

Statistical Precoder Design for Space-Time-Frequency Block Codes in Multiuser MISO-MC-CDMA Systems

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Abstract—In this paper, we present a space-time-frequency joint block coding (STFBC) scheme to exploit the essential space-time-frequency degrees of freedom of multiuser MISO-MC-CDMA systems. Specifically, we use a series of orthogonal random codes to spread the space time code over several sub-carriers to obtain multi-diversity gains, while multiuser parallel transmission is applied over the same sub-carriers by making use of multiple orthogonal code channels. Furthermore, to improve the system performance, we put forward to linear precoding to the predetermined orthogonal STFBC, including transmitting directions selection and power allocation over these directions. We propose a precoder design method by making use of channel statistical information in time domain based on the Kronecker correlation model for the channels, so feedback amount can be decreased largely in multi-carrier systems. In addition, we give the performance analysis from the perspectives of diversity order and coding gain, respectively. Moreover, through asymptotic analysis, we derive some simple precoder design methods, while guaranteeing a good performance. Finally, numerical results validate our theoretical claims.

Index Terms—Multiuser MISO-MC-CDMA, Space-Time-Frequency Block Code, Optimal Precoder Design, Iterative Power Water-Filling, Channel Statistical Information.

I. INTRODUCTION

The initial works of Telatar [1], Foschini and Gans [2], Tarokh *et al.* [3], and Guey *et al.* [4] have spurred considerable research on multiple-input multiple-output (MIMO) systems. As proved by previous literatures, MIMO systems can improve spectrum efficiency, enhance link reliability, or a combination of both by deploying multiple antennas at transmitter or receiver. Meanwhile, with the increase of the demand on data service, orthogonal frequency division multiplexing (OFDM), as a promising transmission technique, has been adopted by several broadband communication standards, such as LTE-A and 802.16m. Therefore, MIMO-OFDM is considered as a potential choice for future wireless broadband communication systems [5] [6].

For MIMO-OFDM for high-speed wireless communications, frequency resource has not been fully exploited. Loosely speaking, the independent fading property of sub-carriers has not been utilized effectively. In addition, for cellular systems, MIMO-OFDM suffers from the interference of the

same sub-carrier from adjacent cells. Fortunately, MIMO-MC-CDMA [7]-[9] has the ability of obtaining frequency gain and mitigating intercell interference simultaneously. Specifically, MC-CDMA spreads each symbol to be transmitted, by an orthogonal random code, over several sub-carriers according to the variable orthogonal spreading factor (VOSF) before inverse fast Fourier transform (IFFT). At the receiver, the same symbols over different sub-carriers are combined by a certain mean to enhance the transmission reliability. Through allocating an orthogonal identified code (OID) for each cell, the interference from adjacent cells can also be avoided directly. Hence, MIMO-MC-CDMA is another feasible choice for broadband wireless communication systems.

Apart from frequency resource, spatial and temporal degrees of freedom for MIMO-MC-CDMA can also be utilized to further improve diversity gain by making use of special space time signaling schemes, such as space time block coding. In [10], the space-time block coded MIMO-MC-CDMA system was studied, and a Bayesian monte carlo multiuser receiver was proposed to improve the overall performance. Moreover, the performance of the space-time block coded MIMO-MC-CDMA system was analyzed, and the closed-form symbol error probabilities over Nakagami- m fading channels was presented in [11]. Furthermore, channel-independent precoding were introduced into MC-CDMA systems to improve the performance [12]-[14].

It is proved that linear precoding by exploiting the channel state information (CSI) at the transmitter can significantly improve system performance and reduce receive complexity. Specifically, linear precoding mainly includes transmission direction selection in spatial domain and power allocation under a given constraint based on the available CSI [15]-[17]. In [18], the transmitter beamforming scheme combining adaptive modulation based on the full CSI is proposed for the space-time block coded OFDM. However, for a FDD system, it is difficult to convey the full CSI from the receiver to the transmitter. Especially in multi-carrier systems, the CSI feedback amount is unbearable. To satisfy the constraint of the amount of feedback, linear precoding based on a finite-resolution quantization codebook, such as Grassmann codebook [19] [20] and vector quantization (VQ) codebook [21] [22], becomes a common solution in single-carrier systems. In [23], the design of Grassmann codebook in the MIMO system employing space-time codes has been well studied. Furthermore, [24] extended Grassmann codebook design to the irregular MIMO system. In [25], the performance of VQ code-

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book in the multiuser multi-antenna system was investigated. Although linear precoding based on quantization codebook can achieve a proper tradeoff between system performance and feedback amount, it is unsuitable to multi-carrier system, since the sum of feedback amount over all sub-carriers is unbearable for a real system. In order to reduce the feedback bits, in [26], an interpolation-based precoding strategy for multi-carrier systems with limited feedback was proposed, which obtains CSI of all sub-carriers via linear weight sum of those of limited sub-carriers according to the correlation of frequency domain. Instead of full feedback, this method reduces the feedback amount at the cost of system performance. In addition, a subspace trace method based on Grassmann manifold theory [27] can be used to solve the problem in multi-carrier systems, but it requires a magnitude of computations to gain the varied direction of subspace. More importantly, codebook based precoding is unfit for the future broadband mobile communication under some conditions, because the CSI may be outdated when the transmitter acquires it through feedback link due to fast channel variation, resulting in performance loss [28]. Fortunately, channel statistical information, e.g. channel mean or correlation, varies slowly and can be used to design the optimal precoder [29] [30]. Moreover, a precoder design method based on channel angular domain for distributed antenna systems was proposed in [31] by minimizing the pairwise error probability of space time codes. However, these works mainly considered the precoder design in single carrier MIMO systems, which is unsuitable for multi-carrier systems. Specifically, precoder design in single-carrier MIMO systems is based on the statistical information in time domain. However, in multi-carrier MIMO systems, such as MIMO-MC-CDMA systems, the precoder is designed using the frequency domain information. Note that it is impractical to estimate the statistical information over all sub-carriers. A feasible way is to build the relation between the time domain and frequency domain statistical information. To the best of the authors' knowledge, it is still an open issue to pose the statistical information of all sub-carrier from those of time domain multi-path channel in multi-carrier systems.

In this paper, we consider a general multiuser MISO-MC-CDMA system. We combine linear precoding and orthogonal STFBC to effectively improve the performance of MISO-MC-CDMA systems. The major contributions of this paper can be summarized as follows:

- 1) We build an analytical framework for channel statistical information in time domain over each sub-carrier based on Kronecker correlation model.
- 2) We obtain the optimal precoder design method based on the statistical information by minimizing the PEP of orthogonal STFBC. Meanwhile, we also present an iterative water-filling power allocation algorithm, which can further optimize the performance with respect to equal power allocation and single beam allocation.
- 3) We get simple precoder design methods without performance loss in some special scenarios through asymptotic analysis as follows:
 - a) When the number of delay taps, the length of spread

codes and the transmit SINR are large, equal power allocation can achieve the asymptotic optimal performance.

- b) When the SINR is low, distributing all the power to the spatial sub-channel with the largest gain is asymptotically optimal.

