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Sensitivity of HBT Interferometry to the Microscopic Dynamics of Freeze-out.

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We study the HBT interferometry of ultra-relativistic nuclear collisions using a freezeout model in which free pions emerge in the course of the last binary collisions in the hadron gas. We show that the HBT correlators of both identical and non-identical pions change with respect to the case of independent pion production. Practical consequences for the design of the event generator with the built in Bose-Einstein correlations are discussed. We argue that the scheme of inclusive measurement of the HBT correlation function does not require the symmetrization of the multi-pion transition amplitudes (wave-functions).

25.75.Gz, 12.38.Mh, 24.85.+p, 25.75.Ld

I. INTRODUCTION

Pion interferometry is expected to provide important information about the space-time picture of ultrarelativistic nuclear collisions. It has already proved to be a sensitive tool for the detection of the collective motion of the matter created in the course of ultrarelativistic nuclear collisions [1–4]. The primary goal of this paper is to show that under favorable circumstances, interferometry is capable of detecting the difference between various mechanisms of freezeout which is the last transient phase before the regime of free streaming of hadrons. The effect we rely upon is entirely due to real interactions at the freezeout stage, and its magnitude may serve as a measure of these interactions. We predict that the normalized two-particle correlator of *non-identical* pions (and even for pairs like πp, pn, etc.) must differ from the reference unit value at any difference Δk of the pions momenta. The normalized correlator of identical pions does not approach unity at large Δk and slightly exceeds value of 2 at $\Delta k = 0$. (In the idealized model which is used in this paper to *demonstrate* these effects, they are small.) The second goal is to discuss the effect of the multiparticle final states on the one- and two-pion inclusive spectra. We demonstrate that the contribution of these states depends on the microscopic dynamical mechanism of freezeout. We explicitly show that the effect of the multi-particle final states is limited by the actual range of the interactions at the freezeout stage. No particles, which are causally or (and) dynamically disconnected from the two-pion inclusive probe, can contribute this effect. Our general conclusion is that the problem of the analysis of HBT data cannot even be posed without reference to an explicit dynamical model. Finally, our model may serve as a prototype of a realistic dynamical mechanism of freezeout, provided that the preceding stage of evolution is a hot gas of hadrons. Our results can be used for the simulation of truly quantum two-pion distributions with the input from the event generators based on semi-classical dynamics.

The paper is organized as follows: In Sec. II, the effect is explained at the introductory level. We pose the problem of interferometry (in its most rigorous form) as a problem of a quantum transition specified by the observables and the initial data in Sec. III. Sec. IV deals with the formulation of the pion interferometry problem and the results of calculations for the hydrodynamic theory of multiple production. A more realistic model of freezeout, which incorporates some elements of hadron kinetics, is considered in Sec. V. The analytic answer for the model-based calculations is obtained by the end of this section. In Sec. VI, we discuss the effect of multi-pion final states on the inclusive two-pion spectrum and speculate about physical phenomena that may lead to the increase of the effect. A practical issue of the event generator, with the built-in Bose-Einstein correlations, is discussed in Sec. VII.

II. PHYSICAL MOTIVATION

The simplest and most popular introductory explanation of the principles of HBT interferometry is as follows: The amplitude to emit ("prepare") the pion at the point x_N and to detect it with the momentum k_1 is $a_N(k_1)e^{-ik_1x_N}$.[1] For a system of two pions prepared at points x_N and x_M, and detected with the momenta k_1 and k_2, the transition amplitude (wave function) is a superposition of two indistinguishable amplitudes,

$$\mathcal{A}_{NM}(k_1,k_2) = a_N(k_1)e^{-ik_1x_N}a_M(k_2)e^{-ik_2x_M} + a_N(k_2)e^{-ik_2x_N}a_M(k_1)e^{-ik_1x_M} \ , \tag{2.1}$$

and for a system of distributed pion sources, the two-particle inclusive spectrum is

$$\frac{dN^{(2)}}{d\boldsymbol{k}_1 d\boldsymbol{k}_2} = 2\sum_{N,M}\left\{|a_N(k_1)a_M(k_2)|^2 + \mathrm{Re}\left[(a_N(k_1)a_M(k_2)a_N^*(k_2)a_M^*(k_1)e^{-i(k_1-k_2)(x_N-x_M)}\right]\right\} \ . \tag{2.2}$$

This scheme carries an implicit assumption that the pions are truly independently *created* in the state of free propagation. Unfortunately, in the literature, this property is mostly taken for granted, though in the real world, it takes place only in very special occasions. For example, independence of pion production in nuclear collisions has been used to describe Bose-Einstein correlations at Bevalac energies when the excited nuclear matter is dominated by nucleons [5]. In that case, the bremsstrahlung of pions, that accompanies scattering of nucleons, was suggested to provide independent pion sources.

At RHIC energies (100 GeV/nucleon), we anticipate that the nuclear matter before the freezeout is totally dominated by pions and only very few nucleons are expected in the central rapidity region. There is a long-standing conjecture that the last form of the matter, before the freezeout, is a hot expanding hadronic gas. If this is indeed the case, then the main type of interaction at the freezeout stage is the binary collisions of pions. Therefore, the free propagation of any pion starts after it has experienced the "last" collision in the hadronic gas, and there is no really independent free pion production. Indeed, in this case, at least four particles are involved in the interference process, because at least two particles appear in the final state in each binary collision. If two pions (say, $\pi^+\pi^+$) are detected, then there are at least two more undetected particles emerging from two different collisions, and these particles may be identical as well. Thus, we encounter an additional (hidden) interference which affects the measured inclusive two-particle cross section.[2] Moreover, even if the detected particles are different (e.g., $\pi^+\pi^-$), their partners still may be identical (e.g., the process $\pi^+\pi^0 \to \pi^+\pi^0$ takes place at the coordinate x_N, and $\pi^-\pi^0 \to \pi^-\pi^0$ takes place at the coordinate x_M). Therefore, if the last interaction is the binary collision in the pion gas, the naïve scheme (2.1) has to be modified. In this modified case, there are four interfering amplitudes,

$$\begin{aligned}\mathcal{A}_{NM}(k_1,k_2;q_1,q_2) = {} & a_N(k_1,q_1)a_M(k_2,q_2)e^{-i(k_1+q_1)x_N}e^{-i(k_2+q_2)x_M} \\ & + a_N(k_1,q_2)a_M(k_2,q_1)e^{-i(k_1+q_2)x_N}e^{-i(k_2+q_1)x_M} \\ & + a_N(k_2,q_1)a_M(k_1,q_2)e^{-i(k_2+q_1)x_N}e^{-i(k_1+q_2)x_M} \\ & + a_N(k_2,q_2)a_M(k_1,q_1)e^{-i(k_2+q_2)x_N}e^{-i(k_1+q_1)x_M} \ .\end{aligned} \tag{2.3}$$

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[1] We essentially base this on the Dirac definition of the wave function as a transition amplitude, $\psi_k(x) \equiv \langle k|x\rangle = \psi_x^*(k)$, which allows for two complementary interpretations: (i) the particle is *prepared* with momentum k and *detected* at the point x, or, equivalently, (ii) the particle is *prepared* at the point x and *detected* with the momentum k. The same interpretation remains valid for the entire hierarchy of the multi-particle wave functions, $\langle k_1, k_2|x_1, x_2\rangle$, etc.

[2] A similar mechanism is known to provide corrections to the collision term in kinetic equations as well as the first virial corrections to the equation of state of quantum gases [6].

The expression for the two-particle spectrum thus becomes more complicated,

$$\frac{dN^{(2)}}{d\boldsymbol{k}_1 d\boldsymbol{k}_2} = \sum_{N,M} \sum_{q_1,q_2} |\mathcal{A}_{NM}(k_1,k_2;q_1,q_2)|^2 = 4 \sum_{N,M} \sum_{q_1,q_2} \Big\{ \; |a_N(k_1,q_1) a_M(k_2,q_2)|^2$$
$$+ \mathrm{Re} \Big[\; a_N(k_1,q_1) a_M(k_2,q_2) a^*_N(k_2,q_1) a^*_M(k_1,q_2) e^{-i(k_1-k_2)(x_N-x_M)}$$
$$+ a_N(k_1,q_1) a_M(k_2,q_2) a^*_N(k_1,q_2) a^*_M(k_2,q_1) e^{-i(q_1-q_2)(x_N-x_M)}$$
$$+ a_N(k_1,q_1) a_M(k_2,q_2) a^*_N(k_2,q_2) a^*_M(k_1,q_1) e^{-i(k_1+q_1-k_2-q_2)(x_N-x_M)} \; \Big] \Big\} \; , \tag{2.4}$$

and contains *three* interference terms (see Fig. 1). The first interference term corresponds to the familiar "opened" interference in the subsystem of the detected pions. It is present only if the detected pions are identical. The two other terms describe the "hidden" interference in the subsystem of undetected pions which, however, are unavoidable partners of the detected pions in the process of their creation in the final states. These terms are present even if the detected pions are different. In fact, we deal with the *dynamically generated* situation when the system of two pions is described by a density matrix and not by the wave function. When the dynamics of the system is driven by binary collisions, the two pions just cannot be found in a pure state![3]

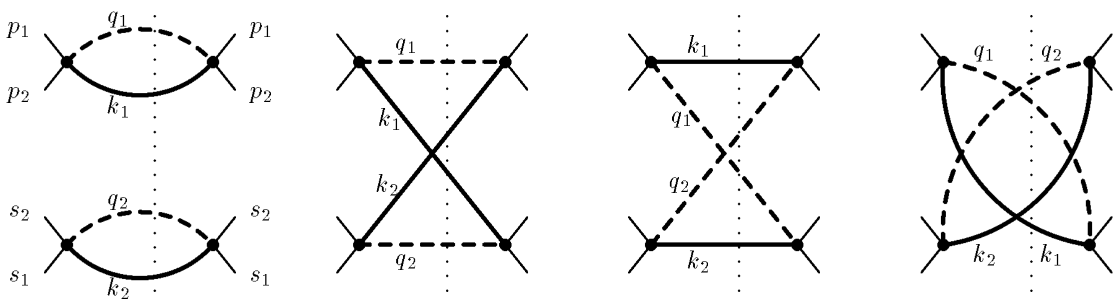

FIG. 1. Diagrams for four terms in Eq. (2.4). Bold solid and dashed lines correspond to the detected and non-detected pions in the final state, respectively. The second graph is absent when the detected pions are non-identical. The vertical dotted lines cut the diagrams through the on-mass-shell lines corresponding to the final-state pions. The external legs correspond to the initial-state pions.

A full scenario of ultrarelativistic heavy-ion collisions is still absent and an understanding of the quantum kinetics from microscopic quantum field theory, rather than from semi-classical approach, is only beginning to emerge. Currently, the most self-consistent scenario is based on an assumption that equilibration happens very early. Then it becomes possible to appeal to the equation of state of nuclear matter at various stages of the scenario, starting from the quark-gluon plasma (QGP) and ending up with the hadronic gas (as is done, e.g., in Ref. [7].) If this is not the case (no hadronic gas occurs), the pions may be created in the course of hadronization and immediately freely propagate. In this situation, it is more likely that the pions are created independently. Thus, the difference between Eqs. (2.1), (2.2) and (2.3), (2.4) becomes of practical importance. Pion interferometry provides a tool which is capable of distinguishing between these two scenaria. In Sec. VI, we argue that the effect of hidden interference may become even stronger if the system approaches the freezeout stage with the "soft" equation of state when the correlation length is long (as is, e.g., in the vicinity of a phase transition).

Furthermore, it is easy to understand that the correlations due to the hidden interference exist even, e.g., between pions and protons, protons and neutrons, etc. All these effects, which may carry significant information about the

[3] At high pion multiplicity, we can neglect an exceptional case when two detected pions come from the same binary collision.

freeze-out dynamics, require (and deserve) special study, which is beyond the scope of this paper. We insist that it is highly desirable (though not easy) either to measure all these correlation effects or to establish their absence at a high level of confidence. Both results are important since they would allow one to constrain the full self-consistent scenario of heavy-ion collisions which is absent now. These constraints may also affect theoretical predictions of the photon and dilepton yields in heavy-ion collisions.

