

**Predictions for single spin asymmetries in  $\ell p^\uparrow \rightarrow \pi X$  and  $\gamma^* p^\uparrow \rightarrow \pi X$** M. Anselmino<sup>1</sup>, M. Boglione<sup>2</sup>, J. Hansson<sup>3</sup> and F. Murgia<sup>4</sup><sup>1</sup> Dipartimento di Fisica Teorica, Università di Torino and  
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C.P. 170, I-09042 Monserrato (CA), Italy**Abstract:**

Predictions for the single transverse spin asymmetry  $A_N$  in semi-inclusive DIS processes are given; non negligible values of  $A_N$  may arise from spin effects in the fragmentation of a polarized quark into a final hadron with a transverse momentum  $\mathbf{k}_\perp$  with respect to the jet axis, the so-called Collins effect. The elementary single spin asymmetry of the fragmenting quark has been fixed in a previous paper, by fitting data on  $p^\uparrow p \rightarrow \pi X$ , and the predictions given here are uniquely based on the assumption that the Collins effect is the only cause of the observed single spin asymmetries in  $p^\uparrow p \rightarrow \pi X$ . Eventual spin and  $\mathbf{k}_\perp$  dependences in quark distribution functions, the so-called Sivers effect, are also discussed.

## 1. Introduction

A phenomenological description of single spin asymmetries in the inclusive production of hadrons in high energy processes, within the realm of pQCD, seems now indeed possible after the extensive amount of work which has been recently done [1]-[14]. Novel, hopefully universal, distribution and fragmentation functions [1, 2, 3, 8, 12] may give origin to sizeable single spin hadronic observables; such functions can be measured in some processes and then used to give theoretical predictions in other cases.

This phenomenological program, based on data on  $p^\uparrow p \rightarrow \pi X$  and  $\bar{p}^\uparrow p \rightarrow \pi X$  [15], was initiated in Refs. [3, 4] and continued in Ref. [5]; a similar attempt, together with a comprehensive theoretical discussion, is considered in Ref. [8].

Several possible origins of single spin asymmetries can be at work in  $p^\uparrow p$  or  $\bar{p}^\uparrow p$  processes: spin and intrinsic  $\mathbf{k}_\perp$  dependences in the distribution functions of unpolarized quarks inside transversely polarized nucleons [1, 3]; spin and intrinsic  $\mathbf{k}_\perp$  dependences in the fragmentation functions of transversely polarized quarks into pions or other hadrons [2, 5]; very recently also possible spin and intrinsic  $\mathbf{k}_\perp$  dependences in the distribution functions of transversely polarized quarks inside unpolarized nucleons have been suggested [12].

The first two possibilities above have been considered respectively in Refs. [4] and [5] and explicit expressions of the relevant elementary functions have been obtained:

$$\Delta^N f_{a/p}(x_a, \mathbf{k}_{\perp a}) = \hat{f}_{a/p^\uparrow}(x_a, \mathbf{k}_{\perp a}) - \hat{f}_{a/p^\downarrow}(x_a, \mathbf{k}_{\perp a}), \quad (1)$$

*i.e.* the difference between the density numbers of partons  $a$ , with all possible polarization, longitudinal momentum fraction  $x_a$  and intrinsic transverse momentum  $\mathbf{k}_{\perp a}$ , inside a transversely polarized proton, and

$$\Delta^N D_{h/a}(z, \mathbf{k}_{\perp h}) = \hat{D}_{h/a^\uparrow}(z, \mathbf{k}_{\perp h}) - \hat{D}_{h/a^\downarrow}(z, \mathbf{k}_{\perp h}), \quad (2)$$

*i.e.* the difference between the density numbers of hadrons  $h$ , with longitudinal momentum fraction  $z$  and transverse momentum  $\mathbf{k}_{\perp h}$  inside a jet originated by the fragmentation of a transversely polarized parton  $a$ .

These two functions are strictly related to the functions denoted by  $f_{1T}^\perp$  and  $H_1^\perp$  in Refs. [9], as it is further discussed in Ref. [14].

We consider here the DIS processes  $\ell p^\uparrow \rightarrow \pi X$  and  $\gamma^* p^\uparrow \rightarrow \pi X$ , with unpolarized leptons; however, the same results hold in case of longitudinally polarized leptons, it is only crucial for the proton spin to be orthogonal to the scattering plane. As it was remarked in Refs. [5] and [16] we do not expect any spin effect in the nucleon distribution functions in such DIS processes, because these should be suppressed by necessary initial state interactions which mean higher powers of  $\alpha_{em}$ . We remain then with the only possibility of the quark fragmentation single spin asymmetry,  $\Delta^N D_{h/a}(z, \mathbf{k}_\perp) \neq 0$ , the so-called Collins effect [2].

Such an effect was studied in Ref. [5] where, assuming that it is the only cause of the observed single spin asymmetry in  $p^\uparrow p \rightarrow \pi X$ , an explicit expression of

$\Delta^N D_{h/a}(z, \mathbf{k}_\perp)$ , based on a simple parametrization, was obtained. By using this expression here we are able to give predictions for the single spin asymmetry

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{2 d\sigma^{unp}} \quad (3)$$

in DIS processes.

In the next Section we will consider the process  $\ell p^\uparrow \rightarrow \pi X$ , in which the final lepton need not be detected, in complete analogy with the  $p^\uparrow p \rightarrow \pi X$  process; parity invariance allows  $A_N$  to be different from zero only if the nucleon spin has a component perpendicular to the scattering plane. This is not the case for longitudinally polarized protons. However, also for such spin configurations, one could have a single spin asymmetry by looking at the semi-inclusive process  $\ell p^\uparrow \rightarrow \ell \pi X$  or  $\gamma^* p^\uparrow \rightarrow \pi X$ ; also a longitudinally polarized proton can have a non-zero spin component perpendicular to the scattering plane, which is now the  $\gamma^*$ - $\pi$  plane. Predictions for such a process will be given in Section 3.

In Section 4 we consider the hypothetical possibility that Sivers effect, *i.e.* the function  $\Delta^N f$  of Eq. (1), might be at work also in DIS processes [19], despite the argument given above about initial state interactions, and compute  $A_N$  in  $\ell p^\uparrow \rightarrow \pi X$  due to such an effect using the expressions of  $\Delta^N f$  derived in Ref. [4]; we do this in order to compare the size and  $x_F$  dependence of  $A_N$  eventually originating from Sivers effect with those of  $A_N$  originating from Collins effect. Comments and conclusions are presented in Section 5.

## 2. Single spin asymmetry $A_N$ for $\ell p^\uparrow \rightarrow \pi X$

The single transverse spin asymmetry (3) for the high energy process  $\ell p^\uparrow \rightarrow \pi X$  in which a final pion is detected with a large  $p_T$  is given, assuming a straightforward generalization of the QCD factorization theorem [2] to the case in which parton transverse motion is taken into account and considering  $\mathbf{k}_\perp$  dependences only in the fragmentation process, by [5, 16]

$$\begin{aligned} 2 A_N \frac{E_\pi d^3 \sigma^{unp}}{d^3 \mathbf{p}_\pi} &= \frac{E_\pi d^3 \sigma^{\ell+p^\uparrow \rightarrow \pi+X}}{d^3 \mathbf{p}_\pi} - \frac{E_\pi d^3 \sigma^{\ell+p^\downarrow \rightarrow \pi+X}}{d^3 \mathbf{p}_\pi} \\ &= \sum_q \int \frac{dx}{\pi z} d^2 \mathbf{k}_\perp \Delta_T q(x) \Delta_{NN} \hat{\sigma}^q(x, \mathbf{k}_\perp) \left[ \hat{D}_{\pi/q^\uparrow}(z, \mathbf{k}_\perp) - \hat{D}_{\pi/q^\uparrow}(z, -\mathbf{k}_\perp) \right], \end{aligned} \quad (4)$$

where  $\Delta_T q$  is the polarized number density for transversely spinning quarks  $q$ , also denoted by  $h_1$  [17], and  $\Delta_{NN} \hat{\sigma}^q$  is the elementary cross-section double spin asymmetry

$$\Delta_{NN} \hat{\sigma}^q = \frac{d\hat{\sigma}^{\ell q^\uparrow \rightarrow \ell q^\uparrow}}{d\hat{t}} - \frac{d\hat{\sigma}^{\ell q^\uparrow \rightarrow \ell q^\downarrow}}{d\hat{t}}. \quad (5)$$

Here  $\uparrow$  and  $\downarrow$  refer to directions “up” and “down” with respect to the scattering plane:  $\uparrow$  is parallel to  $\mathbf{p}_\ell \times \mathbf{p}_\pi$ . Notice that Eq. (4) holds both for unpolarized and longitudinally polarized leptons.

