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ANALYSIS OF TIMOSHENKO BEAM RESTING ON NONLINEAR COMPRESSIONAL AND FRICTIONAL WINKLER FOUNDATION

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ABSTRACT

This paper deals with linear elastic behavior of deep beams resting on linear and nonlinear Winkler type elastic foundations with both compress ional and tangential resistances. The basic or governing equations of beams on nonlinear elastic Winkler foundation are solved by finite difference method. The finite element method in Cartesian coordinates is formulated using two dimensional plane stress isoparametric finite elements to model the deep beam and elastic springs to model the foundation. Two computer programs coded in fortran_77 for the analysis of beams on nonlinear elastic foundations are developed. Comparisons between the two methods and other studies are performed to check the accuracy of the solutions. Good agreement was found between the solutions with percentage difference of 3%. Several important parameters are incorporated in the analysis, namely, the vertical subgrade reaction, horizontal subgrade reaction and beam depth to trace their effects on deflections, bending moments and shear forces.

Keywords: beams, Winkler foundation, finite difference, finite elements, friction, nonlinear.

INTRODUCTION

In the analysis of elastically supported beams, the elastic support is provided by a load-bearing medium, referred to as the 'foundation' along the length of the beam. Such conditions of support can be found in a large variety of geotechnical problems. There are two basic types characterized by the fact that the pressure in the foundation is proportional at every point to the deflection occurring at that point and is independent of pressure or deflection produced at other point (Figure-1). The second type is furnished by elastic solid, which in contrast to the first one represents the case of complete continuity in the supporting medium.

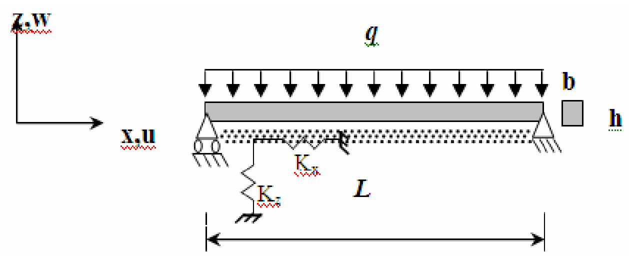

Figure-1. Simply supported beam under external load and foundation resistances.

Al-Hachmi (1997) presented a theoretical analysis for predicting the large displacement elastic stability analysis of plane and space structures subjected to general static loading. The beam-column theory was used in this analysis, taking into accounts both bowing and axial force effects. The general equations of fixed end moments of a beam subjected to lateral loads were also derived. The work employed this analysis to study the behavior of beams with elastic foundations, piles driven into soil and large displacements of submarine pipelines.

Onu (2000) derived a formulation leading to an explicit free-of meshing stiffness matrix for a beam finite element foundation model. The shear deformation contribution was considered and the formulation was based on exact solution of the governing differential equation.

Aristizabal-Ochoa (2001) developed, in a simplified manner**,** a nonlinear large deflection-small strain analysis of a slender beam-column of symmetrical cross section with semi rigid connections under end loads (conservative and no conservative), including the effects of axial load eccentricities and out-of-plumpness.

Guo and Weitsman (2002) made an analytical method, accompanied by a numerical scheme, to evaluate the response of beams on no uniform elastic formulation, where the foundation modulus is $K_z = K_z\,(x)$. The method employed Green's foundation formulation, which results in a system of nonsingular integral equations for the distributed reaction p(x).

Lazem (2003) presented a theoretical analysis for large displacement elastic stability of in-plane structures where some members were embedded into or resting on elastic foundations. The analysis was based on Eulerian formulation, which was developed initially for elastic structures and was extended to include soil-structure interaction.

Al-Azzawi and Al-Ani (2004) studied the linear elastic behavior of thin or shallow beams on Winkler foundations with both normal and tangentional frictional resistances. The finite difference method was used to solve the governing differential equations and good results were obtained with the exact solutions for different load cases and boundary conditions.

Al-Musawi (2005) studied the linear elastic behavior of deep beams resting on elastic foundations. The finite element method in Cartesian coordinates is formulated using different types of one, two and three dimensional isoparametric elements to compare and check the accuracy of the solutions.

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Al-Azzawi (2010) used the finite difference method for solving the basic differential equation for the elastic deformation of a thin beam supported on a nonlinear elastic foundation. A tangent approach was used to determine the modulus of subgrade reaction after constructing a second degree equation for load-deflection diagram. Results of plate loading test of soil obtained in Iraq were used in the analysis. An iterative approach is used for solving the nonlinear problem until the convergence of the solution.

Al-Azzawi and Theeban (2010) studied the geometric nonlinear behavior of beams resting on Winkler foundation. Timoshenko's deep beam theory is extended to include the effect of large deflection theory using finite differences. In the finite element method (ANSYS program), the element SHELL 43was used to model the beam.

Al-Azzawi, Mahdy and Farhan (2010) studied the nonlinear material and geometric behaviors of reinforced concrete deep beams resting on linear and nonlinear Winkler foundations. The finite elements through ANSYS (Release-11, 2007) computer software were used. The reinforced concrete deep beam is molded using (SOLID 65) 8 node brick element and the soil is molded using linear spring (COMBIN 14) element or using nonlinear Winkler spring (COMBIN39) element.

ELASTIC FOUNDATION

Winkler model for both compressional and frictional resistances are used to model the elastic foundations. This model assumes that the base is consisting of closely spaced independent linear springs, consequently as shown in Figure-2.

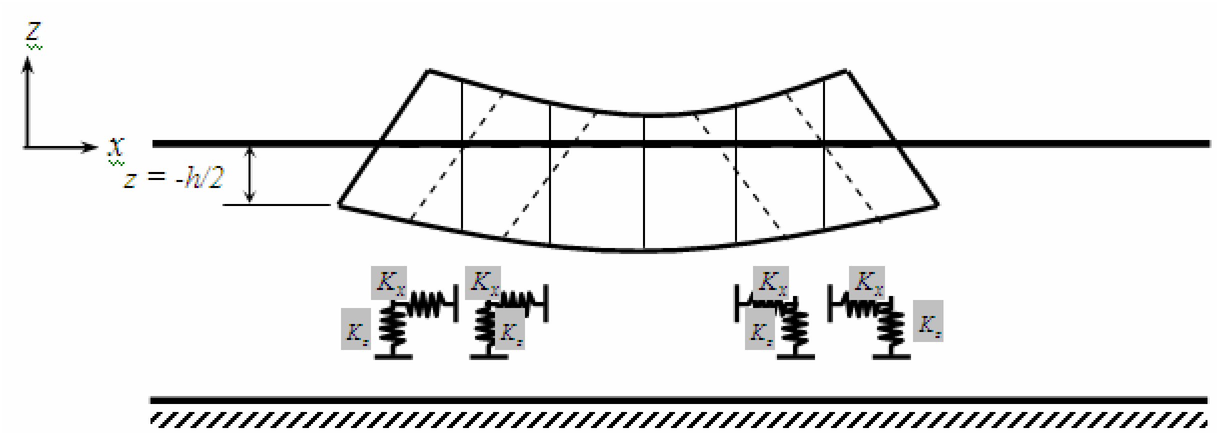

Figure-2. Winkler compress ional and frictional model**.**

Modulus of subgrade reaction is a conceptual relationship between soil pressure and deflection. It can be measured by using plate-loading test. Using this test, a load-deflection curve is adopted. The modulus of subgrade reaction K_z can be calculated using:

$$K_z = \frac{p}{w} \qquad (1)$$

where:

K_z is the modulus of subgrade reaction,
p is the applied pressure and
w is the deflection.

The value of K_z is obtained from the concept of tangent approach as shown in Figure-3.