III. THEORETICAL BACKGROUND: DEFINITION OF OBSERVABLES

A precise definition of observables is extremely important, because the HBT interferometry does not allow one to pose the mathematically unambiguous inverse problem. In general, we have to formulate a model and compare the solution with the data, relying on common sense and physical intuition. The full set of assumptions that accompany the formulation of the model is never articulated in full. Particularly, the question about the nature of states in which the particles are created is never discussed. However, this is the key issue. Interferometry is a consequence of the interference, and, in quantum mechanics, the latter cannot be even addressed without direct reference to the quantum states. Indeed, interference takes place every time when, with a given initial state, there are at least two alternative histories of evolution to a given final state. The wave function is nothing but a transition amplitude between the two states. HBT interferometry studies the two-particle wave functions (or, more precisely, the two-particle density matrix). Therefore, the most natural way to avoid any ambiguity is to pose the whole problem as a problem of a quantum transition.[4]

Let $|\mathrm{in}\rangle$ be one of the possible initial states of the system emitting a pion field which has three isospin components

$$\boldsymbol{\pi}(x) \equiv \{\pi_i(x)\} = \{\pi_+(x), \pi_0(x), \pi_-(x)\} \ , \tag{3.1}$$

where $\pi_-(x) = \pi_+^\dagger(x)$, and $\pi_0(x) = \pi_0^\dagger(x)$. At the moment of the measurement ($t_f \to +\infty$) each component of the pion field can be decomposed into the system of analyzer eigenfunctions $f_{\boldsymbol{k}}(x)$,

$$\pi_i(x) = \int d\boldsymbol{k}[A_i(\boldsymbol{k}) f_{\boldsymbol{k}}(x) + A_i^\dagger(\boldsymbol{k}) f_{\boldsymbol{k}}^*(x)], \quad f_{\boldsymbol{k}}(x) = (2\pi)^{-3/2}(2k_0)^{-1/2} e^{-i\,k\cdot x} \ . \tag{3.2}$$

This expansion holds only after freezeout. In general, $\boldsymbol{\pi}(x)$ is a multiplet of Heisenberg operators driven by the evolution operator S. The annihilation operators $A_i(\boldsymbol{k})$ of the pion field are given by

$$A_i(\boldsymbol{k}) \ = \ \int_{x^0=t_f} d^3x \ f_{\boldsymbol{k}}^*(x) \ i\overset{\leftrightarrow}{\partial_x^0} \ \pi_i(x) \ , \qquad A_i^\dagger(\boldsymbol{k}) \ = \ \int_{x^0=t_f} d^3x \ \pi_i^\dagger(x) \ i\overset{\leftrightarrow}{\partial_x^0} \ f_{\boldsymbol{k}}(x) \ . \tag{3.3}$$

The operator $A_i(\boldsymbol{k})$ describes the effect of a detector (analyzer) far from the point of emission, so, by definition, the pion is detected on mass-shell, $k^0 = (\boldsymbol{k}^2 + m^2)^{1/2}$. The inclusive amplitudes to find one pion with momentum $\boldsymbol{k}$ and two pions with the momenta $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ in the final state are

$$\langle X|A_i(\boldsymbol{k})S|\mathrm{in}\rangle \quad \text{and} \quad \langle X|A_i(\boldsymbol{k}_1)A_j(\boldsymbol{k}_2)S|\mathrm{in}\rangle \ , \tag{3.4}$$

respectively. Here, the states $|X\rangle$ form a complete set of all possible secondaries. Summing the squared moduli of these amplitudes over all (undetected) states $|X\rangle$, and averaging over the initial ensemble, we find the one-particle inclusive spectrum

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[4] We closely follow (especially, in Sec. V) Ref. [8], where this approach was first used for photon interferometry of the QGP. There, the photon emission was due to the processes $q\bar{q} \to \gamma g$ and $qg \to \gamma q$, and the identical gluons or quarks of the final state would interfere as well, if they had not become the constituents of the thermalized system.

$$\frac{dN_i^{(1)}}{d\boldsymbol{k}} = \mathrm{Tr}\hat{\rho}_{\rm in} \sum_X S^\dagger A_i^\dagger(\boldsymbol{k})|X\rangle\langle X|A_i(\boldsymbol{k})S = \mathrm{Tr}\hat{\rho}_{\rm in} S^\dagger A_i^\dagger(\boldsymbol{k})A_i(\boldsymbol{k})S \ , \tag{3.5}$$

where the density operator $\hat{\rho}_{\rm in}$ describes the emitting system. Of these two equations, the second one describes the algorithm of the measurement. In other words, we deal with the operator $N_i(\boldsymbol{k}) = A_i^\dagger(\boldsymbol{k})A_i(\boldsymbol{k})$, which gives the number of pions of the i'th kind detected by an analyzer tuned to momentum $\boldsymbol{k}$. The first equation indicates that all effects of multiparticle production are accounted for in this *basic definition* of the inclusive one-particle spectrum. The multiparticle states are encoded in a sum over the complete set of the unobserved states,

$$\sum_X |X\rangle\langle X| = 1 \ , \tag{3.6}$$

where they enter with an *a priori* assumption that they are *identically weighted*, and all together form a unit operator. In the same way, one may obtain the inclusive two-pion spectrum

$$\frac{dN_{ij}^{(2)}}{d\boldsymbol{k}_1 d\boldsymbol{k}_2} = \mathrm{Tr}\hat{\rho}_{\rm in} \sum_X S^\dagger A_i^\dagger(\boldsymbol{k}_1)A_j^\dagger(\boldsymbol{k}_2)|X\rangle\langle X|A_j(\boldsymbol{k}_2)A_i(\boldsymbol{k}_1)S = \mathrm{Tr}\hat{\rho}_{\rm in}\, S^\dagger A_i^\dagger(\boldsymbol{k}_1)A_j^\dagger(\boldsymbol{k}_2)A_j(\boldsymbol{k}_2)A_i(\boldsymbol{k}_1)S \ . \tag{3.7}$$

Once again, this equation defines the number of pairs,

$$N_{ij}(\boldsymbol{k}_1,\boldsymbol{k}_2) = A_i^\dagger(\boldsymbol{k}_1)A_j^\dagger(\boldsymbol{k}_2)A_j(\boldsymbol{k}_2)A_i(\boldsymbol{k}_1) = N_i(\boldsymbol{k}_1)[N_j(\boldsymbol{k}_2) - \delta_{ij}\delta(\boldsymbol{k}_1 - \boldsymbol{k}_2)] \ , \tag{3.8}$$

as the observable, and incorporates all multiparticle effects by its derivation. This is a function of actual dynamics which is driven by the evolution operator S to select the states which physically contribute to the process of the measurement, and to assign them the *dynamically generated* weights.

Equations (3.5) and (3.7) are universal in a sense that the standard observables of interferometry are expressed in terms of their Heisenberg operators. In order to make them useful, we have to specify both the evolution operator S and the initial data embodied in the density matrix $\rho_{\rm in}$. In other words, the theory that claims to have any predictive power must incorporate physical information about the freezeout dynamics. Two models which reflect our current vision of the heavy-ion scenario, are explored in the next two sections.

IV. NAïVE FREEZEOUT.

The simplest and the most naïve model of freezeout has been explained in detail in Ref. [1]. In order to have a reference point for the discussion of the more involved situation detailed in the next section, we review this model below with new updated emphases. In fact, this model is not dynamical. No microscopic mechanism of free pion production is specified. We just declare the pion field to become free after some time (or, in a more intelligent way, starting from some space-like surface Σ_c). This is what is done technically, but certain assumptions must be kept in mind. First, up to the freezeout surface Σ_c, the system is assumed to be a continuous medium which obeys relativistic hydrodynamic equations. The hypersurface Σ_c corresponds to some critical temperature T_c at which all interactions that maintained the local thermal equilibrium in the expanding medium are switched off. Consequently, we assume the pion distribution over the momenta (not in phase space!) at this moment to be (almost) thermal. It is incorporated into the initial data for the future free propagation.

Second, the correlation length along Σ_c is finite and much less than the total size of the system which is a dynamical consequence of the interactions before the freezeout. In order to incorporate this property into the naïve picture of the instantaneous freezeout, we must consider the whole system as a collection of boxes (fluid elements) filled by free particles which are opened when their world lines reach Σ_c.

Third, even though we may wish to disregard the quantum nature of the pions in the fluid expansion phase, we have to account for it when we pose the problem of interferometry. In other words, the pions have to be produced in certain *states*. This (perhaps, most important) goal is also achieved if we mimic freezeout by the model of the opening boxes.

Practically, we act as follows: The field $\pi_i(x)$ in Eqs.(3.3) has to be evolved in time starting from the initial data on the hypersurface Σ_c, i.e.,

$$\pi_i(x) = \int d\Sigma_\mu(y)\, G_{\rm ret}(x-y)\, \overleftrightarrow{\partial_y^\mu}\, \pi_i(y)\ . \tag{4.1}$$

The space of states in which the density matrix acts is also defined on this surface. Substituting Eqs. (4.1) to (3.3), we find the pion Fock operators, expressed in terms of the initial fields,

$$S^\dagger A_i(\boldsymbol{k})\, S = \int d^3x\, f^*_{\boldsymbol{k}}(x)\ i\overleftrightarrow{\partial_x^0} \int d\Sigma_\mu(y)\, G_{\rm ret}(x-y)\ \overleftrightarrow{\partial_y^\mu}\ \pi_i(y)\ . \tag{4.2}$$

This equation may be simplified using the explicit form of the free pion propagator,

$$S^\dagger A_i(\boldsymbol{k}) S = \theta(x^0-y^0) \int d\Sigma_\mu(y) f^*_{\boldsymbol{k}}(y)\ i\overleftrightarrow{\partial_y^\mu}\ \pi_i(y)\ . \tag{4.3}$$

The answer is simple because the whole problem of evolution is reduced to just the free propagation of the pion field. Now, we have to incorporate the idea of the absence of long-range order on Σ_c. This is done in three steps. First, we replace the continuous integral over Σ_c by the sum of integrals over the "cells" labeled by a discrete index N,

$$\int_{\Sigma_c} d\Sigma(y) \to \sum_N \int_{V_N^\#} d^3y\ , \tag{4.4}$$

where $V_N^\#$ is the three-dimensional volume of the N-th cell on the hypersurface Σ_c. (Hereafter, the #-labeled quantities are related to the local reference frame with the time-axis normal to the hypersurface Σ_c.) The pion field within the N-th cell also acquires an additional label N. Next, we perform the second quantization of the pion field within each cell independently,

$$\begin{aligned} \pi_N(y) &= \sum_{\boldsymbol{p}} [a_N(\boldsymbol{p})\phi_{\boldsymbol{p}}(y) + a_N^\dagger(\boldsymbol{p})\phi^*_{\boldsymbol{p}}(y)]\ , \\ \phi_{\boldsymbol{p}}(y) &= (2\ V_N^* p_0)^{-1/2} e^{-i\,p\cdot x}\ , \qquad p_0^2 = \boldsymbol{p}^2 + m^2\ , \end{aligned} \tag{4.5}$$

where the components of vector $\boldsymbol{p}$ take discrete values defined by the boundary conditions on the walls of each box, and the $*$-labeled quantities are related to the local rest-frame of a fluid element. Finally, we impose the commutation relations on the pion field,

$$[a_N(\boldsymbol{p}_1), a_M^\dagger(\boldsymbol{p}_2)] = \delta_{NM}\delta_{\boldsymbol{p}_1\boldsymbol{p}_2}\ , \tag{4.6}$$

which is equivalent to the dynamical independence of the fields belonging to different cells. Each pion is created and propagates independently of all others. Thus, the model is completely defined. It is mathematically very simple and reflects the main features of the physical process. There are several scales in this model, the size ℓ of the elementary cell, the freezeout temperature, T_c, and the geometric parameters L of the flow. (The correlation length is of the order $\ell \gtrsim 1/T_c$ and, in general, we must require that $\ell \ll L$.) The interplay of these parameters should be explicitly accounted for in the course of calculations. The one-particle spectrum of pions is

$$\frac{dN^{(1)}}{d\boldsymbol{k}} = \sum_N \sum_{\boldsymbol{p}} \langle a_N(\boldsymbol{p}) a_N^\dagger(\boldsymbol{p})\rangle \frac{(k_0^\# + p_0^\#)^2}{4V_N^* p_0^*} \int_{V_N^\#} d^3y e^{-i(k-p)y} \int_{V_N^\#} d^3y e^{+i(k-p)y} \ . \tag{4.7}$$