Following the same procedure as in Ref. [5] we write Eq. (4) as

$$\begin{aligned} & \frac{E_\pi d^3\sigma^{\ell+p^\uparrow \rightarrow \pi+X}}{d^3\mathbf{p}_\pi} - \frac{E_\pi d^3\sigma^{\ell+p^\downarrow \rightarrow \pi+X}}{d^3\mathbf{p}_\pi} = \\ & = \sum_q \int \frac{dx}{\pi z} d^2\mathbf{k}_\perp P^{q/p^\uparrow} f_{q/p}(x) [\Delta_{NN}\hat{\sigma}^q(x, \mathbf{k}_\perp) - \Delta_{NN}\hat{\sigma}^q(x, -\mathbf{k}_\perp)] \Delta^N D_{\pi/q}(z, \mathbf{k}_\perp) \end{aligned} \quad (6)$$

where we have used Eq. (2) and where the integration on  $\mathbf{k}_\perp$  now runs only over one half-plane of its components. We have also used  $\Delta_T q(x) = h_1(x) = P^{q/p^\uparrow} f_{q/p}(x)$  where  $P^{q/p^\uparrow}$  is the polarization of the quark  $q$  inside the transversely polarized proton  $p^\uparrow$ ; it is twice the average value of the  $\uparrow$  component of the quark spin.

We now proceed as in Ref. [5] with the assumption that the dominant contribution is given by valence quarks, which is certainly correct for large  $x_F$  pions which originate from large  $x$  quarks; for this reason we also assume values of  $P^{q/p^\uparrow}$  which do not depend on  $x$ . We take into account only the leading  $\mathbf{k}_\perp$  contributions which originate from  $\mathbf{k}_\perp = \mathbf{k}_\perp^0$  where  $\mathbf{k}_\perp^0$  lies in the overall scattering plane and its magnitude equals the average value of  $\langle \mathbf{k}_\perp^2 \rangle^{1/2} = k_\perp^0(z)$ . That is, we have:

$$\begin{aligned} & \frac{E_\pi d^3\sigma^{\ell+p^\uparrow \rightarrow \pi+X}}{d^3\mathbf{p}_\pi} - \frac{E_\pi d^3\sigma^{\ell+p^\downarrow \rightarrow \pi+X}}{d^3\mathbf{p}_\pi} \\ & \simeq \sum_q \int \frac{dx}{\pi z} P^{q/p^\uparrow} f_{q/p}(x) [\Delta_{NN}\hat{\sigma}^q(x, +\mathbf{k}_\perp^0) - \Delta_{NN}\hat{\sigma}^q(x, -\mathbf{k}_\perp^0)] \Delta^N D_{\pi/q}(z, k_\perp^0). \end{aligned} \quad (7)$$

The value of  $z$  is fixed as function of  $x$  and  $\mathbf{k}_\perp$  by energy-momentum conservation in the elementary scattering.

The average value  $k_\perp^0$  of the transverse momentum of charged pions inside jets does in general depend on  $z$ ; the  $z$  dependence was measured at LEP [18] and we use a fit to their data points [5]:

$$\frac{k_\perp^0(z)}{M} = 0.61 z^{0.27} (1-z)^{0.20}, \quad (8)$$

with  $M = 1 \text{ GeV}/c^2$ .

The explicit expression of  $\Delta^N D_{\pi/q}$  obtained in Ref. [5] by fitting the data on  $p^\uparrow p \rightarrow \pi X$  [15] is, for valence quarks:

$$\begin{aligned} \Delta^N D_{\pi^+/u}(z, k_\perp^0) &= \Delta^N D_{\pi^-/d}(z, k_\perp^0) = \Delta^N D_{val}(z, k_\perp^0) \\ &= 2\Delta^N D_{\pi^0/u}(z, k_\perp^0) = 2\Delta^N D_{\pi^0/d}(z, k_\perp^0) = -\frac{k_\perp^0(z)}{M} 0.22 z^{2.33} (1-z)^{0.24}, \end{aligned} \quad (9)$$

for  $z \leq 0.97742$  and

$$\Delta^N D_{val}(z, k_\perp^0) = -2D_{val}(z) = -2 \times 1.102 z^{-1} (1-z)^{1.2} \quad \text{for } z > 0.97742. \quad (10)$$

The values of  $P^{u/p^\uparrow}$  and  $P^{d/p^\uparrow}$ , always from the fit of Ref. [5], are

$$P^{u/p^\uparrow} = \frac{2}{3} \quad P^{d/p^\uparrow} = -0.88; \quad (11)$$

notice that only their ratio  $P_{u/d} = P^{u/p^\uparrow} / P^{d/p^\uparrow} = -0.76$  has a physical meaning in that the overall normalization of the quark polarizations can always be absorbed by the overall normalization of  $\Delta^N D_{\pi/q}$ .

As we commented in Ref. [5] both the expression of  $\Delta^N D_{val}$  and the value of  $P_{u/d}$  resulting from fitting the  $p^\uparrow p \rightarrow \pi X$  data – assuming spin and  $\mathbf{k}_\perp$  effects only in the fragmentation process – are somewhat surprising and a bit extreme:  $\Delta^N D_{val}$  has to saturate at large  $z$  the necessary inequality  $|\Delta^N D_{val}(z, k_\perp^0)| \leq 2D_{val}(z)$  and  $P_{u/d}$  shows an opposite (as expected) but larger in magnitude (unexpected) polarization of the  $d$  valence quark with respect to the  $u$  valence quark inside a polarized proton. It might be that relevant or even dominant contributions to  $A_N$  in  $p^\uparrow p \rightarrow \pi X$  processes originate from quark distribution functions, Eq. (1), the so-called Sivvers effect [1, 3, 4].

Nevertheless, we assume here that Collins effect alone is responsible for the observed single spin asymmetries and use Eqs. (8)-(11) into Eq. (7) to obtain predictions for  $A_N$ , Eq. (3). The unpolarized cross-section

$$\frac{E_\pi d^3\sigma^{unp}}{d^3\mathbf{p}_\pi} = \sum_q \int \frac{dx}{\pi z} f_{q/p}(x) \frac{d\hat{\sigma}^{\ell q \rightarrow \ell q}}{d\hat{t}}(x) D_{\pi/q}(z) \quad (12)$$

is computed using the same distribution and fragmentation functions as in Ref. [5].

Our results are shown in Figs. 1-4, where we plot  $A_N$  as a function of  $x_F$  at different centre of mass energies typical of experiments running or planned at Jefferson Lab (CEBAF), DESY (HERMES), SLAC (E155) and CERN (SMC, COMPASS) respectively;  $p_T$  is fixed and ranges, according to the different cases, from 1 to 2 GeV/c. The asymmetry turns out to be large and sizeable at large values of  $x_F$  in all cases: we can unambiguously conclude that, if Collins effect is the main cause of the single transverse spin asymmetries observed in  $p^\uparrow p \rightarrow \pi X$  [15], a similarly large asymmetry should be observed in lepto-production processes,  $\ell p^\uparrow \rightarrow \pi X$ .

Data on  $A_N$  in such processes are not available yet, although they might be soon obtainable from SMC and SLAC where DIS processes with transversely polarized protons are studied. Protons are longitudinally polarized at HERMES and Jlab experiments and parity invariance does not allow a single proton spin asymmetry in  $\ell p^\rightarrow \rightarrow \pi X$ ; however, as we said in the introduction and will discuss in the next session, even in such a case one could have a single transverse spin asymmetry looking at the  $\gamma^* p^\uparrow \rightarrow \pi X$  reaction.

### 3. Single spin asymmetry $A_N$ for $\gamma^* p^\uparrow \rightarrow \pi X$

Let us consider now the double-inclusive process  $\ell p^\uparrow \rightarrow \ell \pi X$  in which also the final lepton is detected, which allows to fix the energy and momentum of the virtual photon exchanged between the lepton and the nucleon; one can then consider the centre of mass process  $\gamma^* p^\uparrow \rightarrow \pi X$ . A longitudinally polarized nucleon, *i.e.* a nucleon with its spin along the lepton direction in the laboratory frame, may have a transverse spin component with respect to the  $\gamma^*$ - $\pi$  plane.

We fix our kinematics in Fig. 5; we have defined the  $\gamma^* p^\uparrow \rightarrow \pi X$  scattering plane as the  $xz$ -plane; the spin  $\uparrow$  ( $\downarrow$ ) is then along the  $+y$  ( $-y$ ) direction. The angle between the  $\ell$ - $\ell'$  and the  $\gamma^*$ - $\pi$  planes is  $\beta$ . We shall give values of the single spin asymmetry  $A_N$ , Eq. (3), for this configuration.