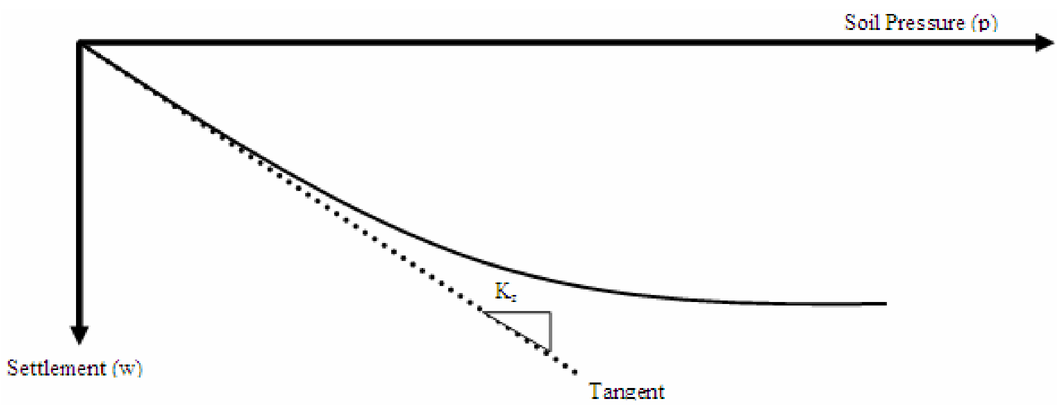

Figure-3. Typical soil pressure-settlement curve.

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There are a wide range of K_z values for different types of soil. In the present study, a quasi linearization method or iteration procedure to get the value of K_z is used. This leariazation by iteration method was developed using the tangent method as a basic approach.

In this study, the linear and nonlinear behaviors are adopted. The nonlinear behavior is modeled using iterative values of K_z. A typical p-w diagram was taken from a plate loading test which was carried out on a soil in Baghdad. The result of this test is shown in Figure-4. The Consultant Engineering Bureau in the University of Baghdad had carried out this test in Al-Muthana airport region for the Big Baghdad Mosque project.

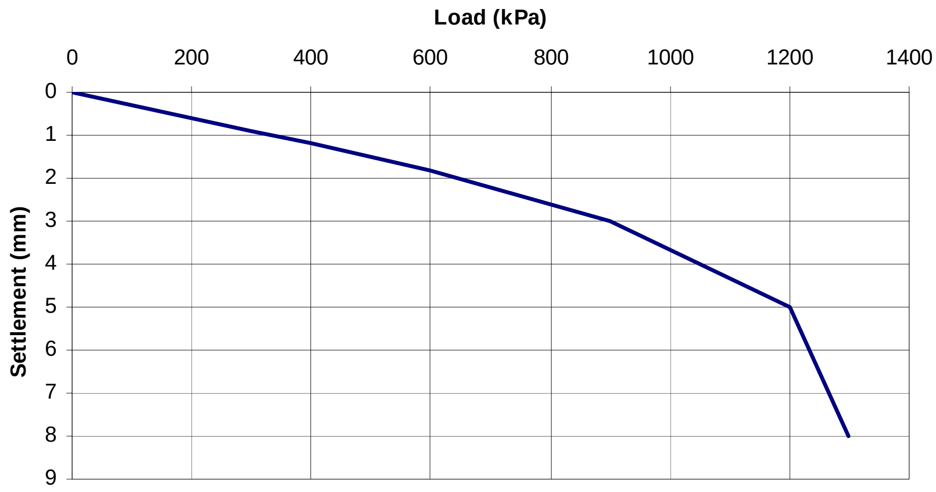

Figure-4. Plate loading test data [consultant engineering bureau / university of Baghdad].

The data shown in the load-deflection curve is used to obtain the following second degree polynomial equation:

$$K_z(w) = 280000 + 1.962 * 10^7 w - 8.021 * 10^9 w^2 \quad (2)$$

which gives, the initial modulus of subgrade reaction= 280000 kN/m^3 and the final modulus of subgrade reaction = 171429 kN/m^3 for $w \geq 0.0051$ m. In the present study, the horizontal subgrade reaction is assumed to have the same values and behavior of vertical subgrade reaction.

ASSUMPTIONS AND GOVERNING EQUATIONS FOR DDEEP BEAMS

The main assumptions are:

a) Plane cross sections before bending remain plane after bending.
b) The cross section will have additional rotation due to transverse shear. Warping of the cross section by transverse shear will be taken into consideration by introducing a shear correction factor (c^2).

The governing equations of deep beams on elastic foundations characterized by Winkler model for compressional and frictional resistances could be obtained [deep beam with uniform subgrade by Al-Jubori (1992)]:

$$Gc^2 A\left(\frac{d\psi}{dx} + \frac{d^2 w}{dx^2}\right) + K_z(w)w = -q \quad (3)$$

$$EI\frac{d^2\psi}{dx^2} - Gc^2 A\left[\psi + \frac{dw}{dx}\right] = K_x(w) \times \frac{h}{2} \times \psi \quad (4)$$

where G is the shear modulus , c^2 is the shear correction factor (c^2 =5/6 for rectangular cross sections and c^2=1 for I-sections), A is the cross-sectional area of the beam, ψ is the rotation of the transverse sections in *xz*-plane of the beam, w is the transverse deflection, E is the modulus of elasticity of the beam material, q is the transverse load per unit length, K_z and K_x are the linear or nonlinear moduli of subgrade reaction in z and x directions and I is the moment of inertia of the beam section. In case of the depth of the beam decreases (thin beam) the shear modulus becomes infinite and equation 3 vanishes as $\psi = -\frac{dw}{dx}$ and equation 1 reduces to the case of small deflection of thin beams.

FINITE DIFFERENCE METHOD

The finite difference method is one of the most general numerical techniques. In applying this method, the derivatives in the governing differential equations under consideration are replaced by differences at selected points. These points or nodes are making the finite difference mesh. In the analysis of deep beams by this method, the differential equations at each point (or node) are replaced by difference equations. By assembling the difference equations for all nodes, a number of simultaneous algebraic equations are obtained and solved by Gauss-Jordan method.

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The beam is divided into intervals of (Δx) in the (x) direction as shown in Figure-5, assuming (n) to represent the number of nodes and (i) the node number under consideration. In the finite difference method, the curve profile of the beam deflection is approximated by a straight line between nodes for the finite difference expressions of the first derivatives and by a parabola for the second derivatives.

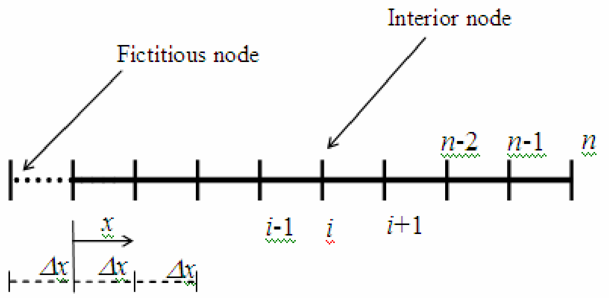

Figure-5. Finite difference mesh for the deep beam.

The governing equations are rewritten in finite differences and are produced for an interior node (i):

$$Gc^2A\left[\frac{\psi_{i+1}-\psi_{i-1}}{2\Delta x}+\left(\frac{w_{i+1}-2w_i+w_{i-1}}{(\Delta x)^2}\right)\right]+q_i-K_z(w_i).w_i=0 \tag{5}$$

$$EI\left[\frac{\psi_{i+1}-2\psi_i+\psi_{i-1}}{\Delta x^2}\right]-Gc^2A\left[\psi_i+\left(\frac{w_{i+1}-w_{i-1}}{2\Delta x}\right)\right]=[K_x=K_z(w_i)]\times\frac{h}{2}\times\psi_i \tag{6}$$

The solution of the governing differential equations of deep beams must simultaneously satisfy the differential equations and the boundary conditions for any given beam problem.

