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Glueball Spectra of SU(2) Gauge Theories in 3 and 4 Dimensions:

A Comparison with the Isgur-Paton Flux Tube Model

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Abstract We use the results of recent lattice calculations to obtain (part of) the mass spectrum of continuum SU(2) gauge theory in both 2+1 and 3+1 dimensions. We compare these spectra to the predictions of the Isgur-Paton flux tube model for glueballs. We use this comparison to test the reliability of different aspects of the model and also to learn which aspects of the lattice calculations it is important to improve upon.

Current lattice calculations[1-4] of the SU(2) spectrum in 3+1 dimensions are accurate, have a good control of finite-volume corrections, and have been performed down to very small values of the lattice spacing, a. This should encourage us to see whether an accurate extrapolation to the continuum limit is possible. Similar calculations[5] in 2+1 dimensions are even more accurate and some such extrapolations have already been made in that case.

In both dimensions the theories are believed to possess linear confinement at large distances and they become free at short distances. These similarities suggest that a comparison of the respective mass spectra should be informative. However there is a limit as to how focussed our comparison of the spectra can be if posed in the abstract. To do better it will clearly be very useful to have a model framework within which to couch the discussion. We can then hope to pin-point which dynamical features of the model work in practice and which do not. Using two different spatial dimensions provides an extended lever arm for the comparison. In return we can expect that the model will focus our attention on what are the weak points of current lattice calculations.

Glueball models are more speculative than models for the usual hadrons, where at least the basic starting point, valence (constituent) quarks, is well established. We are aware of two models that might be useful for our purposes here: the bag model[6] and the flux tube model[7]. In this paper we shall work with the flux tube model. A detailed comparison with the bag model would also be of interest. However in that case there is a complication - the lightest 0^+ state has an imaginary mass once the centre of mass motion is subtracted - and it is not clear to us to what extent the usual resolution of this problem should be considered uncontroversial.

We turn now to the lattice glueball spectra. Lattice masses always come in lattice units and so, to remove the lattice spacing, we will consider ratios of masses. In general the most accurately calculated physical quantity is the string tension, σ. Therefore the mass ratios that we shall extrapolate to the continuum limit will be of the form $m/\sqrt{\sigma}$ where m is a glueball mass. Now the leading lattice correction to such a mass ratio is known to be of the form[8] $O((a/\xi)^2)$ where ξ is some physical length scale. We choose to use the length scale $\xi_\sigma = 1/\sqrt{\sigma}$ in which case this leading correction is proportional to $a^2\sigma$. (This correction may also contain a dependence on g^2 but this varies weakly, if at all, with a and so we neglect it.) We now fit the D=4 lattice results[1-4] in the range $\beta \equiv 4/g^2 \leq 2.85$ with a formula that incorporates this correction and we obtain the continuum mass ratios shown in the relevant column of Table 1. For D=3 we perform an identical analysis on the results of ref(5), for the range $\beta \equiv 4/ag^2 \leq 14.5$, and we obtain the second column of Table 1.

To be confident in these results requires a rather detailed analysis of the mass calculations at each value of β. This will be provided in a longer paper[9]. That paper will also contain a careful description of the flux-tube model, which we only sketch here, and it will contain a much more comprehensive comparison of spectra than we are able to provide in this Letter.

One point of detail that we are forced to address here is the spin assignment. The glueball

wave-functionals are constructed using only the lattice rotational symmetries. Thus a state that we call $J = 0$ may actually be $J = 4, 8, ...$ The usual practice is to label the state by the lowest allowed value of J on the assumption that that is presumably the lightest of the contributing states. Our analysis of the flux-tube model will show that this assumption breaks down too often for it to be really useful. Fortunately one can easily show[9] that where the basic components used in constructing the glueball wavefunctionals are broad and smooth - as are the smeared operators used in those lattice calculations whose results we employ - the lattice operator will have its largest individual projection onto the state with the lowest allowed J. One can therefore infer that when we extract the lightest mass contributing to the correlation function of such smeared operators, if the normalised projection of the operator onto this lightest mass state is much greater than 0.5, then it is extremely unlikely that that state possesses anything other than the lowest allowed value of J. By this criterion all the spin assignments in Table 1 are correct and should be taken at face value.

