Thermodynamic transitions in inhomogeneous cuprate superconductors

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Scanning tunneling spectroscopy studies on $Bi_2Sr_2CaCu_2O_{8+\delta}$ suggest the presence of electronic inhomogeneity with a large spatial variation in gap size. Andersen *et al* have modelled this variation by assuming a spatially-varying pairing interaction. We show that their calculated specific heat is incompatible with the experimental data which exhibit narrow transitions. This calls into question the now-common assumption of gap and pairing inhomogeneity

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In a recent paper Andersen and coworkers[1] attempt to model the spectroscopic inhomogeneity inferred from scanning tunneling spectroscopy (STS) studies on $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Bi-2212)[2,3]. These STS studies suggest a $\sim \pm 25\%$ variation in both the gap magnitude and the local density of states (LDOS), on a length scale of the order of one or two coherence lengths, ξ_0. The locations with large spectral gaps lack well-developed coherence peaks and, based on the absence of Ni resonances there, they are suggested to be non-superconducting[3]. These conclusions have become highly influential.

To model these effects Andersen *et al.*[1] adopt a spatially-inhomogeneous pairing interaction in a *d*-wave BCS pairing model and solve self-consistently for the local gap magnitude in order to map the spatially varying interaction onto the observed variation in gap magnitude. They then compute the specific heat and show that the breadth of the anomaly is similar to the spread in local gap magnitude. This breadth, they claim, is "comparable to experiments on this material", and conclude that "substantial nanoscale electronic inhomogeneity is characteristic of the bulk BSCCO system".

However our data[4] (see Fig. 1), which they use for their comparison, points to the very opposite conclusion. They erroneously equate the extended fluctuation region above T_c with broadening of their mean-field (MF) transition over a 40K range. Strong fluctuations are an intrinsic property of this highly-anisotropic material[5] but are not included in their calculations. In fact it is the narrow region of strong negative curvature close to each peak (see arrows in Fig. 1) that reveals the true extent of extrinsic broadening. Such a strong T-dependence over a very restricted T-range would not be possible in a material with a broad distribution of T_c values.

Moreover, the locations where their gap is a maximum correspond to the maximal local pairing interaction i.e. where SC is strongest. The STS studies[3] show the opposite: SC is weakest and perhaps absent at the points where the supposed gap is maximal. Their model also implies that there are local regions where SC persists well above T_c, as much as 30K for Bi-2212. Such regions are not observed in STS[6,7].

Here we consider the experimental specific heat data in more detail and show that the transitions are not strongly broadened as suggested by Andersen *et al.*[1], thus potentially reversing their inference of inhomogeneous pairing in the bulk material. This concurs with several other studies which imply the absence of gross inhomogeneity in cuprate superconductors[8,9].

The specific heat near T_c consists of a MF step at T_c and a (nearly symmetrical) fluctuation contribution above and below T_c[5]. More generally, T_c may be broadened out into a distribution of T_c values. The separate contributions of fluctuations and transition broadening may seem similar well away from the mean T_c, but nearby they are quite distinctive and easily separated. Andersen et al.[1] fail to recognize this distinction.

Let us consider first fluctuations where there is a sharply defined T_c. The fluctuation specific heat should diverge at T_c but is cut-off due to the inhomogeneity length scale. For Bi-2212 we have previously analyzed the fluctuation contribution[4] and deduced transition half-widths as small as $\Delta T_c/T_c \sim 0.014$, consistent with an inhomogeneity length scale as large as $16\xi_0$, much greater than ξ_0. The cutoff is reflected in the narrow region of negative curvature between the inflexion points in the specific heat coefficient, $\gamma(T)$, just above and below T_c as shown by the arrows in Fig. 1(a).

In Fig. 1(b) we show the derivative $\partial\gamma/\partial T$ from our data and compare it with that from Andersen's model calculation. The data curves correspond to doping states of $p = 0.16$, 0.19, 0.20 and 0.215. The inflexion points are located at the maxima and minima below and above T_c, and between them $\partial\gamma/\partial T$ changes sign. For $p = 0.19$ the inflexion points are just 3.3K apart. For the model calculation they are up to 40K apart (dashed curve and arrows), just as would be expected for a $\pm 25\%$ spread of gap values. This directly illustrates that we observe rather sharp thermodynamic transitions in Bi-2212.