The analogue of Eq. (4) is now

$$\begin{aligned}
2 A_N \frac{d\sigma^{\gamma^* p \rightarrow \pi X}}{dx dQ^2 dz d^2 p_T} &= \frac{d\sigma^{\gamma^* p^\uparrow \rightarrow \pi X}}{dx dQ^2 dz d^2 p_T} - \frac{d\sigma^{\gamma^* p^\downarrow \rightarrow \pi X}}{dx dQ^2 dz d^2 p_T} \\
&= \sum_q P^{q/p^\uparrow} f_{q/p}(x) \left[ \frac{d\hat{\sigma}^{\gamma^* q^\uparrow \rightarrow q^\uparrow}}{dQ^2} - \frac{d\hat{\sigma}^{\gamma^* q^\uparrow \rightarrow q^\downarrow}}{dQ^2} \right] \left[ \hat{D}_{\pi/q^\uparrow}(z, \mathbf{p}_T) - \hat{D}_{\pi/q^\uparrow}(z, -\mathbf{p}_T) \right],
\end{aligned} \tag{13}$$

where  $x$  and  $Q^2$  are the usual DIS variables and where the unpolarized cross-section is:

$$\frac{d\sigma^{\gamma^* p \rightarrow \pi X}}{dx dQ^2 dz d^2 p_T} = \sum_q f_{q/p}(x) \frac{d\hat{\sigma}^{\gamma^* q \rightarrow q}}{dQ^2} \hat{D}_{\pi/q}(z, \mathbf{p}_T). \tag{14}$$

Notice that in the  $\gamma^* p$  c.m. frame, neglecting the transverse motion of the quarks inside the polarized proton – that is, taking into account only Collins effect – the transverse momentum  $\mathbf{p}_T$  of the produced pion is the same as the intrinsic  $\mathbf{k}_\perp$  of the pion relative to fragmenting quark direction. The angle between the quark transverse spin direction and  $\mathbf{k}_\perp$  – sometimes referred to as Collins angle – is fixed and equals  $\pi/2$  in the configuration of Fig. 5.

The cross-section for the  $\gamma^* q \rightarrow q$  process, with unpolarized quarks, is given by:

$$\begin{aligned}
\frac{d\hat{\sigma}^{\gamma^* q \rightarrow q}}{dQ^2} &= C \frac{1}{2} \sum_{\lambda_q, \lambda_{q'}} \sum_{\lambda_\gamma, \lambda'_\gamma} M_{\lambda_{q'}; \lambda_\gamma, \lambda_q} M_{\lambda_{q'}; \lambda'_\gamma, \lambda_q}^* \rho_{\lambda_\gamma, \lambda'_\gamma}(\gamma^*) \\
&\equiv C \frac{1}{2} \sum_{\lambda_q, \lambda_{q'}} \sum_{\lambda_\gamma, \lambda'_\gamma} \langle \lambda_{q'} | \lambda_\gamma, \lambda_q \rangle \langle \lambda_{q'} | \lambda'_\gamma, \lambda_q \rangle^* \rho_{\lambda_\gamma, \lambda'_\gamma}(\gamma^*),
\end{aligned} \tag{15}$$

where the  $M$ 's are the helicity amplitudes for the  $\gamma^* q \rightarrow q$  process,  $C$  contains kinematical and flux factors which cancel out in the ratio giving  $A_N$  and  $\rho(\gamma^*)$  is the helicity density matrix of the virtual photon emitted by the lepton; its explicit expression depends on the DIS variables and can be found, for example, in Ref. [20]. Analogously one has, for transversely polarized quarks:

$$\frac{d\hat{\sigma}^{\gamma^* q^\uparrow \rightarrow q^{\uparrow(\downarrow)}}}{dQ^2} = C \sum_{\lambda_\gamma, \lambda'_\gamma} \langle \uparrow(\downarrow) | \lambda_\gamma, \uparrow \rangle \langle \uparrow(\downarrow) | \lambda'_\gamma, \uparrow \rangle^* \rho_{\lambda_\gamma, \lambda'_\gamma}(\gamma^*). \tag{16}$$

In terms of helicity states the transverse spins “up” and “down” are given, for the initial ( $q$ ) and final ( $q'$ ) quarks, by:

$$|q; \uparrow\rangle = \frac{1}{\sqrt{2}} (|+\rangle - i|-\rangle) \quad |q'; \uparrow, \downarrow\rangle = \frac{1}{\sqrt{2}} (\pm|+\rangle + i|-\rangle). \tag{17}$$

One finds:

$$\frac{d\hat{\sigma}^{\gamma^*q \rightarrow q}}{dQ^2} = C e^2 e_q^2 Q^2 \frac{1 + (1 - y)^2}{(2 - y)^2} \quad (18)$$

and

$$\frac{d\hat{\sigma}^{\gamma^*q^\uparrow \rightarrow q^\uparrow}}{dQ^2} - \frac{d\hat{\sigma}^{\gamma^*q^\uparrow \rightarrow q^\downarrow}}{dQ^2} = 2 C e^2 e_q^2 Q^2 \frac{1 - y}{(2 - y)^2} \cos 2\beta \quad (19)$$

where  $y = Q^2/xs$ ; notice that a dependence on the angle  $\beta$  (see Fig. 5) remains in the elementary polarized process.

Insertion of Eqs. (18) and (19) into Eqs. (13) and (14) obtains:

$$A_N^{\gamma^*p^\uparrow \rightarrow \pi X}(x, Q^2, z, p_T) = \frac{(1 - y) \cos 2\beta}{[1 + (1 - y)^2]} \frac{\sum_q e_q^2 P^{q/p^\uparrow} f_{q/p}(x) \Delta^N D_{\pi/q}(z, p_T)}{\sum_q e_q^2 f_{q/p}(x) D_{\pi/q}(z)}. \quad (20)$$

We give numerical estimates of  $A_N/|\cos 2\beta|$  using  $\Delta^N D_{\pi/q}$  and  $P^{q/p^\uparrow}$  from Eqs. (9)-(11). The average  $p_T$  value is given, at any  $z$ , by Eq. (8); the  $p_T$  dependence in the fragmentation function in the denominator of Eq. (20) is neglected.

The actual asymmetry  $A_N$  measurable in processes initiated by longitudinally polarized protons – *i.e.* protons with spin parallel to the lepton direction – is smaller than that given in Eq. (20), which assumes a  $P = 100\%$  nucleon polarization perpendicular to the  $\gamma^*$ - $\pi$  scattering plane. The real kinematical situation is shown in Fig. 6, from which one sees that the observed asymmetry is given by Eq. (20) multiplied by  $P \sin \theta_\gamma \sin \Phi$  (the component of the actual target polarization  $P$  perpendicular to the production plane) and in which one sets  $\beta = \Phi$ . Some preliminary data from HERMES and SMC show the  $\Phi$  dependence of  $A_N$  [21, 22].

In Figs. 7-8 we plot the value of  $A_N(x, Q^2, z)$  for CEBAF and HERMES planned or running energies, as a function of  $z$ , for several values of  $x$  and at fixed values of  $Q^2$ . These plots show that the asymmetry is large, for large  $x$  values, but only at very large  $z$  values, which might be difficult to reach experimentally.

The  $z$  dependence of  $A_N$  is dominated by the  $z$  dependence of  $\Delta^N D_{\pi/q}/D_{\pi/q}$ ; actually, in all cases in which only one flavour  $q$  contributes dominantly, Eq. (20) simplifies to

$$A_N^{\gamma^*p^\uparrow \rightarrow \pi X}(y, z, p_T) \simeq \frac{(1 - y) \cos 2\beta}{[1 + (1 - y)^2]} \frac{P^{q/p^\uparrow} \Delta^N D_{\pi/q}(z, p_T)}{D_{\pi/q}(z)}. \quad (21)$$

It is then clear why  $A_N$  is small at small  $z$  values and is large and constant for  $z \geq 0.977$ : this is due to the behaviour of  $\Delta^N D_{val}$  resulting from fitting the  $p^\uparrow p \rightarrow \pi X$  data, see Fig. 3 of Ref. [5]. In particular the saturated behaviour of  $\Delta^N D_{val}$  at large  $z$ , Eq. (10), forces the large value of  $A_N$  when  $z \rightarrow 1$ . All these are typical features of the Collins effect, assuming that it is the only origin of single transverse spin asymmetries in  $p^\uparrow p \rightarrow \pi X$  processes, and it would be very interesting to check them experimentally.