At this point, we must treat one of the integrals as a delta-function which sets the momenta $\boldsymbol{p}^\#$ and $\boldsymbol{k}^\#$ equal, while the second integral becomes just the volume $V_N^\#$ of the fluid cell. This procedure requires that $|\boldsymbol{k}|,\ |\boldsymbol{p}| \ \gg\ 1/\ell \sim T_c$, i.e., the measured momenta should be sufficiently high. In the same way, we obtain the expression for the two-particle spectrum. Since by virtue of Eq. (4.6) we have

$$\langle a_N(\boldsymbol{p}_1) a_M^\dagger(\boldsymbol{p}_2) a_{M'}(\boldsymbol{p'}_2) a_{N'}^\dagger(\boldsymbol{p}'_1)\rangle =$$
$$= (\delta_{NN'}\delta_{MM'}\delta_{\boldsymbol{p}_1\boldsymbol{p'}_1}\delta_{\boldsymbol{p}_2\boldsymbol{p'}_2} + \delta_{NM'}\delta_{MN'}\delta_{\boldsymbol{p}_1\boldsymbol{p'}_2}\delta_{\boldsymbol{p}_2\boldsymbol{p'}_1})\langle a_N(\boldsymbol{p}_1) a_N^\dagger(\boldsymbol{p}_1)\rangle\langle a_M(\boldsymbol{p}_2) a_M^\dagger(\boldsymbol{p}_2)\rangle,$$

the two-pion inclusive spectrum becomes,

$$\frac{dN_{ij}^{(2)}}{d\boldsymbol{k}_1 d\boldsymbol{k}_2} = \frac{dN^{(1)}}{d\boldsymbol{k}_1}\frac{dN^{(1)}}{d\boldsymbol{k}_2} + \sum_{NM}\sum_{\boldsymbol{p}_1,\boldsymbol{p}_2} \langle a_N(\boldsymbol{p}_1) a_N^\dagger(\boldsymbol{p}_1)\rangle\langle a_M(\boldsymbol{p}_2) a_M^\dagger(\boldsymbol{p}_2)\rangle$$
$$\times \frac{(k_1^{0\#} + p_1^{0\#})(k_1^{0\#} + p_2^{0\#})(k_2^{0\#} + p_2^{0\#})(k_2^{0\#} + p_1^{0\#})}{4^2 V_N^* V_M^* p_1^{0*} p_2^{0*}} \cos(k_1 - k_2)(x_N - x_M)$$
$$\times \int_{V_N^\#} d^3y e^{-i(k_1-p_1)y} \int_{V_N^\#} d^3y e^{+i(k_1-p_2)y} \int_{V_M^\#} d^3y e^{-i(k_2-p_2)y} \int_{V_M^\#} d^3y e^{+i(k_2-p_1)y} \ . \tag{4.8}$$

Thus, we have reproduced Eq. (2.2) which followed from an intuitive conjecture (2.1) about the interference of two indistinguishable amplitudes. These amplitudes, by their design, reflect all relevant properties of the interaction which prepare the two-pion system and a device which detects this system. This *single device* consists of two detectors (e.g., two tracks) tuned to the pions with momenta $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$. Since, by the definition of the inclusive measurement, nothing else is measured, there are two and only two interfering amplitudes. Regardless of how large the total number, $\mathcal{N}$, of pions produced in a particular event is, only the two-pion transition amplitude has to be symmetrized since the remaining $\mathcal{N} - 2$ pions are not measured.

In order to avoid any misunderstanding, we remind the reader that the quantum-mechanical measurement of some observable, by definition, includes the procedure of averaging over an ensemble. A single element of this ensemble carries no quantum-mechanical information. Following Ref. [9], we can argue that the inclusive measurement explores all quantum mechanical fluctuations which can dynamically develop before the moment of measurement and are consistent with the detector response. From this point of view, nothing but a causal chain of real interactions can affect the detector response, and only dynamical histories can interfere. There is a significant difference between the cases when one-, two-, or three-particle inclusive distributions are measured. Each of these measurements is unique in a sense that they are all *mutually exclusive* even if they are obtained from the same ensemble of multiparticle events. For example, if the three-pion distribution is measured (i.e., the average over an ensemble is accomplished), then the integration over the momentum of the third pion does not result in the two-pion inclusive distribution [12]. This example reflects a qualitative difference between classical and quantum distributions. The former are defined immediately in terms of *probabilities* while the later are defined via *transition amplitudes* with the probabilities playing the secondary role.

The same conclusions follow from the field-theory formalism we employ. The interference emerges as a strict consequence of the commutation relations (4.6) which incorporate a distinctive property of the hydrodynamic model to localize the freezeout point for each pion independently. We start from the quantum operator (3.8) of the measured observable and trace the two-pion signal back to its origin. Eventually, we arrive at the square of the symmetrized two-pion amplitude, once again, regardless of the total number $\mathcal{N}$of pions in a particular event. The states with many pions are completely accounted for in Eqs. (3.4) and (3.7) and, consequently, in (4.7) and (4.8). However, they can contribute to the one- and two-pion observables only via real interactions which are absent in our oversimplified model of a naïve freezeout. A more realistic example with the interaction is considered in the next section.

In Eq.(4.8), we seemingly encounter a problem. If, e.g., the first of the integrals is considered as the delta-function which sets momenta $\boldsymbol{p}_1^{\#}$ and $\boldsymbol{k}_1^{\#}$ equal, then the value of the second one is not obvious. It occurs that the second integral over the volume $V_N^{\#}$ becomes

$$\int_{V_N^{\#}} d^3y e^{+i(k_1-k_2)y} = V_N^{\#}. \tag{4.9}$$

To prove this, we need more physical information. The interferometry would have been impossible if we could have traced each pion back to the coordinate of its emission or, in other words, if we could have built an "optical image" of the source. According to the Rayleigh criterion, the latter is possible only if $|\boldsymbol{k_1}-\boldsymbol{k_2}|\ L\ \gtrsim 1$. Therefore, we may observe interference only provided the last inequality does not hold. Thus we have $|\boldsymbol{k_1}-\boldsymbol{k_2}|\ \lesssim 1/L\ \ll\ 1/\ell$, which proves (4.9).

In the continuous limit, by introducing an auxiliary "emission function" $J(k_1,k_2)$,

$$J(k_1,k_2)\ =\ \int_{\Sigma_c} d\Sigma_\mu(x)\,\frac{k_1^\mu+k_2^\mu}{2}\,n(k_1\cdot u(x))\,e^{-i(k_1-k_2)x}\ , \tag{4.10}$$

the expressions for the one- and two-particle inclusive spectra can be rewritten in a form which is convenient for numerical computations:

$$k^0\,\frac{dN_1}{d\vec{k}}\ =\ J(k,k)\ \ , \tag{4.11}$$

and

$$k_1^0k_2^0\,\frac{dN_2}{d\vec{k}_1 d\vec{k}_2}\ =\ J(k_1,k_1)\,J(k_2,k_2)\ +\ \mathrm{Re}\Big[J(k_1,k_2)\,J(k_2,k_1)\Big]\ \ , \tag{4.12}$$

respectively.

In Ref. [2], these equations were used to study interferometry for several types of one–dimensional flow. Here, we are interested only in the case of the boost-invariant geometry which must be very close to the reality of RHIC. Indeed, any scenario initiated with a strong Lorentz contraction (up to 0.1 fm!) of the nuclei cannot possess a scale associated with the initial state. Hence, both at classical and quantum levels, the system must evolve with the preserved boost-invariance. In terms of the hydrodynamic theory of multiple production [10], the absence of scale in initial data of the relativistic hydrodynamic equations immediately leads to the Bjorken self-similar solution [11] as the only possible solution. Recent analysis of quantum fluctuations at the earliest stage of heavy-ion collisions [9] indicates the same. The overlap of Lorentz-contracted nuclei converts them into a system of modes of *expanding plasma* with the global boost-invariant geometry. This is a single quantum transition and it is not compatible with the picture of gradually developing parton cascade [13].

Theoretical analysis of two-pion correlations, in the picture of "naïve freezeout," from the phase of expanding hot pion gas was done in Refs. [1,2]. One of the predictions of these studies, the so-called m_t-scaling of the pion and kaon correlators, was confirmed by the data of the NA35 and NA44 collaborations [3,4] obtained from Au-Au collisions at SPS energies (∼10 GeV/nucleon). These data carry two important messages. First, even when the nuclei are Lorentz-contracted only up to the size of 1 fm, this is almost enough to bring about the boost invariant regime of collective flow. There is no doubts that this picture will be even more pronounced at RHIC. The second, less trivial consequence of the observed m_t-scaling is that *the freezeout is sharp*; if the creation of the final-state pions were extended in (local) time, the m_t-scaling would vanish [14]. Thus, addressing the pion production at RHIC, we have every reason to rely on two facts: (i) the hadronic matter before the freezeout forms a collective system, and (ii) this system is in the state of the self-similar boost-invariant expansion.

For the immediate goals of this study, an advantage of the intensive longitudinal flow is that it provides a window in the phase-space of the two pions where the effect of the hidden interference (explained in Sec. II, and computed in more detail in Sec. V) is most visible. This window corresponds to the measurement of the transverse size of the longitudinally-expanding pipe using pairs of pions with the same rapidity. In this way, we employ a distinctive feature of the hydrodynamic-type sources to localize the emission spectrum. The required localization in rapidity is achieved by choosing pions with large transverse momenta ($p_t \gtrsim 3T \sim 3m_\pi$). This has a very simple physical explanation. If the emitting system is kinetically equilibrated (or even sufficiently chaotized) then the mean energy per particle is limited from above. Thus, in the rest-frame of a fluid element, the particles with large transverse momenta can have, on average, only small longitudinal momenta. In other words, such particles are effectively frozen into the collective hydrodynamic motion in the laboratory frame.[5] Measuring there longitudinal rapidity after freezeout, we are most likely to measure the longitudinal rapidity of the fluid at the emission site.

The parameters of the model with the Bjorken geometry are the critical temperature, T_c ($\sim m_\pi$), and the space–like freeze–out hypersurface, defined by $t^2 - z^2 = \tau^2 =$ const. The coordinates and the four-velocity of the fluid are parameterized as $x^\mu = (\tau \cosh\eta, \vec{r}, \tau \sinh\eta)$, and $u^\mu(x) = (\cosh\eta, \vec{0}, \sinh\eta)$, respectively. The rapidity, η, of a fluid cell is restricted to $\pm Y$ in the center-of-mass frame. We assume an axially-symmetric distribution of hot matter in a pipe with area $S_\perp = \pi R_\perp^2$. The particles are described by their momenta, $k_i^\mu = (k_i^0, \vec{k}_i, k_i^z) \equiv (m_i \cosh\theta_i, \vec{k}_i, m_i \sinh\theta_i)$, where $\vec{k}_i$ is the two-dimensional vector of the transverse momentum, θ_i is the particle rapidity in z-direction, and $m_i^2 \equiv m_{\perp i}^2 = m^2 + \vec{k}_i^2$ is the transverse mass. Let us introduce $2\alpha = \theta_1 - \theta_2$, $2\theta = \theta_1 + \theta_2$. The one-particle distribution is expected to be close to a thermal distribution of pions at temperature $T = T_c$. Since the $m_\perp$ values of interest are larger than T_c, we may take this distribution in a Boltzmann form, and use the saddle-point method to estimate the integrals (4.12). For the one-particle distribution these two steps yield

$$\frac{dN}{d\theta_1 d\vec{k}_1} \approx \tau S_\perp m_1 \int_{-Y}^{Y} d\eta \cosh(\theta_1 - \eta) e^{-m_1 \cosh(\theta_1-\eta)/T_c} \approx \tau S_\perp m_1 \sqrt{\frac{2\pi T_c}{m_1}}\, e^{-m_1/T_c} \ . \tag{4.13}$$

The general expression for $J(k_1, k_2)$ is

$$\begin{aligned} J(k_1,k_2) = \frac{1}{2}\int d\vec{r}_1 f(r_1) e^{i(\vec{k}_1-\vec{k}_2)\vec{r}_1}\tau \int_{-\eta}^{\eta} d\eta \, \Big[(m_1+m_2)\cosh(\theta-\eta)\,\cosh\alpha - (m_1-m_2)\sinh(\eta-\theta)\sinh\alpha\Big] \\ \times \exp\Big\{ -\frac{1}{T_c}\Big[(m_1 + iF(m_1-m_2))\cosh(\eta-\theta)\cosh\alpha - (m_1 + iF(m_1+m_2))\sinh(\eta-\theta)\sinh\alpha\Big]\Big\} \ , \end{aligned} \tag{4.14}$$

where $F = \tau T_c$. Once we consider only those pions with large transverse momenta, the integral over the rapidity η can (and should) be computed in the saddle-point approximation. This yields

$$\begin{aligned} R(k_1,k_2) = \frac{dN_2}{dy_1 d\vec{p}_1 dy_2 d\vec{p}_2} \Big/ \left[\frac{dN_1}{dy_1 d\vec{p}_1}\frac{dN_1}{dy_2 d\vec{p}_2}\right] - 1 = \frac{1}{4}\, f(|\vec{q}_\perp| R_\perp)\, \frac{g(z)\, g(1/z)}{[h(z)\, h(1/z)]^{3/2}} \\ \times \exp\left\{ -\frac{\mu}{T_c}\left[h(z)\cos\frac{H(z)}{2} + h(\frac{1}{z})\cos\frac{H(1/z)}{2} - z - \frac{1}{z}\right]\right\} \end{aligned}$$