The rest of this paper is organized as follows. Section II provides a general overview of the MISO-MC-CDMA system under consideration. In Section III, we focus on the precoder design using channel statistical information. For some special cases, we give the corresponding precoder design methods with low complexity in Section IV. Simulation results are discussed and analyzed in Section V and we conclude the whole paper in Section VI.

Notation: We use bold upper (lower) letters to denote matrices (column vectors), $(\cdot)^*$ to denote conjugate transpose, $(\cdot)^T$ to denote matrix transpose, \mathbf{I}_k to denote the $k \times k$ identity matrix, $E[\cdot]$ to denote expectation, $\lambda_i\{\mathbf{A}\}$ to denote the i th largest singular values of \mathbf{A} , $\|\cdot\|_2$ to denote the matrix l_2 -norm, $\|\cdot\|_F$ to denote the matrix Frobenius norm, $\text{tr}(\cdot)$ to denote matrix trace, \sim to denote equality in distribution, \otimes to denote Kronecker product operator, and \succeq to denote the matrix positive semi-definite relation. The acronym i.i.d. means “independent and identically distributed” and pdf means “probability density function”.

II. SYSTEM MODEL

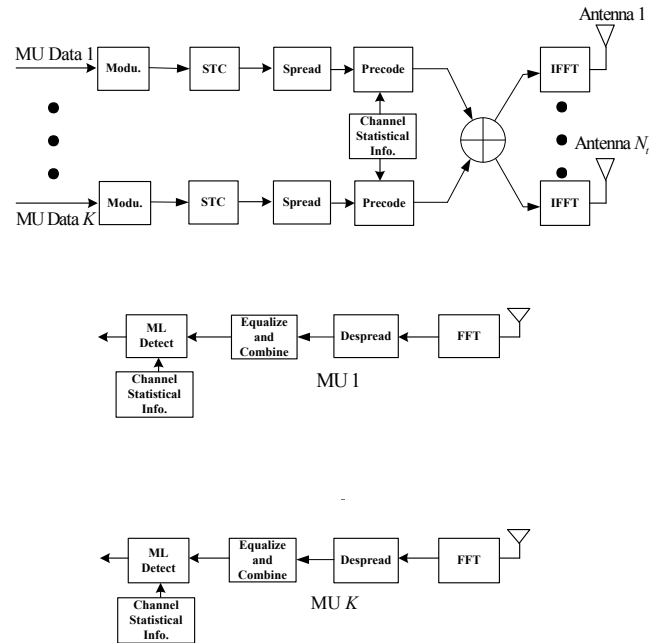


Fig. 1. A block diagram for MIMO-MC-CDMA systems with STFBC.

Consider a multiuser MISO-MC-CDMA downlink, where the base station (BS) has N_t transmit antennas, and K mobile users (MU) equip with one receive antenna each, as illustrated in Fig.1. MISO-MC-CDMA is a multiplexing technique which permits multiple users to access the wireless channel simultaneously by modulating and spreading

their input data signals across the frequency domain using different spreading sequences. Therefore, MISO-MC-CDMA combines the advantages of MISO, OFDM and CDMA indeed. For the k th MU, its data is first mapped into a series of modulated symbols by a modulator. Then, the consecutive M modulated symbols are used to construct a space time code \mathbf{X}_k of size $M \times T$. For analytical convenience, we consider orthogonal space block codes in this paper. Next, the space time code is spread to N_v parallel codes by a VOSF $\mathbf{c}_k = \{c_{k,1}, c_{k,2}, \dots, c_{k,N_v}\}$ of length N_v . After the OFDM modulation with N_c sub-carriers, the N_v spread space time codes are mapped into N_v different sub-carriers with large spacing. Based on the statistical information of the sub-carrier, the corresponding spread space time code is precoded by the precoding matrix $\mathbf{F}_{k,i}$ for $i = 1, \dots, N_v$. Due to the VOSF of MIMO-MC-CDMA, multiple MUs' precoded space time codes can be transmitted on a carrier simultaneously, so the spectrum efficiency can be improved effectively. In this paper, for ease of analysis, we assume each sub-carrier carries N_u MUs' precoded space time codes. Finally, the space time codes are transmitted by using T time slots. Assume the channel is block fading and ergodic, in other words, the channel keeps constant during a space time code of length T time slots and fades independently between codes. It is noticed that, due to the homogeneous channels of different MUs, we normalize the path-dependent fading, including path loss and shadow fading. Without loss of generality, we focus on investigating the design of optimal precoding matrix of MU 0 and analyzing the corresponding performance throughout this paper. Thus, the baseband frequency domain output signal of the 0th MU on the i th sub-carrier can be expressed as

$$\mathbf{Y}_{0,i} = \sqrt{\rho} \mathbf{H}_{0,i} \mathbf{F}_{0,i} c_{0,i} \mathbf{X}_0 + \sqrt{\rho} \sum_{k=1}^{N_u-1} \mathbf{H}_{0,i} \mathbf{F}_{k,i} c_{k,i} \mathbf{X}_k + \mathbf{N}_{0,i}, \quad (1)$$

where ρ is the total transmit power, $\mathbf{F}_{0,i}$ is the corresponding precoding matrix designed based on channel statistical information. As mentioned above, $\mathbf{F}_{0,i}$ can be expressed as the product of a $N_t \times M$ precoding direction matrix $\mathbf{V}_{0,i}$ that is used to specify the waveform of space time code and a $M \times M$ positive semi-definite diagonal power allocation matrix $\mathbf{D}_{0,i}$, \mathbf{X}_0 and $\mathbf{Y}_{0,i}$ are $M \times T$ transmitted and $1 \times T$ received space time codes (STC), respectively. $\mathbf{N}_{0,i}$ is $1 \times T$ received noise vector with i.i.d entries $\sim \mathcal{CN}(0, \sigma_n^2)$. $c_{0,i}$ is the spread code, and N_u is the number of users on one sub-carrier. Assume the VOSF for all MU is N_v , viz. the same STC is spread over N_v sub-carriers with orthogonal random code of length N_v . Thus, we have

$$\sum_{i=1}^{N_v} c_{k,i} c_{l,i} = \begin{cases} 0, & k \neq l, \\ N_v, & k = l, \end{cases}$$

subject to that N_c/N_v is an integer in order to effectively utilize all the sub-carriers. $\mathbf{H}_{0,i}$ is the N_t dimensional channel response vector from the BS to MU 0 over the i th sub-carrier. In this paper, we consider a frequency-selective MIMO channel with L resolvable delay taps in time domain, which can

be expressed as

$$\mathbf{G}_0(t) = \sum_{l=0}^{L-1} \mathbf{G}_{0,l} \delta(t - \tau_l), \quad (2)$$

where $\mathbf{G}_{0,l}$ is the N_t dimensional channel response vector of the l th path in time domain, which has the corresponding propagation delay τ_l . In particular, $\mathbf{G}_{0,l}$ is a zero mean complex Gaussian random vector with the correlation matrix \mathbf{R}_0 , namely, $\mathbf{G}_{0,l}$ can be expressed as

$$\mathbf{G}_{0,l} = \mathbf{G}_{\text{i.i.d}} \mathbf{R}_0^{1/2}, \quad l = 0, \dots, L-1,$$