#### 4. Single spin asymmetry $A_N$ for $\ell p^\uparrow \rightarrow \pi X$ with Sivers effect

Sivers effect is forbidden by time reversal invariance for free particles [2]; however, one can avoid this conclusion by invoking some soft initial state interactions between the incoming particles [3, 12]. This is plausible for  $pp$  interactions or, in general, hadron-hadron interactions where many soft gluons might be exchanged, or some external gluonic fields should be present; the assumption of the validity of the factorization theorem also in this case allows then to compute  $A_N$  due to Sivers effect [3, 4].

We cannot use such an argument in  $\ell N$  processes because initial state interactions would now lead to strong suppressions of  $\mathcal{O}(\alpha_{em})$ . It has been suggested [19] that some spin-isospin interactions might avoid the time reversal problem in chiral models, with quarks moving in a background of chiral fields, with a spin-isospin interaction which mixes states of different flavours under time reversal.

We do not claim here the definite validity of the above suggestion, which would require further developments and for which we have no explicit model; we simply assume that Sivers effect can be at work also in  $\ell p$  interactions and consistently compute  $A_N$  in  $\ell p^\uparrow \rightarrow \pi X$  processes due solely to this origin. Our aim is that of comparing the values and features of  $A_N$  due to Sivers effect to the values and features of  $A_N$  due to Collins effects, as computed in Section 1, in order to eventually distinguish between the two cases, independently of their theoretical plausibilities.

If we assume that only Sivers effect is at work, Eq. (4) changes into:

$$\begin{aligned} 2 A_N \frac{E_\pi d^3 \sigma^{unp}}{d^3 \mathbf{p}_\pi} &= \frac{E_\pi d^3 \sigma^{\ell+p^\uparrow \rightarrow \pi+X}}{d^3 \mathbf{p}_\pi} - \frac{E_\pi d^3 \sigma^{\ell+p^\downarrow \rightarrow \pi+X}}{d^3 \mathbf{p}_\pi} \\ &= \sum_q \int \frac{dx}{\pi z} d^2 \mathbf{k}_\perp \Delta^N f_{q/p}(x, \mathbf{k}_\perp) \frac{d\hat{\sigma}^{\ell q \rightarrow \ell q}}{d\hat{t}}(x, \mathbf{k}_\perp) D_{\pi/q}(z) \end{aligned} \quad (22)$$

where  $d\sigma^{unp}$  is the same as in Eq. (12). Again, Eq. (22) holds both for unpolarized and longitudinally polarized leptons: the proton spin has to be orthogonal to the scattering plane.

We proceed in the same way as in Section 1, and Refs. [3, 4], to obtain, in analogy to Eq. (7):

$$\begin{aligned} &\frac{E_\pi d^3 \sigma^{\ell+p^\uparrow \rightarrow \pi+X}}{d^3 \mathbf{p}_\pi} - \frac{E_\pi d^3 \sigma^{\ell+p^\downarrow \rightarrow \pi+X}}{d^3 \mathbf{p}_\pi} \\ &\simeq \sum_q \int \frac{dx}{\pi z} \Delta^N f_{q/p}(x, k_\perp^0) [d\hat{\sigma}^{\ell q \rightarrow \ell q}(x, +\mathbf{k}_\perp^0) - d\hat{\sigma}^{\ell q \rightarrow \ell q}(x, -\mathbf{k}_\perp^0)] D_{\pi/q}(z). \end{aligned} \quad (23)$$

The function  $\Delta^N f_{q/p}(x, k_\perp^0)$  can be found in Refs. [4], where it was derived for  $u$  and  $d$  valence quarks by fitting the data on  $p^\uparrow p \rightarrow \pi X$  (notice that, as commented in Ref. [5], the values of  $N_a$  given in Eq. (9) of Ref. [4] should be multiplied by a factor 4):

$$\Delta^N f_{u/p}(x, k_\perp^0) = \frac{k_\perp^0(x)}{M} 14.72 x^{1.34} (1-x)^{3.58} \quad (24)$$



$$\Delta^N f_{d/p}(x, k_{\perp}^0) = -\frac{k_{\perp}^0(x)}{M} 4.96 x^{0.76} (1-x)^{4.14} \quad (25)$$

with

$$\frac{k_{\perp}^0(x)}{M} = 0.47 x^{0.68} (1-x)^{0.48}. \quad (26)$$

From Eqs. (22)-(26) and (12) we can compute  $A_N$  as hypothetically given by Siverts effect alone; the results, at the same energy and  $p_T$  values of the similar results obtained from Collins effects, Figs. 1-4, are shown in Figs. 9-12.

Also in this case  $A_N$  turns out to be large and detectable. The  $x_F$  dependences and the maximum values are clearly different from those generated by Collins effect:  $A_N$  is smoother and less pronounced for  $\pi^+$  and  $\pi^0$  and steeper and larger for  $\pi^-$ , the opposite of what is shown in Figs. 1-4 where  $\pi^+$  and  $\pi^0$  have a stronger dependence on  $x_F$  and higher absolute values than  $\pi^-$ . In particular Siverts effect would produce the biggest (in magnitude) asymmetries for  $\pi^-$ 's, whereas Collins effect does it for  $\pi^+$ 's.

## 5. Comments and conclusions

Single transverse spin asymmetries have been observed in  $p^{\uparrow}p \rightarrow \pi X$  and  $\bar{p}^{\uparrow}p \rightarrow \pi X$  processes [15] in a kinematical region where the unpolarized cross-section is well described by pQCD and the factorization theorem at leading twist: these measured large values of  $A_N$  are very surprising because the same pQCD and factorization theorem at leading twist predict negligible values of  $A_N$ . They have to be understood and checked with further experiments.

Several possible explanations have been suggested within QCD [1]-[14] or even classical models [23], and indeed phenomenological consistent descriptions of single spin asymmetries in several processes seem now possible and should be pursued in order to understand the basic underlying mechanisms which originate them. We have studied two possible mechanisms [3]-[5], the so-called Siverts and Collins effects, respectively described by the new  $\mathbf{k}_{\perp}$  and spin dependent functions of Eqs. (1) and (2). By fitting the available data we have deduced simple explicit expressions for  $\Delta^N f_{q/p}(x, k_{\perp}^0)$  and  $\Delta^N D_{\pi/q}(z, k_{\perp}^0)$ .

We have considered here the inclusive production of pions in lepton-nucleon interactions in the process  $\ell p^{\uparrow} \rightarrow \pi X$  in complete analogy to  $p^{\uparrow}p \rightarrow \pi X$ . Independently of the details of the proposed mechanisms to generate single spin asymmetries, the mere fact of measuring  $A_N$  in such a process is significant and important. As we said, for several reasons we expect that single spin asymmetry in  $\ell N$  interactions can only originate from single spin effects in the fragmentation of a transversely polarized quark into the final pion; if this is true also for  $p^{\uparrow}p \rightarrow \pi X$ , as some authors think, then  $A_N$  should be of similar nature in  $\ell p^{\uparrow}$  and  $pp^{\uparrow}$  (or  $p^{\uparrow}p$ ) interactions. The different elementary dynamics in the two cases –  $\ell q \rightarrow \ell q$  versus QCD dynamics like  $qq \rightarrow qq$  or  $qg \rightarrow qg$  – should result in (very interesting) differences in the shape of  $A_N$ , but not in the size of its magnitude.

In Figs. 1-4 we have presented our predictions for  $A_N$  in  $\ell p^\uparrow \rightarrow \pi X$  processes exactly under the above assumption: that only Collins effect can be responsible for some single transverse spin dependence. If data agree with our predictions it would confirm this assumption and the consistency of our phenomenological approach to the computations of inclusive single spin asymmetries; if not, it might be that other effects, like Sivers effect, are responsible for  $A_N$  in hadron-hadron collisions. Data might already be available from SMC and SLAC experiments, where nucleons can be transversely polarized, and will be available in the near future from other experiments which have now only longitudinally polarized nucleons.

Single transverse spin asymmetries may be measurable also in the case of longitudinally polarized nucleons provided one looks at the double inclusive process,  $\ell p^\uparrow \rightarrow \ell \pi X$  from which one can reconstruct the  $\gamma^* p^\uparrow \rightarrow \pi X$  reaction, which, in general, occurs in a plane different from the  $\ell\text{-}\ell'$  plane where the longitudinal nucleon spin lies (Fig. 5). This further selection of events - to those such that the proton polarization vector has a significant component perpendicular to the  $\gamma^*\text{-}\pi$  scattering plane - might greatly reduce the experimental available information; also, our predictions, Figs. 7-8, show sizeable values of  $A_N$  only at large  $z$  values, which might be difficult to reach.