Boundary conditions are represented in finite difference form by replacing the derivatives in the mathematical expressions of various boundary conditions by their finite difference approximations. When central differences are used at the boundary nodes, fictitious points outside the beam are required. These may be defined in terms of the inside points when the behavior of the beam functions are known at the boundary nodes.

FINITE ELEMENT ANALYSIS

The finite element method is an approximate method for the analysis of framed and continuum structures. The basic philosophy of this method is that the structures or the continuum is divided into small elements of various shapes and types, which are assembled together to form an approximate mathematical model. In this paper, the finite element method in Cartesian coordinate is used to solve the problems of deep beams resting on Winkler type elastic foundations with both normal and frictional restraints. The two dimensional isoparametric plane stress elements are used, each node have two degrees of freedom (the deflection w and displacement u) as shown in Figure-6.

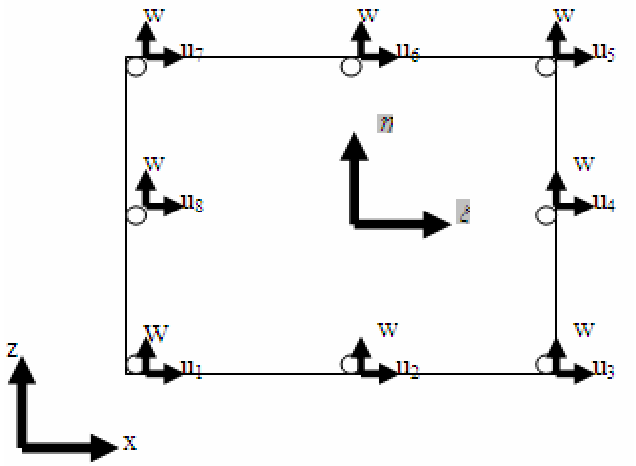

Figure-6. Plane stress element.

The two dimensional element in local coordinates ξ and η has eight nodes as shown in Figure-6 (Hinton and Owen 1977).

Each node i in a plane stress element has two degrees of freedom. They are u_i and w_i. Thus, the element degrees of freedom may be listed in the vector (or row matrix):

$$\{\delta_e\}=[w_1,u_1,.........,w_8,u_8]^T$$

Shape functions

For the four-node isoparametric quadrilateral element, the shape functions are:

$N_1=(1-\xi)(1-\eta)/4,\quad N_2=(1+\xi)(1-\eta)/4$

$N_3=(1+\xi)(1+\eta)/4$ and $N_4=(1-\xi)(1+\eta)/4$ (7)

For the eight-node isoparametric quadrilateral element, the shape functions are:

$N_1=(1-\xi)(1-\eta)(1+\xi+\eta)/4$

$N_2=(1-\xi^2)(1-\eta)/2$

$N_3=(1+\xi)(1-\eta)(\xi-\eta-1)/4$

$N_4=(1+\xi)(1-\eta^2)/2$

$N_5=(1+\xi)(1+\eta)(\xi-\eta-1)/4$

$N_6=(1-\xi^2)(1+\eta)/2$

$N_7=(1+\xi)(1+\eta)(-\xi+\eta-1)/4$

$N_8=(1-\xi)(1-\eta^2)/2$ (8)

Thus, the degrees of freedom w and u can be defined in terms of the shape functions:

$$w(\xi,\eta)=\sum_{i=1}^{n}N_i\cdot w_i \tag{9a}$$

$$u(\xi,\eta)=\sum_{i=1}^{n}N_i\cdot u_i \tag{9b}$$

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The x and y coordinates can be defined in the same manner:

$$x(\xi,\eta)=\sum_{i=1}^{n} N_i \cdot x_i \qquad (10)$$

$$y(\xi,\eta)=\sum_{i=1}^{n} N_i \cdot y_i \qquad (11)$$

Thus, the geometry and the assumed displacement field are described in a similar fashion using the shape functions and the model values (thus, the name of isoparametric element is given).

Jacobian matrix-[J]

The Jacobian matrix [J] is obtained from the following expression:

$$[J]=\begin{bmatrix}\frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi}\\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta}\end{bmatrix}=\sum_{i=1}^{n}\begin{bmatrix}\frac{\partial N_i}{\partial \xi}x_i & \frac{\partial N_i}{\partial \xi}y_i\\ \frac{\partial N_i}{\partial \eta}x_i & \frac{\partial N_i}{\partial \eta}y_i\end{bmatrix} \qquad (12)$$

The inverse of the Jacobian matrix $[J]^{-1}$ can be readily obtained using standard matrix inversion technique:

$$[J]^{-1}=\begin{bmatrix}\frac{\partial \xi}{\partial x} & \frac{\partial \eta}{\partial x}\\ \frac{\partial \xi}{\partial y} & \frac{\partial \eta}{\partial y}\end{bmatrix}=\frac{1}{\det J}\begin{bmatrix}\frac{\partial y}{\partial \eta} & \frac{-\partial y}{\partial \xi}\\ -\frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \xi}\end{bmatrix} \qquad (13)$$

The shape function derivatives are calculated from the expressions as:

$$\frac{\partial N_i}{\partial x}=\frac{\partial N_i}{\partial \xi}\frac{\partial \xi}{\partial x}+\frac{\partial N_i}{\partial \eta}\frac{\partial \eta}{\partial x}$$

$$\frac{\partial N_i}{\partial y}=\frac{\partial N_i}{\partial \xi}\frac{\partial \xi}{\partial y}+\frac{\partial N_i}{\partial \eta}\frac{\partial \eta}{\partial y} \qquad (14)$$

where $\frac{\partial \xi}{\partial x},\frac{\partial \eta}{\partial x},\frac{\partial \xi}{\partial y}$ and $\frac{\partial \eta}{\partial y}$ and obtained from $[J]^{-1}$.

Strain matrix-[B]

The strains are defined in terms of the nodal displacements and shape function derivatives by the expression:

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_z\\ \gamma_{xz}\end{Bmatrix}=\begin{bmatrix}0 & \frac{\partial N_i}{\partial x}\\ \frac{\partial N_i}{\partial z} & 0\\ \frac{\partial N_i}{\partial x} & \frac{\partial N_i}{\partial z}\end{bmatrix}\begin{Bmatrix}w_i\\ u_i\end{Bmatrix} \qquad (15)$$

The strain matrix $[B_i]$ contains shape function derivatives which may be calculated from the expression (14) and the coordinates x and y which may be calculated at the Gauss point coordinates from the expressions (10) and (11).

Matrix of elastic constants-[D]

The generalized stress–strain relationship for a beam of isotropic elastic material may be written as:

$$\begin{Bmatrix}\sigma_x\\ \sigma_z\\ \tau_{xz}\end{Bmatrix}=\begin{bmatrix}\frac{E}{(1-\nu^2)} & \frac{\nu E}{(1-\nu^2)} & 0\\ \frac{\nu E}{(1-\nu^2)} & \frac{E}{(1-\nu^2)} & 0\\ 0 & 0 & \frac{E}{2(1+\nu)}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_z\\ \gamma_{xz}\end{Bmatrix} \qquad (16)$$

or

$$\{\sigma_e\}=[D]\{\varepsilon_e\} \qquad (17)$$

where $\{\sigma_e\}$ is the element stresses and [D] is the matrix of elastic constants for the isotropic elastic material.

