=1}^N (q_k = p)$, which is 1. Also $\sum_{i=1}^N p_i = 1$. Thus, we get the value of the constant C to be 1.

$$\Rightarrow endk - begk = p_i$$

Figure 1 shows a sample map fulfilling these properties. Figure 1(a) shows the full map with different parts q_1, q_2, \dots, q_N present, while Figure 1(b) zooms into individual linear part q_k . The maps are placed adjacent to each other so that each input point is mapped into an output point in the range $[0, 1)$. The total number of distinct ways of arranging N maps to obtain ρ fulfilling the properties mentioned above is given by $N! = N \times (N - 1) \times (N - 2) \dots 3 \times 2 \times 1$, where ! denotes the factorial sign. It is same as arranging these N maps in a sequence, one after another, with the end interval of one map touching the beginning interval of another.

However, there are N different piece-wise maps, each with two possible orientations (with positive or negative slope). Thus, the number of total permutations possible is given by $N!2^N$ which is independent of unique symbol probability. Thus, for N-ary arithmetic coding or arithmetic coding with N symbols, it is possible to have $N!2^N$ different mappings each leading to same compression efficiency. Since we can arbitrarily choose any 1 of the $N!2^N$ maps, the key space for encoding a single bit of data. It can have a positive or negative slope depending on choice is $\lceil \log_2(N!2^N) \rceil$ bits, where $\lceil \cdot \rceil$ represents the greatest integer function. For $N = 2$, it gives 8 mappings. If we increase N to 4 this value increases to 384.

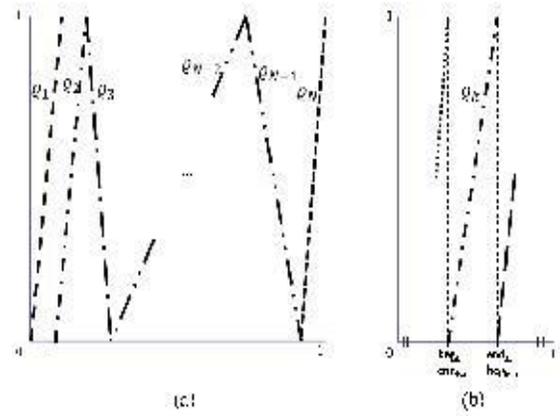


Figure 1: A sample piece-wise linear map for arithmetic coding like compression (a) The entire map is shown (ρ) (b) A single linear part of the map (q_k) is zoomed

The equation for individual maps can be derived as follows:

$$y' = q_k(x') = \left(\frac{x' - begk}{endk - begk} \right) \text{ or } \left(1 - \frac{x' - begk}{endk - begk} \right)$$

The equation for the full map is given by

$$y = \rho(x) = q_k(x): begqk \leq x < endk$$

The coding procedure, correspondence to arithmetic coding and compression efficiency of basic chaotic coding is explained in [13]. The compression efficiency of this scheme is explained in [11] to be same as arithmetic coding i.e. it achieves Shannon efficiency for large numbers.

III. BINARY CHAOTIC ARITHMETIC CODING (BCAC)

In the previous section we explained how arithmetic coding can be viewed as re-iteration on chaotic maps. AC is more commonly implemented in binary mode to reduce the computational requirements of video coders. For same considerations, we discuss implementation and security issues with BCAC in this paper after introducing CAC in last section. The Binary CAC (or BCAC) uses either of the eight equivalent skewed binary maps (shown in Figure 2) based on an input key. These maps differ from each other in the way input is mapped into the chaotic orbit - differ in the interval in which the arithmetic code must lie for a symbol '0' or '1' but the width of interval remains the same. In next section, we will formulate a mathematical procedure to generate the eight maps and choose between them using the parameter.