Although Sivers effect is not expected to contribute to  $A_N$  in lepton-nucleon interactions, unless some really new and important mechanism allows avoiding problems with time reversal invariance, we have computed  $A_N$  assuming only Sivers effect to be at work, Figs. 9-12; a comparison with the corresponding results originated by Collins effect, Figs. 1-4, shows interesting differences: in particular  $|A_N|$  is biggest for  $\pi^+$  according to Collins mechanism, whereas it is biggest for  $\pi^-$  according to Sivers.

To conclude, we believe that the measurement of  $A_N$  - possibly in the simple and more direct channel  $\ell p^\uparrow \rightarrow \pi X$  - can and should be performed by several experiments; the comparison with existing values of  $A_N$  from  $p^\uparrow p \rightarrow \pi X$  and  $\bar{p}^\uparrow p \rightarrow \pi X$  processes would immediately allow to draw conclusions on possible origins of single spin effects. Once more, spin dependent measurements probe the hadronic structure and the corresponding theories and models at deeper levels than usual unpolarized quantities.

## Acknowledgements

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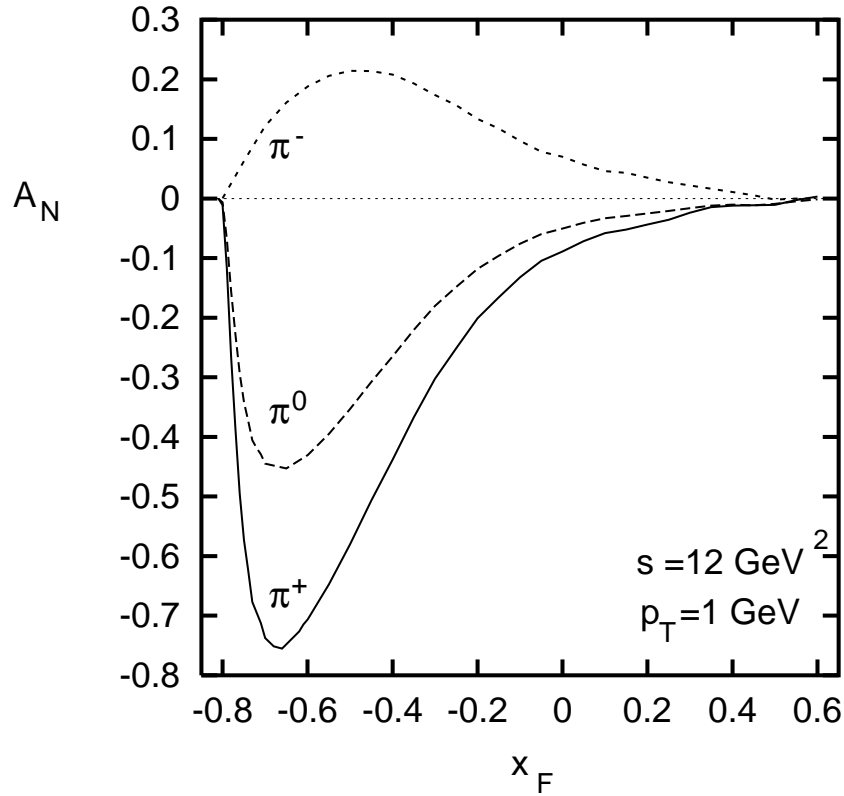


Figure 1: Predictions for the single spin asymmetry  $A_N$  in the DIS process  $lp^\uparrow \rightarrow \pi X$  as a function of  $x_F$ : the solid line refers to  $\pi^+$ , the dashed line to  $\pi^0$  and the dotted line to  $\pi^-$ . We assume that only Collins effect is active and the function  $\Delta^N D(z, k_\perp)$  needed to calculate  $A_N$  is derived from a fit to E704 experimental data on  $p^\uparrow p \rightarrow \pi X$  [5]. The value of  $s$  is chosen as a planned CEBAF value,  $s = 12 \text{ GeV}^2$ , and the transverse momentum of the pion is fixed to  $p_T = 1.0 \text{ GeV}$ .

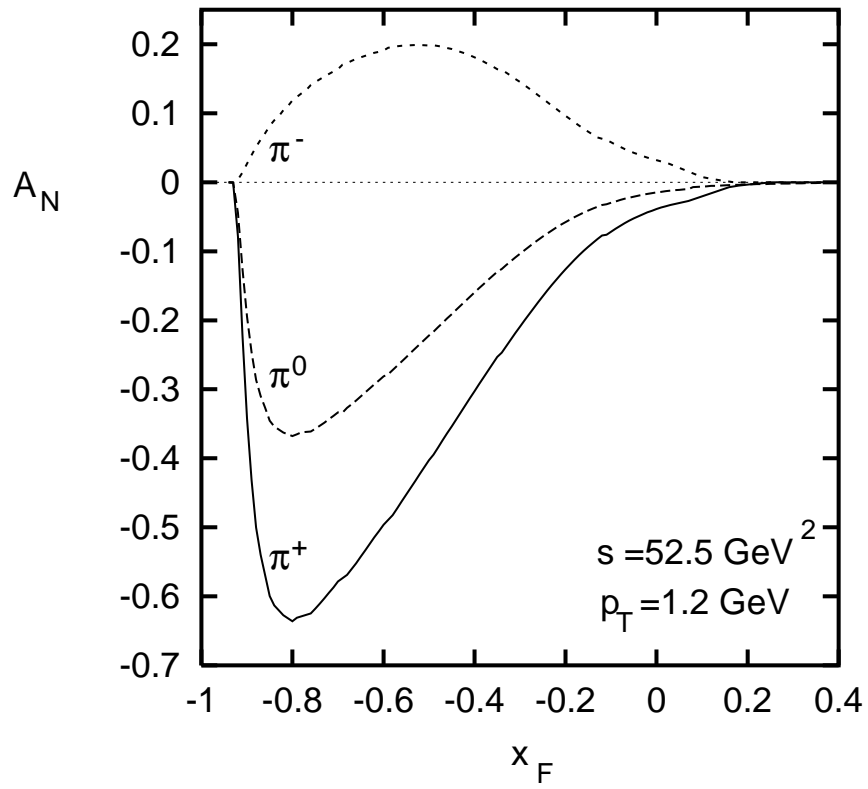


Figure 2: The same as in Fig. 1 for typical kinematical values of HERMES experiment,  $s = 52.5 \text{ GeV}^2$ , and  $p_T = 1.2 \text{ GeV}$ .

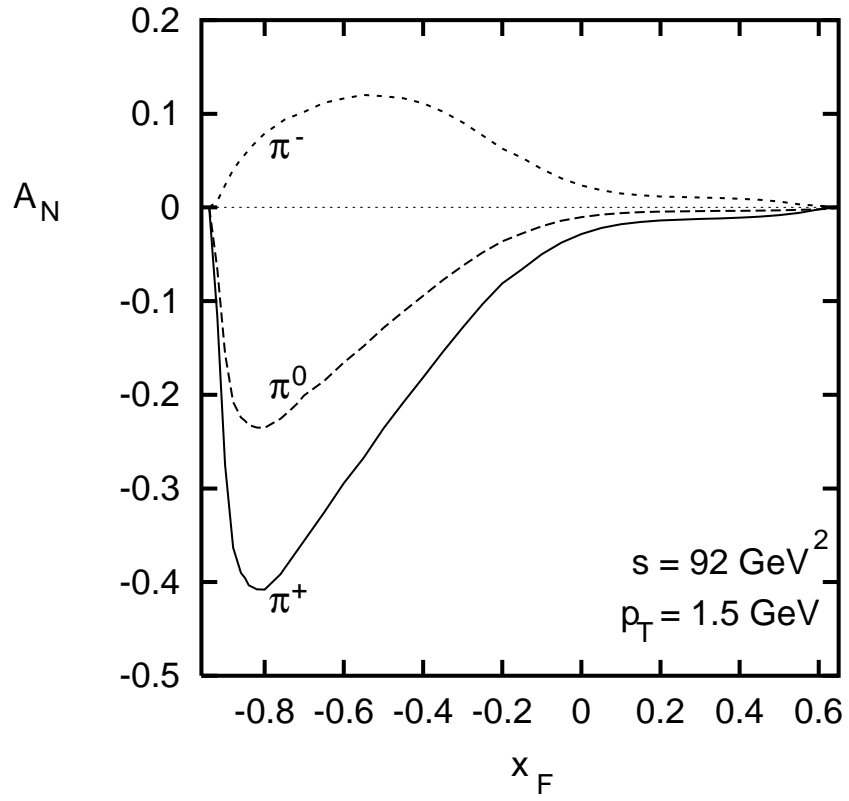


Figure 3: The same as in Fig. 1 at SLAC energy,  $s = 92 \text{ GeV}^2$ , with the transverse momentum of the pion fixed to  $p_T = 1.5 \text{ GeV}$ .

