Model dependence of isospin sensitive observables at high densities

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Within two different frameworks of isospin-dependent transport model, i.e., Boltzmann-Uehling-Uhlenbeck (IBUU04) and Ultrarelativistic Quantum Molecular Dynamics (UrQMD) transport models, sensitive probes of nuclear symmetry energy are simulated and compared. It is shown that neutron to proton ratio of free nucleons, π^-/π^+ ratio as well as isospin-sensitive transverse and elliptic flows given by the two transport models with their "best settings", all have obvious differences. Discrepancy of numerical value of isospin-sensitive n/p ratio of free nucleon from the two models mainly originates from different symmetry potentials used and discrepancies of numerical value of charged π^-/π^+ ratio and isospin-sensitive flows mainly originate from different isospin-dependent nucleon-nucleon cross sections. These demonstrations call for more detailed studies on the model inputs (i.e., the density- and momentum-dependent symmetry potential and the isospin-dependent nucleon-nucleon cross section in medium) of isospin-dependent transport model used. The studies of model dependence of isospin sensitive observables can help nuclear physicists to pin down the density dependence of nuclear symmetry energy through comparison between experiments and theoretical simulations scientifically.

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I. INTRODUCTION

The equation of state of isospin asymmetric nuclear matter, i.e. the density dependence of the symmetry energy especially its high-density behavior, is still one of the open questions in nuclear physics. In recent years, many nuclear physicists have made great efforts to explore the density-dependent nuclear symmetry energy, which has significant ramifications in understanding the structure of rare isotopes, heavy-ion nuclear reactions induced by radioactive beam [1–3], and also in astrophysics [4–6]. Around normal density, the symmetry energy has been roughly constrained from, e.g., studying isospin diffusion [7–10] and isoscaling [11] in heavy-ion reactions, the size of neutron skin in heavy nuclei [12], and isotope dependence of the giant monopole resonances in even-A Sn isotopes [13]. However, there is a high degree of uncertainty for constraints of nuclear symmetry energy in the high-density areas. Different transport models give practically opposite conclusions for the high-density dependence of the symmetry energy, e.g., a very soft symmetry energy at the supra-saturation density was indicated by fitting the FOPI data [14] based on the the isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU04) model [15, 16] whereas Feng et al. obtained a stiff result using the LQMD model [17]. Other similar studies were also obtained in the QMD framework [18, 19]. This situation calls for the studies of model dependence of probing the symmetry energy by using heavy-ion collisions. It was exciting to note that a so-called model-independent constraint of the high-density dependence of the symmetry energy was recently obtained by Cozma *et al.* [20].

In fact, there are many factors affecting nuclear reaction transport simulation, such as the initialization of colliding nuclei, the nucleon-nucleon interaction potential, nucleon-nucleon elastic and inelastic scattering cross sections, and the designs of the framework of transport model codes [19, 21–25]. So it is very necessary to make a dialogue between different models, to see how large the differences are on the values of isospin sensitive observables. To give the model error estimation of observable actually involves different transport calculations and one by one examining of the effects of the uncertainties caused by different model inputs. This is boring but important to read the experiment data "correctly". In this study, within the frameworks of isospin-dependent transport models Boltzmann-Uehling-Uhlenbeck (IBUU04) and Ultrarelativistic Quantum Molecular Dynamics (UrQMD), we investigated the model dependences of some frequently used isospin-sensitive observables π^-/π^+ ratio and n/p ratio of free nucleons and isospin-sensitive directed and elliptic flows, which have been predicted to be sensitive to nuclear symmetry energy [2, 3].

II. THE TRANSPORT MODELS

To simulate nuclear collisions, transport model that one frequently utilized is the Boltzmann-Uehling-Uhlenbeck (BUU) equation, which provides an approximate Wigner transform of the one-body density matrix

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as its solution [26]. The BUU transport model is usually used to describe one-body observable although some afterburner can be added to predict many-body correlation [27]. The other frequently utilized approaches is the Molecular Dynamics Model (QMD), which represents the individual nucleons as Gaussian "wave-packet" with mean values that move according to the Hamilton's equations [28]. QMD model has advantage over many-body correlation and thus frequently used to predict cluster production in heavy-ion collisions [29]. In the following discussions, we use the isospin-dependent IBUU04 and UrQMD transport models to discuss the model dependence of isospin sensitive observables at high densities.