To show that the transition is indeed narrow we plot in Fig. 2 the field dependence of γ for a Bi-2212 sample with doping $p \approx 0.21$. Firstly it is clear from this plot that $T_c(H = 0)$ is close to the peak (as expected if the MF step is small). The narrow peak is progressively

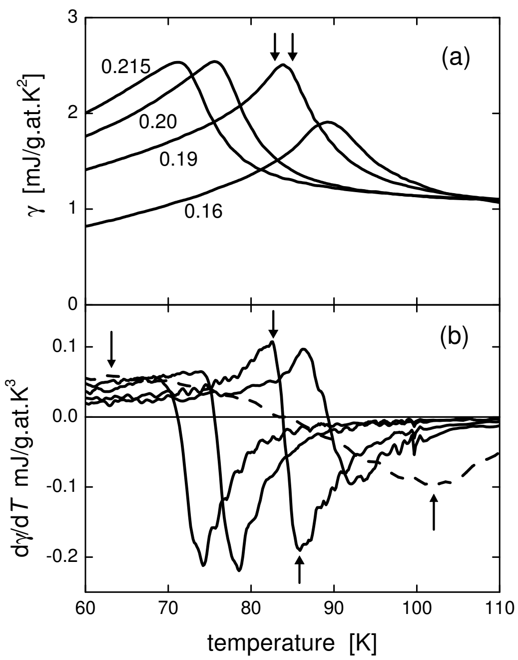

FIG. 1: (a) The specific heat coefficient, γ, for Bi-2212 with p= 0.16, 0.19, 0.20 and 0.215, respectively. (b) The derivative $\partial\gamma/\partial T$. The dashed curve is $\partial\gamma/\partial T$ from the Andersen *et al.* calculation. Arrows indicate inflexion points in $\gamma(T)$.

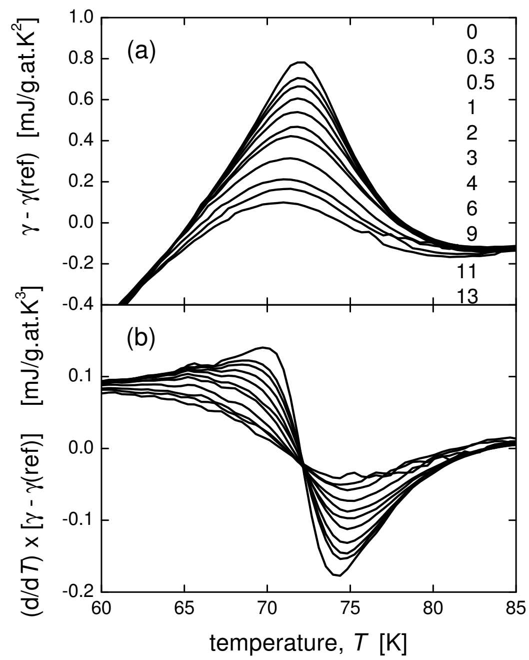

FIG. 2: (a) The field dependence of the specific heat coefficient γ-γ(ref) for Bi-2212. The field is shown in units of Tesla. (b) the derivative, $\frac{d}{dT}\times(\gamma-\gamma(\mathrm{ref}))$ showing the field-broadening of the inflexion points.

suppressed and broadened by the field, with a marked effect even for fields as low as 0.3T. The vortex separation $\sqrt{\phi_0/H}\sim 45$ nm$/\sqrt{H(\mathrm{Tesla})}$, which acts as an inhomogeneity length scale, is very large at such low fields and the sensitivity of the transition to a field as low as 0.3T supports our conclusion that the order parameter is rather homogeneous. Calculated transitions based on an inhomogeneity length scale of 1.6 - 2.5 nm would be totally insensitive to such low fields.

To gain more insight into the shape of the anomaly we have simulated the effects of broadening by integrating sharp specific heat transitions over gaussian distributions of T_c. The data are compared with similar plots for some of our Bi-2212 data. To cover a wide range of situations we have considered (a) transitions with a simple MF step, (b) pure $\ln t$ fluctuations with no MF step, and (c) admixtures of the two. In all following cases C corresponds to the difference $C - C_{normal}$.