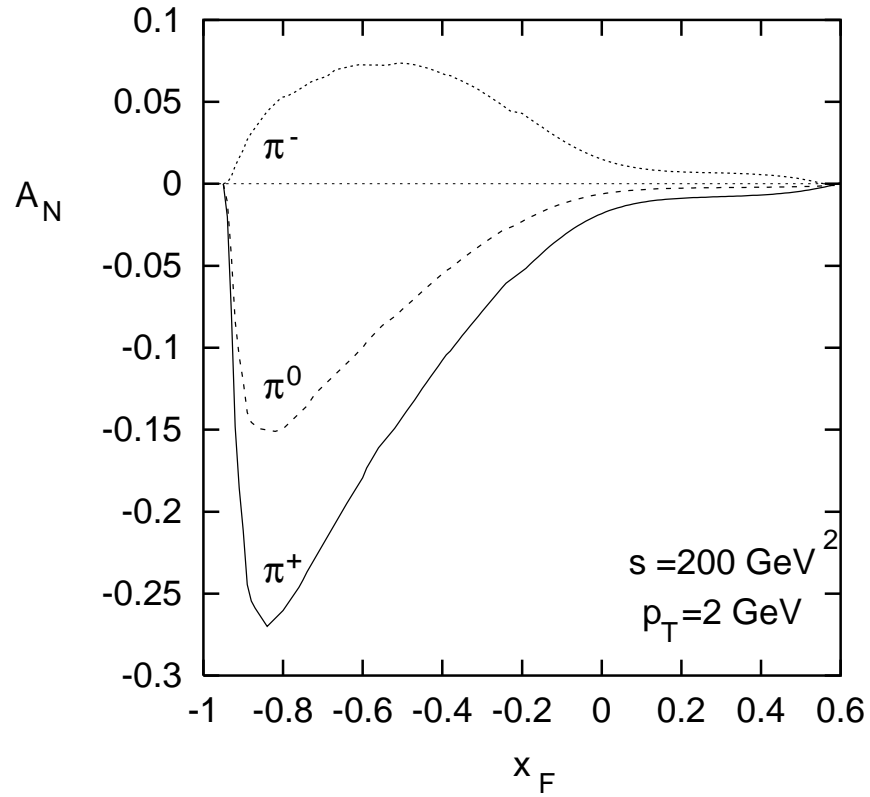


Figure 4: The same as in Fig. 1 for the SMC energy value  $s = 200 \text{ GeV}^2$ , and a transverse momentum of the pion  $p_T = 2 \text{ GeV}$ . Notice that  $s = 400 \text{ GeV}^2$  would yield a very similar result.

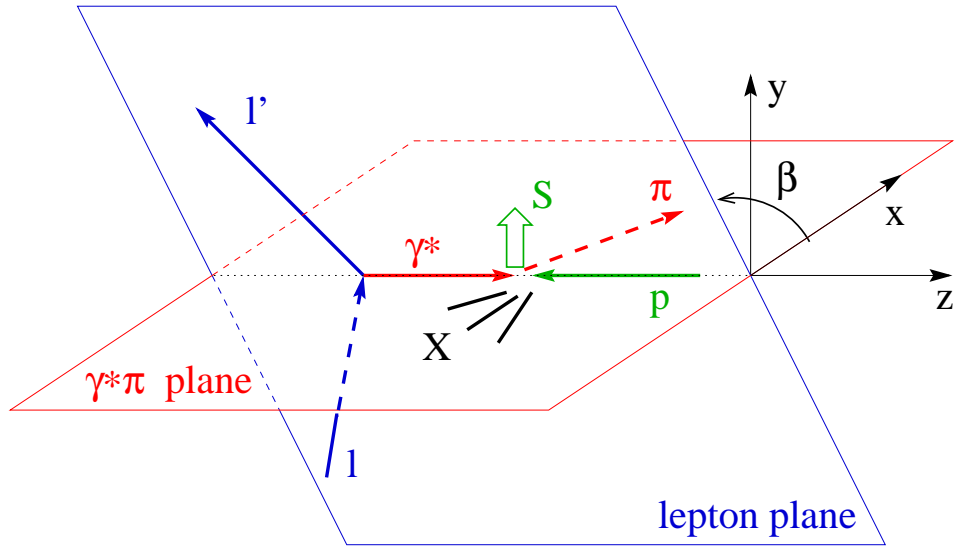


Figure 5: Kinematical and spin configurations adopted for the computation of  $A_N$  in  $\gamma^* p^\uparrow \rightarrow \pi X$  processes, Eq. (20) of text.



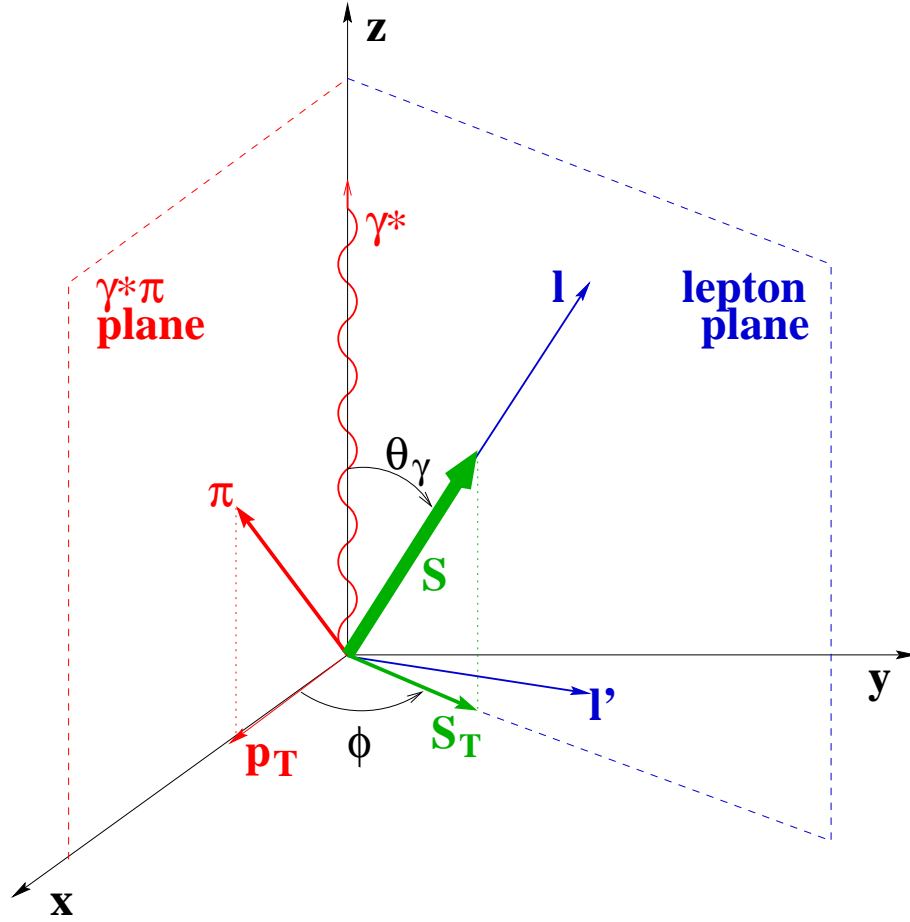


Figure 6: The planes  $\gamma^*$ - $\pi$  and  $l$ - $l'$  in the reference frame where the  $\gamma^*$  moves along the  $z$ -axis and the nucleon at rest is longitudinally polarized, *i.e.* its spin is parallel to the  $l$ -direction of motion. The nucleon spin component perpendicular to the  $\gamma^*$ - $\pi$  plane is  $S \sin \theta_\gamma \sin \Phi = S_T \sin \Phi$  and the Collins angle between  $\mathbf{S}_T$  and  $\mathbf{p}_T$  is  $\Phi$ .