Stress matrix-[BD]

Similarly, the stress at any point within the element for a beam of isotropic material can be expressed as:

$$\{\sigma_e\}=[D][B]\{\delta_e\}=[S]\{\delta_e\} \qquad (18)$$

Stiffness matrix for the plane stress element-[Kp]

The stiffness matrix for a beam with isotropic elastic material is given as:

$$[K_p]=\sum_{i=1}^{n}\int_{-1}^{+1}\int_{-1}^{+1}[B_i]^T.[D][B_i].\det J d\xi d\eta \qquad (19)$$

where [D] is given in equation (16) for isotropic elastic martial. Numerical integration can be used to evaluate the above integration using Gauss-Legendre quadrature rule**.**

Stiffness matrix for the foundation-[K_f]

For a foundation represented by Winkler model for both compressional and frictional resistances, the stiffness matrix is:

$$[K_f]=\begin{bmatrix}[R_W] & 0 & 0 & 0 & 0\\ 0 & [R_W] & 0 & 0 & 0\\ 0 & 0 & [R_W] & 0 & 0\\ 0 & 0 & 0 & [R_W] & 0\\ 0 & 0 & 0 & 0 & [R_W]\end{bmatrix}_{nxn} \qquad (20)$$

where

$$[R_W]=\begin{bmatrix}K_{f1} & 0\\ 0 & K_{f2}\end{bmatrix}$$

Here,

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$$K_{f_1} = \int_{-1}^{+1} N'_i . K_z . \det J' . d\xi. \tag{21a}$$

$$K_{f_2} = \int_{-1}^{+1} N'_i . K_x . \det J' . d\xi. \tag{21b}$$

where N'_i are the shape functions for the two or three node one dimensional beam element, J' is the Jacobian matrix for the one dimensional beam element and K_z and K_x are the moduli of subgrade reactions in z and x directions.

Computer program

In the present study, the computer program (DBNEF) (Deep Beams on Nonlinear Elastic Foundations) was formed. The program is developed to deal with any boundary condition, such as simply supported, fixed, and free-ends deep beams. In the program, the solution is divided into two steps. The first step is to find the governing equations of deflections and rotations and then these governing differential equations for deep beams on elastic foundation (in terms of w and ψ) are converted into finite differences. After writing the finite difference equations for each boundary and interior nodes, assembling for these equations must be made to form a system of simultaneous algebraic equations. Gauss-Jordan method is used in the program to solve the system of equations to obtain deflections and rotations at each node. The obtained deflections are compared with deflections of the previous iteration after changing the value of subgrade reaction and the procedure is repeated until convergence is obtained. The second step is to find the moment and shear at each node. Also, the computer program presented by Hinton and Owen (1977) is modified in this work to be capable of solving the problem of beams on nonlinear elastic foundations. Two-dimensional plane stress elements resting on Winkler type compress ional and frictional foundations have been used. The numerical results obtained from the finite difference and finite element methods have been compared with available exact and other analytical and numerical results.

APPLICATIONS AND DISCUSSIONS

Verifications

Simply supported beam on linear compressional Winkler foundation

The problem of simply supported beam resting on linear Winkler foundation solved by Al-Azzawi & Al-Ani, (2004) using finite difference method and analytical solution by Hetenyi, (1974) (thin beam theory) is considered. The same application is solved by using ten eight node plane stress finite elements and finite differences (deep beam theory). All information and finite element mesh for plane stress elements over half of the beam is shown in Figure-7.

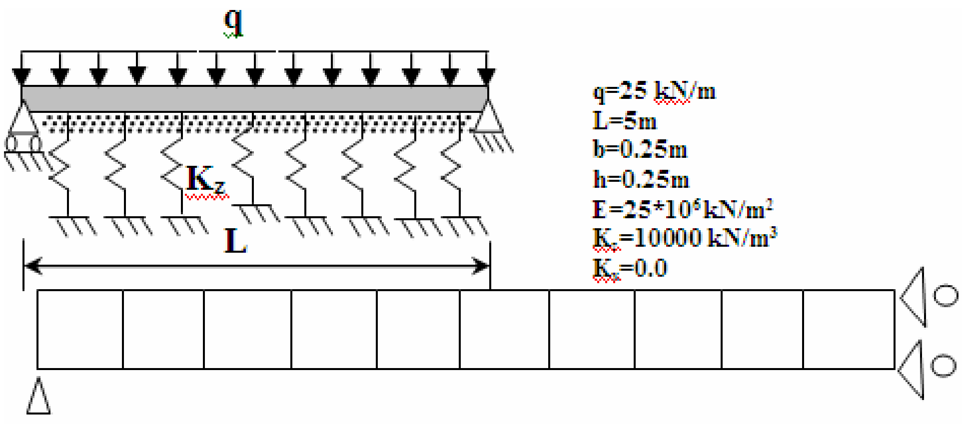

Figure-7. Beam on Winkler foundation and finite element mesh over half of the beam.

Figures 8, 9 and 10 show the deflection profiles, bending moment and shear force diagrams in x-direction for numerical, exact and the present study (finite difference and finite element methods). The results show good agreements by the used methods with percentage difference of 3%.

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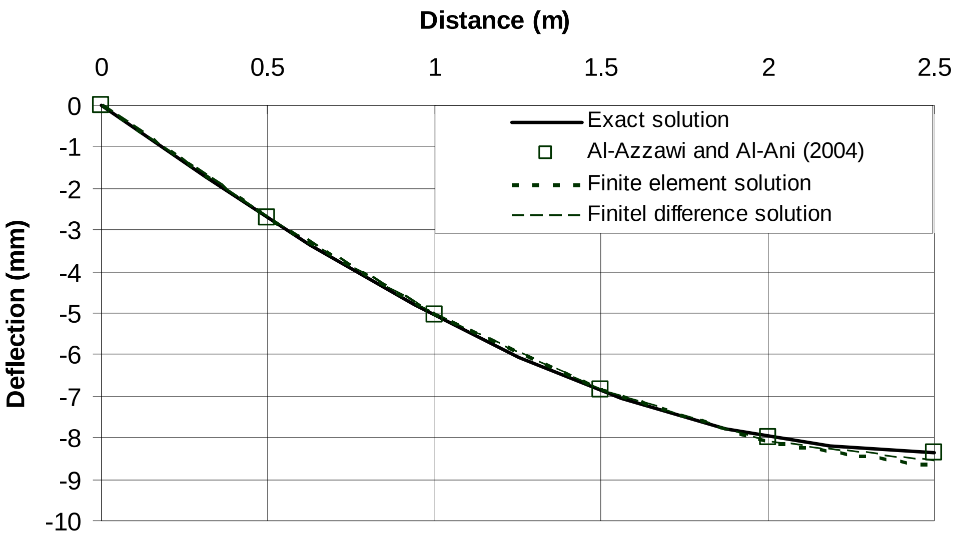

Figure-8. Deflection curves for simply supported beam resting on Winkler foundation.