For the simulated MF specific heat function we adopt a form which conserves entropy at T_c, and has a step $\Delta C_{mf} = 2$ at T_c:

$$\begin{aligned} C_{mf}(x) &= x(3x^2-1) \quad \text{for } x = T/T_c \leq 1, \\ C_{mf}(x) &= 0 \qquad\qquad\quad\;\; \text{for } x > 1. \end{aligned} \tag{1}$$

For the fluctuation specific heat we assume

$$C_{fluc}(x) = \ln(1/\mid x-1\mid) \text{ for all } x \tag{2}$$

This has a symmetrical divergence at T_c. For the distribution of T_c values we assume a Gaussian function centered on T_{co} with half-width w = 0.001, 0.02, 0.05, 0.1, 0.15 and 0.2:

$$P(y) = \frac{1}{w\sqrt{2\pi}} \times \exp(-\frac{y^2}{2w^2}); \quad y = T_c/T_{co}. \tag{3}$$

The specific heat resulting from this T_c distribution is

$$C(T) = \int_0^\infty C(x)P(y)dy \tag{4}$$

Results. For a range of widths w, plots of C and dC/dT versus $t = (T/T_{co}) - 1$ are shown in Fig. 3 for C_{mf} and in Fig. 4 for C_{fluc} . Admixtures $C_{mix} = C_{mf} + mC_{fluc}$ with m = 0, 0.2, 0.4, 0.6, 0.8 and 1 are plotted versus t in Fig. 5(a) and (b) for $w = 0.02$.

It is important, firstly, to note that in spite of the very different T-dependencies of the unbroadened anomalies, the region over which broadening effects are important is very similar for C_{mf}, C_{fluc} and C_{mix}. This shows that, though the Andersen model does not include fluctuations, the addition of fluctuations would not alleviate the disparity between their model and the experimental data. In particular, inclusion of fluctuations will not result in a more narrow negative-curvature region around T_c as seen in the data. We also find that the relative positions of key features of the T-derivatives of the broadened curves are insensitive to the detailed T-dependencies assumed for C_{mf} and C_{fluc}. From these plots we find the

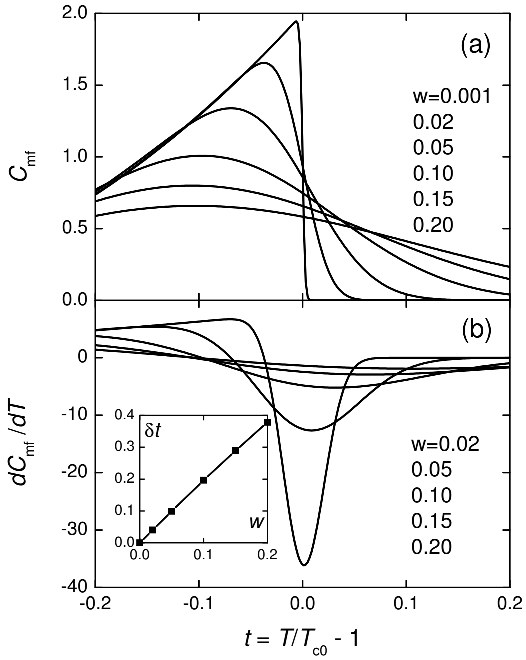

FIG. 3: (a) The MF specific-heat anomaly modelled using eqn. (1) with a normal distribution of T_c values with half width, $w = 0.001$, 0.02, 0.05, 0.10, 0.15 and 0.20. (b) The derivative $\mathrm{d}C_{mf}/\mathrm{d}T$. Inset: δt (defined in text) versus w.

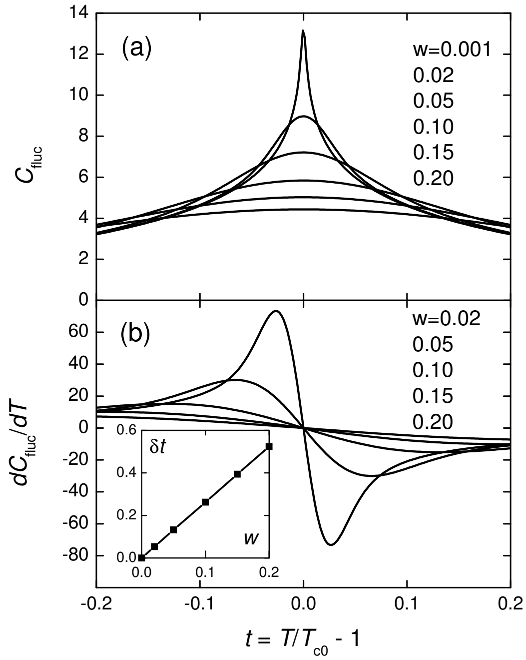

FIG. 4: (a) The fluctuation specific-heat anomaly modelled using eqn. (2) with a normal distribution of T_c values with half width, $w = 0.001$, 0.02, 0.05, 0.10, 0.15 and 0.20. (b) The derivative $\mathrm{d}C_{fluc}/\mathrm{d}T$. Inset: δt (defined in text) versus w.

locations of key features and determine their relation to the half-width w of the distribution of T_c values.