A. The isospin-dependent BUU transport model

In the used IBUU04 model, an isospin- and momentum-dependent mean-field potential [30] is used, i.e.,

$$\begin{aligned} U(\rho,\delta,\mathbf{p},\tau) &= A_u(x)\frac{\rho_{\tau'}}{\rho_0}+A_l(x)\frac{\rho_\tau}{\rho_0} \\ &+B(\frac{\rho}{\rho_0})^\sigma(1-x\delta^2)-8x\tau\frac{B}{\sigma+1}\frac{\rho^{\sigma-1}}{\rho_0^\sigma} \\ &+\frac{2C_{\tau,\tau}}{\rho_0}\int d^3\,\mathbf{p}'\frac{f_\tau(\mathbf{r},\mathbf{p}')}{1+(\mathbf{p}-\mathbf{p}')^2/\Lambda^2} \\ &+\frac{2C_{\tau,\tau'}}{\rho_0}\int d^3\,\mathbf{p}'\frac{f_{\tau'}(\mathbf{r},\mathbf{p}')}{1+(\mathbf{p}-\mathbf{p}')^2/\Lambda^2}, \quad (1) \end{aligned}$$

where $\delta=(\rho_n-\rho_p)/(\rho_n+\rho_p)$ is the isospin asymmetry, and ρ_n, ρ_p are neutron ($\tau=1/2$) and proton ($\tau=-1/2$) densities, respectively. Detailed parameter settings can be found in Ref. [31]. The parameter x is used for simulating different density dependences of the symmetry energy $E_{sym}(\rho)$ predicted by microscopic and phenomenological many-body approaches [32], but in our present work we just choose the parameter $x=0$. Shown in Fig. 1 is the density dependence of nuclear symmetry energy used in the IBUU04 transport model and the following UrQMD transport model. From Fig. 1 we can see that nuclear symmetry energies used in the IBUU04 model (with $x=0$) and the UrQMD model (with $\gamma=0.5$) are almost the same. However, the same density-dependent symmetry energy does not mean the used symmetry potential is also the same. The derived symmetry potential from Eq. 1 is shown in the left panel of Fig. 2, it is clearly seen that the symmetry potential in the IBUU04 model is a density- and momentum-dependent symmetry potential. For small momentum's nucleons, strength of the symmetry potential increases with density. However, for large momentum's nucleons strength of the symmetry potential decreases with density. And we can clearly see that the symmetry potential of the IBUU04 used is different from that used in the UrQMD model for nucleons with nonzero momenta.

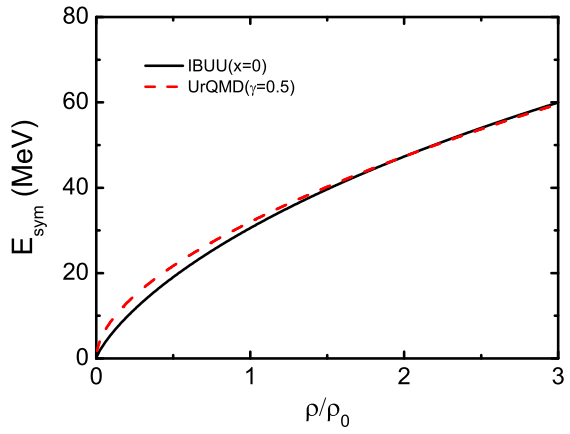

FIG. 1: Density dependent nuclear symmetry energies used in the IBUU04 and the isospin-dependent UrQMD transport models.

For the IBUU04 calculations, we also adopted an isospin-dependent in-medium reduced NN (nucleon-nucleon) elastic scattering cross section, which originating from the scaling model according to nucleon effective mass [10, 33–35], i.e., based on the assumption that in-medium NN scattering transition matrix is the same as that in vacuum [34], the elastic NN scattering cross section in medium σ_{NN}^{medium} is reduced by a factor of

$$\begin{aligned} R_{medium}(\rho,\delta,\mathbf{p}) &= \sigma_{NN_{elastic}}^{medium}/\sigma_{NN_{elastic}}^{free} \\ &= (\mu_{NN}^*/\mu_{NN})^2, \quad (2) \end{aligned}$$

where μ_{NN} and μ_{NN}^* are the reduced masses of the colliding nucleon pair in free space and medium, respectively. Momentum- and density-dependent reduced factors of NN scattering cross sections R_{nn}, R_{np}, R_{pp} are shown in the right panel of Fig. 2. It is seen that the momentum- and density-dependent reduced factor of NN scattering cross section used in the IBUU04 model demonstrates evident momentum- and isospin-dependence. For in-medium NN inelastic scattering cross section, we use the experimental free space NN inelastic scattering cross section in the two transport models since the medium effect of NN inelastic scattering cross section is still an open question.