Before turning to the model it is interesting to compare the two spectra in Table 1. What is most striking is how similar they are, taking into account that in 3 dimensions $J \neq 0$ states of opposite parity should be degenerate - which is what we observe (within errors). For both dimensions the lightest state is the 0^+ and the next lightest is the 2^+, with the 2^- and 0^- not too far away. Then come the heavier states of spin $J = 1, 3$. Even the ratios of the 2^+ to 0^+ masses are not very different. The 0^+ glueball is heavier in D=3 than D=4 when expressed in units of the string tension and we might wonder whether this can be simply related to the change in the number of spatial dimensions. As we shall see below, the flux-tube model does provide an explanation of this kind.

In theories with linear confinement one expects the flux between well separated fundamental sources to be localised in a stringlike flux-tube. If one considers a flux-tube that closes upon itself rather than ending on quarks, then we have a colourless, quarkless 'excitation' which we can imagine providing us with the basic component of a glueball state. This is the starting point of the Isgur-Paton flux-tube model[7].

We shall begin with the D=2+1 case. Consider as our starting point a circular flux-loop of radius ρ lying in the spatial plane. If we ignore the internal structure of the loop, then its oscillations are of two types. First there are the periodic oscillations about this circle. Secondly there are the 'collective' radial oscillations of the whole loop. The oscillations of the first type, upon quantisation, are described by phonons, of frequency m/ρ. We can define them so that they have angular momentum $\pm m$. Let n_m^+, n_m^- be the number of phonons with frequency m/ρ and with $J = \pm m$ respectively. Then the total angular momentum contributed by the phonons is

$$J = \sum_m m \left(n_m^+ - n_m^- \right) \tag{1}$$

and the total excitation energy will be $E = M/\rho$ where

$$M = \sum_m m \left(n_m^+ + n_m^- \right) \tag{2}$$

In D=3 parity flips angular momentum so it corresponds to interchanging the + and − phonons. Now, infinitesimal $m = 1$ oscillations are easily seen to be equivalent to infinitesimal translations and rotations. Thus we shall follow ref(7) and exclude these modes from consideration. This is analogous to the cm momentum subtraction in the bag model that we referred to earlier. Since these are simple harmonic oscillators they contribute a zero point energy, and this clearly diverges. The divergent piece is proportional to the string length and can be absorbed into the bare string tension to produce the observed string tension, σ. For long strings the leading remaining piece is simply $13\pi(D-2)/6l$ where l is the string length $2\pi\rho$. We recognise this as being the usual Lüscher universal string correction[10,11] with the contribution of the $m = 1$ mode removed. Finally we put in a factor to soften the $1/\rho$ behaviour because we know that the string is really a flux-tube with a width of order $1/\sqrt{\sigma}$. Putting all this together we can write the total energy of the string as

$$E_s^M(\rho) = 2\pi\rho\sigma + \frac{M-\gamma}{\rho}\left(1 - e^{-f\sqrt{\sigma}\rho}\right) \tag{3}$$

with M related to the phonon content of the string by eqn(2). While we shall keep in mind both that the theoretically preferred value of γ is the string value, $13(D-2)/12$, and that we expect $f \sim 1$, we shall treat both of these as free parameters in our calculations below (again following ref(7)).

The second type of oscillation consists of variations in ρ. If we think of this as being a collective 'slow' oscillation, then we can simplify the problem of quantising the flux-tube by making an adiabatic approximation[7] where these 'slow' oscillations take place in a potential provided by the 'fast' phonon oscillations. Then the energy eigenstates of the flux loop are characterised by the numbers of phonons of various types and a radial wavefunction that is a solution of the radial Schrodinger equation

$$\{\frac{-9}{16\pi\sigma}\frac{d^2}{d\xi^2} + E_s^M(\xi^{3/2})\}\psi(\xi) = E\psi(\xi) \tag{4}$$

using the variable $\xi = \rho^{3/2}$ which turns out[7] to be what is appropriate here. We solve this equation for the energies and corresponding eigenstates using the numerical method of ref(12).

Before turning to the detailed glueball mass spectra thus obtained, it is worth reconsidering the adiabatic approximation used above. The model, like the theory, possesses only an overall mass scale which we may choose to be $\sqrt{\sigma}$. In units of this mass scale the whole

spectrum is then fixed (for fixed values of γ and f). So there is no obvious 'limit' where the adiabatic assumption becomes manifestly accurate. This is unlike the case with quarks where we can consider the limit of large quark masses. It is more-or-less obvious that in such a one-scale problem we should not expect the adiabatic assumption to be very good and so it would be pointless to look for a perfect fit between the model predictions and the real world of Table 1. What we should rather do is to identify those parts of the spectrum where there is reasonable agreement and to try and relate any (dis)agreement to particular aspects of the model's dynamics.