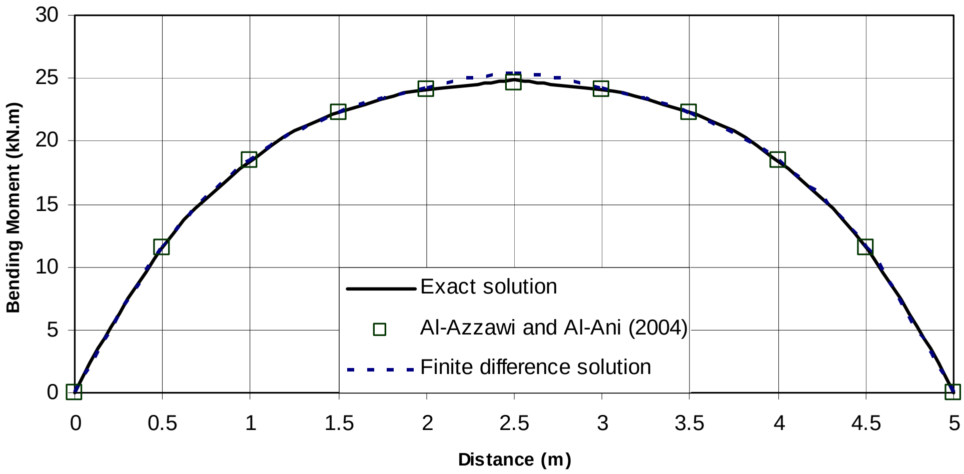

Figure-9. Bending moment curves for simply supported beam resting on Winkler foundation.

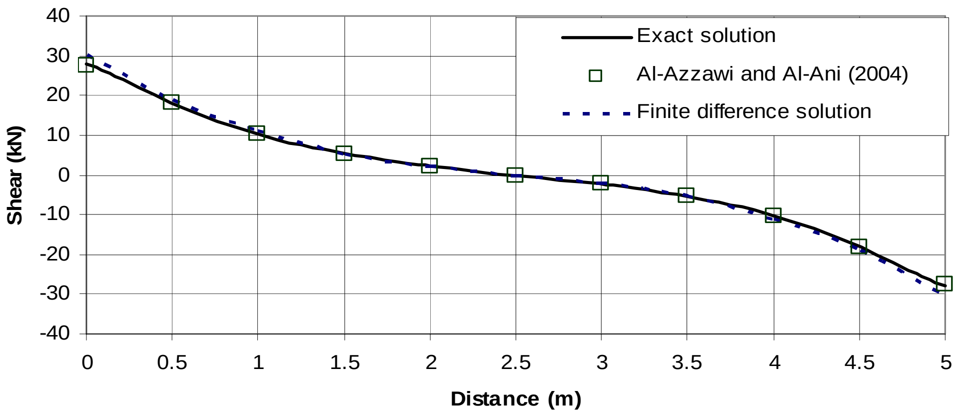

Figure-10. Shear force curves for simply supported beam resting on Winkler foundation.

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Simply supported beam on nonlinear compressioinal Winkler foundation

The problem of simply supported beam resting on nonlinear Winkler foundation solved by Al-Azzawi, (2010) using finite difference method (thin beam theory) is considered. The problem is solved by using ten eight node plane stress finite elements and finite differences (deep beam theory). All information and finite element mesh for plane stress elements over half of the beam is shown in Figure-11.

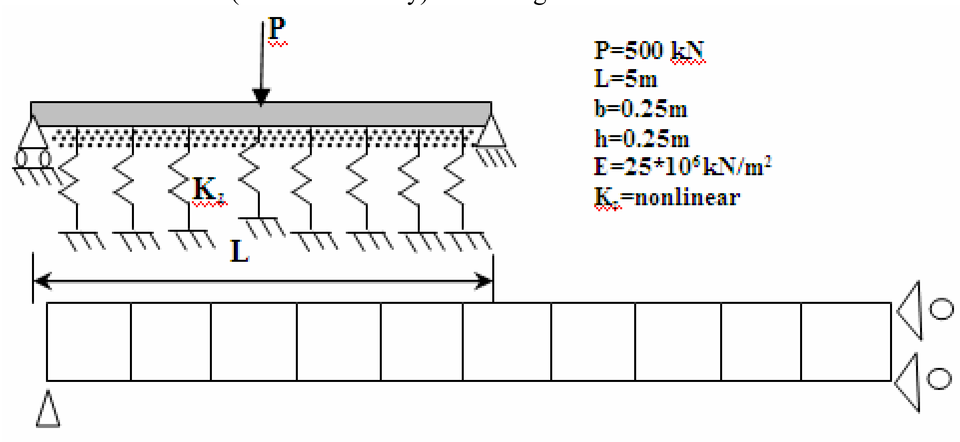

Figure-11. Beam on Winkler foundation and finite element mesh over half of the beam.

Figure-12 shows the deflection profile along x-direction for the nonlinear elastic Winkler foundation while Figures 13 and 14 show the bending moment and shearing force along x-direction. The results show good agreement for the different solutions with percentage difference of 3%.

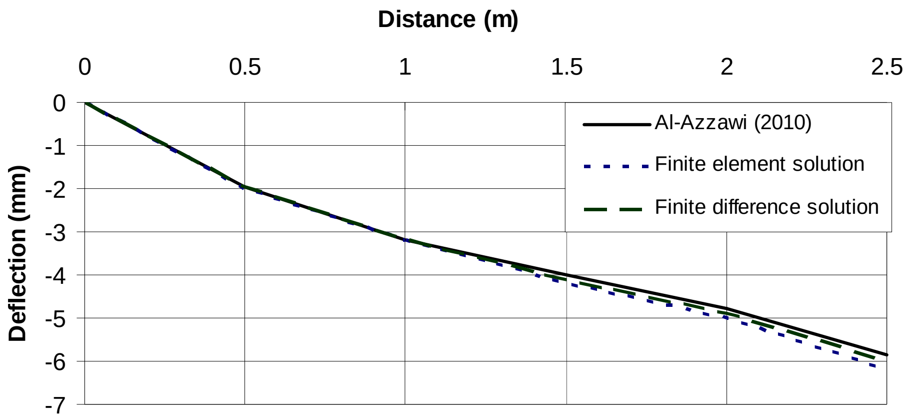

Figure-12. Deflection curves for simply supported beam resting on nonlinear Winkler foundation.

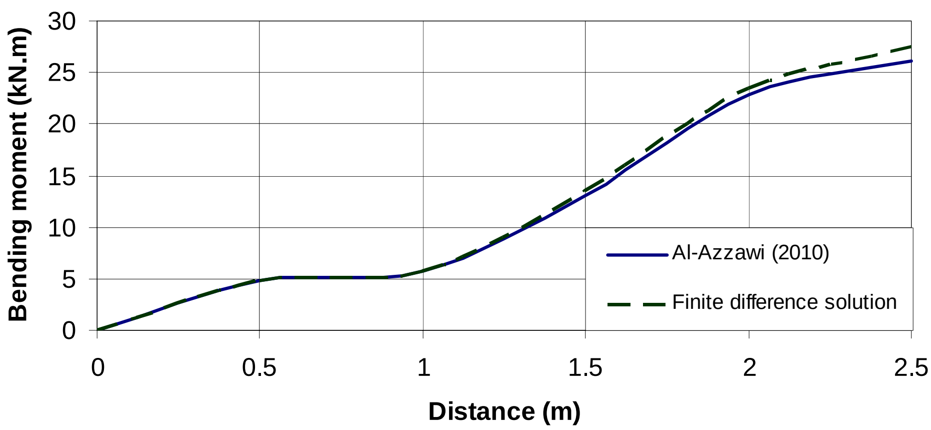

Figure-13. Bending moment curves for simply supported beam resting on nonlinear Winkler foundation.