(i) For the MF anomaly, shown in Fig. 3, the most useful features are the positions of the points of maximum negative and positive curvature d^2C/dT^2 at t_- and t_+ respectively, and the difference $\delta t = t_+ - t_-$. In the inset to Fig. 3(b) we show δt plotted as a function of the half-width w. Over most of the range of w, $\delta t \approx 2.0w$.

(ii) For the symmetric fluctuation anomaly, shown in Fig. 4, the most useful features are the positions of the points of maximum positive and negative slope dC/dT at t_- and t_+ respectively, and the difference $\delta t = t_+ - t_-$. In the inset to Fig. 4(b) we show δt plotted as a function of w. Over the entire range, $\delta t \approx 2.63w$.

(iii) For the admixture of a MF anomaly and fluctuations, Fig. 5 shows plots of $C_{mix} = C_{mf} + mC_{fluc}$ for w=0.02 with $m = 0$, 0.2, 0.4, 0.6, 0.8 and 1. These are useful for comparison with typical cuprate specific heat data and cover the range from asymmetric (m=0) to almost symmetric (m=1) anomalies. A value of $m \approx 0.8$ is appropriate for weakly-overdoped Bi-2212. From Fig. 5(b) we obtain the positions of the points of maximum positive and negative slope at t_- and t_+, respectively, and the difference $\delta t = t_+ - t_-$. For larger values of m, typical of our Bi-2212 samples, we find $\delta t \approx 2.7w$. For m=∞ (pure fluctuations) we found, above, $\delta t \approx 2.63w$.

Now we compare this model data with measured data for Bi-2212. Plots of dC/dT versus $\tau = T/T_p - 1$ are shown in Fig. 5(c) for eight values of p (ranging from well underdoped to well overdoped). Comparison with Fig. 5(b) shows that T_c is only 1-2% above T_p, so separations of peaks in t and in τ are almost identical. When underdoped the anomaly is pure fluctuations with no MF step while the increase in MF step is evident for overdoping from the increasingly different relative magnitudes of the positive and negative peaks in dC/dT.

We estimate the transition width, w, from the separation of the positive and negative peaks in dC/dT. For all samples with $p \geq 0.169$ the separation is $\delta t \approx 0.05$ - 0.06. Taking $\delta t \approx 2.7w$ for $m \approx$1 (see Fig. 5(b)) gives $w \approx 0.019$ - 0.021 for the half width of the distribution of T_c values. For p=0.162 we have $\delta t \approx 0.073$ or $w \approx$ 0.028, and for p=0.138 we have $\delta t \approx 0.14$ or $w \approx 0.056$. All these values are in good agreement with the values of the half-width Δt shown in Fig. 4 of our previous work[4].

As a check on our previous method[4] for estimating these half-widths, Δt, we show in Fig. 6 plots of C_{mix} versus $\log_{10}(t)$ and $\log_{10}(t^*)$, respectively, for $w = 0.02$ and $m = 0$ to 1, where $t^* = \sqrt{t^2 + \Delta t^2}$. Our normal procedure is to choose a value of Δt that just averts the negative curvature close to T_c seen in plots of C_{mix} versus $\log_{10}(t)$, and it is evident from the solid curves that the choice $\Delta t = w$ achieves this result. This confirms that estimates of the broadening Δt from plots of ΔC versus $\log_{10}(t^*)$ give reliable values comparable to the true half-width w. The plots in Fig. 6 also show that this procedure provides a reliable estimate of the MF step $\Delta C_{mf} \approx 1.5$ even in the presence of large fluctuations.