where $\mathbf{G}_{\text{i.i.d}}$ is a random vector $\sim \mathcal{CN}(0, \mathbf{I}_{N_t})$. Thus, the frequency response of the n th sub-carrier ($0 \leq n \leq N_c$) can be written as

$$\begin{aligned} \mathbf{H}_{0,n} &= \sum_{l=0}^{L-1} \mathbf{G}_{0,l} \exp(-j2\pi n l / N_c) \\ &= \mathbf{G}_{0,\text{path}} \cdot (\mathbf{W}_n \otimes \mathbf{I}_{N_t}), \end{aligned} \quad (3)$$

where $\mathbf{G}_{0,\text{path}} = [\mathbf{G}_{0,0}, \dots, \mathbf{G}_{0,L-1}]$ and $\mathbf{W}_n = [\exp(-j2\pi n \times 0 / N_c), \dots, \exp(-j2\pi n \times (L-1) / N_c)]^T$. As seen in (3), $\mathbf{H}_{0,n}$ is a sum of $\mathbf{G}_{0,l}$ with linear weight, so that $\mathbf{H}_{0,n}$ is also a zero mean complex Gaussian random vector, whose correlation matrix can be casted as

$$\begin{aligned} \mathbf{R}_{0,n} &= E[\mathbf{H}_{0,n}^H \mathbf{H}_{0,n}] \\ &= (\mathbf{W}_n \otimes \mathbf{I}_{N_t})^H E[\mathbf{G}_{0,\text{path}}^H \mathbf{G}_{0,\text{path}}] (\mathbf{W}_n \otimes \mathbf{I}_{N_t}) \quad (4) \\ &= (\mathbf{W}_n \otimes \mathbf{I}_{N_t})^H (\mathbf{R}_{0,\text{tap}}^T \otimes \mathbf{R}_0) (\mathbf{W}_n \otimes \mathbf{I}_{N_t}) \quad (5) \\ &= (\mathbf{W}_n^H \mathbf{R}_{0,\text{tap}}^T \mathbf{W}_n) \otimes \mathbf{R}_0 \quad (6) \\ &= \mathbf{W}_n^H \mathbf{R}_{0,\text{tap}}^T \mathbf{W}_n \mathbf{R}_0, \quad (7) \end{aligned}$$

where $\mathbf{R}_{0,\text{tap}}$ is the tap correlation matrix. In this paper, we assume that tap correlation is independent of transmit correlation. Equation (6) uses the Kronecker product property that $(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = (\mathbf{AC}) \otimes (\mathbf{BD})$. For analytical simplicity, we assume that the taps are independent of each other, namely, $\mathbf{R}_{0,\text{tap}} = \mathbf{I}_L$. In fact, if each path has its unique propagation environment, it is reasonable to assume the taps are uncorrelated. Under this condition, we get an interesting result that the correlation matrices of all sub-carrier are identical for an arbitrary MU. Therefore, the distribution of the n th sub-carrier can be written as

$$\mathbf{H}_{0,n} \sim \mathcal{CN}(0, L\mathbf{R}_0). \quad (8)$$

Based on channel statistical information in time domain, such as channel mean, channel covariance, and tap correlation matrices, which can be obtained by averaging a number of channel realization, the statistical information of all sub-carriers can be figured out easily. Due to its slow variety, channel statistical information keeps constant during a relatively long time, which promises its advantage over instantaneous channel information for future broadband wireless communication systems with limited feedback. In this paper, we assume perfect channel statistical information at the BS and MUs.

As seen in (1), the first term in R.H.S is the expected received signal, the middle term is the other $N_u - 1$ MUs'

interference on the same sub-carrier, and the third term is the received noise. To enhance the transmission reliability, namely increasing received SINR, it is imperative to suppress the multi-user interference (MUI). Inspired by the orthogonality of spread code between MUs on the same set of sub-carriers, we first multiply the received signal by the corresponding MU's spread code to filter MUI, then combine the same transmitted signals on N_v different sub-carriers. Thus, the combined signal of the 0th MU can be written as

$$\begin{aligned} \bar{\mathbf{Y}}_0 &= \sqrt{\rho} \mathbf{X}_0 \sum_{i=0}^{N_v-1} \mathbf{H}_{0,i} \mathbf{F}_{0,i} \\ &+ \sqrt{\rho} \sum_{k=1}^{N_u-1} \mathbf{X}_k \sum_{i=0}^{N_v-1} \mathbf{H}_{0,i} \mathbf{F}_{k,i} c_{k,i} c_{0,i} + \bar{\mathbf{N}}_0, \end{aligned} \quad (9)$$

where $\bar{\mathbf{N}}_0 = \sum_{i=0}^{N_v-1} c_{0,i} \mathbf{N}_{0,i}$ is the combined noise with i.i.d. entries $\sim \mathcal{CN}(0, N_v \sigma_n^2)$. Because of frequency selective fading, the orthogonality of code channel is destroyed, which results in that MUI can not be canceled completely. One feasible solution to combat residual MUI is to adopt equalization technique before combination, in (9), we use equal gain combination (EGC) instead of other complicated methods due to its simplicity.

After EGC in frequency domain, maximal-likelihood (ML) detection is performed to obtain

$$\hat{\mathbf{X}}_0 = \arg \min_{\mathbf{X}_0 \in \mathcal{X}} \left\| \bar{\mathbf{Y}}_0 - \sqrt{\rho} \mathbf{X}_0 \sum_{i=0}^{N_v-1} \mathbf{H}_{0,i} \mathbf{F}_{0,i} \right\|_F, \quad (10)$$

where \mathcal{X} is the STC codebook. According to the law of large number (LLN), if the term $N_v N_u$ is large enough, the second term in (9), namely MUI, can be considered as a zero mean Gaussian random variable with the covariance of σ_{MUI}^2 . Intuitively, σ_{MUI}^2 is a function of N_u , N_v and ρ . σ_{MUI}^2 can be computed by statistical average on a large number of samples, for given N_u , N_v and ρ . Thus, for a given channel realization $\mathbf{H}_0 = [\mathbf{H}_{0,1}, \dots, \mathbf{H}_{0,N_v}]$, with ML detection in (10) and applying the Chernoff bound, the PEP of precoded STFBC in MIMO-MC-CDMA systems can be tightly upper-bound [32] by (11) at the top of next page.

The focus of this paper is on the design of the optimal precoder $\mathbf{F}_{0,i}, i = 0, \dots, N_v$ at the BS based on channel statistical information. The precoder can be divided into two components: the $N_t \times M$ unitary matrix $\mathbf{V}_{0,i}$ and the $M \times M$ diagonal positive semi-definite matrix $\mathbf{D}_{0,i}$. The matrix $\mathbf{V}_{0,i}$, as the precoding direction information, serves to adapt the predetermined space time codeword to the current channel condition. Given the optimization goal, e.g. minimizing the upper bound of PEP in this paper, the matrix $\mathbf{V}_{0,i}$ is a function of channel statistical information on the i th sub-carrier. As another important precoding information, the matrix $\mathbf{D}_{0,i}$ is used to allocate transmit power for the basis-beams of the precoding direction matrix to further improve system performance.