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[5]This is the physical origin of the so-called m_t-scaling and the qualitative basis for the saddle-point calculations below.

$$\times \cos\left\{\frac{\mu}{T_c}\left[h(z)\sin\frac{H(z)}{2} + h(\frac{1}{z})\sin\frac{H(1/z)}{2}\right] + \frac{3}{4}\left[H(z) + H(\frac{1}{z})\right] + G(z) + G(\frac{1}{z})\right\} , \tag{4.15}$$

where $\mu = (m_1 m_2)^{1/2}$ and $z = (m_1/m_2)^{1/2}$, and we have introduced the functions

$$h(z) = \left\{\left[z^2 - F^2(z - \frac{1}{z})^2 + 4F^2\sinh^2\alpha\right]^2 + 4F^2(z^2 - \cosh 2\alpha)^2\right\}^{1/4} ;$$

$$g(z) = \left[(z^2 + \cosh 2\alpha)^2 + F^2(z^2 - \frac{1}{z^2})^2\right]^{1/2} ;$$

$$\tan H(z) = \frac{2F(\cosh 2\alpha - z^2)}{z^2 - F^2(z - \frac{1}{z})^2 + 4F^2\sinh^2\alpha}; \qquad \tan G(z) = \frac{F(z^2 - 1/z^2)}{z^2 + \cosh 2\alpha} . \tag{4.16}$$

When the transverse momenta of two pions are equal, then $m_1 = m_2$, and Eq. (4.15) reproduces the result of Ref. [2]. The function f depends on how we parameterize the transverse distribution of the matter in the longitudinally expanding pipe. If $f(r) = \theta(R_\perp^2 - r^2)$ then $f(|\vec{\Delta}k_\perp|R_\perp) = [2J_1(|\Delta\vec{k}|R_\perp)/|\Delta\vec{k}|R_\perp]^2$, where J_1 is a Bessel function. This HBT correlator possesses a natural property of any two-point function in the boost-invariant geometry; it depends on the difference of rapidities and not on the difference of the longitudinal momenta. The m_t-scaling of the "visible longitudinal size" is just a synonym for this property, which supports a picture of the sharp freeze-out of matter in the state dominated by the boost-invariant-like longitudinal expansion.

Two remarks are in order:

1. The shape of the correlator given by Eqs. (4.15) and (4.16) is manifestly non-Gaussian. Thus, it is plausible to get rid of the intermediate Gaussian fit and to use these equations to fit the data as the first step.

2. A main deficiency of the correlator (4.15) is that the possible transverse expansion has not been taken into account. This is not easy to do since the initial data in the transverse plane are, as yet, poorly understood and the amount of matter involved in the collective transverse motion at the freezeout stage may vary depending on the initial conditions, equation of state, etc. We consider this issue still open.

V. DYNAMICAL FREEZEOUT.

Now, let us formulate a more realistic model for the freezeout, relying on the following phenomenological input. Let the system before the collision be a pion-dominated hadronic gas. Therefore, since pions are the lightest hadrons, the regime of continuous medium is supported mainly due to $\pi\pi$-scattering. In the framework of chiral perturbation theory there are two main channels contributing to this process. The first channel is s-wave scattering which we shall model using the sigma-model. The second channel is the p-wave $\pi\pi$-scattering via the ρ-meson. We shall use a model interaction Lagrangian of the form

$$\mathcal{L}_{\rm int} = \frac{g_{4\pi}}{4}(\boldsymbol{\pi}^\dagger\cdot\boldsymbol{\pi})^2 + g_\sigma\sigma(\boldsymbol{\pi}^\dagger\cdot\boldsymbol{\pi}) + \frac{f_{\rho\pi\pi}}{2}\boldsymbol{\rho}^\mu\cdot[\partial_\mu\boldsymbol{\pi}^\dagger\times\boldsymbol{\pi} + \boldsymbol{\pi}^\dagger\times\partial_\mu\boldsymbol{\pi}] + \cdots , \tag{5.1}$$

where $\sigma(x)$ is the field of the scalar σ-mesons, and $\boldsymbol{\rho}^\mu = (\rho^-, \rho^0, \rho^+)$ is the field of the ρ-mesons. The reader can easily recognize here that part of the Lagrangian of the sigma-model which is necessary to reproduce the tree-level (or even skeleton) amplitudes of the s- and p-wave $\pi\pi$-scattering. The evolution operator is usually defined as

$$S = T\exp\{i\int\mathcal{L}_{\rm int}(x)d^4x\} . \tag{5.2}$$

Applying the standard commutation formulae [15],

$$A_{[i]}(\boldsymbol{k})S - SA_{[i]}(\boldsymbol{k}) = \int d^4x \frac{\delta S}{\delta \pi_i^\dagger(x)} f_{\boldsymbol{k}}^*(x) \ , \quad S^\dagger A_{[i]}^\dagger(\boldsymbol{k}) - A_{[i]}^\dagger(\boldsymbol{k})S^\dagger = \int d^4x f_{\boldsymbol{k}}(x) \frac{\delta S^\dagger}{\delta \pi_{[i]}(x)} \ , \tag{5.3}$$

to the Eqs. (3.5) and (3.7) we obtain

$$\frac{dN_{[a]}^{(1)}}{d\boldsymbol{k}} = \int d^4y_1 d^4y_2 f_{\boldsymbol{k}}(y_1) \left\langle \frac{\delta S^\dagger}{\delta \pi_{[a]}(y_1)} \frac{\delta S}{\delta \pi_{[a]}^\dagger(y_2)} \right\rangle f_{\boldsymbol{k}}^*(y_2) \ , \tag{5.4}$$

for the one-particle spectrum of the pions of kind a (π^+, π^0, π^-), and

$$\frac{dN_{[ab]}^{(2)}}{d\boldsymbol{k}_1 d\boldsymbol{k}_2} = \int d^4y_1 d^4y_2 d^4y_3 d^4y_4 f_{\boldsymbol{k}_1}(y_1) f_{\boldsymbol{k}_2}(y_3) \left\langle \frac{\delta^2 S^\dagger}{\delta \pi_{[a]}(y_1) \delta \pi_{[b]}(y_3)} \frac{\delta^2 S}{\delta \pi_{[a]}^\dagger(y_2) \delta \pi_{[b]}^\dagger(y_4)} \right\rangle f_{\boldsymbol{k}_1}^*(y_2) f_{\boldsymbol{k}_2}^*(y_4) \ , \tag{5.5}$$

for the spectrum for pairs of pions of kinds a and b. According to our agreement, the angular brackets denote an average weighted by the density matrix $\rho_{\rm in}$. [In order to avoid confusion, we place the isospin indices in square brackets.]

Since the goal of this paper is to demonstrate a physical effect, we shall limit ourselves, in what follows, with the $\boldsymbol{\pi}^4$ interaction term. In this case, we are able to do most calculations analytically, which is necessary in order to understand the interplay of the many parameters. More involved calculations will require a realistic description of the $\pi\pi$-scattering and Monte-Carlo computation of multiple integrals.

Calculation of the one-pion spectrum is relatively simple. A direct computation of the functional derivatives in Eq.(5.4) leads to

$$\frac{dN_{[a]}^{(1)}}{d\boldsymbol{k}} = \frac{g_{4\pi}^2}{4} \int d^4x d^4y \frac{e^{-ik(x-y)}}{2k^0(2\pi)^3} \sum_{i,j} \langle T^\dagger(S^\dagger : \pi^{\dagger[i]}(x)\pi^{[i]}(x)\pi^{\dagger[a]}(x) :) T(: \pi^{[a]}(y)\pi^{\dagger[j]}(y)\pi^{[j]}(y) : S) \rangle \ , \tag{5.6}$$

where the symbols T and $T^\dagger$ denote the time and anti-time orderings, respectively. Coupling the field operators according to the Wick theorem (to the lowest order of the perturbation expansion we have to put $S = 1$), the average in Eq. (5.6) becomes,

$$\langle \ \cdots \ \rangle = i^3 [G_{01}^{[aa]}(y,x) G_{10}^{[ij]}(x,y) G_{01}^{[ji]}(y,x) + G_{01}^{[ja]}(y,x) G_{10}^{[ij]}(x,y) G_{01}^{[ai]}(y,x)] \ , \tag{5.7}$$

where any pair of arguments x and y lie within the same cell of a size defined by the correlation length in the system under consideration. This is a consequence of the cellular structure of the density operator $\hat{\rho}_{\rm in}$ which determines the average $\langle \cdots \rangle$. The correlators $G_{AB}(x,y)$ are defined (using the Keldysh technique [16]; see also Ref. [9]) as

$$G_{AB}^{[ij]}(x,y) = -i \langle T_c(\pi^{[i]}(x_A) \pi^{\dagger[j]}(y_B)) \rangle \ . \tag{5.8}$$

Taking for $a = 1$ (π^+) and using the facts that $G^{[ij]} \propto \delta^{ij}$, $G_{01}^{[33]}(y,x) = G_{10}^{[11]}(x,y)$, and $G_{10}^{[33]}(y,x) = G_{01}^{[11]}(x,y)$ we arrive at

$$\langle \ \cdots \ \rangle = i^3 G_{01}^{[11]}(y,x) [3 G_{10}^{[11]}(x,y) G_{01}^{[11]}(y,x) + G_{10}^{[00]}(x,y) G_{01}^{[00]}(y,x)] \ . \tag{5.9}$$

Next, we have to insert this expression into Eq. (5.6) and pass over to the momentum representation (cell-by-cell) according to

$$\begin{aligned} G_{01}^{[11]}(p) = G_{10}^{[33]}(-p) &= -2\pi i [\theta(p^0) n^{(+)}(p) + \theta(-p^0)] \ , \\ G_{10}^{[11]}(p) = G_{01}^{[33]}(-p) &= -2\pi i [\theta(p^0) + \theta(-p^0) n^{(-)}(p)] \ , \\ G_{01}^{[00]}(p) &= -2\pi i [\theta(p^0) n^{(0)}(p) + \theta(-p^0)] \ , \\ G_{10}^{[00]}(p) &= -2\pi i [\theta(p^0) + \theta(-p^0) n^{(0)}(-p)] \ . \end{aligned} \tag{5.10}$$

These equations reflect a very important feature of the process under investigation (already accounted for in Eqs. (5.4) and (5.5).) Namely, the initial states in the system of colliding pions are populated with the densities $n_N^{(\pm)}(p)$ of charged pions and $n_N^{(0)}(p)$ of neutral pions. All the final states of free propagation *are not occupied*. They are virtually present with an *a priori* unit weight in the expansion (3.6) of the unit operator. Access to the states with many pions is provided dynamically by real interactions, and these states show up in higher orders of the evolution operator expansion. Depending on the type of commutation relations, these states always appear in a properly symmetrized form, and there is no need to start with any *ad hoc* symmetrization. By definition, the wave function is the transition amplitude between the prepared initial state and detected final state. Unless the transition is explicitly measured, the wave function does not exist at all and there is no object for symmetrization. Therefore, there is no statistical correlations associated with the multiple production of pions. All correlations are dynamical (see further discussion in Sec. VI).