We can also test the adiabatic assumption self-consistently. If it held very well then we would normally expect the splittings due to the phonon excitations to be much larger than the splittings due to radial excitations. In Fig.1 we show a typical example of the masses of states of various radial, n, and phonon, M, quantum numbers. We see that the phonon and radial excitations lead to similar energy splittings. This tells us that the adiabatic approximation, while not necessarily very poor, is likely to be rather crude. A similar situation is found to hold in 4 dimensions. In fact in that case we have[13], from ref(4), a mass estimate for the first 0^+ excited state and it is very close in mass to the lightest 2^+, just as predicted by the model. (This is a result at one value of β and so has to be taken cautiously.)

We show in Fig.2 the lowest states of the model as functions of the parameter f and for two values of γ (including the theoretically favoured value). The lightest state is the circular loop with no phonons which is clearly 0^+. The phonon states provide the $J \neq 0$ states which are parity doubled. On each of these states we have a tower of radial excitations with the same spins and parities. Since we exclude phonons with $m = 1$ it is clear that the first $J \neq 0$ states above the 0^+ will have $J = 2$, from one $m = 2$ phonon. To obtain $J = 1$ we need a phonon content of, for example, $n_3^+ = 1$ and $n_2^- = 1$; and so the state will be considerably heavier. All this is qualitatively like the actual spectrum summarised in Table 1. Moreover, as we see in Fig.2, the predicted value of $m(0^+)/\sqrt{\sigma}$ is quite close to its observed value, if we choose f near its expected value of unity. However the 0^- glueball, which requires large M, e.g. $n_4^+ = 1$ and $n_2^- = 2$, is predicted to be considerably heavier than the $J = 1$ states and this is contrary to Table 1. Indeed it is only near $f = 1$ that the $0^+, 0^-$ states are in quantitative agreement with the spectrum in Table 1 and in that case the $J = 1, 2$ states are much too light. However if we look[9] at the wave-functions of the states we see that the $J = 0$ states in particular are localised near small ρ - which is why they are so sensitive to the value of f at small f - and for such states the string picture is likely to be least reliable. Which brings us back to our earlier observation that it surely makes no sense to be searching for an exact agreement between the model and the real world.

In Fig.2 we also show the predictions of the model for the lightest $J = 3$ and $J = 4$ states. We observe that the 4^- is lighter than the 0^- and that the $J = 3$ state is lighter than the $J = 1$ state. This highlights how dangerous is the usual assumption that underlies the choice of labelling of states calculated on the lattice. The fact that the predicted 4^- is so close to the observed '0^-', makes the question of whether, in fact, this latter state might

not be 4^- particularly important. As we said earlier, we believe that there are convincing arguments to show that this is not the case.

We now turn to the model in 4 dimensions. There are two additional types of oscillation. The first consists of periodic vibrations orthogonal to the plane of the loop. This leads, upon quantisation, to an additional kind of phonon. The zero-point energy is doubled and so the theoretically favoured value of γ becomes 13/6 rather than 13/12 as in 3 dimensions. The second additional type of oscillation is associated with the collective rotation of the plane of the loop. In the model this is assumed to be a 'slow' fluctuation and hence one that enters the Schrodinger equation through the addition of a familiar angular momentum term. The states produced purely from phonons will still possess parity doubling and it is these collective angular excitations that allow the D=4 spectrum not to be parity doubled for many values of J. If the adiabatic approximation was very good then the lightest $J \neq 0$ states would consist of angular excitations of the 0^+ ground state and we would expect much smaller energy splittings than in D=3. However in practice the angular excitation energy is comparable to the basic phonon excitation energy, just as we found for radial excitations, and so it is not clear how these new excitations will alter the spectrum.