III. OPTIMAL PRECODING FOR 3D BLOCK CODE

In this section, we investigate the precoder design for 3D block codes (STFBC) in a multiuser MISO-MC-CDMA

system based on channel statistical information in detail. In short, by minimizing the tight upper bound of average PEP of precoded 3D block code, we design the optimal precoding direction matrix and present some effective power allocation algorithms.

A. Optimization Objective

Prior to discussing the design of the optimal precoder, we first put the attention on the performance criterion. As mentioned above, we take the average PEP of precoded 3D block code as the optimization objective. As seen in (11), given the channel realization, the upper bound can be expressed as (12) at the top of next page, where $\eta = \frac{\rho}{4(N_v \sigma_n^2 + \sigma_{MUI}^2)}$, and

$\mathbf{A}_0 = (\mathbf{X}_0 - \hat{\mathbf{X}}_0)(\mathbf{X}_0 - \hat{\mathbf{X}}_0)^H$ is the codeword distance product matrix. (12) results from the fact that the precoder on an arbitrary sub-carrier is just a function of channel statistical information and the space time codeword to the transmitted, thus, based on the obtained conclusion above, the precoders on all sub-carriers $\mathbf{F}_{0,i}$ are identical for a MU, namely \mathbf{F}_0 . As seen in (8), the pdf of sub-carrier i can be expressed as

$$f(\mathbf{H}_{0,i}) = \frac{\exp(-\text{tr}(\mathbf{H}_{0,i} \mathbf{M}_0^{-1} \mathbf{H}_{0,i}^H))}{\pi^{N_t} \det(\mathbf{M}_0)}, i = 0, \dots, N_v - 1, \quad (13)$$

where $\mathbf{M}_0 = L \mathbf{R}_0$ is the common covariance matrix of MU 0 on all N_v chosen sub-carriers and R_0 is the correlation matrix in time domain. Since the frequency interval between adjacent sub-carriers is greater than correlation bandwidth, it is reasonable to assume that the distributions of these sub-carriers are independent of each other. Therefore, the pdf of $\Phi = \sum_{i=0}^{N_v-1} \mathbf{H}_{0,i}$ is also a complex Gaussian pdf, given by

$$f_\Phi(\mathbf{Z}_0) = \frac{\exp(-\text{tr}(\mathbf{Z}_0 \mathbf{P}_0^{-1} \mathbf{Z}_0^H))}{\pi^{N_t} \det(\mathbf{P}_0)}, \quad (14)$$

where $\mathbf{P}_0 = L N_v \mathbf{R}_0$. Averaging (12) over (14), we obtain the following bound on the average PEP:

$$\begin{aligned} \bar{P} &= \int_{\Phi} P(\mathbf{Z}_0) f_\Phi(\mathbf{Z}_0) d\mathbf{Z}_0 \\ &= \frac{\int_{\Phi} \exp(-\text{tr}(\mathbf{Z}_0 (\eta \mathbf{F}_0 \mathbf{A}_0 \mathbf{F}_0^H + \mathbf{P}_0^{-1}) \mathbf{Z}_0^H)) d\mathbf{Z}_0}{\pi^{N_t} \det(\mathbf{P}_0)} \\ &= \frac{\det(\mathbf{Q}_0)}{\det(\mathbf{P}_0)} \int_{\Phi} \frac{\exp(-\text{tr}(\mathbf{Z}_0 \mathbf{Q}_0^{-1} \mathbf{Z}_0^H))}{\pi^{N_t} \det(\mathbf{Q}_0)} d\mathbf{Z}_0 \end{aligned} \quad (15)$$

$$= \det(\mathbf{I}_{N_t} + \eta L N_v \mathbf{F}_0 \mathbf{A}_0 \mathbf{F}_0^H \mathbf{R}_0)^{-1}, \quad (16)$$

where $\mathbf{Q}_0 = (\eta \mathbf{F}_0 \mathbf{A}_0 \mathbf{F}_0^H + \mathbf{P}_0^{-1})^{-1}$, (16) is a result of that $\int_{\Phi} \frac{\exp(-\text{tr}(\mathbf{Z}_0 \mathbf{Q}_0^{-1} \mathbf{Z}_0^H))}{\pi^{N_t} \det(\mathbf{Q}_0)} d\mathbf{Z}_0$ is the integral of a complex Gaussian pdf and thus equals to one. Now, we have got the exact expression of the upper bound of the average PEP. In next sections, we start to investigate the design of optimal precoder in the sense of statistical average.

B. Optimal Precoder Design

With the obtained upper bound of the average PEP in (16), we construct the optimal precoder \mathbf{F}_0 , including

$$P(\mathbf{X}_0 \rightarrow \hat{\mathbf{X}}_0 | \mathbf{H}_0) \leq \exp \left(- \frac{\rho \left\| \sum_{i=0}^{N_v-1} \mathbf{H}_{0,i} \mathbf{F}_{0,i} (\mathbf{X}_0 - \hat{\mathbf{X}}_0) \right\|_F^2}{4(N_v \sigma_n^2 + \sigma_{MUI}^2)} \right). \quad (11)$$

$$\begin{aligned} P \left(\sum_{i=0}^{N_v-1} \mathbf{H}_{0,i} \right) &= \exp \left(- \frac{\rho \left\| \sum_{i=0}^{N_v-1} \mathbf{H}_{0,i} \mathbf{F}_{0,i} (\mathbf{X}_0 - \hat{\mathbf{X}}_0) \right\|_F^2}{4(N_v \sigma_n^2 + \sigma_{MUI}^2)} \right) \\ &= \exp \left(- \eta \text{tr} \left(\left(\sum_{i=0}^{N_v-1} \mathbf{H}_{0,i} \right) \mathbf{F}_0 \mathbf{A}_0 \mathbf{F}_0^H \left(\sum_{i=0}^{N_v-1} \mathbf{H}_{0,i} \right)^H \right) \right). \end{aligned} \quad (12)$$

precoding direction matrix \mathbf{V}_0 and power allocation matrix \mathbf{D}_0 . As seen in (16), the optimal precoder is a function of channel correlation matrix \mathbf{R}_0 and space time codeword distance matrix \mathbf{A} . Thus, the design objective can be expressed as

$$\mathbf{F}_0 = \arg \max_{\mathbf{F}_0 \in \mathbf{C}^{N_t \times M}} \det(\mathbf{I}_{N_t} + \eta L N_v \mathbf{F}_0 \mathbf{A}_0 \mathbf{F}_0^H \mathbf{R}_0), \quad (17)$$

where $\mathbf{C}^{N_t \times M}$ denotes the set of complex matrices of size $N_t \times M$.