After some manipulations, we obtain for the one-particle spectrum,

$$\frac{dN^{(1)}_{\pi^+}}{d\boldsymbol{k}} = \frac{g^2_{4\pi}}{4}\sum_N V_N^{(4)} \int \frac{d^4p_1 d^4p_2 d^4q}{2k^0(2\pi)^8}\delta(k+q-p_1-p_2)\delta(p_1^2-m^2)\delta(p_2^2-m^2)\delta(q^2-m^2)$$
$$\times[3n_N^{(+)}(p_1)n_N^{(+)}(p_2)+6n_N^{(+)}(p_1)n_N^{(-)}(p_2)+2n_N^{(+)}(p_1)n_N^{(0)}(p_2))+n_N^{(0)}(p_1)n_N^{(0)}(p_2))] \ . \tag{5.11}$$

Here, as in the model of the "naïve freezeout" discussed in Sec. IV, one of the integrals over the space-time volume $V_N^{(4)}$ of the domain (where the individual collision occurs), is treated as the delta-function (viz., momentum conservation), while the second integral becomes just the four-volume $V_N^{(4)}$ of the interaction domain. (More details are given in Appendix A.) In what follows, we shall limit ourselves to a simplified picture when all the statistical weights of the initial state are taken in the Boltzmann form and the chemical potential is zero. In this case, we gain an overall factor of 3+6+2+1=12 and arrive at

$$\frac{dN^{(1)}_{\pi^+}}{d\boldsymbol{k}} = \frac{12g^2_{4\pi}}{4}\sum_N V_N^{(4)} \int \frac{d^4p_1 d^4p_2 d^4q}{2k^0(2\pi)^8}\delta(k+q-p_1-p_2)\delta(p_1^2-m^2)\delta(p_2^2-m^2)\delta(q^2-m^2)e^{-\beta_N u_N(p_1+p_2)} \ . \tag{5.12}$$

Now, the integration over $l=p_1-p_2$ can be explicitly carried out (see Eq. (B.1)

$$\frac{dN^{(1)}_{\pi^+}}{d\boldsymbol{k}} = \frac{12g^2_{4\pi}}{4}\sum_N V_N^{(4)} e^{-\beta_N(ku_N)}\int \frac{d^4q}{2k^0(2\pi)^8}\frac{\pi}{4}e^{-\beta_N(qu_N)}\delta(q^2-m^2)\sqrt{1-\frac{4m^2}{(k+q)^2}} \ . \tag{5.13}$$

Introducing an auxiliary function,

$$I(k,N,x_{NM}) = \int d^4q\delta_+(q^2-m^2)e^{-\beta_N(qu)}\ e^{iqx_{NM}}\sqrt{1-\frac{4m^2}{(k+q)^2}} \ , \tag{5.14}$$

which is computed in Appendix B, we may write explicitly,

$$\frac{dN^{(1)}_{\pi^+}}{d\boldsymbol{k}} = \frac{12\pi g^2_{4\pi}}{16}\sum_N \frac{V_N^{(4)}}{2k_0(2\pi)^8}e^{-\beta_N(ku_N)}I(k,N,0) = \frac{12g^2_{4\pi}}{16}2\pi m^2\frac{K_1(m/T)}{m/T}\int\frac{d^4x}{2k_0(2\pi)^8}e^{-ku(x)/T}$$
$$= \frac{12g^2_{4\pi}}{16(2\pi)^8}2\pi m^2\frac{K_1(m/T)}{m/T}\tau\Delta\tau S_\perp\sqrt{\frac{2\pi T}{m_\perp}}e^{-m_\perp/T} \ , \tag{5.15}$$

where, with reference to the data discussed in Sec IV, we assume that the freezeout occurs in a very small interval $\Delta\tau$ of the proper time τ. Additionally, we further simplify the model by taking the temperature the same throughout the entire freezeout domain.

Calculation of the two-pion spectrum is very cumbersome but follows exactly the same guideline,

$$\frac{dN^{(2)}_{\pi^a\pi^b}}{d\boldsymbol{k}_1 d\boldsymbol{k}_2} = \frac{g_{4\pi}^4}{16}\int d^4y_1 d^4y_2 d^4y_3 d^4y_4 \frac{e^{-ik_1(y_1-y_2)}e^{-ik_2(y_3-y_4)}}{4k_1^0k_2^0(2\pi)^6}$$
$$\times \sum_{i,j,l,n} \langle T^\dagger(S^\dagger{:}\pi^{\dagger[i]}(y_1)\pi^{[i]}(y_1)\pi^{\dagger[a]}(y_1){:}\ {:}\pi^{\dagger[l]}(y_3)\pi^{[l]}(y_3)\pi^{\dagger[b]}(y_3){:})$$
$$\times T({:}\pi^{[a]}(y_2)\pi^{\dagger[j]}(y_2)\ \pi^{[j]}(y_2){:}\ {:}\pi^{[b]}(y_4)\pi^{\dagger[n]}(y_4)\pi^{[n]}(y_4){:}S)\rangle\ , \tag{5.16}$$

where, as before, we must put $S=1$ in the lowest order of the perturbation expansion. If the observed pions are identical, then the result of coupling is a very long expression which is given in its full form in Appendix A, Eq. (A.1). The next step is to integrate over the elementary volumes $\mathcal{V}_N$. This is done according to Eq. (A.3). Finally, using the Boltzmann approximation for the statistical weights, we carry out an explicit integration over $l_1 = s_1 - s_2$ and $l_2 = p_1 - p_2$ (see Eq. (B.1)). The result is as follows,

$$\frac{dN^{(2)}_{\pi^+\pi^+}}{d\boldsymbol{k}_1 d\boldsymbol{k}_2} = \frac{dN^{(1)}_{\pi^+}}{d\boldsymbol{k}_1}\frac{dN^{(1)}_{\pi^+}}{d\boldsymbol{k}_2} + \frac{\pi^2 g_{4\pi}^4}{16^2}\sum_{N,M} V_N^{(4)}V_M^{(4)} e^{-\beta_N(k_1u_N)}e^{-\beta_M(k_2u_M)}$$
$$\times \int \frac{d^4q_1 d^4q_2}{4k_1^0k_2^0(2\pi)^{16}}\sqrt{1-\frac{4m^2}{(k_1+q_1)^2}}\sqrt{1-\frac{4m^2}{(k_2+q_2)^2}}e^{-\beta_N(q_1u_N)}e^{-\beta_M(q_2u_M)}\delta_+(q_1^2-m^2)\delta_+(q_2^2-m^2)$$
$$\times \mathrm{Re}\{144\ e^{-i(k_1-k_2)(x_N-x_M)} + 44e^{-i(q_1-q_2)(x_N-x_M)} + 44e^{-i(q_1+k_1-q_2-k_2)(x_N-x_M)}\}\ . \tag{5.17}$$

In this equation, we recognize the symmetrized transition amplitude for all four pions engaged in the process of the two pion production. Each term acquires a factor associated with the isospin algebra. Though all multi-pion states are included in the definitions (5.4) and (5.5) of the observables, it is neither necessary nor even possible to symmetrize them, if only one or two pions are measured. The next orders of the evolution operator expansion will bring in additional final-state particles. They will all be properly symmetrized. However, these terms will be more and more suppressed (discussion in Sec. VI).

In a similar way, the original expression, (A.2), is transformed to the inclusive spectrum of two different pions,

$$\frac{dN^{(2)}_{\pi^+\pi^-}}{d\boldsymbol{k}_1 d\boldsymbol{k}_2} = \frac{dN^{(1)}_{\pi^+}}{d\boldsymbol{k}_1}\frac{dN^{(1)}_{\pi^-}}{d\boldsymbol{k}_2} + \frac{g_{4\pi}^4}{16^2}\sum_{N,M} V_N^{(4)}V_M^{(4)} e^{-\beta_N(k_1u_N)}e^{-\beta_M(k_2u_M)}$$
$$\times \int \frac{d^4q_1 d^4q_2}{4k_1^0k_2^0(2\pi)^{16}}\sqrt{1-\frac{4m^2}{(k_1+q_1)^2}}\sqrt{1-\frac{4m^2}{(k_2+q_2)^2}}e^{-\beta_N(q_1u_N)}e^{-\beta_M(q_2u_M)}\delta_+(q_1^2-m^2)\delta_+(q_2^2-m^2)$$
$$\times \mathrm{Re}[16\ e^{-i(q_1-q_2)(x_N-x_M)} + 26\ e^{-i(q_1+k_1-q_2-k_2)(x_N-x_M)}] \tag{5.18}$$

Next, we have to find the net yield of the interference terms after the momenta of the unobserved pions are integrated out. As a first step, we can use an auxiliary function $I(k,x,\Delta x)$, (5.14) and rewrite Eqs. (5.17) and (5.18), replacing the discrete sums by the integration over the continuous distribution of the collision points,

$$\frac{dN^{(2)}_{\pi^+\pi^+}}{d\boldsymbol{k}_1 d\boldsymbol{k}_2} = \frac{\pi^2 g_{4\pi}^4}{16^2}\int \frac{d^4x_1 d^4x_1}{4k_1^0k_2^0(2\pi)^{16}} e^{-(k_1u(x_1))/T_c}e^{-(k_2u(x_2))/T_c}$$
$$\times \mathrm{Re}\{144 I(k_1,x_1,0)I(k_2,x_2,0)[1+e^{-i(k_1-k_2)(x_1-x_2)}]$$
$$+44 I(k_1,x_1,(x_1-x_2))I^*(k_2,x_2,(x_1-x_2))[1+e^{-i(k_1-k_2)(x_1-x_2)}]\}\ , \tag{5.19}$$

for identical pions, and

$$\frac{dN^{(2)}_{\pi^+\pi^-}}{d\boldsymbol{k}_1 d\boldsymbol{k}_2} = \frac{\pi^2 g_{4\pi}^4}{16^2}\int \frac{d^4x_1 d^4x_1}{4k_1^0k_2^0(2\pi)^{16}} e^{-(k_1u(x_1))/T_c}e^{-(k_2u(x_2))/T_c}$$
$$\times \mathrm{Re}\{144 I(k_1,x_1,0)I(k_2,x_2,0) + I(k_1,x_1,(x_1-x_2))I^*(k_2,x_2,(x_1-x_2))[16+26e^{-i(k_1-k_2)(x_1-x_2)}]\}\ , \tag{5.20}$$

for non-identical pions. These equations are written in the same approximation as the one-pion inclusive spectrum (5.15).

The first two terms of Eq. (5.19) have a well known form of the product of the intensities of two independent sources and an interference function $[1 + cos\Delta k\Delta x]$ and correspond to the standard scheme of HBT interferometry. In these terms, the space-time integration over coordinates x_1 and x_2 is factorized and reproduces the result of the naïve model of freezeout given by Eq.(4.15). The only difference is an inessential kinematic factor and a form-factor

$$I(k_1,x_1,0)I(k_2,x_2,0) = 4\pi^2 m^4 \left[\frac{K_1(m/T)}{m/T}\right]^2 , \tag{5.21}$$

which eventually cancels out in the normalized correlator. In the last two terms of Eq. (5.19), we encounter an additional factor $\mathcal{F}(x_1,x_2) = I(k_1,x_1,\Delta x)I^*(k_2,x_2,\Delta x)$ which, in general, does not allow one to factorize the space-time integrations. In our special geometry of the freezeout, however, some further simplifications are possible, provided the transverse momenta k_{1t} and k_{2t} are large. Indeed, integrating with respect to x_1, we end up with a modified version of the emission function (4.10),

$$J(k_1,k_2;x_2) = \int d^4x_1 e^{-k_1\cdot u(x_1))/T} e^{-i(k_1-k_2)x_1}\mathcal{F}(x_1,x_2)$$

$$= \Delta\tau \int d^2\vec{r}_1 f(\vec{r}_1) e^{i\vec{r}_1(\vec{k}_1-\vec{k}_2)} \int_{-Y}^{Y} \tau d\eta_1 e^{-(m_1/T)\cosh(\eta_1-\theta_1) - i\tau[m_1\cosh(\eta_1-\theta_1) - m_2\cosh(\eta_1-\theta_2)]}\mathcal{F}(\Delta\eta,\Delta\vec{x}) , \tag{5.22}$$

where the form-factor $\mathcal{F}$ can be conveniently rewritten as

$$\mathcal{F}(\Delta\eta,\vec{\rho}) = 4\pi^2 m^4 \left|\frac{K_1(m\sqrt{U^2})}{m\sqrt{U^2}}\right|^2 , \tag{5.23}$$

where

$$U^2 = \frac{1}{T^2} + \vec{\rho}^{\,2} + 4T\tau(T\tau + i)\sinh^2\frac{\eta_1-\eta_2}{2} ,$$

and we denoted $\vec{\rho} = \vec{r}_1 - \vec{r}_2$. The saddle point of the integration over η_1 in Eq. (5.22) is defined by the equation,[6]

$$\tanh(\eta_1 - \theta) = \frac{1 + iT\tau[1 + (m_2/m_1)]}{1 + iT\tau[1 - (m_2/m_1)]} \tanh\alpha , \tag{5.24}$$

and is not affected by the form-factor (5.23). Thus, if we select pions with the same rapidity, then $2\alpha = \theta_1 - \theta_2 = 0$ and the saddle point occurs at the point $\eta_1 = \theta = \theta_1 = \theta_2$. Integrating in the same way with respect to x_2, we come to the conclusion that $\eta_2 = \theta$. The form-factor then becomes independent of rapidities η, and its argument becomes a real function,[7]

$$\mathcal{F}(0,\vec{\rho}) = 4\pi^2 m^4 \frac{K_1^2(m\sqrt{(1/T^2) + \vec{\rho}^{\,2}})}{m^2[(1/T^2) + \vec{\rho}^{\,2}]} . \tag{5.25}$$

From now on, we will deal only with pions of the same rapidity, which means that we physically take aim at two scattering processes which occur in one narrow slice of the longitudinally expanding pipe. In this particular case, the very cumbersome formulae (like (4.15)) become drastically simplified. The spectrum of two identical pions reads as

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[6] The exponential in Eq.(5.22) is the same as in Eq.(4.14). Thus, it is exactly the saddle point (5.24), which leads to Eqs. (4.15) and (4.16).