B. The isospin-dependent UrQMD transport model

The UrQMD model is a microscopic model used to simulate (ultra)relativistic heavy ion collisions and it has also been used as a component of various hybrid transport approaches [36]. In the isospin-dependent UrQMD model, the used momentum-dependent potential was proposed by Bass *et al.* based on the mean field theory and expressed as [37]

$$U_{md}=t_{md}\ln^2[1+a_{md}(\mathbf{p}_i-\mathbf{p}_j)^2]\frac{\rho_i}{\rho_0}, \quad (3)$$

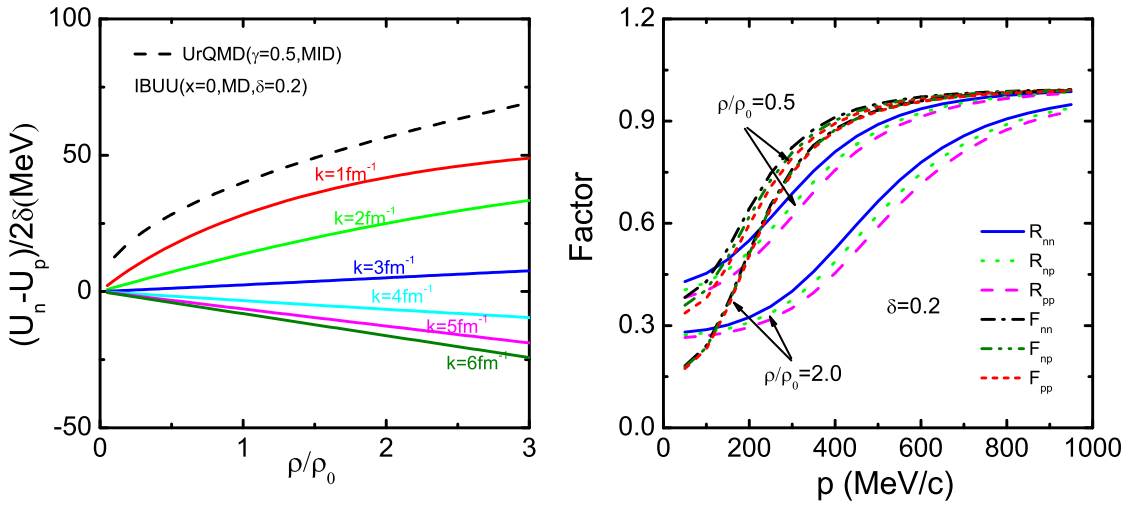

FIG. 2: Symmetry potentials and reduced medium correction factors of NN cross section used in the IBUU04 and the UrQMD transport models. In the left panel, lines labelled by different momenta are the symmetry potentials used in the IBUU04. In the right panel, F_{nn}, F_{np}, F_{pp} denote reduced factors used in the UrQMD, and R_{nn}, R_{np}, R_{pp} are reduced factors used in the IBUU04 model.

where $t_{md} = 1.57\ MeV$ and $a_{md} = 500\ c^2/GeV^2$. For the symmetry potential energy density, we use the form of

$$V_{sym} = (S_0 - \frac{\varepsilon_F}{3})u^\gamma \delta^2 \tag{4}$$

and the parameter settings are the same as in Ref. [38]. S_0 is the symmetry energy at normal nuclear density ρ_0 and its value is about 30∼ 36 MeV [39–41]. Here we choose S_0= 32 MeV. ε_F denotes the Fermi kinetic energy at the normal nuclear density, which is approximately about 38 MeV. $u = \frac{\rho}{\rho_0}$ is the reduced nuclear density, γ is the strength parameter of the density dependence of symmetry potential, and $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$ is the isospin asymmetry. Here we adopt a soft ($\gamma = 0.5$) density-dependent symmetry potential, its corresponding density-dependent symmetry energy is similar with $x = 0$ case used in the IBUU04 model as shown in Fig. 1. From the symmetry potential energy density V_{sym}, one can get the symmetry potential as a function of density as shown in the left panel of Fig. 2. It seems like the symmetry potential used in the IBUU04 model at low ultimate momentum.