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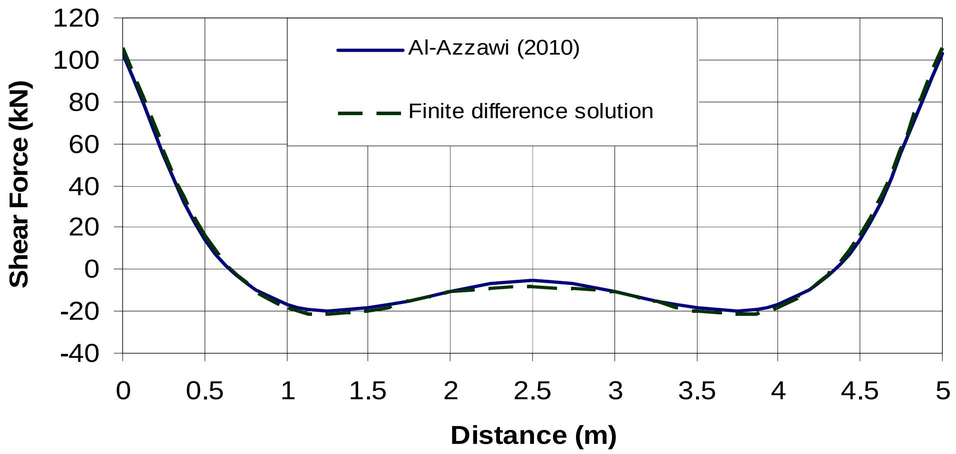

Figure-14. Shear force curves for simply supported beam resting on nonlinear Winkler foundation.

Free ends beam on nonlinear compressional Winkler foundation with end load

A beam of (E=25×10^6 kN/m^2, ν=0.15) and having a length of (1.8m), width (b=0.2m), depth (h=0.45m) and subjected to a concentrated load (P=220.5 kN), is considered as shown in Figure-15. The beam is resting on nonlinear compressional Winkler foundation with initial modulus (Kz=0.2kN/m^3) and this value and other values are obtained from plate-load test. This case was analyzed by Al-Hachmi (1997) by using the beam-column method. In the present study, the finite-element method is used to solve this problem. The present study results of deflections are plotted together with Al-Hachmi (1997) results as shown in Figure-16. The comparisons between the two solutions show good agreement with percentage difference of 3%.

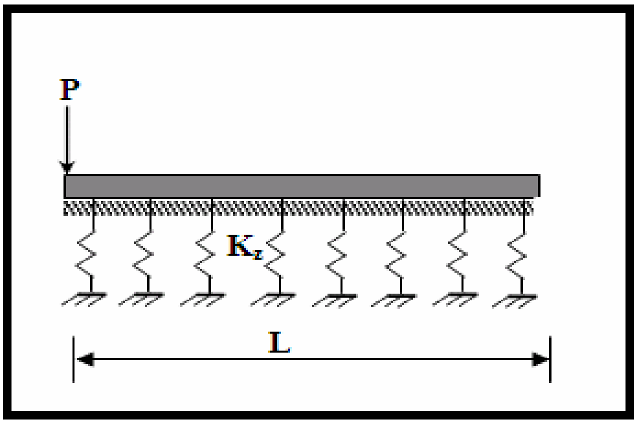

Figure-15. End loaded free-ends deep beam resting on nonlinear compressional Winkler foundation.

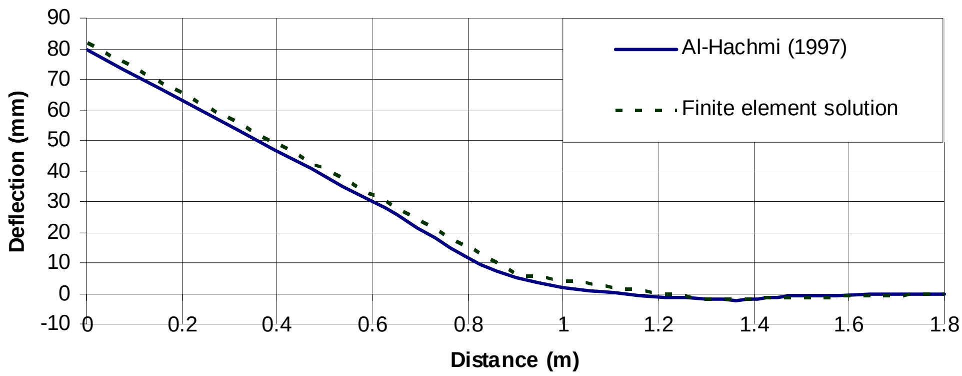

Figure-16. Deflection curves for free ends beam resting on nonlinear compressional Winkler foundation.

Parametric study

Linear compressional and frictional Winkler foundation

The same simply supported beam with same properties shown in Figure-7 is considered here. Figure-17 shows that the mid span deflection decreases as the depth of the beam increases because the stiffness of the beam increases. Figure-18 shows that the mid span moment increases as the depth of the beam increases because the stiffness of the beam increases. Also, Figure-19 shows that the maximum shear increases as the depth of the beam increases.

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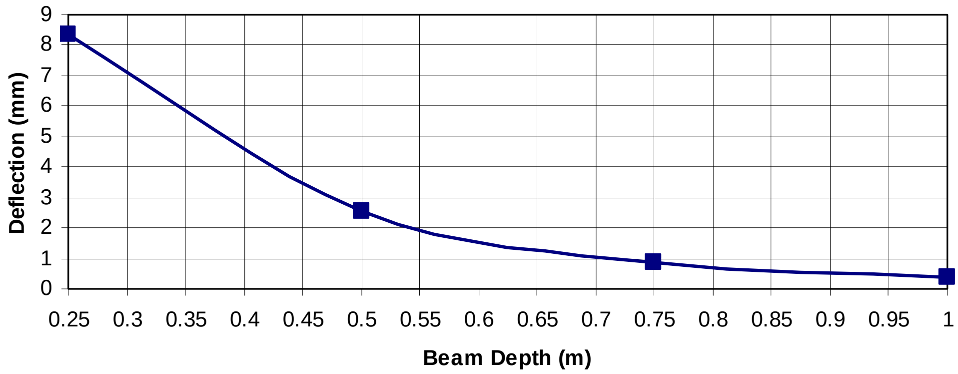

Figure-17. Effect of beam depth on mid span deflection for simply supported beam under uniform load (K_x= 0 and K_z =10000 kN/m^3).

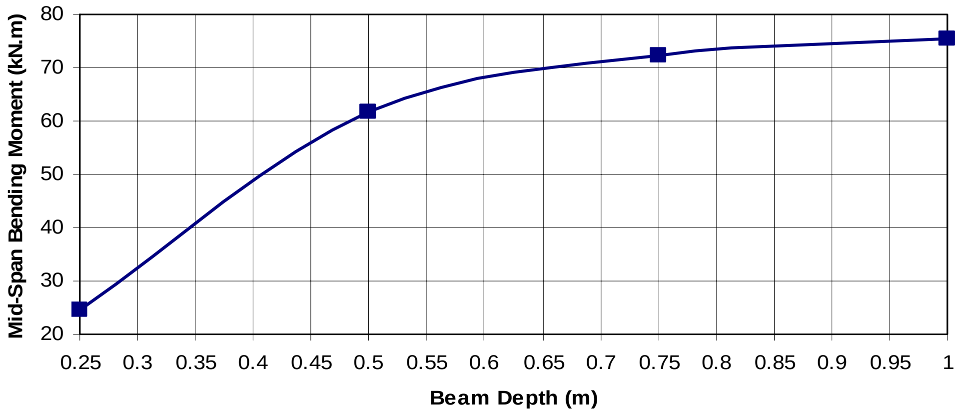

Figure-18. Effect of beam depth on mid span moment for simply supported beam under uniform load (K_x= 0 and K_z =10000 kN/m^3).