[7] When $\eta_1 = \eta_2$, the form-factor $\mathcal{F}$ is a smoothly decreasing function. When $\eta_1 \neq \eta_2$, it acquires an additional oscillatory pattern in the $\Delta\eta$-direction.

$$\frac{dN^{(2)}_{\pi^+\pi^+}}{d\boldsymbol{k}_1 d\boldsymbol{k}_2} = \frac{dN^{(1)}_{\pi^+}}{d\boldsymbol{k}_1}\frac{dN^{(1)}_{\pi^+}}{d\boldsymbol{k}_2} + \frac{\pi^2 g^4_{4\pi}}{16^2}\frac{144}{4k^0_1k^0_2(2\pi)^{16}}4\pi^2 m^4(\Delta\tau)^2\tau^2\frac{2\pi T}{\sqrt{m_1m_2}}e^{-(m_1+m_2)/T_c}$$
$$\times\Bigg\{\mathrm{Re}\frac{\sqrt{m_1m_2}}{[m_1m_2+F(F-i)(m_1-m_2)^2]^{1/2}}\Bigg[\Big(2\pi\int rdrf(r)J_0(|\Delta\vec{k}|r)\Big)^2\Big[\frac{K_1(m/T)}{m/T}\Big]^2$$
$$+\frac{44}{144}2\pi\int r_1dr_1r_2dr_2f(r_1)f(r_2)\int_0^{2\pi}d\phi\,\frac{K_1^2(m\sqrt{(1/T^2)+\vec{\rho}^{\,2}})}{m^2[(1/T^2)+\vec{\rho}^{\,2}]}J_0(|\Delta\vec{k}|\rho)\Bigg]$$
$$+\frac{44}{144}2\pi\int r_1dr_1r_2dr_2f(r_1)f(r_2)\int_0^{2\pi}d\phi\,\frac{K_1^2(m\sqrt{(1/T^2)+\vec{\rho}^{\,2}})}{m^2[(1/T^2)+\vec{\rho}^{\,2}]}\Bigg\}\,, \tag{5.26}$$

where $\rho^2 = r_1^2 + r_2^2 - 2r_1r_2\cos\phi$. In the same way, we obtain the two-particle spectrum of non-identical pions,

$$\frac{dN^{(2)}_{\pi^+\pi^-}}{d\boldsymbol{k}_1 d\boldsymbol{k}_2} = \frac{dN^{(1)}_{\pi^+}}{d\boldsymbol{k}_1}\frac{dN^{(1)}_{\pi^-}}{d\boldsymbol{k}_2} + \frac{\pi^2 g^4_{4\pi}}{16^2}\frac{144}{4k^0_1k^0_2(2\pi)^{16}}4\pi^2 m^4(\Delta\tau)^2\tau^2\frac{2\pi T}{\sqrt{m_1m_2}}e^{-(m_1+m_2)/T_c}$$
$$\times 2\pi\int r_1dr_1r_2dr_2f(r_1)f(r_2)\int_0^{2\pi}d\phi\,\frac{K_1^2(m\sqrt{(1/T^2)+\vec{\rho}^{\,2}})}{m^2[(1/T^2)+\vec{\rho}^{\,2}]}$$
$$\times\Bigg\{\frac{16}{144}+\frac{26}{144}\,\mathrm{Re}\Bigg[\frac{\sqrt{m_1m_2}}{[m_1m_2+F(F-i)(m_1-m_2)^2]^{1/2}}\Bigg]\,J_0(|\Delta\vec{k}|\rho)\Bigg\}\,. \tag{5.27}$$

In order to give a quantitative estimate of the effect (yet with all reservations intact due to the imperfection of our model description of the $\pi\pi$-interaction) we shall assume that the distribution of the longitudinally expanding hot pipe in the transverse direction is homogeneous within a cylinder of radius $R_\perp$. We also neglect the transverse flow. Next, we normalize the two-pion spectrum by the product of the one-pion spectra and introduce

$$C_{ab}(\boldsymbol{k}_1,\boldsymbol{k}_2) = \frac{dN^{(2)}_{ab}/d\boldsymbol{k}_1d\boldsymbol{k}_2}{(dN^{(1)}_a/d\boldsymbol{k}_1)(dN^{(1)}_b/d\boldsymbol{k}_2)}\,. \tag{5.28}$$

Finally, we consider only the case of $|\boldsymbol{k}_1| = |\boldsymbol{k}_2|$. Then, $m_1 = m_2$ and we arrive at

$$C_{\pi^+\pi^+} = 1 + \left(\frac{2J_1(|\Delta\vec{k}|R)}{|\Delta\vec{k}|R}\right)^2 + \frac{11}{36}\int_0^R\frac{r_1dr_1r_2dr_2}{(\pi R)^2}\int_0^{2\pi}d\phi\,\frac{2\pi}{K_1^2(m/T)}\,\frac{K_1^2[(m/T)\sqrt{1+T^2\vec{\rho}^{\,2}}]}{1+T^2\vec{\rho}^{\,2}}[1+J_0(|\Delta\vec{k}|\rho)]\,, \tag{5.29}$$

$$C_{\pi^+\pi^-} = 1 + \int_0^R\frac{r_1dr_1r_2dr_2}{(\pi R)^2}\int_0^{2\pi}d\phi\,\frac{2\pi}{K_1^2(m/T)}\,\frac{K_1^2[(m/T)\sqrt{1+T^2\vec{\rho}^{\,2}}]}{1+T^2\vec{\rho}^{\,2}}\left[\frac{1}{9}+\frac{13}{72}J_0(|\Delta\vec{k}|\rho)\right]\,, \tag{5.30}$$

where we remind the reader that the longitudinal rapidities of the pions are set equal. The remaining integrations in Eqs. (5.29) and (5.30) are performed numerically and the results are plotted in Figs. 2 and 3.

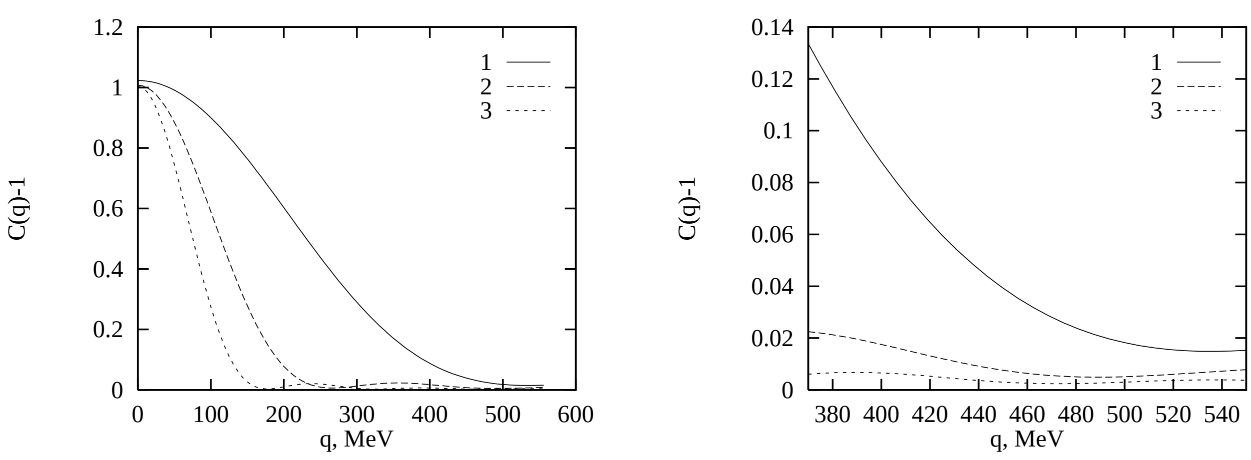

FIG. 2. Normalized correlator of identical pions as a function of $q = (\Delta k)_{\rm side}$ (with $(\Delta k)_{\rm out} = 0$ and $\Delta\theta = 0$): (1) $mR_\perp = 1$; (2) $mR_\perp = 2$; (3)– $mR_\perp = 3$. The right plot is a magnified portion of the left plot which allows one to see the scale of correction due to the hidden interference.

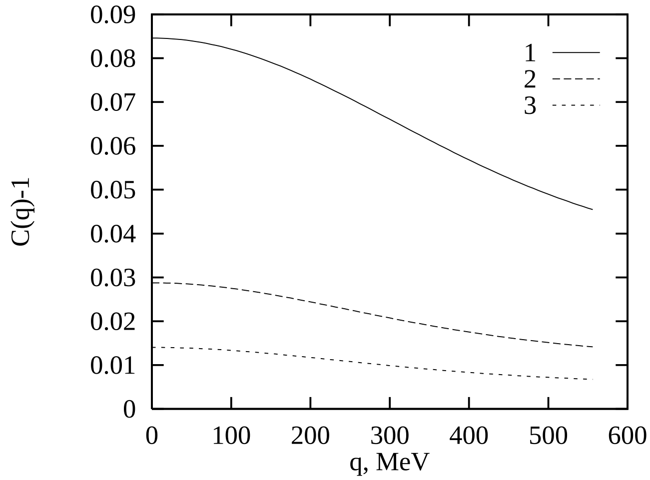

FIG. 3. Normalized correlator of non-identical pions. Notation is the same as in Fig. 2

VI. POSSIBLE CONTRIBUTIONS OF THE MULTIPARTICLE STATES.

As we have seen in the previous section, the multi-pion final states indeed contribute to the one- and two-pion inclusive cross sections. This contribution is due to the real interactions, and therefore, we may speculate about its magnitude in different situations and at different conditions of observation. Within the unrealistic $\boldsymbol{\pi}^4$-model we employed to make analytic solution possible, the effect is understandably small. Indeed, the additional pions emitted in the collision process lead to the form-factor (5.25), $\mathcal{F}(\vec{\rho})$, in the interference term. Let us consider the formal limit of $T \to \infty$. Then the form-factor becomes

$$\mathcal{F}(\vec{\rho}, T \to \infty) = 4\pi^2 m^4 \frac{K_1^2(\, m\sqrt{\vec{\rho}^{\,2}}\,)}{m^2 \vec{\rho}^{\,2}} \ . \tag{6.1}$$

This expression is immediately recognized as the square of the pion vacuum correlator,

$$G_1(\vec{\rho}) = -i\langle 0|\pi(\vec{\rho})\pi^\dagger(0) + \pi^\dagger(0)\pi(\vec{\rho})|0\rangle \ ,$$

which represents the density of states in the pion vacuum. These are exactly the states which are excited as the final states of free propagation in the course of the $\pi\pi$-collision, and the characteristic scale of correlation for these vacuum fluctuations can be nothing but m_π^{-1}. At $T \to \infty$, the colliding pions are (formally) very hard (their thermal wavelength becomes very short) and they cannot explore any scale except this one. Thus, the "hidden interference" is active only if two collisions occur within the range of a typical fluctuation in the "trivial" pion vacuum. Otherwise, these collisions are dynamically disconnected and the interference becomes strongly suppressed. At finite temperatures, the behavior of the form-factor at the origin is less singular, but the main scale for the distance ρ remains m_π^{-1}, which defines the smallness of the effect in the oversimplified model $\boldsymbol{\pi}^4$. The parameter which regulates the magnitude of the excess of the *normalized* correlators (5.29) and (5.30) is the ratio of the effective radius of the dynamical correlations to the full transverse size $R_\perp$ of the expanding pipe. As is seen from the plot of Fig. 3, at $T = m_\pi$ the excess reaches 0.08 when $R_\perp \simeq m_\pi^{-1}$.

This understanding of the nature of the scale of correlation shows that the magnitude of the effect may be quite different when the $\pi\pi$-interaction is treated more adequately. For example, with properly fitted parameters, the σ-model provides a good description of many data which depend on $\pi\pi$-interaction. Most of low-energy interactions of elementary particles are mediated via wide resonances like f_0 (which is sometimes identified with σ), a_1, ρ, etc. In this picture, the $\pi\pi$-interaction is not as local as in the $\boldsymbol{\pi}^4$-model, and more scales show up resulting in the dependence of the $\pi\pi$-cross section on the momentum transfer. The low-energy dynamics is rich and includes a chiral phase transition. In the vicinity of this phase transition the correlation radii should increase. Thus, on the one hand, we may speculate about various dynamical phenomena which may lead to an increase of the correlation radii. On the other hand, the system may develop a strong transverse hydrodynamic motion. In this case, the "measured $R_\perp$" also becomes a dynamically defined effective quantity, which is (though not as much as in the longitudinal direction) smaller than the natural transverse size.