We show in Fig.3 the calculated spectrum in 4 dimensions as a function of f and for two values of γ, including the theoretically favoured value of $\gamma = 13/6$. For values of f close to unity the spectrum resembles, at least qualitatively, the observed spectrum. That is, the lightest state is a 0^+ with the $2^+, 2^-, 0^-$ states next, and $J = 1, 3$ forming a heavier cluster of states. Indeed at $f \simeq 1$ the agreement is quite good even at the quantitative level. In D=4 the two kinds of phonons allow us to have a lighter 0^- than in D=3. A striking prediction of the model that seems to hold for all values of f and γ is that the $J = 3$ states are parity-doubled, unlike the $J = 1$ states that are somewhat heavier in the model. Unfortunately these states possess large errors in the current lattice calculations and so we cannot test these predictions at present.

We return now to our earlier observation that the mass ratio $m(0^+)/\sqrt{\sigma}$ is larger by about 25% in D=3 than in D=4. As we see in Table 1 there seems to be some such upward rescaling for the low-lying spectrum as a whole. It turns out that this phenomenon has a very simple origin in the flux-tube model. This can be seen if we compare the D=3 and D=4 spectra for the string values of γ, as in Fig.2b and Fig.3b respectively. We observe that the 0^+ mass ratio is indeed larger in D=3 and that for $f \simeq 1$ the values are close to those in Table 1. If on the other hand we compare the D=3 and D=4 spectra at the *same* value of γ we see very little difference. Thus the origin of the difference lies in the factor of two difference between the values one calculates for γ in 3 and 4 dimensions. This arises from a factor of two between the zero-point energies in 3 and 4 dimensions. This in turn follows from the fact that in 3 spatial dimensions there are twice as many types of transverse fluctuation for the string as compared to 2 spatial dimensions. So the point is that the zero-point energy reduces the energy density of a string of finite length, as in a glueball, from the value of σ, and the reduction is twice as great in D=4 because there are twice as many transverse directions in which the string can oscillate. This argument is a rigorous one for the lowest-lying 0^+ state because for that state it is only through the

value of γ that the dimensionality enters the calculation. Moreover it is clear that this is an effect that will apply to any glueball state that is composed of a small flux loop.

The flux-tube model does surprisingly well in reproducing the overall features of the spectra, in both 3 and 4 dimensions. Perhaps its main weakness is that it predicts the 0^- to be always heavier than the $1^\pm$ in D=3. On the other hand one might regard the fact that the $J = 1$ states are heavier than the $J = 2$ states as a surprise and so the fact that the model reproduces this feature is a success for it. In the model it arises from the absence of phonons with $m = 1$ and from the fact that the flux loop has no direction for SU(2). The latter is no longer the case for SU(3) and will pose a problem for the model because the D=4 lattice SU(3) spectra do not show any light $J = 1$ states. (However we leave the SU(3) comparison till the D=3 spectra become available.)

In the model it is the phonon excitations that naturally produce parity doubling. So the D=4 spectrum will display parity doubling for those quantum numbers where the phonon states are the lightest ones - such as $J = 3$. This prediction tests the fundamental dynamical assumptions of the model. It is unfortunate that the lattice results are still too poor, for such heavier glueballs, to test this prediction. Of course the basic assumption of the model is that glueballs are essentially finite loops of flux. One may view it as significant that this provides a very simple explanation for the observed fact that, when expressed in units of the string tension, the lowest lying glueballs are lighter in 4 than in 3 dimensions.

The comparisons with the flux-tube model have focussed attention on features of the spectrum that might otherwise have appeared to be without any special significance. In particular it has focussed attention on some things that lattice calculations need to do better. For example it is crucial that we be confident in the continuum spin assignments for states on the lattice; and it is also important to calculate the masses of states such as the $J = 4$ etc. Indeed the time has come to construct lattice wave-functionals that are good approximations, up to corrections of $O(a^2)$ say, to states of a particular continuum J. There is no real difficulty in doing so. It is also clear that it is not only the lowest-lying states of any given J^P that are important; the excitations carry valuable information, in this model, about the splittings due to the 'collective' radial and angular excitations. The energies of such states are easily calculable in lattice calculations although it has usually not been thought important to do so and almost no published results exist. Finally heavier states, such as the $J = 3$ states in 4 dimensions, can also be important, as was pointed out in the previous paragraph. On the lattice these are most easily calculated by working with small lattice spacings. This will be time-consuming but not outrageously so.

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Figure Captions

Fig.1: The lowest few $0^{+,-}$ and $2^{+,-}$ states as functions of f in 3 dimensions. M is the phonon quantum number and n is the radial quantum number. The points display the actual calculated spectra and the lines are just to guide the eye. Dashed lines connect radially excited states.