As for two-body scattering cross section in medium, it is somewhat complicated than that used in the IBUU04 model. In-medium NN elastic cross section is modified by nuclear medium according to the QHD theory [42–44]. In the present work, the in-medium NN elastic cross section $\sigma^{medium}_{NN_{elastic}}$ comes from the free space elastic scattering cross section $\sigma^{free}_{NN_{elastic}}$ multiplied by a medium correction factor $F(u, \delta, p)$. It is formulated as

$$\sigma_{NN_{elastic}} = F(u, \delta, p) \times \sigma^{free}_{NN_{elastic}} \tag{5}$$

$$= F^p_\delta \times F^p_u \times \sigma^{free}_{NN_{elastic}}. \tag{6}$$

The medium correction factor $F(u, \delta, p)$ is consist of the momentum-dependent isospin-scalar density effect F^p_u and the momentum-dependent isospin-vector mass-splitting effect F^p_δ. Here the used non-relativistic neutron mass is larger than that of proton in the neutron-rich medium, which is consistent with the results of the Dirac-Brueckner-Hartree-Fock (DBHF) theory or the extended Brueckner-Hartree-Fock (BHF) theory. The isospin-dependent splitting effect on NN elastic cross section which represented by the F_δ factor has been studied in Ref. [45, 46]. The momentum-dependent reduced factors F^p_u and F^p_δ are expressed in one formula as

$$F^p_{\delta,u} = \begin{cases} 1, & p_{NN}{>}1\ \text{GeV/c}; \\ \frac{F_{\delta,u}-1}{1+(p_{NN}/0.225)^3} + 1, & p_{NN}{\leq}1\ \text{GeV/c}. \end{cases} \tag{7}$$

$$F_u = \frac{1}{6} + \frac{5}{6}e^{-3u}, \tag{8}$$

and isospin-dependent

$$F_\delta = \begin{cases} 1 - \frac{0.85}{1+3.25u}\delta, & \text{pp}; \\ 1 + \frac{0.85}{1+3.25u}\delta, & \text{nn}; \\ 1, & \text{np}. \end{cases} \tag{9}$$

Here p_{NN} is the relative momentum of the two colliding nucleons in the NN center-of-mass system [46]. Momentum- and density-dependent reduced factors of NN scattering cross sections F_{nn}, F_{np}, F_{pp} used in the UrQMD model are shown in the right panel of Fig. 2. It is seen that the momentum- and density-dependent reduced factor of NN scattering cross section used here demonstrates weak isospin dependence. Compared with that used in the UrQMD model, the reduced factor of NN scattering cross section used in the IBUU04 model shows more density- and isospin-dependent.

III. RESULTS AND DISCUSSIONS

Generally speaking, the strength of symmetry potential (which relates to the symmetry energy directly) is much weaker than the strength of isoscalar potential. The other characteristic is that the symmetry potential has opposite actions for neutrons and protons. Therefore one always constructs observables of the symmetry energy using differences or ratios of isospin multiplets of baryons, mirror nuclei and mesons [47]. In the following we mainly discuss the frequently used observables of nuclear symmetry energy, i.e., nucleon or π meson emissions and nucleonic collective flow.

A. nucleon and π meson emissions

Neutron to proton ratio of free nucleons as a probe of nuclear symmetry energy in heavy-ion collisions was first proposed by Li *et al.* in 1997 [48]. And π^-/π^+ ratio was first proposed in 2002 as a probe of nuclear symmetry energy by Li [49]. Double neutron to proton ratio from isotopic reaction systems was also proposed in 2006 as a probe of nuclear symmetry energy [47] and t/^{3}He was proposed as a similar probe as n/p in 2003 [50]. Later on double π^-/π^+ ratio as a probe of the high-density behavior of the nuclear symmetry energy was proposed in 2006 [51]. After that, a lot of studies on such similar probes were carried out in recent years [52–60].