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TABLE I. PARAMETER LIST FOR THE EIGHT POSSIBLE CHOICES OF CHAOTIC ENCODER

Parameter	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
M1	p	p	-p	-p	p	-p	-p	p
B1	0	0	p	p	1 - p	1	1	1 - p

M2	1-p	p-1	p-1	1-p	1-p	1-p	p-1	p-1
B2	p	1	1	p	0	0	1-p	1-p
N1	1/p	1/p	-1/p	-1/p	1/(1-p)	1/(1-p)	-1/(1-p)	-1/(1-p)
C1	0	0	1	1	0	0	1	1
N2	1/(1-p)	-1/(1-p)	-1/(1-p)	1/(1-p)	1/p	-1/p	-1/p	1/p
C2	-p/(1-p)	1/(1-p)	1/(1-p)	-p/(1-p)	(p-1)/p	1/p	1/p	(p-1)/p
I1	0	0	0	0	(1-p)	(1-p)	(1-p)	(1-p)
I2	p	p	p	p	1	1	1	1
I3	p	p	p	p	0	0	0	0
I4	1	1	1	1	1-p	1-p	1-p	1-p
K	p	p	p	p	1-p	1-p	1-p	1-p

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We define the generalized skewed binary map with the following equations:

$$y = \begin{cases} n_1x + c_1 & \text{when } x \leq k \\ n_2x + c_2 & \text{when } x > k \end{cases}$$

$$\text{Decode } \begin{cases} m_1y + b_1 & \text{when '0'} \\ m_2y + b_2 & \text{when '1'} \end{cases}$$

Then, the iteration on skewed binary map is defined by the following equation :

$$x = \begin{cases} m_1y + b_1 & \text{when '0'} \\ m_2y + b_2 & \text{when '1'} \end{cases}$$

Where $n_1, n_2, c_1, c_2, m_1, m_2, b_1$ and b_2 values can be precomputed for different maps and stored in the table for look up for fast access. Table 1 gives the value of these parameter for all eight chaotic maps.

Grangetto et al. [4] present a Randomized Binary Arithmetic Coding (RBAC) scheme where they change the ordering of '0' and '1' intervals in a Binary Arithmetic Coder (BAC) based on a key. RBAC can be seen as a special case of

BCAC where only two of the eight modes of BCAC are used for encryption purposes. Similarly, KSAC [6] can be represented in terms of piece-wiselinear maps by removing the condition of continuity of individual maps ($j(x)$). Each part_j maps a discontinuous interval on x-axis to the interval [0,1).

IV. SECURE VIDEO MULTICAST

The BCAC schemes allows a given input string to be decoded by more than one key for correct reconstruction. This is attributed to the fact, that for a given bit - two of the eight maps will give the same output interval. For example - encoding '0' with either of the maps given in Figure2(a) and Figure 2(b) will lead to same result. The same is true if we encode '1' with maps given in Figure 2(a) and Figure 2(d).

Thus, we can infer that "Of the 8 possible maps, two maps will give the same reconstruction for a single bit but the choice of bit depends on its value being '0' or '1'." For a N bit message, we have a pool of 2^N choices out of the total keyspace of

2^{3N} . We illustrate this with the example of a three bit message '001' which is encoded with the maps '1', '3' and '6' or simply '136'. For bit '0', map '2' will decode similar to map '1'. For bit '0', map '4' will decode similar to map '3', and for bit '1', map '5' will decode similar to map '6'. Thus, different users can download the same encoded message and decrypt them on their machine using different keys chosen from the pool of eight keys - '245', '246', '235', '236', '135', '136', '145' and '146'.

Thus, the same compressed (and encrypted) data can be sent to all the users in the multicast, while a unique key can be sent to each user. In this example (of three bits compression), the key space is $8^3 = 512$, of which only 8 keys are correct.