As defined above, the precoder \mathbf{F}_0 can be written as the following form via singular value decomposition (SVD)

$$\mathbf{F}_0 = \mathbf{V}_0 \mathbf{D}_0 \mathbf{U}_0^H, \quad (18)$$

where \mathbf{U}_0 is an $M \times M$ unitary matrix. Similarly, we give the SVD terms of \mathbf{A}_0 and \mathbf{R}_0 as

$$\mathbf{A}_0 = \mathbf{V}_{\mathbf{A},0} \mathbf{D}_{\mathbf{A},0} \mathbf{V}_{\mathbf{A},0}^H, \quad (19)$$

$$\mathbf{R}_0 = \mathbf{V}_{\mathbf{R},0} \mathbf{D}_{\mathbf{R},0} \mathbf{V}_{\mathbf{R},0}^H, \quad (20)$$

where $\mathbf{V}_{\mathbf{A},0}$ and $\mathbf{V}_{\mathbf{R},0}$ are $M \times M$ and $N_t \times N_t$ unitary matrices, respectively. $\mathbf{D}_{\mathbf{A},0}$ and $\mathbf{D}_{\mathbf{R},0}$ are the corresponding $M \times M$ and $N_t \times N_t$ positive semi-definite diagonal matrices, respectively. It is well known that orthogonal space time block code (OSTBC) is appealing in multi-antenna system due to its lower decoding complexity. In what follows, we discuss the design of optimal precoder based on such a code.

For OSTBC, it has the appealing property that

$$\mathbf{A}_0 = \mu_0 \mathbf{I}_{N_t}, \quad (21)$$

where μ_0 is a distance factor which depends on the modulation mode of the symbol to be transmitted. Substituting (18), (20) and (21) into (17) yields the corresponding optimization objective for OSTBC

$$\begin{aligned} (\mathbf{V}_0, \mathbf{D}_0, \mathbf{U}_0) &= \arg \max_{\mathbf{V}_0 \in \mathbf{U}^{N_t \times N_t}, \mathbf{D}_0 \in \mathbf{D}^{M \times M}, \mathbf{U}_0 \in \mathbf{U}^{M \times M}} \det(\mathbf{I}_{N_t} \\ &\quad + \eta \mu_0 L N_v \mathbf{V}_0 \mathbf{D}_0 \mathbf{U}_0^* \mathbf{U}_0 \mathbf{D}_0 \mathbf{V}_0^H \mathbf{V}_{\mathbf{R},0} \mathbf{D}_{\mathbf{R},0} \mathbf{V}_{\mathbf{R},0}^H) \\ &= \arg \max_{\mathbf{V}_0 \in \mathbf{U}^{N_t \times N_t}, \mathbf{D}_0 \in \mathbf{D}^{M \times M}, \mathbf{U}_0 \in \mathbf{U}^{M \times M}} \det(\mathbf{I}_{N_t} \\ &\quad + \eta \mu_0 L N_v \mathbf{D}_0^2 \mathbf{V}_{\mathbf{R},0}^H \mathbf{V}_{\mathbf{R},0} \mathbf{D}_{\mathbf{R},0} \mathbf{V}_{\mathbf{R},0}^H \mathbf{V}_0), \end{aligned} \quad (22)$$

where $\mathbf{U}^{N_t \times N_t}$ denotes the set of unitary matrices of size $N_t \times N_t$ and $\mathbf{D}^{M \times M}$ denotes the set of positive semi-definite

diagonal matrices of size $M \times M$. According to Hadamard's inequality [33], for a positive semi-definite matrix \mathbf{Q} , its determinant satisfies $\det(\mathbf{Q}) \leq \prod_i \|\mathbf{q}_i\|_2$, where \mathbf{q}_i is the i th column of \mathbf{Q} . The equality holds if and only if \mathbf{Q} is a diagonal matrix. Therefore, in order to maximize (22), $\mathbf{D}_0^2 \mathbf{V}_{\mathbf{R},0}^H \mathbf{V}_{\mathbf{R},0} \mathbf{D}_{\mathbf{R},0} \mathbf{V}_{\mathbf{R},0}^H \mathbf{V}_0$ must be a diagonal matrix, namely $\mathbf{V}_0 = \mathbf{V}_{\mathbf{R},0}$. As seen in (22), another optimization matrix \mathbf{U}_0 is independent of the objective function, so it can be an arbitrary unitary matrix, such as \mathbf{I}_M . Thus, the optimization objective reduces to

$$\mathbf{D}_0 = \arg \max_{\mathbf{D}_0 \in \mathbf{D}^{M \times M}} \det(\mathbf{I}_{N_t} + \eta \mu_0 L N_v \mathbf{D}_0^2 \mathbf{D}'_{\mathbf{R},0}), \quad (23)$$

where $\mathbf{D}'_{\mathbf{R},0}$ is a diagonal matrix composed of the first M diagonal elements of $\mathbf{D}_{\mathbf{R},0}$. Let $\mathbf{D}_0 = \text{diag}\{d_1, \dots, d_i, \dots, d_M\}$ and $\mathbf{D}'_{\mathbf{R},0} = \text{diag}\{d_{\mathbf{R},1}, \dots, d_{\mathbf{R},i}, \dots, d_{\mathbf{R},M}\}$, where $d_i \geq 0$ and $d_{\mathbf{R},i} \geq 0$ for $1 \leq i \leq M$, then the power allocation matrix \mathbf{D}_0 can be obtained by solving the following optimization problem

$$\begin{aligned} J_1 : \min_{\mathbf{D}_0} & - \prod_{i=1}^M \left(1 + \eta \mu_0 L N_v d_{\mathbf{R},i} d_i^2 \right) \\ \text{s.t.} & \sum_{i=1}^M d_i^2 = 1 \\ & d_i \geq 0, 1 \leq i \leq M. \end{aligned}$$

Clearly, both the objective function and constraint conditions are convex functions of optimization variables d_i , so J_1 is a standard convex optimization problem. Form the Lagrangian of J_1 as

$$\begin{aligned} L(\nu, \lambda_1, \dots, \lambda_M) &= - \prod_{i=1}^M \left(1 + \eta \mu_0 L N_v d_{\mathbf{R},i} d_i^2 \right) \\ &\quad + \nu \left(\sum_{i=1}^M d_i^2 - 1 \right) - \sum_{i=1}^M \lambda_i d_i, \end{aligned} \quad (24)$$

where ν is the Lagrange multiplier associated with the equality constraint $\sum_{i=1}^M d_i^2 = 1$, and λ_i is the Lagrange multiplier associated with the inequality constraint $d_i \geq 0$. Since J_1 is a

convex optimization problem, strong duality holds. Therefore, the primal and dual optimal points of J_1 satisfy the following Karush-Kuhn-Tucher (KKT) conditions

$$\begin{aligned} \sum_{i=1}^M (d_i^*)^2 &= 1 \\ d_i^* &\geq 0 \\ \lambda_i &\geq 0 \\ \lambda_i d_i^* &= 0 \\ \left. \frac{\partial L(\nu, \lambda_1, \dots, \lambda_M)}{\partial d_i} \right|_{d_i^*, \nu^*, \lambda_i^*} &= 0, \end{aligned}$$

where $i = 1, \dots, M$, and $(\cdot)^*$ denotes the optimal value. By solving the above KKT conditions, we have

$$d_i^* = \sqrt{\max \left(\frac{\eta \mu_0 L N_v d_{\mathbf{R},i} - \nu^*}{\eta \mu_0 L N_v d_{\mathbf{R},i} \nu^*}, 0 \right)}, \quad (25)$$

where ν^* can be derived by solving the equality $\sum_{i=1}^M (d_i^*)^2 = 1$ through numerical iterative method. Intuitively, the above power allocation follows the water-filling principle [34], we name it water-filling power allocation in the rest of this paper.