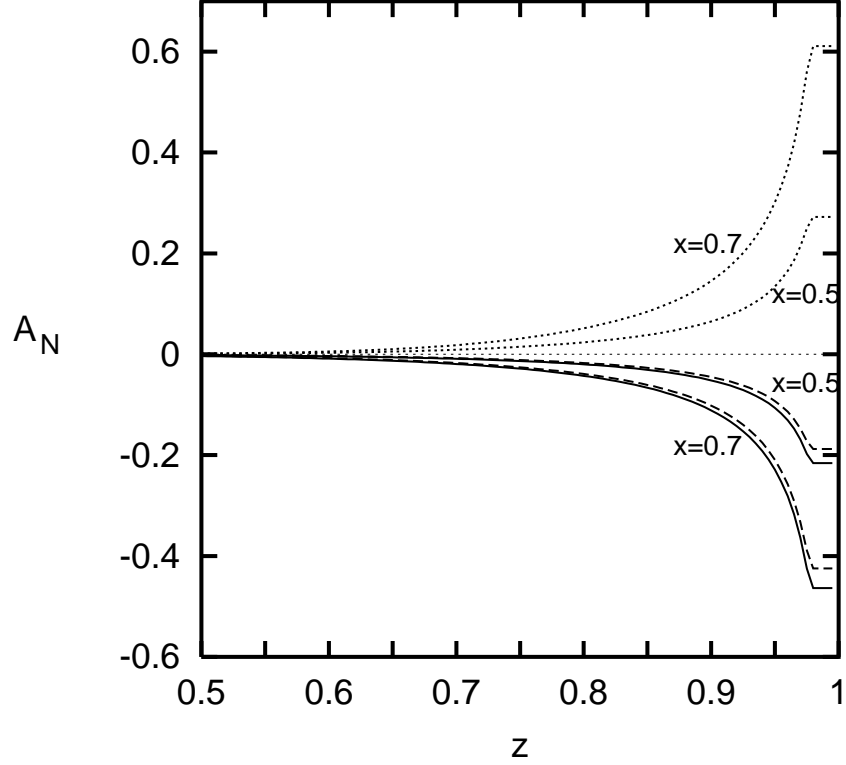


Figure 7: Prediction for the single spin asymmetry  $A_N/|\cos 2\beta|$ , Eq. (20) of text, for pion production in the process  $\gamma^* p^\uparrow \rightarrow \pi X$  as a function of  $z$ . Here we have taken  $Q^2 = 5.0 \text{ GeV}^2$  and  $s = 12 \text{ GeV}^2$ . The two different sets of curves correspond to either  $x = 0.5$  or  $x = 0.7$ . The solid and dashed lines in the negative plane are for  $\pi^+$  and  $\pi^0$  respectively. The dotted curves in the positive plane correspond to  $\pi^-$ . For  $z \geq 0.97742$  the single spin asymmetry is constant because of the constraint  $\Delta^N D = -2D$  [5]. The  $z$ -range has been restricted to  $0.5 - 1.0$  for aesthetic reason only ( $A_N$  is zero between  $z = 0.0$  and  $z = 0.5$ ). Notice how the single spin asymmetry for  $\pi^0$  is almost the same as that for  $\pi^+$ .

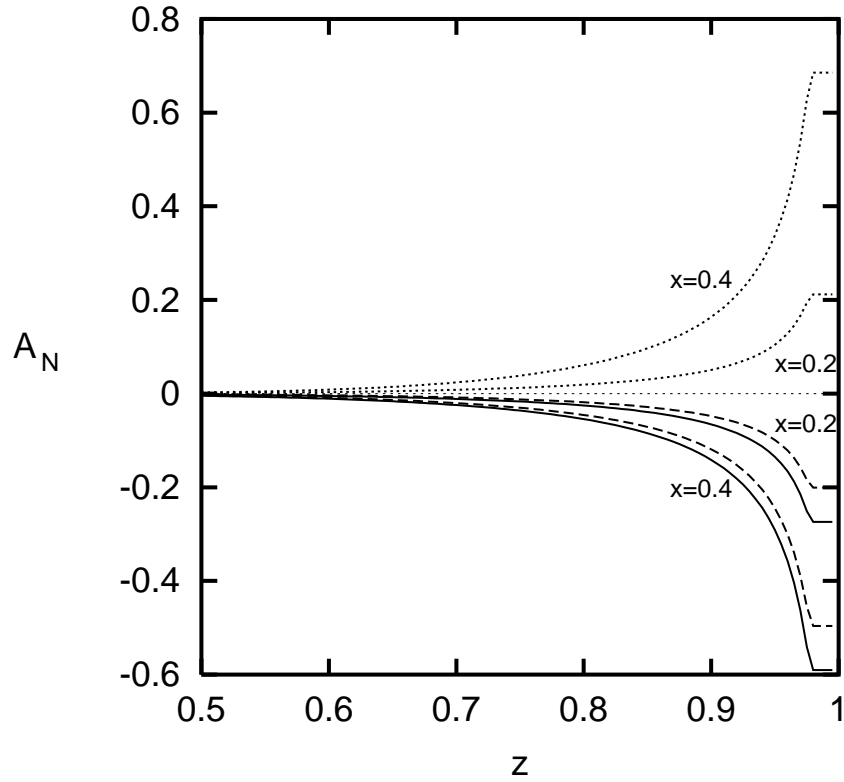


Figure 8: The same as in Fig. 7 with typical HERMES kinematical values:  $Q^2 = 8.0$   $\text{GeV}^2$  and  $s = 52.4$   $\text{GeV}^2$ . The two different sets of curves correspond to either  $x = 0.2$  or  $x = 0.4$  (notice that the cut in  $x$  for HERMES is  $0.02 \leq x \leq 0.4$ ).

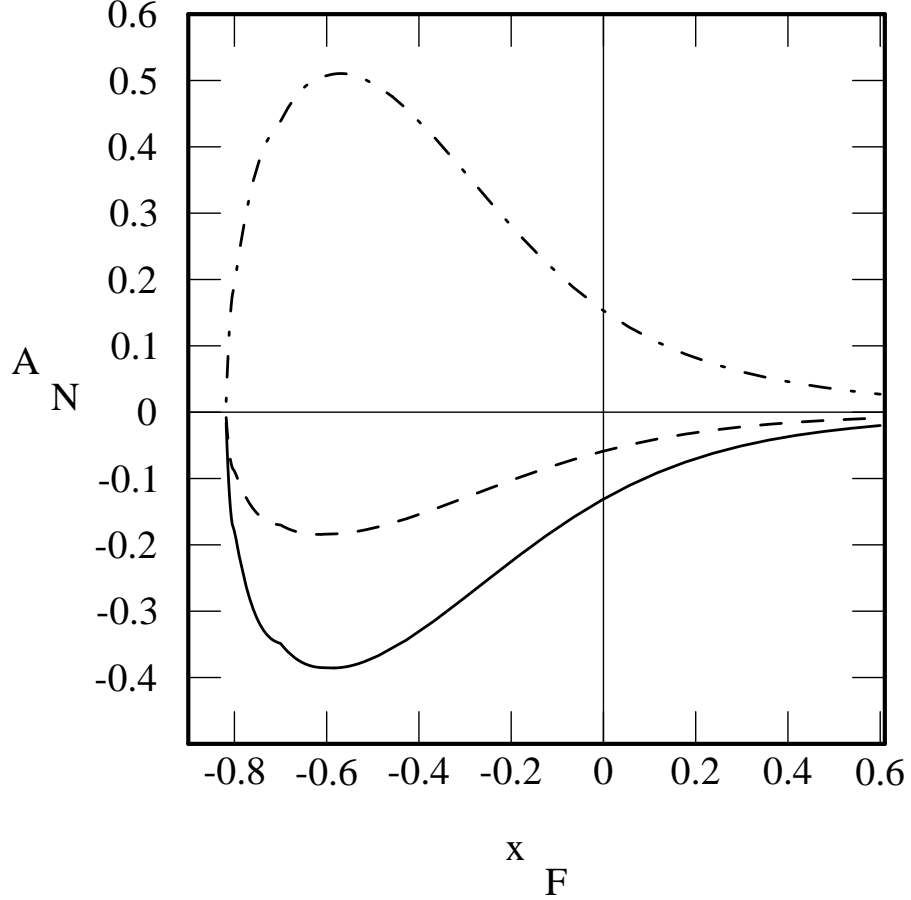


Figure 9: Predictions for the single spin asymmetry  $A_N$  in the DIS process  $lp^\uparrow \rightarrow \pi X$  as a function of  $x_F$ : the solid line refers to  $\pi^+$ , the dashed line to  $\pi^0$  and the dotted-dashed line to  $\pi^-$ . We hypothetically assume that only Siverts effect is active and the function  $\Delta^N f(x, k_\perp)$  needed to calculate  $A_N$  is derived from a fit to E704 experimental data on  $p^\uparrow p \rightarrow \pi X$  [4]. The value of  $s$  is chosen as a planned CEBAF value,  $s = 12 \text{ GeV}^2$ , and the transverse momentum of the pion is fixed to  $p_T = 1.0 \text{ GeV}$ .