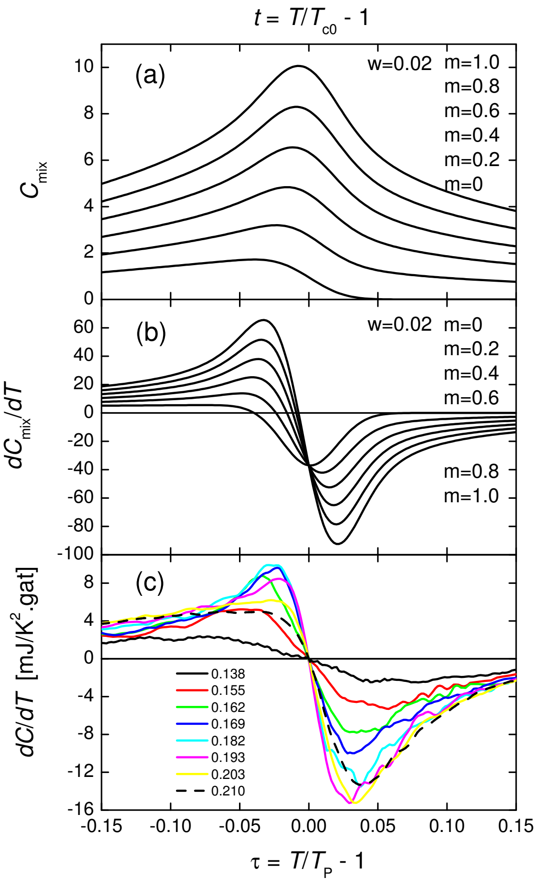

FIG. 5: (a) The admixture specific-heat anomaly $C_{mix} = C_{mf} + mC_{fluc}$ with half width, $w = 0.02$ and various mixing ratios, m, plotted versus $t = (T/T_{co}) - 1$. (b) The derivative dC_{mix}/dT vs t. The zero crossing occurs at T_p and the curves intersect at T_{c0}. (c) dC/dT vs τ for Bi-2212 at different doping levels (annotated) showing a crossover from pure fluctuations for $p < 0.162$ to an admixture with a MF step for $p > 0.162$.

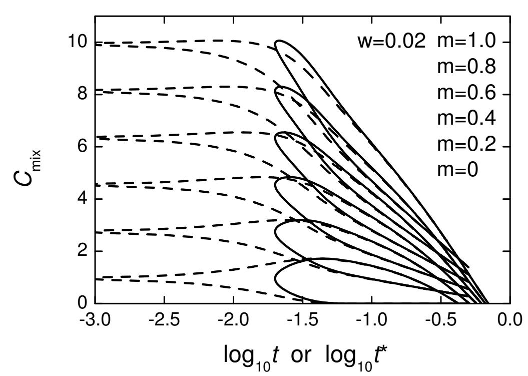

FIG. 6: The admixture specific-heat anomaly $C_{mix} = C_{mf} + mC_{fluc}$ with half width, $w = 0.02$, plotted against $\log_{10}(t)$ (dashed curves) and against $\log_{10}(t^*)$ (solid curves) with $t^* = \sqrt{t^2 + \Delta t^2}$ and setting $\Delta t = w$.

Turning to the MF calculation of the specific-heat anomaly by Andersen *et al.*[1], and recalling that their model does not include fluctuations, we compare their results (shown in Fig. 4(b) of their paper) with our plots for a fluctuation-free broadened MF step, shown here in Fig. 3. From the location of the temperatures T_- and T_+ of their maximum negative and positive curvature, we obtain $\delta t = (T_+ - T_-)/T_{av} \approx 0.146$, 0.204 and 0.232 for their curves for $\delta g/t = 1.0$, 1.5 and 2.0, respectively. Taking $\delta t = 2w$ from Fig. 3(b) we find $w \approx 0.073$, 0.102 and 0.116, respectively, as the half-widths for the three curves. This agrees with the $\approx 20\%$ breadth of the transition that they quoted for $\delta g/t = 1.5$, the value which best accounts for the gap-maps. This may be compared with the similar analysis of our Bi-2212 data from which we obtained $w \approx 0.02$ over most of the doping range, a factor of five, or more, lower than the Andersen estimate based on gap-maps. It surely cannot be claimed that the spectroscopic and thermodynamic data are consistent. Indeed, given our evidence that the spread of T_c values is rather narrow, their calculations show plainly that the inhomogeneity in the gap-maps cannot result from gross pairing energy disorder in the bulk.

To conclude, we have shown that the inference of pairing inhomogeneity from STS gap maps, and the resultant transition broadening, is inconsistent with the specific heat data which exhibit sharp features with transition widths of the order of 3K in Bi-2212. It is not possible with any broad spread of SC gaps to have strong T-dependences over a narrow T-range. This concurs with a recent analysis of quasi-particle scattering seen in spatial modulations of STS data. The Fourier transform of these patterns reveals spots, corresponding to scattering q-vectors, that are far too narrow for the presumed broad distribution of SC gaps[9]. The inference of large-scale gap and pairing inhomogeneity at (π,0) does not appear to be supported by the wider thermodynamic[4], NMR[8], ARPES[10] and tunneling[9,11] data.

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