C. Performance Analysis

Diversity order and coding gain are two most important indicatives of SNR performance of various transmission schemes. In this section, we derive the diversity order and coding gain of the precoded space-time-frequency block code in multiuser MISO-MC-CDMA systems. Diversity order G_d and coding advantage G_c are respectively defined as the slope and offset of average PEP curves when SNR η approaches infinity as expressed below

$$G_d = - \lim_{\eta \rightarrow \infty} \frac{\log(\bar{P}(\eta))}{\log(\eta)}, \quad (26)$$

and

$$G_c = \lim_{\eta \rightarrow \infty} \frac{\bar{P}^{-1/G_d}(\eta)}{\log(\eta)}. \quad (27)$$

Theorem 1: Precoded space-time-frequency block code in multiuser MISO-MC-CDMA systems based on channel statistical information can achieve full diversity order N_t .

Proof: Substituting the optimal precoder into the upper bound of the average PEP in (16), we have

$$\begin{aligned} \bar{P} &= \det \left(\mathbf{I}_{N_t} + \eta L N_v \mathbf{D}_0^2 \mathbf{D}_{\mathbf{A},0} \mathbf{D}'_{\mathbf{R},0} \right)^{-1} \\ &\leq \det \left(\eta L N_v \mathbf{D}_0^2 \mathbf{D}_{\mathbf{A},0} \mathbf{D}'_{\mathbf{R},0} \right)^{-1} \end{aligned} \quad (28)$$

$$= \eta^{-N_t} \det \left(L N_v \mathbf{D}_0^2 \mathbf{D}_{\mathbf{A},0} \mathbf{D}'_{\mathbf{R},0} \right)^{-1}, \quad (29)$$

where (28) results from the fact that $\mathbf{D}_0^2 \mathbf{D}_{\mathbf{A},0} \mathbf{D}'_{\mathbf{R},0}$ is a positive semi-definite matrix. Hence the upper bound of the average PEP decays as η^{-N_t} , which proves the claim about full diversity order in theorem 1. In this paper, we consider the case with single receive antenna, however, our proposed transmission scheme can be extended to the scenario with

N_r receive antennas easily. Then, the diversity order is $N_t N_r$ provably. In addition, multiuser diversity seems to be another available resource that could be exploit through MU selection to increase diversity gains [35]. ■

Theorem 2: Precoded space-time-frequency block code in multiuser MISO-MC-CDMA system based on channel statistical information can pose $L N_v$ times coding gain than conventional precoding space-time block code.

Proof: According to the definition of coding gain, the corresponding coding gain of precoded space-time-frequency block code in multiuser MIMO-MC-CDMA system based on channel statistical information can be expressed as [36]

$$\begin{aligned} G_c &= \lim_{\eta \rightarrow \infty} \frac{\bar{P}^{-1/G_d}(\eta)}{\log(\eta)} \\ &= L N_v \det \left(\mathbf{D}_0^2 \mathbf{D}_{\mathbf{A},0} \mathbf{D}'_{\mathbf{R},0} \right)^{1/G_d} \end{aligned} \quad (30)$$

$$= L N_v \left(\prod_{m=1}^M d_m^2 d_{\mathbf{A},m} d'_{\mathbf{R},m} \right)^{1/G_d}. \quad (31)$$

Built on the above analysis, it is clearly that, for single carrier system, the coding gain is $\det \left(\mathbf{D}_0^2 \mathbf{D}_{\mathbf{A},0} \mathbf{D}'_{\mathbf{R},0} \right)^{1/G_d}$. Thereby, through space-time-frequency coding, the coding gain is improved $L N_v$ times. ■

In addition, from (31), it is found that, when $M = N_t$, lower transmit correlation denotes higher coding gain. This is because that, given $\text{tr}(\mathbf{R}_0) = \sum_{m=1}^{N_t} d'_{\mathbf{R},m} = N_t$, $\prod_{m=1}^{N_t} d'_{\mathbf{R},m} \leq \left(\frac{\sum_{m=1}^{N_t} d'_{\mathbf{R},m}}{N_t} \right)^{N_t} = 1$ and the equality holds if and only if $d'_{\mathbf{R},1} = \dots = d'_{\mathbf{R},N_t} = 1$. In other words, the closer the eigenvalues are, the larger the coding gain is, which means lower correlation is preferred. On the other hand, for $M < N_t$, the best condition is $d'_{\mathbf{R},1} = \dots = d'_{\mathbf{R},M} = \frac{N_t}{M}$. Therefore, the high correlation may perform better than lower correlation, since the lower correlation would waste partial gain when the number of nonzero eigenvalues is larger than M .

For MISO-MC-CDMA systems with precoded STFBC, there exist space, time, frequency and multiuser resources available to promise system reliable. Remarkably, adaptive precoding and optimal power allocation can provide a better performance over traditional fixed transmission strategies under the same condition, especially at lower SINR, which will be confirmed later by the simulation results.

IV. THE SPECIAL TRANSMISSION SCENARIOS

Although the closed-form solutions of the optimization problem given by (17) have been obtained, the power allocation matrix of the optimal precoder should be solved through iterative water-filling, which arises high computational complexity at BS, especially with numerous MUs. For some special scenarios, we can adopt simple algorithms to achieve optimal or near optimal performance, which promise our proposed transmission schemes more feasible. In what follows, we discuss these special scenarios in detail, respectively.

A. Numerous Delay Taps, Long Spread Code and high SINR

It is shown in (17) that the number of delay taps L and the length of spread code N_v have a positive impact on the

upper bound of the average PEP. As defined above, L is represented as the number of resolvable transmission paths. The number of resolvable path deeply depends on the propagation environment, for example, typical urban channel model (TUX) for UMTS specified by 3GPP has 20 delay taps and channel no. 6 for DRM system produced by ETSI has 5 resolvable paths. Similarly, the length of spread code N_v may be large according to the system requirement. In addition, the transmit SINR η can be also very high due to large transmit power. We define $\Gamma = \eta L N_v$ as the equivalent transmit SINR. With the L , N_v and η , Γ becomes quite large. In this context, we have the following theorem:

Theorem 3: In multiuser MISO-MC-CDMA systems, at high effective SINR region due to large L , N_v or η , for the optimal precoder, $\mathbf{V}_0 = \mathbf{V}_{\mathbf{R},0}$ is used to adapt to the channel condition, $\mathbf{U}_0 = \mathbf{V}_{\mathbf{A},0}$ makes the precoder matched with space time codeword to be transmitted, and $\mathbf{D}_0 = \sqrt{1/M} \mathbf{I}_M$, namely equal power allocation, can achieve near optimal performance.