Fig.2: The D=3 glueball spectrum as a function of f and for the values of γ indicated. The column of points at the far right comes from Table 1. Other points display the values obtained with the model. The lines are to guide the eye. The dashed line is for the 0^-.

Fig.3: The D=4 glueball spectrum as a function of f and for the values of γ indicated. The column of points at the far right comes from Table 1. Other points display the values obtained with the model. The lines are to guide the eye. Full lines connect positive parity states, dashed lines negative parity.

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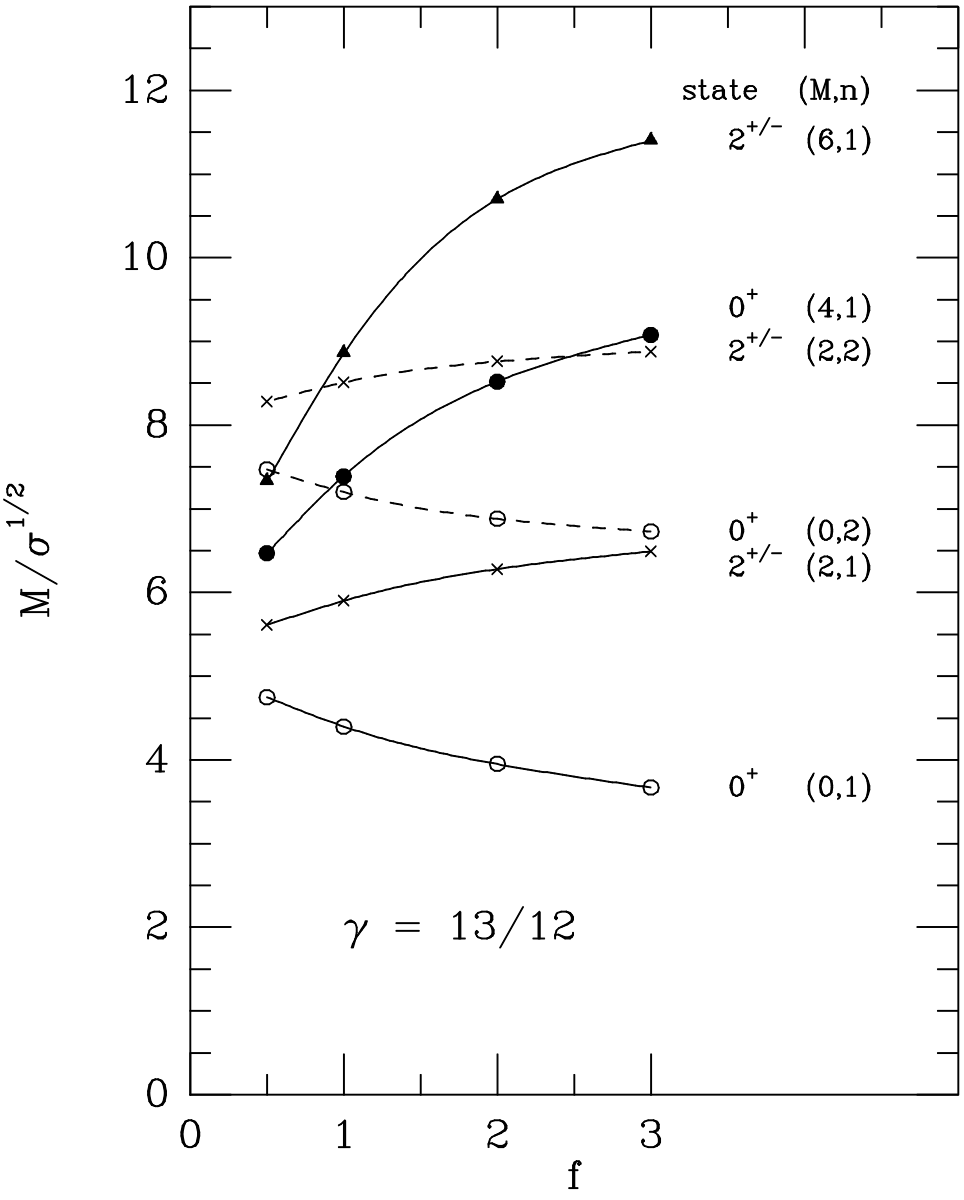
state (M,n)
$2^{+/-}$ (6,1)
0^+ (4,1)
$2^{+/-}$ (2,2)
0^+ (0,2)
$2^{+/-}$ (2,1)
0^+ (0,1)
$\gamma = 13/12$
$M/\sigma^{1/2}$
f
0
1
2
3
0
2
4
6
8
10
12