Until now, we have considered the lowest order (with respect to the interaction) contributions to the one- and two-pion inclusive distributions. The next orders can be accounted for by the expansion of the evolution operator in the Eqs. (5.6) and (5.16). Even without explicit calculations, it is clear that there will be two types of corrections. The virtual radiative corrections should be absorbed into the definitions of the dressed propagators and vertices. Within the framework of our phenomenological approach, these are of no interest or value. The real corrections are connected with the production of additional pions and involve integration over the positions of the additional points of emission (collision) and over the momenta of the additional non-observed pions. The corresponding complicated transition amplitudes will be properly symmetrized. However, the result of the integration over the unobserved momenta will again be a set of form-factors which will cut off all distances between the points of scattering which exceed the correlation length. Thus, the higher order effects will always be suppressed with respect to the lowest order.

Our last remark is about the possibility of the induced emission of pions. To address this question, one has to realize that the so-called Bose-enhancement has two different (and mutually exclusive) aspects. If we deal with the measured states, then the symmetrization is solely due to the measurement. This is the essence of the HBT effect. If the states are not measured, then we indeed deal with the induced emission which is due to the factor $\sqrt{n+1}$ in the equation $a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$, which requires that the process of the emission of the $(n+1)$-th quantum physically occurs in the field of all n *identical* quanta emitted before. This is the basic idea behind the laser. If the pions are produced with space-like separation, then stimulated emission is impossible.

VII. BOSE-EINSTEIN CORRELATIONS IN EVENT GENERATORS

Our estimates show that the effect of hidden interference is not expected to be large unless the system undergoes the freezeout starting from a soft mode with long-range correlations. This possibility is very appealing due to its possible physical richness. Careful simulations will be required in order to estimate the signal-to-noise ratio in a real detector. This job is usually done with the aid of the so-called event generators. A complexity stemming from an additional interference makes simulations more challenging. We address this issue below.

Over the last decade, many computer codes which are supposed to mimic the particle yields in heavy-ion collisions have been developed. These codes rely on classical propagation of the particles in hadronic cascades. They do not describe the quantum properties of the measurement which lead to the interference of amplitudes. At best, the output of an event generator is the list of particles with one-to-one correspondence between their momenta and the coordinates of their emission. Such a list is equivalent to a complete optical image of the source. Therefore, there are no alternative histories in the free propagation. An "afterburner" has to be added to incorporate quantum effects.

In order to recover the alternative histories and thus simulate the two-pion correlations in a model with independent pion production, one has to know four one-particle amplitudes, $a_N(\boldsymbol{k})$. They are necessary to recover the two-pion amplitude $\mathcal{A}_{NM}(k_1,k_2)$ (see Eq. (2.1)). Hence, one has to store some information about the pion sources before the emission which allows for such a reconstruction. In the model of binary collisions, the parameters of parents (their ID, momenta, and the coordinates of the reaction volume) have to be stored for each pion from the detected pair. Otherwise, it is impossible to reconstruct all four amplitudes which interfere in the process of the two-pion measurement.

Eventually, one has to average the square of the inclusive amplitude over an ensemble of all events. Then, this procedure has to be repeated for every point in the two-particle momentum space. In more sophisticated models, with the pions coming from decays of resonances, we may have even more interfering amplitudes and each case requires special consideration [17].

The requirement that the quantum mechanics of the last interaction, which forms the two-pion spectrum, must be recovered, follows from first principles and cannot be removed. In some particular cases, one may take a short cut and weaken the requirements of the complete description. However, every precaution should be taken that the initial state of the interfering system is described as a *quantum state*.

Finally, we have to mention that in the literature, another kind of correlation function, which has been introduced [18] (see also Refs. [19,20]) in connection with the problem of simulation of Bose-Einstein correlations in multi-pion events, is widely discussed. The algorithmic definition of this object (which is different from the inclusive differential distribution used in this paper) has several steps: (i) The events are sorted by the total number $\mathcal{N}$ of pions in the event. (ii) The correlation function $C_{\mathcal{N}}(k_1, ..., k_{\mathcal{N}})$ is *measured*, i.e., the $\mathcal{N}$-pion amplitude is fully symmetrized and its squared modulus is averaged over the subset of events with this given $\mathcal{N}$. (iii) All but two arguments of $C_{\mathcal{N}}(k_1, ..., k_{\mathcal{N}})$ are integrated out, which results in a set of two-pion correlators $C_{\mathcal{N}}(k_1, k_2)$, still dependent on $\mathcal{N}$. (iv) This set is averaged over $\mathcal{N}$, in order to obtain $C(k_1, k_2)$. We could not find a quantum observable which would correspond to this correlator. We also could not establish a direct connection between this correlator and the parameters of the emitting system.

VIII. CONCLUSIONS

In this paper, we show that precise measurements of two-particle correlators, both for identical and non-identical particles, may uncover important information about the freezeout dynamics. A reliable theoretical prediction for the magnitude of these correlations can be made only within the framework of an elaborated scenario of the heavy-ion collision which is currently absent. Data may be very helpful for the theoretical design of the fully self-consistent scenario.

It is equally important to measure the magnitude of the effect of hidden interference or to establish its absence at a high level of confidence. From this point of view, our calculations carry an important message that there are no theorems which require the normalized two-pion correlator to approach unity for large differences of momenta. Neither should it take the value of 2 at $\Delta k = 0$. The latter value is the norm (charge) of the state with two identical pions in a *pure* state only. In general, two final-state pions are described by a density matrix. For the same reason, the normalized correlator of non-identical pions should not equal 1. Moreover, the correlator should not equal 1 even for the couples like πp, pn, etc. Therefore, the technique of “mixing of events” should be used with extreme caution, if used at all. The idea of a universal Gaussian fit of the HBT correlators can hardly be fruitful as well. (The authors fully realize that there may be many technical difficulties in the practical implementation of this advice.)

One more practical lesson of our analysis concerns the theory of multi-pion correlations. This issue cannot even be addressed without an explicit account of the dynamical mechanism responsible for the creation of multi-particle states. Our general conclusion is that HBT interferometry cannot be model-independent.

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APPENDIX A.

Here, we reproduce lengthy expressions for the two-particle inclusive spectra as they appear after performing all couplings. We do not use them in this form throughout the paper. However, they are the starting point, would we wish to design a code for the simulation of the two-pion correlations. In this code, the particle densities $n_N(p)$ will serve as the frequencies for the initial-state pions to appear within the range of the last interaction. At least some part of the integrations with respect to the momenta of the unobserved final-state pions is preferable to do analytically in order to simplify the remaining Monte-Carlo integrations.

The full expression for the two-particle spectrum of identical pions as it appears after performing all couplings in Eq.(5.16), is

$$\frac{dN^{(2)}_{\pi^+\pi^+}}{d\boldsymbol{k}_1 d\boldsymbol{k}_2} = \frac{dN^{(1)}_{\pi^+}}{d\boldsymbol{k}_1}\frac{dN^{(1)}_{\pi^+}}{d\boldsymbol{k}_2} + \frac{g^4_{4\pi}}{16}\sum_{N,M}\int \frac{d^4s_1 d^4s_2 d^4p_1 d^4p_2 d^4q_1 d^4q_2}{4k_1^0 k_2^0 (2\pi)^{24}}$$
$$\times\delta_+(q_1^2-m^2)\delta_+(q_2^2-m^2)\delta_+(s_1^2-m^2)\delta_+(s_2^2-m^2)\delta_+(p_1^2-m^2)\delta_+(p_2^2-m^2)$$
$$\times\Big\{[3n_N^{(+)}(s_1)n_N^{(+)}(s_2)+6n_N^{(+)}(s_1)n_N^{(-)}(s_2)+2n_N^{(+)}(s_1)n_N^{(0)}(s_2)+n_N^{(0)}(s_1)n_N^{(0)}(s_2)]$$
$$\times[3n_M^{(+)}(p_1)n_M^{(+)}(p_2)+6n_M^{(+)}(p_1)n_M^{(-)}(p_2)+2n_M^{(+)}(p_1)n_M^{(0)}(p_2)+n_M^{(0)}(p_1)n_M^{(0)}(p_2)]$$
$$\times e^{-i(k_1-k_2)x_{NM}}\int_{\mathcal{V}_N}dy_1 e^{-i(k_1+q_1-s)y_1}\int_{\mathcal{V}_N}dy_2 e^{i(k_2+q_1-s)y_2}\int_{\mathcal{V}_M}dy_3 e^{-i(k_2+q_2-p)y_3}\int_{\mathcal{V}_M}dy_4 e^{i(k_1+q_2-p)y_4}$$
$$+[26n_N^{(+)}(s_1)n_N^{(-)}(s_2)n_M^{(+)}(p_1)n_M^{(-)}(p_2)+5n_N^{(+)}(s_1)n_N^{(+)}(s_2)n_M^{(+)}(p_1)n_M^{(+)}(p_2)$$
$$+5n_N^{(+)}(s_1)n_N^{(-)}(s_2)n_M^{(0)}(p_1)n_M^{(0)}(p_2)+5n_N^{(0)}(s_1)n_N^{(0)}(s_2)n_M^{(+)}(p_1)n_M^{(-)}(p_2)$$
$$+2n_N^{(0)}(s_1)n_N^{(+)}(s_2)n_M^{(0)}(p_1)n_M^{(+)}(p_2)+n_N^{(0)}(s_1)n_N^{(0)}(s_2)n_M^{(0)}(p_1)n_M^{(0)}(p_2)]$$
$$\times\Big[e^{-i(q_1-q_2)x_{NM}}\int_{\mathcal{V}_N}dy_1 e^{-i(k_1+q_1-s)y_1}\int_{\mathcal{V}_N}dy_2 e^{i(k_1+q_2-s)y_2}\int_{\mathcal{V}_M}dy_3 e^{-i(k_2+q_2-p)y_3}\int_{\mathcal{V}_M}dy_4 e^{i(k_2+q_1-p)y_4}$$
$$+e^{-i(q_1+k_1-q_2-k_2)x_{NM}}\int_{\mathcal{V}_N}dy_1 e^{-i(k_1+q_1-s)y_1}\int_{\mathcal{V}_N}dy_2 e^{i(k_2+q_2-s)y_2}\int_{\mathcal{V}_M}dy_3 e^{-i(k_2+q_2-p)y_3}\int_{\mathcal{V}_M}dy_4 e^{i(k_1+q_1-p)y_4}\Big]\Big\}\ , \quad \text{(A.1)}$$

where $s = s_1 + s_2$, $p = p_1 + p_2$; x_N and x_M are the "central" coordinates of two cells, and $x_{NM} = x_N - x_M$. A quite long expression emerges for the correlator of two different pions as well.

$$\frac{dN^{(2)}_{\pi^+\pi^-}}{d\boldsymbol{k}_1 d\boldsymbol{k}_2} = \frac{dN^{(1)}_{\pi^+}}{d\boldsymbol{k}_1}\frac{dN^{(1)}_{\pi^-}}{d\boldsymbol{k}_2} + \frac{g^4_{4\pi}}{6}\sum_{N,M}\int \frac{d^4s_1 d^4s_2 d^4p_1 d^4p_2 d^4q_1 d^4q_2}{4k_1^0 k_2^0 (2\pi)^{24}}$$
$$\times\delta_+(q_1^2-m^2)\delta_+(q_2^2-m^2)\delta_+(s_1^2-m^2)\delta_+(s_2^2-m^2)\delta_+(p_1^2-m^2)\delta_+(p_2^2-m^2)$$
$$\times\Big[e^{-i(q_1-q_2)x_{NM}}[7n_N^{(+)}(s_1)n_N^{(+)}(s_2)n_M^{(+)}(p_1)n_M^{(+)}(p_2)$$
$$+6n_N^{(+)}(s_1)n_N^{(-)}(s_2)n_M^{(-)}(p_1)n_M^{(-)}(p_2)+n_N^{(+)}(s_1)n_N^{(-)}(s_2)n_M^{(0)}(p_1)n_M^{(0)}(p_2)+n_N^{(0)}(s_1)n_N^{(0)}(s_2)n_M^{(0)}(p_1)n_M^{(0)}(p_2)]$$
$$\times\int_{\mathcal{V}_N}dy_1 e^{-i(k_1+q_1-s)y_1}\int_{\mathcal{V}_N}dy_2 e^{i(k_1+q_2-s)y_2}\int_{\mathcal{V}_M}dy_3 e^{-i(k_2+q_2-p)y_3}\int_{\mathcal{V}_M}dy_4 e^{i(k_2+q_1-p)y_4}$$

$$+e^{-i(q_1+k_1-q_2-k_2)x_{NM}}[17n_N^{(+)}(s_1)n_N^{(-)}(s_2)n_M^{(+)}(p_1)n_M^{(-)}(p_2)$$
$$+4n_N^{(+)}(s_1)n_N^{(-)}(s_2)n_M^{(0)}(p_1)n_M^{(0)}(p_2)+4n_N^{(0)}(s_1)n_N^{(0)}(s_2)n_M^{(+)}(p_1)n_M^{(-)}(p_2)+n_N^{(0)}(s_1)n_N^{(0)}(s_2)n_M^{(0)}(p_1)n_M^{(0)}(p_2)]$$
$$\times\int_{\mathcal{V}_N}dy_1e^{-i(k_1+q_1-s)y_1}\int_{\mathcal{V}_N}dy_2e^{i(k_2+q_2-s)y_2}\int_{\mathcal{V}_M}dy_3e^{-i(k_2+q_2-p)y_3}\int_{\mathcal{V}_M}dy_4e^{i(k_1+q_1-p)y_4}\Bigg] \ . \qquad (A.2)$$

In Sec. V, we proceed with a simplified version of these equations which emerges when the distributions $n_N^{(j)}(p)$ are taken in Boltzmann form with an additional assumption that the system is locally neutral. This results in a common weight function and coefficients which are just the numbers of terms in different groups in Eqs. (A.1) and (A.2).