Proof: The objective function in (17) can be written as $\det(\mathbf{I}_{N_t} + \Gamma \mathbf{F}_0 \mathbf{A}_0 \mathbf{F}_0^H \mathbf{R}_0)$. Note that, when the effective SINR $\Gamma \rightarrow \infty$, then the term \mathbf{I}_{N_t} in the objective function can be omitted with respect to the dominating term $\Gamma \mathbf{F}_0 \mathbf{A}_0 \mathbf{F}_0^H \mathbf{R}_0$. Thus in the limit, the optimization problem is equivalent to maximizing $\det(\mathbf{I}_{N_t} + \Gamma \mathbf{F}_0 \mathbf{A}_0 \mathbf{F}_0^H \mathbf{R}_0)$, subject to the power constraint, which can be written as (32) at the top of next page. According to Hadamard's inequality, in order to maximize the above objective function, the following conditions must be satisfied: $\mathbf{V}_0 = \mathbf{V}_{\mathbf{R},0}$ and $\mathbf{U}_0 = \mathbf{V}_{\mathbf{A},0}$. So the objective function reduces to

$$\mathbf{D}_0 = \arg \max_{\mathbf{D}_0 \in \mathbf{D}^{M \times M}} \det(\Gamma \mathbf{D}_0^2 \mathbf{D}_{\mathbf{A},0} \mathbf{D}'_{\mathbf{R},0}),$$

whereafter, let $\mathbf{D}_0 = \text{diag}\{d_1, \dots, d_i, \dots, d_M\}$, the power allocation problem is equivalent to the following optimization problem:

$$J_2 : \max_{\mathbf{D}_0} \det(\Gamma \mathbf{D}_0^2 \mathbf{D}_{\mathbf{A},0} \mathbf{D}'_{\mathbf{R},0}) = \det(\Gamma \mathbf{D}_{\mathbf{A},0} \mathbf{D}'_{\mathbf{R},0}) \prod_{i=1}^M d_i^2$$

$$\text{s.t.} \quad \sum_{i=1}^M d_i^2 = 1$$

$$d_i \geq 0, \quad i = 1, \dots, M.$$

This optimization problem J_2 can be solved based on the fact that $\prod_{i=1}^M d_i^2 \leq \left(\sum_{i=1}^M \frac{d_i^2}{M}\right)^M = M^{-M}$, and the equality holds if and only if $d_1^2 = \dots = d_M^2 = 1/M$, so we have $\mathbf{D}_0 = \sqrt{1/M} \mathbf{I}_M$. ■

B. Low SINR

By using Taylor expansion, $\det(\mathbf{I}_{N_t} + \eta L N_v \mathbf{F}_0 \mathbf{A}_0 \mathbf{F}_0^H \mathbf{R}_0)$ can be expressed as $1 + \eta L N_v \text{tr}(\mathbf{D}_0^2 \mathbf{D}_{\mathbf{A},0} \mathbf{D}'_{\mathbf{R},0}) + \mathcal{O}(\eta)$, where $\mathcal{O}(\eta)$ denotes the high order term of the SINR η . When η is sufficiently small, the high order term $\mathcal{O}(\eta)$ becomes negligible. Then, the objective function is reduced to $1 + \eta L N_v \text{tr}(\mathbf{D}_0^2 \mathbf{D}_{\mathbf{A},0} \mathbf{D}'_{\mathbf{R},0})$. In this context, the power allocation

can be described as the following optimization problem:

$$J_3 : \max_{\mathbf{D}_0} \text{tr}(\mathbf{D}_0^2 \mathbf{D}_{\mathbf{A},0} \mathbf{D}'_{\mathbf{R},0})$$

$$\text{s.t.} \quad \text{tr}(\mathbf{D}_0^2) = 1$$

$$\mathbf{D}_0 \succeq 0.$$

By solving this problem, we could get the following theorem:

Theorem 4: In multiuser MISO-MC-CDMA systems, at low SINR region, for the optimal precoder, $\mathbf{V}_0 = \mathbf{V}_{\mathbf{R},0}$ is used to adapt space time codeword to the channel condition, $\mathbf{U}_0 = \mathbf{V}_{\mathbf{A},0}$ makes the precoder matched with space time codeword to be transmitted, and $\mathbf{D}_0 = \text{diag}\{0, \dots, 1, \dots, 0\}$, where the position of 1, represented as m^* , satisfies the below condition: $m^* = \arg \max_{1 \leq m \leq N_t} [\mathbf{D}_{\mathbf{A},0} \mathbf{D}'_{\mathbf{R},0}]^{(m)}$, $[\mathbf{B}]^{(m)}$ represents the m th diagonal element of \mathbf{B} . Generally speaking, distributing all transmit power to the m^* th spatial subchannel achieves the optimal performance asymptotically.

Proof: Let $\mathbf{D}_{\mathbf{A},0} = \text{diag}\{d_{\mathbf{A},1}, \dots, d_{\mathbf{A},m}, \dots, d_{\mathbf{A},M}\}$ and $\mathbf{D}'_{\mathbf{R},0} = \text{diag}\{d_{\mathbf{R},1}, \dots, d_{\mathbf{R},m}, \dots, d_{\mathbf{R},M}\}$. The optimization problem J_3 can be rewritten as

$$J_4 : \max_{\mathbf{D}_0} \sum_{m=1}^M d_m^2 d_{\mathbf{A},m} d_{\mathbf{R},m}$$

$$\text{s.t.} \quad \sum_{m=1}^M d_m^2 = 1$$

$$d_m \geq 0, \quad m = 1, \dots, M.$$

Clearly, the optimization problem J_4 is a linear programming problem. Considering the feasible set is a polyhedron, the basic feasible solution must be one of the vertices of the polyhedron. For this problem, there are M poles in the form of $(0, \dots, 1, \dots, 0)$ comprising one 1 and $M - 1$ zeros. Obviously, only when $d_{m^*} = 1, d_1 = \dots = d_{m^*-1} = d_{m^*+1} = \dots = d_{N_t} = 0$, where $m^* = \arg \max_{1 \leq m \leq N_t} d_{\mathbf{A},m} d_{\mathbf{R},m}$, the objective function is maximal. ■

V. SIMULATION RESULTS

In order to examine the performance of the proposed system framework, we present some representative simulation results in various scenarios. The main simulation parameters, referred to the IEEE 802.16e system simulation requirements [6], are configured as seen in Tab.1. Moreover, we adopt the ITU channel model suggested by the draft. Data is modulated by 16-QAM and transmitted frame by frame. In the duration of a data frame, the channel is assumed to keep constant and fades in inter-frames according to the famous Jakes model. Note that we set the maximal doppler spread as 50Hz, then the channel variety is not so fast and the statistical information can be estimated perfectly. In what follows, we construct the transmit correlation matrix as follows: $\mathbf{R}_0(m, n) = \kappa^{|m-n|}$, where $\mathbf{R}_0(m, n)$ is the (m, n) th element of correlation matrix \mathbf{R}_0 and κ is the so called correlation coefficient. For convenience of implementation, Alamouti code is used in all simulation cases.

$$(\mathbf{V}_0, \mathbf{D}_0, \mathbf{U}_0) = \arg \max_{\mathbf{V}_0 \in \mathbf{U}^{N_t \times N_t}, \mathbf{D}_0 \in \mathbf{D}^{M \times M}, \mathbf{U}_0 \in \mathbf{U}^{M \times M}} \det(\Gamma \mathbf{V}_0 \mathbf{D}_0 \mathbf{U}_0^H \mathbf{V}_{A,0} \mathbf{D}_{A,0} \mathbf{V}_{A,0}^H \mathbf{U}_0 \mathbf{D}_0 \mathbf{V}_0^H \mathbf{V}_{R,0} \mathbf{D}_{R,0} \mathbf{V}_{R,0}^H). \quad (32)$$