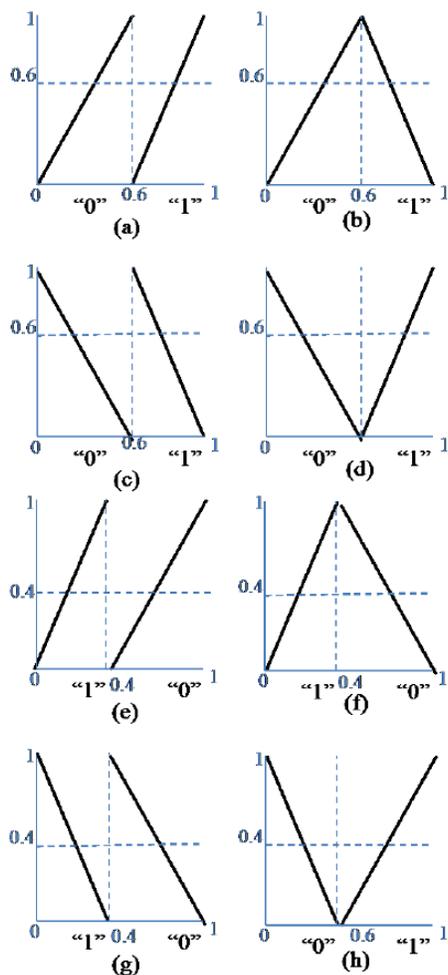


Figure 2: Eight modes of Binary Chaotic Arithmetic Coding

Let us say that we choose to encrypt only first M bits of a bitstream of length N . Typically, we may expect $N=1000 \sim 100,000$. However, we can choose small values of M , say $M=128$. For this choice, we have 2128 correct decryption keys in a key space of 2384 bits. Thus, the probability of guessing a right key is 2^{-256} and an exhaustive search of key space will not be helpful. The same compressed video string can be distributed to different users (upto 2128), each obtaining his own unique key to decode/decrypt it. Identifying the keys can help us to trace users who may be involved in illegal or unauthorized distribution of keys. Therefore, a single encoded stream can be multicasted to different users who can decode video content using their own keys.

If a user relays his key to an unauthorized person, as in the case of content piracy, and the unauthorized person is caught for Digital Rights Infringement, the relaying user can be directly identified and blacklisted/penalized for his improper action. It is not possible to generate a random correct key, as the key is 2128 bits. But, an attacker may use the knowledge that

every three bits of the key are correlated to each other to reduce the attack space to 128. Still, the key space is large enough to guarantee success.

A. Limitations

A possible limitation of this feature is that we need to send a unique key for each plaintext. In many encryption mechanisms, the same key is used for multiple encodes or sessions. This is not possible in our scenario, because multiple CAC multicast keys are generated from one original key based on the encoded content. With changing content (plain text), the same original key will lead to different multicast keys. This means that, for every string of N bits (where we encrypt first M bits), we need to transmit $3M$ bits of key information, to every user in the multicast.

However, by choosing $M \ll N$, we can easily overcome this limitation. The overhead of key distribution will be very low. Plus, re-distributing keys adds to the security of the multicast system.

Another possible limitation is the possibility of collusion attack. Multiple (say K) users may collude together to mix their keys and generate new keys which are then relayed to illegitimate users. The generated new keys can be different than the original keys. As K increases, it becomes more and more difficult to recognize the original colluding users.

B. Modification for Secure Multicast

To address the collusion attack scenario, we propose a modification to the key distribution scheme where keys are first encrypted by the coder using an asymmetric encryption scheme and then distributed to different users. Thus, we need to provision cryptographic encryption of the keys used by the system. These can be decoded and then used.

In this scenario, we incur the extra cost of cryptographic encryption of the keys ($3M$ bits). By choosing $M \ll N$, we can ensure that this overhead is negligible compared to the overall cost of re-encryption for every client.

CONCLUSION

In this paper, we discussed how chaotic arithmetic coding can be used for secure multicast of multimedia data. We found that it is possible for the users to collude if they exchange their keys with each other, making it difficult for DRM authorities to detect the user who leaked the key to an unlicensed user. However, we proposed to encrypt these user keys and decode them inside the decoder using an asymmetric system.

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