Shown in Fig. 3 are the evolutions of n/p ratio of free nucleons and $(\pi^-/\pi^+)_{like}$ ratio in the central reaction of ^{197}Au+^{197}Au at a beam energy of 400 MeV/nucleon simulated by isospin-dependent IBUU04 and UrQMD models. With the dynamics of pion resonance productions and decays, the $(\pi^-/\pi^+)_{like}$ ratio naturally becomes π^-/π^+ ratio at final stage [61]. The large difference of n/p of free nucleons emitted at the beginning from the two models is due to different initializations of colliding nuclei. Compared with the UrQMD's result, the small value of n/p ration from the IBUU04 at final stage is caused by its smaller value of the momentum-dependent symmetry potential than the momentum-independent symmetry potential used in the UrQMD as shown in the left panel of Fig. 2. And the smaller value of π^-/π^+ ratio from the IBUU04 calculation than that from the UrQMD is due to its observable smaller pp elastic cross section than that of nn (as shown in the right panel of Fig. 2, which causing relatively larger pp in-elastic cross section than that of nn), thus relatively larger number of π^+ mesons are produced than π^-, which giving a smaller value of π^-/π^+ ratio than that of UrQMD. In addition, different methods of constructing cluster [62] and NN inelastic cross section [63] are also the reasons of model dependence. From Fig. 3 we can clearly see that the n/p ratio of free nucleons and π^-/π^+ ratio given by UrQMD are, respectively, 14.3% and 30% larger than that of IBUU04 model, degrees of model uncertainty of isospin sensitive observables n/p ratio of free nucleons and π^-/π^+ ratio are thus larger than corresponding effects of nuclear symmetry energy on these two probes [61].

Shown in Fig. 4 is the n/p ratio of free nucleons and π^-/π^+ ratio as a function of kinetic energy in the central reaction of ^{197}Au +^{197} Au at a beam energy of 400 MeV/nucleon simulated by the IBUU04 and the UrQMD models. For the n/p ratio of free nucleons, we can see that both models give the same trend of n/p ratio as a function of nucleonic kinetic energy. Again, the result of the UrQMD model is overall larger than that of the IBUU04 model due to their different strengths of symmetry potential as shown in the left panel of Fig. 2. Because the difference of the two symmetry potentials used in the two models becomes larger and larger with increase of nucleon's momentum, the difference of the values of n/p ratios of free nucleons given by the two models also becomes larger with nucleon's kinetic energy. From the right panel of Fig. 4, we can see that there is a cross between the π^-/π^+ ratios from the UrQMD model and that from the IBUU model. At lower kinetic energy part, the value of π^-/π^+ ratio from the UrQMD is much larger than that from the IBUU model, but at high kinetic energy the value of π^-/π^+ ratio from the IBUU is larger than that from the UrQMD model. This is caused by different Coulomb action treatments in the two models. Because most pion mesons are from resonance's decays, most pion mesons are distributed at low energies. The result shown in the right panel of Fig. 4 is consistent with the result shown in the right panel of Fig. 3. Over all, the large model-dependence shown in Fig. 3 and Fig. 4 should be kept into mind while comparing model calculations with experimental data.

B. nucleonic collective flows

Difference of neutron and proton collective flows as a probe of nuclear symmetry energy was first proposed by Greco *et al.* in 2003 [64]. A lot of studies on such probes were carried out in recent years [19, 65–69]. Later on, difference of collective flows of light clusters as a probe of nuclear symmetry energy was proposed in 2009 [70]. In Fig. 5, we show the reduced rapidity distributions of neutron and proton directed flows and $n-p$ directed flow difference $v_1^n - v_1^p$. Here $v_1 = < \frac{p_x}{p_t} >$ and v_1^n is the directed flow for neutrons, v_1^p is the directed flow for protons. The reduced rapidity is $y_0 = y/y_b$ and $y_b = 0.8935$ is the projectile rapidity. From Fig. 5 we can clearly see that the effects of isospin on directed nucleonic flow given by the IBUU04 model is obviously larger than that of the UrQMD model with the same symmetry energy selection $x = 0$ ($\gamma = 0.5$). Therefore the slope of the $n-p$ directed flow $v_1^n - v_1^p$ given by the IBUU04 is also evidently larger than that of the UrQMD model. This large model dependence inevitably affects obtaining the information of density-dependent symmetry energy from reading related experimental data by theoretical transport model.