TABLE I
SYSTEM PARAMETER TABLE

Parameter	Description	Value
f_c	Carrier Frequency	2.5 GHz
BW	Total Bandwidth	10 MHz
N_c	Number of Sub-Carriers	1024
F_s	Sampling Frequency	11.2 MHz
Δf	Sub-Carrier Space	10.9375 KHz
CP	Cycle Prefix Length	128
T_s	OFDM Symbol Length	102.86 us
T_F	Frame Length	5 ms
N_v	Orthogonal Spread Factor	8
N_t	Number of Transmit Antennas	4
N_u	Number of MUs on Each Carrier	4

A. Different Transmission Schemes

At first, we compare the SER performance of four different transmission schemes, namely open loop, antenna selection, statistical precoding and ideal precoding with $\kappa = 0.3$. As the names imply, open loop means that the transmitter has no channel statistical information and the Alamouti code are directly transmitted without precoding, antenna selection chooses two optimal antennas from all transmit antenna over all carriers for the transmission of Alamouti code based on the feedback information from the MUs, statistical precoding is the proposed precoding scheme, and ideal precoding denotes that the precoding direction matrix and power allocation matrix over each sub-carrier are designed based on full channel instantaneous information. As seen in Fig.2, in comparison with open loop, statistical precoding has an obvious performance gain. For example, at the SER of 10^{-2} , statistical precoding provides more than 1.5dB gains. Moreover, with the increase of SNR, the performance gain increases accordingly. Although antenna selection performs better than statistical precoding significantly, it is difficult to apply in practical multi-carrier systems, because it needs to update selection information within a channel correlation time over all subcarriers, which is quite unbearable. Fortunately, statistical precoding can asymptotically approach the performance of antenna selection in high SNR region. From the perspectives of feedback amount and computational complexity, open loop does not require any feedback information and has the lowest complexity. Antenna selection needs to convey the selection information during each correlation time and has a lower complexity. With respect to antenna selection, the proposed statistical precoding has a relatively higher complexity, but requires less feedback amount, since channel statistical information varies very slowly. Additionally, it does not need to convey the information over all sub-carriers as mentioned earlier. Ideal precoding has the highest complexity and requires infinite feedback amount. Thereby, statistical precoding is a preferred choice in multi-carrier systems.

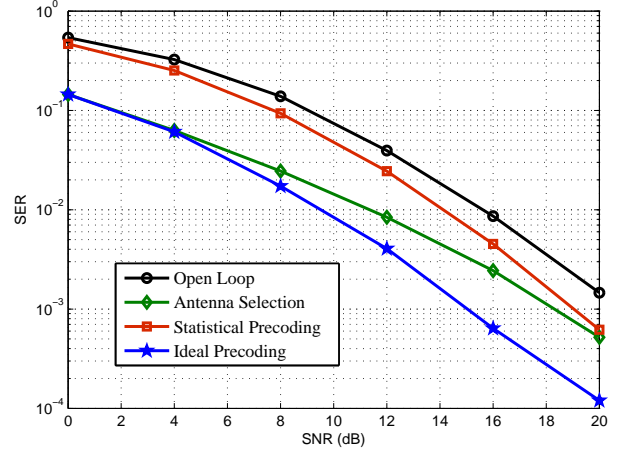


Fig. 2. SER performance comparison with different transmission schemes.

B. Different Power Allocation

In this subsection, we investigate the effect of power allocation methods on the SER performance with $\kappa = 0.1$. As seen in Fig.3, at low SNR, under 8dB, single beam allocation nearly performs the same as water-filling allocation, but there is a large gap at high SNR. For example, water-filling allocation has more than 3dB gain at the SER of 10^{-2} . On the other hand, with the increase of SNR, the performance of equal allocation is asymptotically close to that of water-filling allocation, which is consistent with our theoretical claim. Therefore, we can replace water-filling allocation with single beam allocation in low-SNR regime and equal allocation in high-SNR regime, so that computation complexity can be reduced significantly, while there is no obvious SER performance loss.

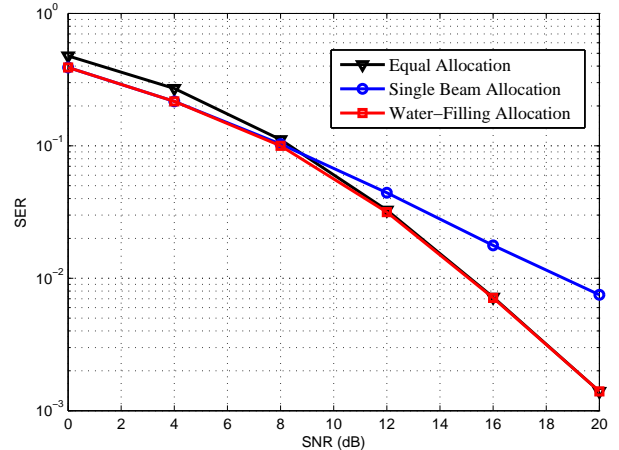


Fig. 3. SER performance comparison with different power allocation methods.

C. Different Correlation Coefficients

Fig.4 illustrates the SER performance with different correlation coefficients. For $M < N_t$, the performance with $\kappa = 0.1$ is clearly worse than that with $\kappa = 0.4$. For example, there is about 1dB performance gap at the SER of 10^{-2} . It is because partial channel gain is wasted under low correlation condition, which confirms our theoretical claim earlier. With the increase of κ , there is still a performance gain in low-SNR regime, but not in high-SNR regime, because the coding gain from $\prod_{m=1}^M d'_{\mathbf{R},m}$ can be neglected when SNR is large enough.

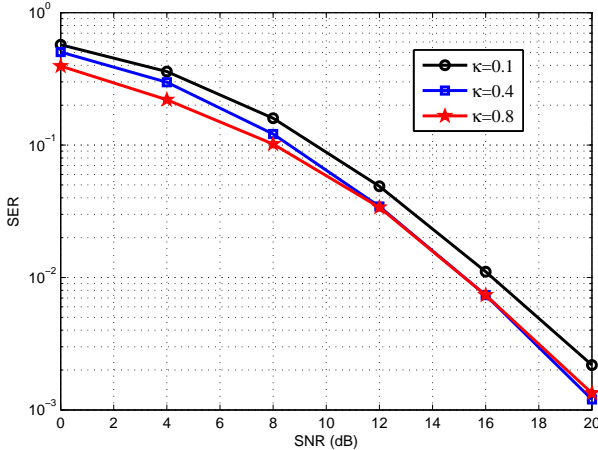


Fig. 4. SER performance comparison with different correlation coefficients.

VI. CONCLUSIONS

In this paper, we proposed a multiuser MISO-MC-CDMA system framework for broadband wireless communication systems. This configuration utilizes the space-time-frequency signal to obtain multi-diversity gains, so that it can provide the higher transmission reliability. To further improve the system performance and reduce receive complexity, we exploit the channel information to linear precode the space-time codeword on each sub-carrier. Under the consideration of the amount of feedback, we propose the design method of the optimal precoder, including precoding direction matrix and power allocation matrix, in the sense of statistical average. For some special transmission scenarios, we have given the corresponding simple design method to reduce computational complexity at BS. Numerous simulation results confirm our theoretical claims.

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