Fig. 1

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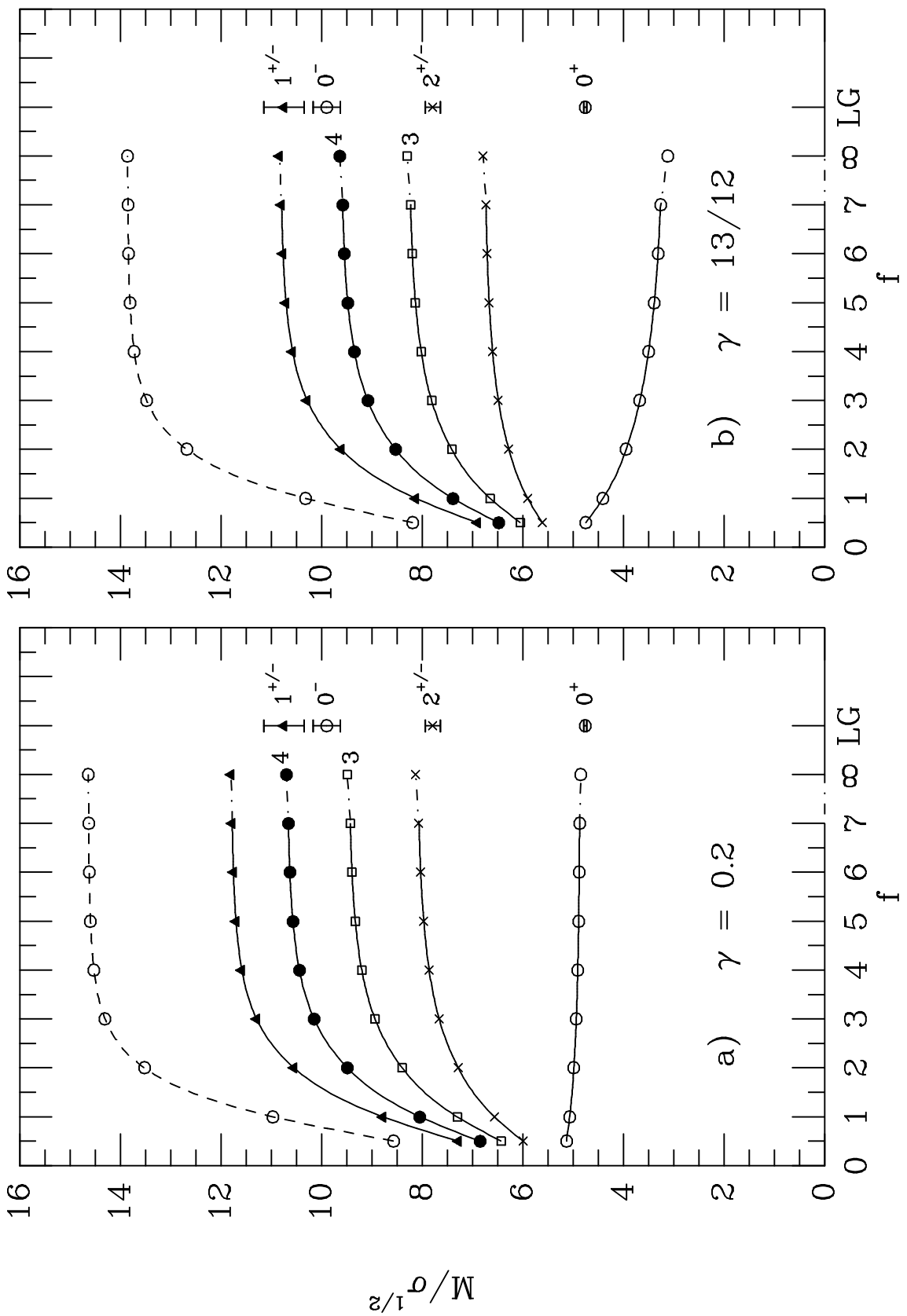