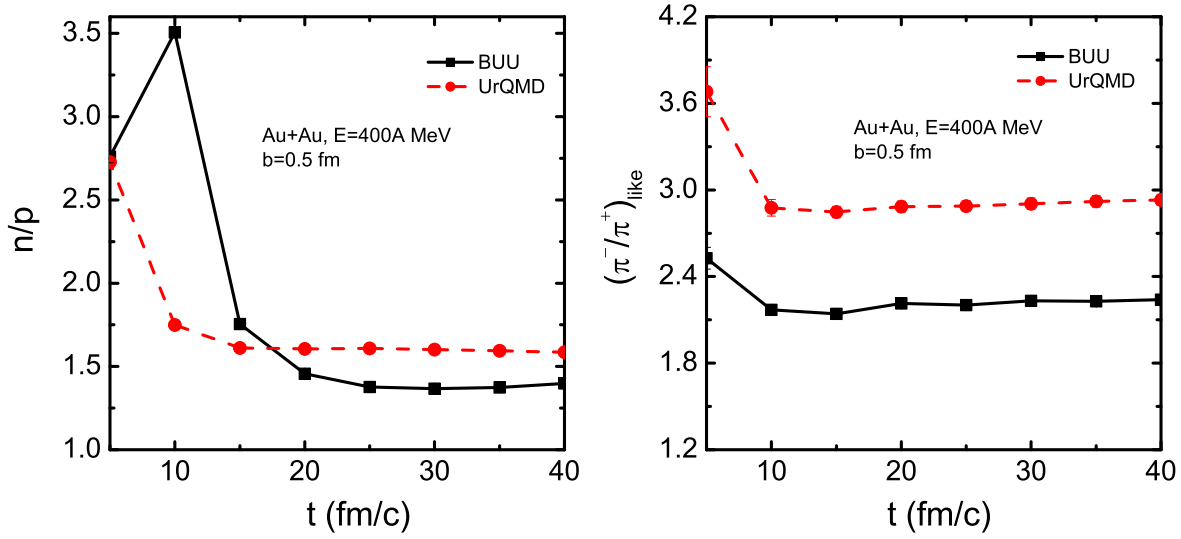

FIG. 3: Evolution of n/p ratio of free nucleons and $(\pi^-/\pi^+)_{like}$ ratio in the central reaction of $^{197}Au+^{197}Au$ at a beam energy of 400 MeV/nucleon. The black solid line and red dashed line denote results of IBUU04 model (t_{max}= 40 fm/c) and UrQMD model (t_{max}= 150 fm/c), respectively.

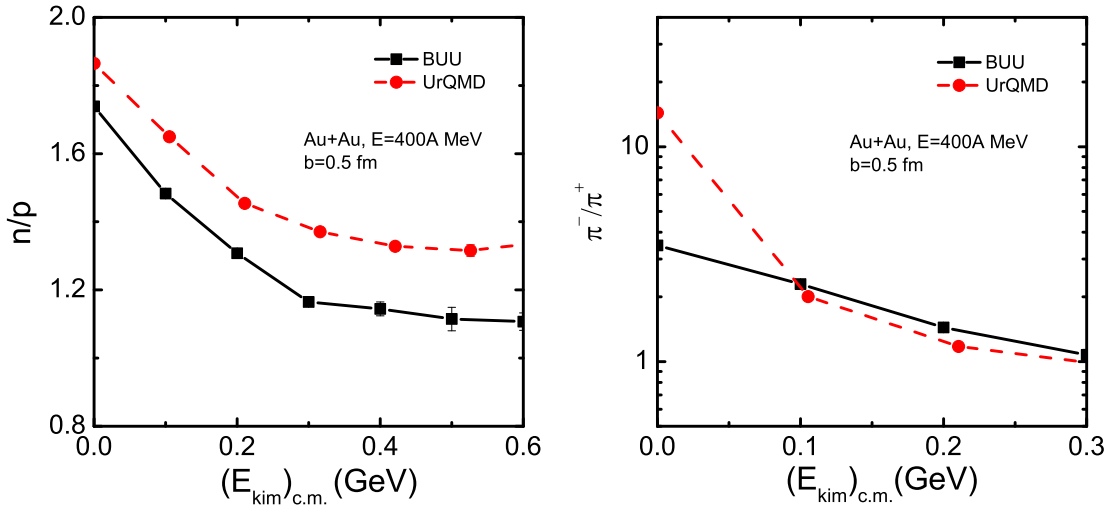

FIG. 4: n/p ratio of free nucleons and π^-/π^+ ratio as a function of kinetic energy in the central reaction of $^{197}Au+^{197}Au$ at a beam energy of 400 MeV/nucleon. The black solid line and red dashed line represent the results of IBUU04 model (t_{max}= 40 fm/c) and the UrQMD model (t_{max}= 150 fm/c), respectively.

The reason why the nucleonic collective flow given by the IBUU04 model show large isospin effect is that the used in-medium NN cross section in the IBUU04 model show large isospin effects as shown in the right panel of Fig. 2. The symmetry potential here in fact does not affect the strength of nucleonic collective flow evidently [71]. Thus although the value of the symmetry potential used in the UrQMD model is larger than that used in the IBUU04 model, isospin effect on nucleonic collective flow is still smaller than that calculated by the IBUU04 model since the symmetry potential has minor effect [71].