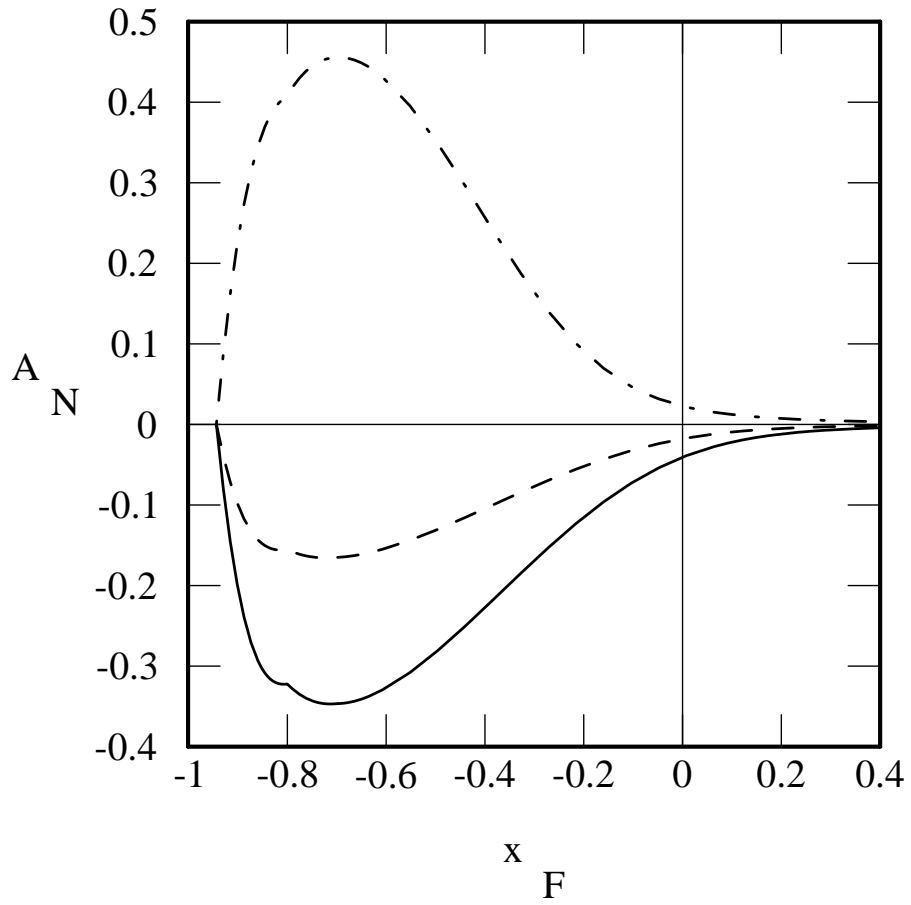


Figure 10: The same as in Fig. 9 for typical kinematical values of HERMES experiment,  $s = 52.5 \text{ GeV}^2$ , and  $p_T = 1.2 \text{ GeV}$ .

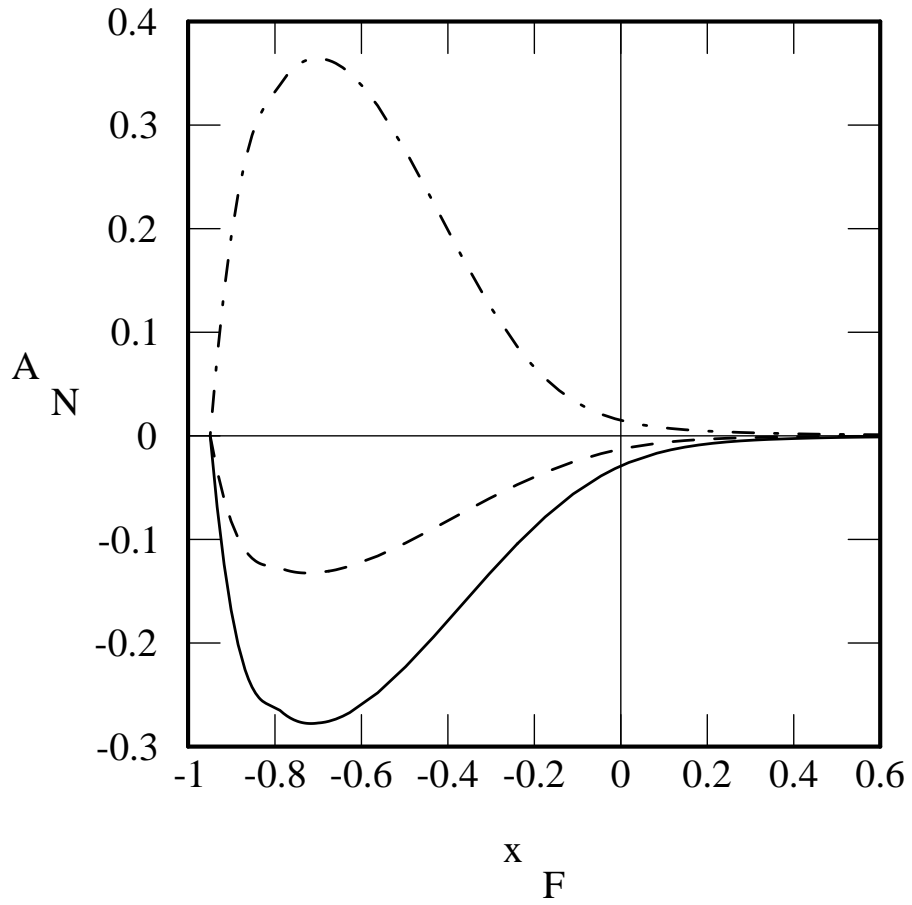


Figure 11: The same as in Fig. 9 at SLAC energy,  $s = 92 \text{ GeV}^2$ , with the transverse momentum of the pion fixed to  $p_T = 1.5 \text{ GeV}$ .

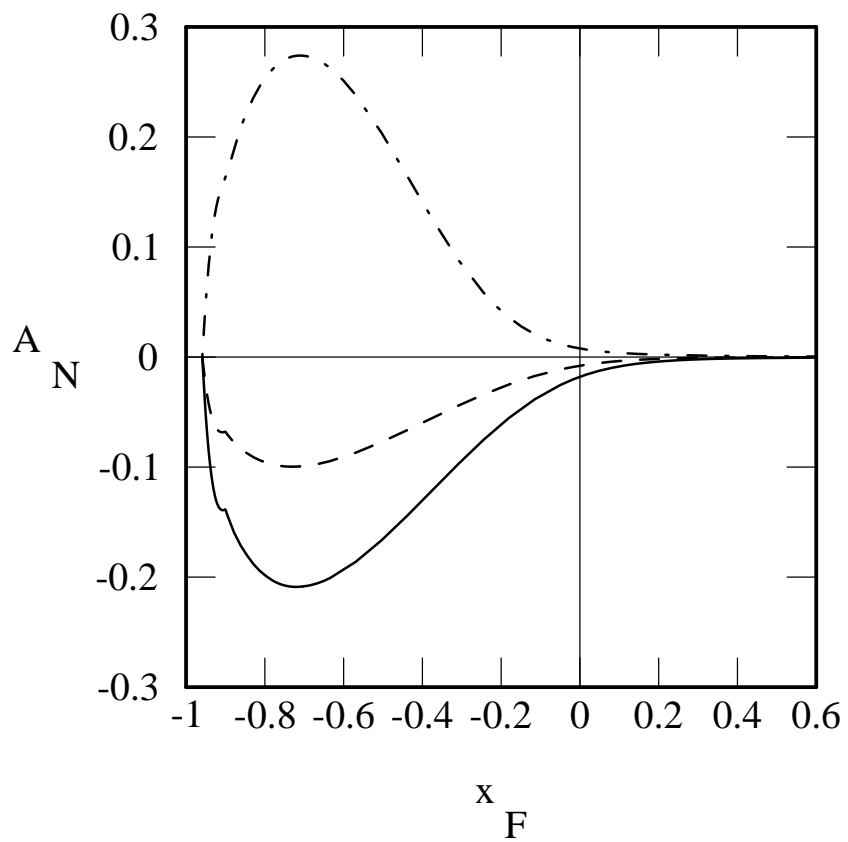


Figure 12: The same as in Fig. 9 for the SMC energy value  $s = 200 \text{ GeV}^2$ , and a transverse momentum of the pion  $p_T = 2 \text{ GeV}$ .