Fig. 2

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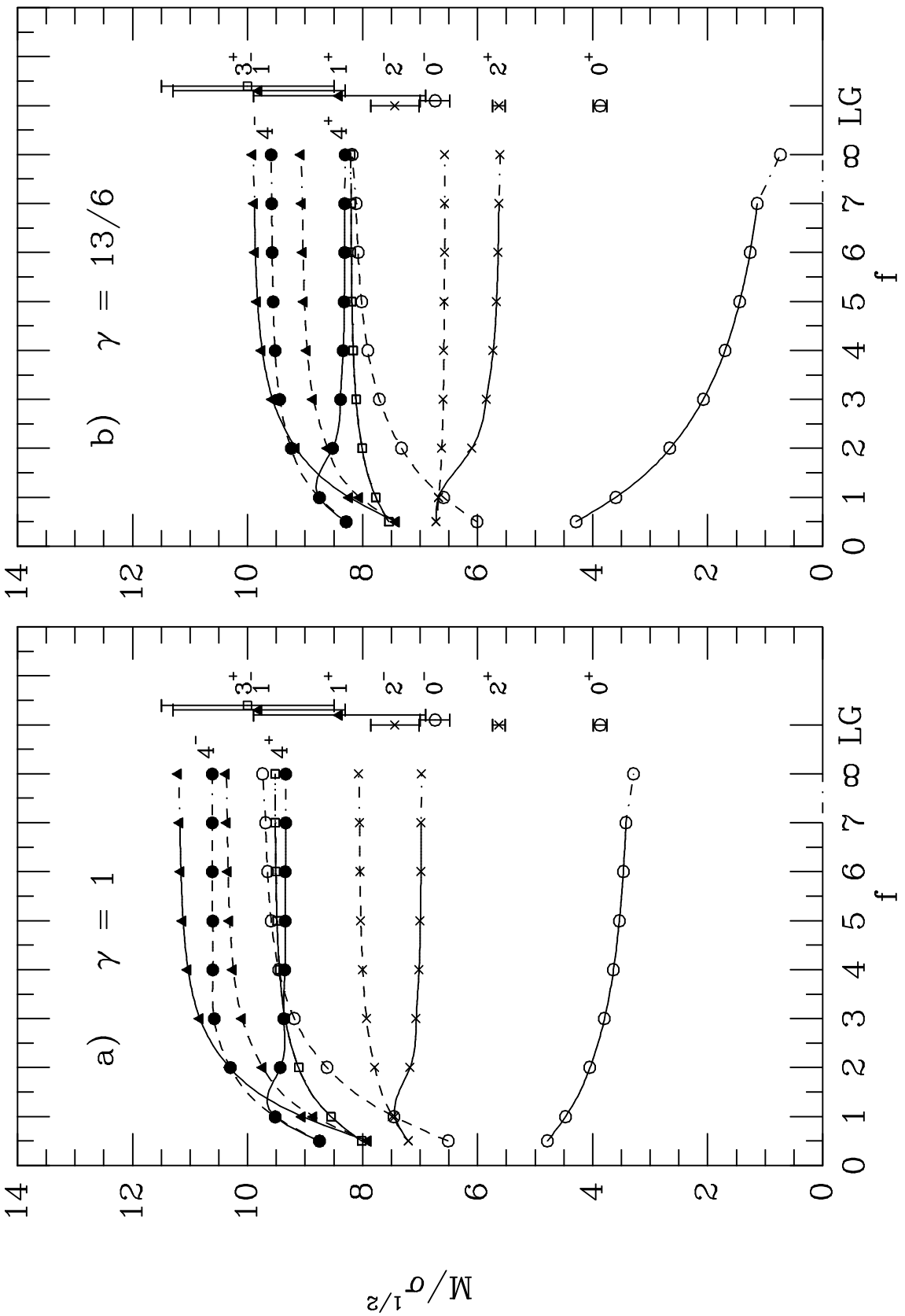

Fig. 3

$m/\sqrt{\sigma}$		
State	$D=3$	$D=4$
0^+	4.763(31)	3.87(12)
2^+	7.88(12)	5.63(11)
2^-	7.68(15)	7.44(42)
0^-	9.90(27)	6.74(26)
1^+	10.34(36)	8.4(15)
1^-	11.16(40)	9.8(15)
3^+	$--$	10.0(15)
3^-	$--$	$--$

Table 1: Continuum glueball spectrum for SU(2) in 3 and 4 dimensions.