The neutron-proton differential flow was first proposed

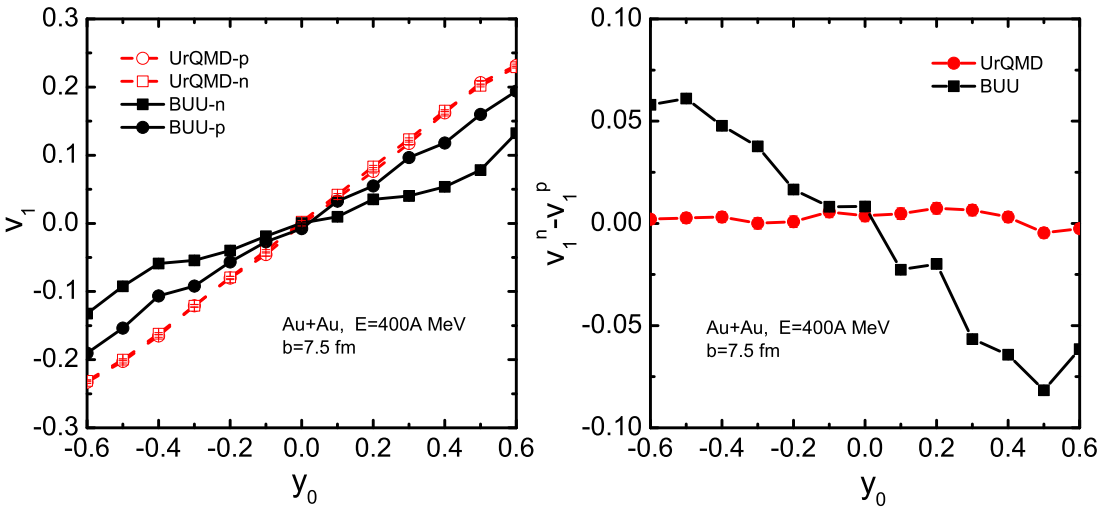

FIG. 5: Rapidity distributions of the neutron and proton directed flows $< \frac{p_x}{p_t} >$ and $n-p$ directed flow difference $v_1^n - v_1^p$ simulated respectively by IBUU04 model (t_{max}= 40 fm/c) and UrQMD model (t_{max}= 150 fm/c) in the semi-central reaction of $^{197}Au + ^{197}Au$ at a beam energy of 400 MeV/nucleon.