Integration over the elementary volumes in Eqs. (A.1) and (A.2) requires special discussion: There is an important difference between the volumes V_N in equations of Sec. IV and the volumes $\mathcal{V}_N$ here. The former are related to the elementary fluid cells where the distributions $n_N(p)$ of particles over their momenta are established due to many collisions inside the N'th cell , and we do not (and even are prohibited to) localize the coordinates of individual collisions within the cell explicitly. The latter correspond to the opposite case, when we locate the space-time coordinates of individual collisions. Therefore, the volumes $\mathcal{V}_N$ correspond to the actual range of the pion interaction potential and are much smaller than the volumes V_N of the fluid cells required by the hydrodynamic picture. Hence, the meaning of the distribution functions $n_N(p)$ changes as well; now they correspond to the probability for the pion with momentum $\boldsymbol{p}$ to penetrate the "reaction domain" near the space-time point x_N. In fact, the volume $\mathcal{V}_N$ is defined by the cross section of the $\pi\pi$-interaction itself. This picture is consistent only if the pions themselves are considered as wave packets and not as plane waves. The volume $\mathcal{V}_N$ is that volume where the incoming packets are identified by the interaction and where the outgoing packets are completely formed. With this picture in mind, we can integrate, e.g.,

$$\int_{\mathcal{V}_N}dy_1e^{-i(k_1+q_1-s)y_1}\int_{\mathcal{V}_N}dy_2e^{i(k_1+q_2-s)y_2}=(2\pi)^4\delta(k_1+q_1-s)\int_{\mathcal{V}_N}dy_2e^{-i(q_1-q_2)y_2}=(2\pi)^4\delta(k_1+q_1-s)\mathcal{V}_N \ . \qquad (A.3)$$

In the first equation, we assume that the relation between the volume of integration and momenta allows one to verify the conservation of momentum in the collision. In the second equation, we took into account that the actual range for the momenta q_1 and q_2 is of the order of $2T\sim 2m_\pi$, while the length of the π-π scattering is $\sim 1/5m_\pi$. In all subsequent calculations we can rescale $\mathcal{V}_N$ back to the elementary volume V_N and thus restore the status of $n_N(p)$ as the distribution function.

APPENDIX B. MISCELLANEOUS FORMULAE.

1. If we adopt the Boltzmann shape of the distribution function, then the product of the statistical weights $n(p_1)n(p_2)$ becomes a function of $p=p_1+p_2$ and it is possible to integrate $l=p_1-p_2$ out. We have an integral,

$$\int d^4p_1d^4p_2f(p)\delta_+(p_1^2-m^2)\delta_+(p_2^2-m^2)=\frac{1}{8}\int d^4pf(p)\int d^4l\delta(p^2+l^2-4m^2)\delta(pl)$$
$$=\int d^4pf(p)\frac{\pi}{2p^0}\int_0^\infty\delta(p^2-4m^2+\boldsymbol{l}^2)\ \boldsymbol{l}^2\ d|\boldsymbol{l}|=\frac{\pi}{4}\int d^4pf(p)\sqrt{1-\frac{4m^2}{p^2}}\ , \qquad (B.1)$$

where in the second equation we integrate using a special reference frame where $\boldsymbol{p}=0$, and restore the invariant form in the final answer.

2. The correlation functions of Sec. V are expressed via the emission function $I(k,N,x_{NM})$ which is computed below:

$$I(k,N,x_{NM}) = \int d^4q \delta_+(q^2-m^2) e^{-\beta(qu)}\ e^{iqx}\sqrt{1-\frac{4m^2}{(k+q)^2}}\ , \tag{B.2}$$

where $\beta = \beta_N$, $u = u_N$, and $x = x_{NM}$. The integral is convenient to compute in a special reference frame $\overset{\circ}{\mathcal{R}}$ where the time-like vector k has no spatial components, $\overset{\circ}{\boldsymbol{k}} = 0$, $\overset{\circ}{k}_0 = m$. Using the delta-function to integrate out q^0, we arrive at

$$I(k,N,x_{NM}) = \int_0^{2\pi} d\phi \int_0^{\pi} \sin\alpha d\alpha \int_0^{\infty} \frac{qdq}{2\sqrt{q^2+m^2}}$$
$$\times \exp\{-\sqrt{q^2+m^2}(\beta - i\overset{\circ}{x}_0) + q[\beta|\overset{\circ}{\boldsymbol{u}}|\cos\alpha - i|\overset{\circ}{\boldsymbol{x}}|(\cos\alpha\cos\psi - \sin\alpha\sin\psi\cos\phi)]\}\ . \tag{B.3}$$

The first step is to integrate over ϕ which results in the Bessel function, $2\pi J_0(|\overset{\circ}{\boldsymbol{x}}|q\sin\alpha\sin\psi)$. The next integration follows a known formula,

$$\int_0^{\pi} d\alpha\ \sin\alpha\ e^{ib\cos\alpha} J_0(a\sin\alpha) = 2\frac{\sin\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}}\ , \tag{B.4}$$

which holds for arbitrary complex a and b. In our case, $a^2+b^2 = -(\beta\overset{\circ}{\boldsymbol{u}} - i\overset{\circ}{\boldsymbol{x}})^2 = -\overset{\circ}{\boldsymbol{U}}{}^2$ where we have introduced the complex four-vector $U^\mu = \beta u^\mu - ix^\mu$. Finally, changing the variable of integration, $q = m\sinh v$, we arrive at

$$I(k,N,x_{NM}) = \frac{2\pi m}{|\overset{\circ}{\boldsymbol{U}}|}\int_0^{\infty} (\cosh v - 1)dv\ e^{-m\overset{\circ}{U}_0\cosh v}\sinh(m|\overset{\circ}{\boldsymbol{U}}|\sinh v)$$
$$= \frac{2\pi m}{\sqrt{U^2}\sinh\chi}\left[\int_0^{\infty} dv\ (\cosh\chi\cosh v - 1)e^{-m\sqrt{U^2}\cosh v} - \int_0^{\infty} dv\ (\cosh v - 1)e^{-m\sqrt{U^2}\cosh(v+\chi)}\right]\ . \tag{B.5}$$

The Lorentz-invariant expressions for the quantities defined in the reference frame $\overset{\circ}{\mathcal{R}}$ are

$$m\overset{\circ}{U}_0 = (kU) = m\sqrt{U^2}\cosh\chi,\quad m|\overset{\circ}{\boldsymbol{U}}| = \sqrt{(kU)^2 - m^2U^2} = m\sqrt{U^2}\sinh\chi\ . \tag{B.6}$$

The first of the integrals in Eq. (B.5) is calculated exactly. The second one can be estimated by means of integration by parts. Since the integrand has the root of the second order at $v = 0$, we have to integrate by parts three times before the first non-vanishing term shows up. The net yield of this procedure is

$$-\frac{4\pi m}{|\overset{\circ}{\boldsymbol{U}}|}\int_0^{\infty} dv\ \sinh^2\frac{v}{2}e^{-m\sqrt{U^2}\cosh(v+\chi)} = -\frac{4\pi m^2}{m^4|\overset{\circ}{\boldsymbol{U}}|^4}e^{-m\overset{\circ}{U}_0} + \cdots\ . \tag{B.7}$$

Since $m\overset{\circ}{U}_0 = (kU) \equiv (ku)/T - i(kx_{NM})$, this term is strongly suppressed (both by the exponent and by at least the fourth power of the small number m/k_t). Finally, we have to high accuracy,

$$I(k,N,x_{NM}) = \frac{2\pi m}{\sqrt{U^2}}\left[\coth\chi K_1(m\sqrt{U^2}) - \frac{1}{\sinh\chi}K_0(m\sqrt{U^2})\right]\ . \tag{B.8}$$

Here, $K_n(z)$ is the standard notation for the modified Bessel function, and we remind the reader that U^2 and χ (as well as $\overset{\circ}{U}_0$ and $|\overset{\circ}{\boldsymbol{U}}|$) all are complex quantities. When $x \equiv x_{NM} = 0$, we have $U^2 = \beta^2$, $\cosh\chi = (ku)/m = (m_t/m)\cosh(\theta-\eta) \gg 1$, and consequently, $\sinh\chi \gg 1$, $\coth\chi \sim 1$. Therefore,

$$I(k,N,0) = 2\pi m^2\frac{K_1(\beta m)}{\beta m}\ . \tag{B.9}$$

In the particular cases considered in Sec. V, the second term in Eq. (B.8) is always smaller than the first one by the factor m/k_t. An important property of the final answer (B.8) is that the quantity

$$U^2 = \beta^2(x_1) - (x_1 - x_2)^2 - 2i\beta(x_1)(u(x_1) \cdot (x_1 - x_2))$$

in the argument of function K_1, does not depend on the large parameter k_t/m. Therefore, the saddle point of the integration over the coordinate rapidities η is never affected by this function.

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[1] A. Makhlin, and Yu. Sinyukov, Sov. J. Nucl. Phys. **46**, 354 (1987).

[2] V. Averchenkov, A. Makhlin, and Yu. Sinyukov, Sov. J. Nucl. Phys. **46** (1987) 905; A. Makhlin, and Yu. Sinyukov, Z. Phys., **C39**, 69 (1988)

[3] The NA35 collaboration: M. Gazdzicki *et al.*, in Proceedings of Quark Matter '95, Nucl. Phys. **A590**, 197c(1995).

[4] The NA44 collaboration: B.V. Jacak *et al.*, in Proceedings of Quark Matter '95, Nucl. Phys. **A590**, 215c(1995).

[5] M. Gyulassy, S.K. Kauffmann, and L.W. Wilson, Phys.Rev.**C20**, 2267 (1979)

[6] Yu.L. Klimontovich, Kinetic theory of non-ideal gases and non-ideal plasmas, Oxford, New York : Pergamon Press, 1982.

[7] C.M. Hung and E.V. Shuryak Phys.Rev.**C57**, 1891 (1998)

[8] A. Makhlin, Sov. J. Nucl. Phys. **49**, 238 (1989).

[9] A. Makhlin, E. Surdutovich, Phys. Rev. **C58**, 389 (1998).

[10] L.D. Landau, Izv.Akad.Nauk Ser.Fiz. **17**, 51 (1953):

[11] J.D. Bjorken, Phys. Rev. **D27**, 140 (1998)

[12] M.I. Podgoretskii, Sov. J. Part. Nucl. **20**, 266 (1989).

[13] K. Geiger, Phys. Rev. **D 46**, 4965 and 4986 (1992).

[14] A. Makhlin, E. Surdutovich, G. Welke, in Proceedings of HIPAGS '96, p.173, Detroit, 1996.

[15] N.N. Bogolyubov, D.V. Shirkov, Introduction to the theory of quantized fields, Interscience, NY,1959

[16] L.V. Keldysh, Sov. Phys. JETP **20**, 1018 (1964); E.M. Lifshits, L.P. Pitaevsky, Physical kinetics, Pergamon Press, Oxford, 1981.

[17] A. Makhlin, E. Surdutovich, in preparation

[18] W.A. Zajc, Phys. Rev. **D35**, 3396 (1987).

[19] S. Pratt, Phys. Lett. **B301**, 159 (1993).

[20] U.A. Wiedemann, Phys. Rev. **C57**, 3324 (1998); R.L. Ray, Phys. Rev. **C57**, 2523 (1998)