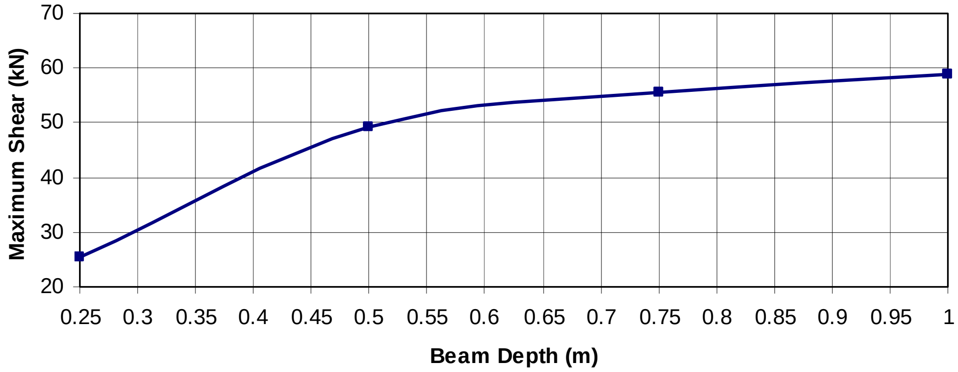

Figure-19. Effect of beam depth on maximum shear for simply supported beam under uniform load (K_x= 0 and K_z =10000 kN/m^3).

Figures 20, 21 and 22 show that as the subgrade reaction coefficient increased the mid span deflection, mid span moment and maximum shear decreased because the stiffness of the foundation increases.

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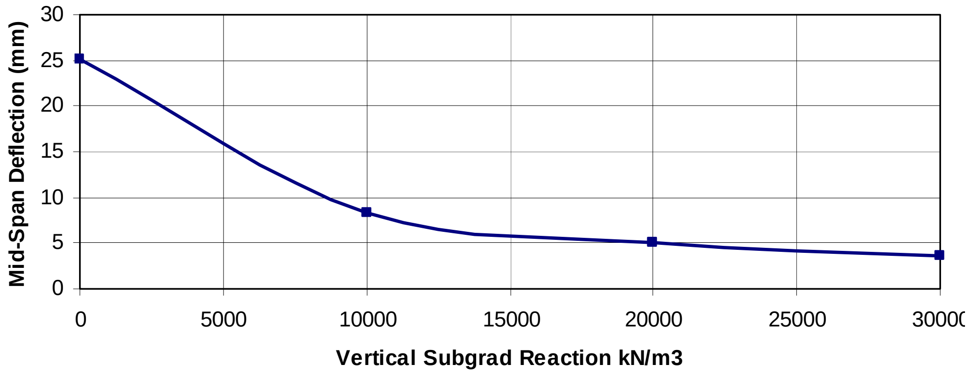

Figure-20. Effect of vertical subgrade reaction on mid-span deflection for simply supported beam under uniform load (K_x= 0 and beam depth=0.25m).

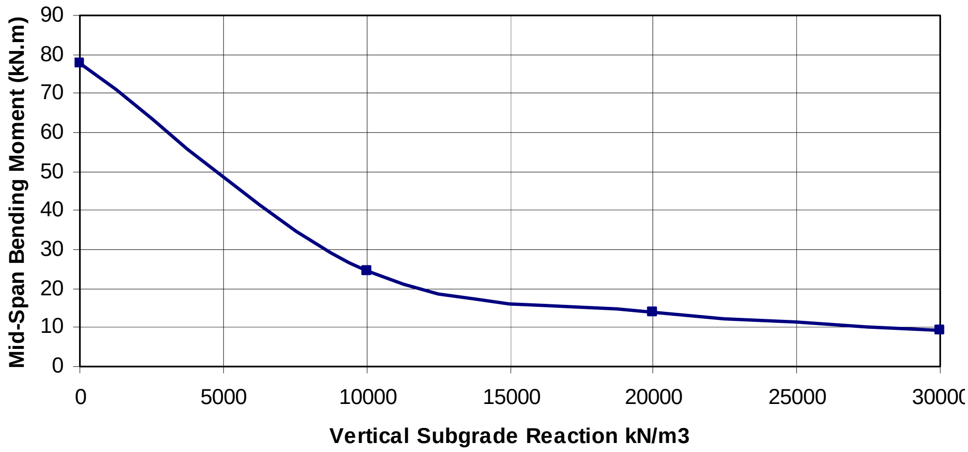

Figure-21. Effect of vertical subgrade reaction on mid-span moment for simply supported beam under uniform load (K_x= 0 and beam depth=0.25m).

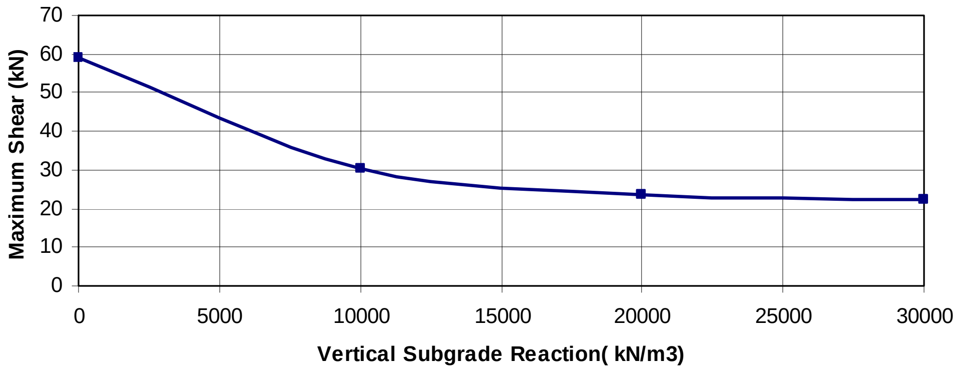

Figure-22. Effect of vertical subgrade reaction on maximum shear for simply supported beam under uniform load (K_x= 0 and beam depth=0.25).

Figure 23, 24 and 25 show that as the horizontal subgrade reaction coefficient increased the mid span deflection, mid span moment and maximum shear decreased because the stiffness of the foundation increases.

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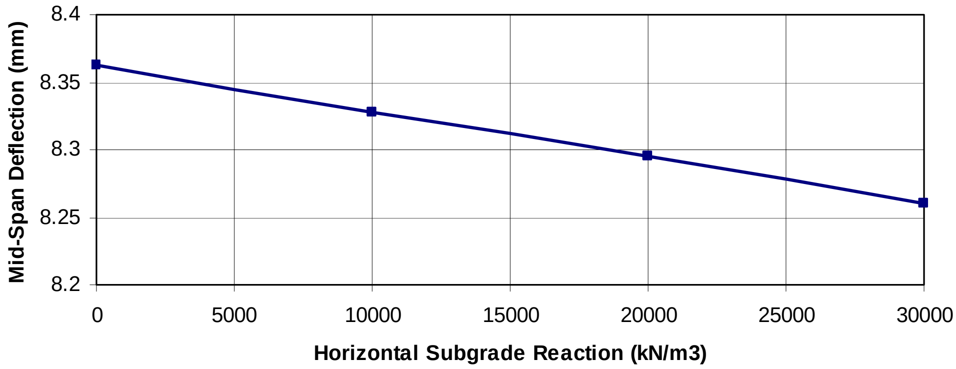

Figure-23. Effect of horizontal subgrade reaction on mid span deflection for simply supported beam under uniform load (K_z =10000 kN/m^3 and beam depth=0.25m).

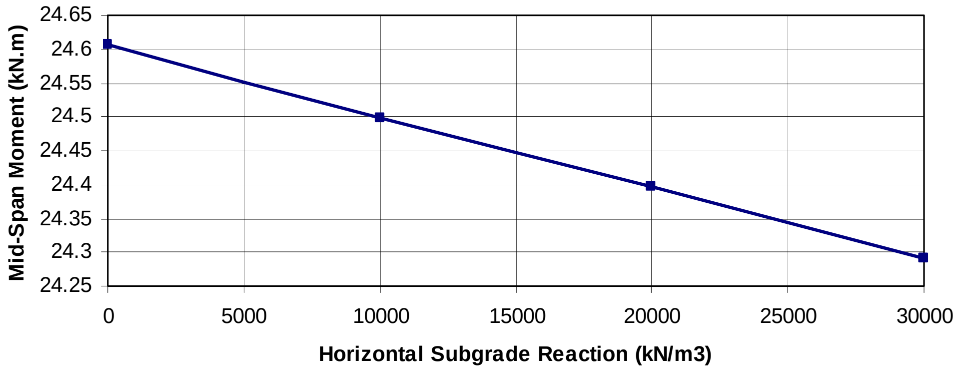

Figure-24. Effect of horizontal subgrade reaction on mid span moment for simply supported beam under uniform load (K_z =10000 kN/m^3 and beam depth=0.25m).

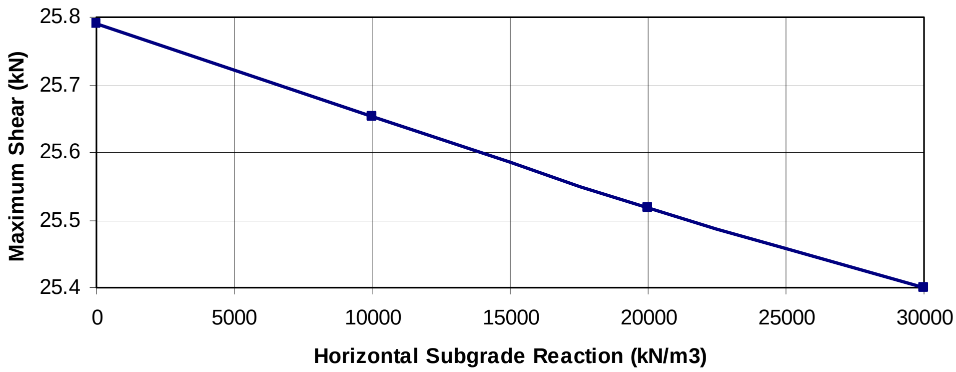

Figure-25. Effect of horizontal subgrade reaction on maximum shear for simply supported beam under uniform load (K_z =10000 kN/m^3 and beam depth=0.25m).

Nonlinear compressional and frictional Winkler foundation

The same simply supported beam with same properties shown in Figure-11 is considered. Figure-26 shows that the mid-span deflection for the linear and nonlinear modulus decreases as the depth of the beam increases because the section flexural rigidity EI of the beam increases for both linear and nonlinear foundations.

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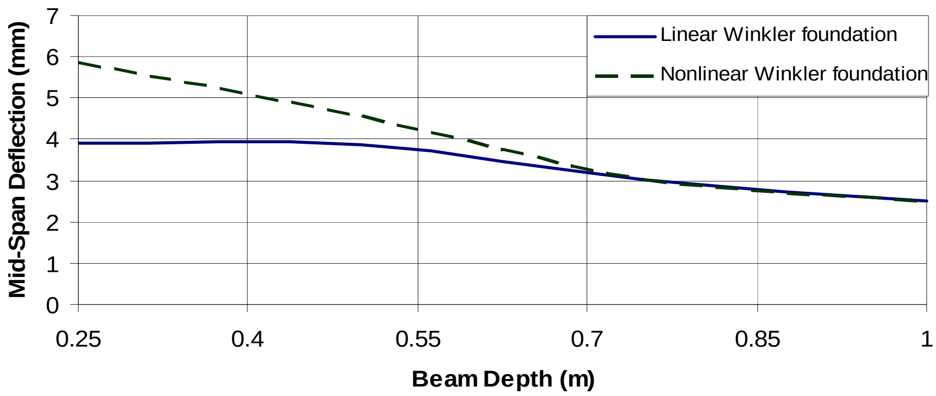

Figure-26. Effect of beam depth on mid-span deflection.

Figures 27 and 28 show that the mid-span moment and maximum shear force increases as the depth of the beam increases also, because the section flexural rigidity EI of the beam increases for both linear and nonlinear foundations.

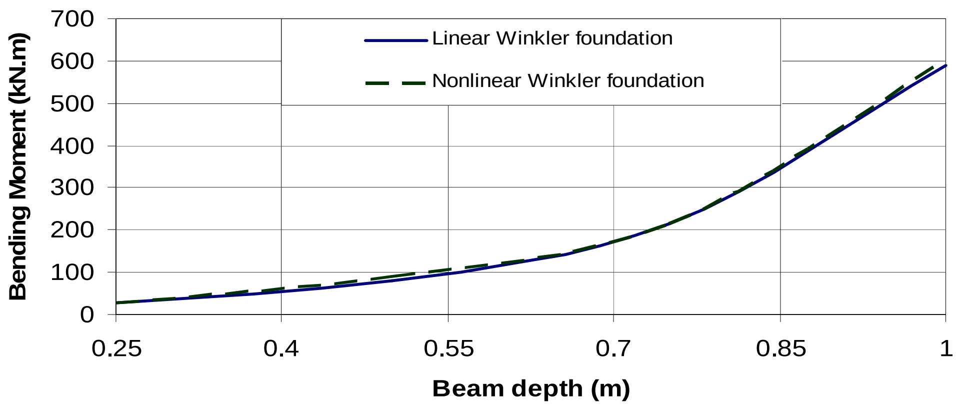

Figure-27. Effect of beam depth on mid-span bending moment.

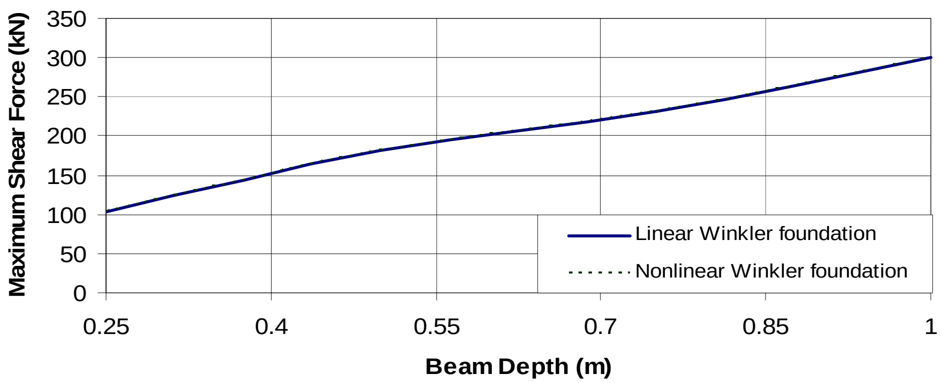

Figure-28. Effect of beam depth on maximum shear force.

CONCLUSIONS

From this study, the main conclusions are given below:

a) The results obtained from the exact, finite difference and finite element solutions check the accuracy of the method used in this paper in which they are in good agreement.
b) The effect of beam depth is significant on the results and increasing beam depth will decrease the mid span deflection and increase the moment and shear resistances.
c) The effect of friction at the beam-foundation interface is found to be small on the deflection, moment and shear.
d) The effect of varying vertical modulus of elastic foundation on deflection, moment and shear is significant.
e) The obtained results show different values for both deflection and bending moment but rather close values

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for shearing force for high values of applied loads on the beam, which is resting on linear or nonlinear elastic Winkler foundation. The nonlinear behavior of soil was obtained by using high-applied loads (to make the difference in results much obvious). This study shows that the elastic method for analyzing beam resting on Winkler foundation is still valid for ordinary applied loading on beams. The effect of beam depth on maximum beam deflection and bending moment is found to be significant but not much on shearing force for nonlinear foundations.

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