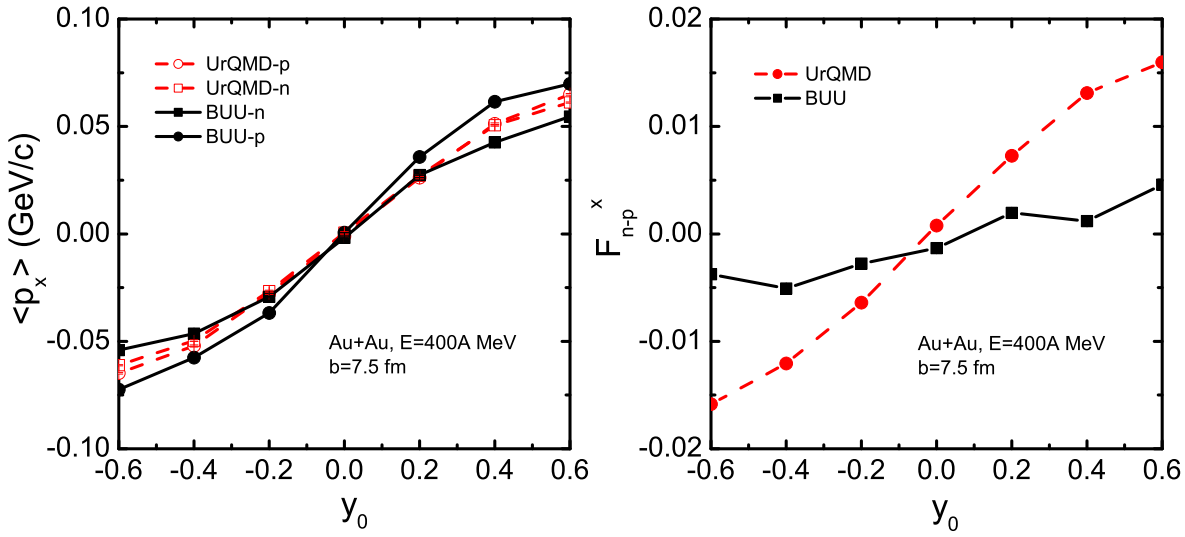

FIG. 6: Rapidity distributions of the transverse flow $< p_x(y) >$ for neutrons and protons and the neutron-proton differential transverse flow F^x_{n-p} simulated by the IBUU04 (t_{max}= 40 fm/c) and the UrQMD (t_{max}= 150 fm/c) models in the semi-central reaction of $^{197}Au + ^{197}Au$ at a beam energy of 400 MeV/nucleon.

as a probe of nuclear symmetry energy in 2000 by Li [72]. This approach utilizes constructively both the isospin fractionation and the nuclear collective flow as well as their sensitivities to the isospin-dependence of the nuclear equation of state. Later on, this approach was extended to two reaction systems using different isotopes of the same element in 2006 [73]. Rapidity dependences of the transverse flow $< p_x(y) >$ of the neutrons and protons and the neutron-proton differential transverse flows are shown in Fig. 6. The neutron-proton differential trans-

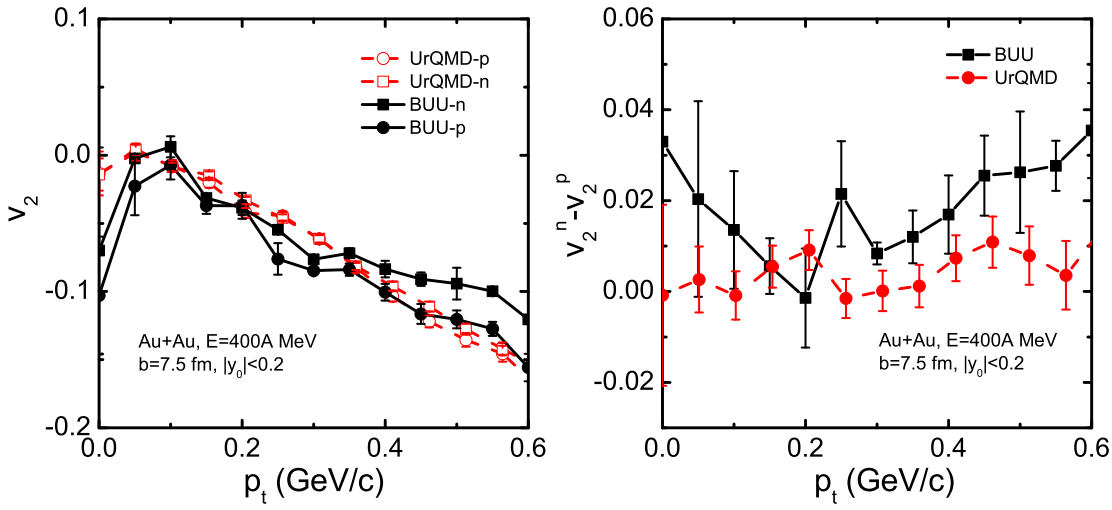

FIG. 7: Transverse momentum distributions of the neutron and proton elliptic flows and the $n-p$ elliptic flow difference $v_2^n - v_2^p$ given by the IBUU04 (t_{max}= 40 fm/c) and the UrQMD (t_{max}= 150 fm/c) models in the semi-central reaction $^{197}Au + ^{197}Au$ at a beam energy of 400 MeV/nucleon.

verse flow is expressed as [73]

$$F^x_{n-p}(y) = \frac{N_n(y)}{N(y)} < p^n_x(y) > - \frac{N_p(y)}{N(y)} < p^p_x(y) >, \quad (10)$$

where $N(y)$, $N_n(y)$ and $N_p(y)$ denote the number of free nucleons, neutrons and protons at rapidity y, respectively. And $< p^n_x(y) >$ and $< p^p_x(y) >$ are the average transverse momenta of neutrons and protons at rapidity y, respectively. From the left panel of Fig. 6, we can see that with the same symmetry energy, nucleonic transverse flow given by the IBUU04 model shows large isospin effect whereas the result of the UrQMD model does not. This is understandable since the NN cross section used in the IBUU04 model shows larger isospin effect as discussed in Fig. 5. It is noted that the slope of neutron-proton differential flow is larger for the UrQMD model than that for the IBUU04 model, this is understandable since the isospin fractionation is larger for the UrQMD model as shown in the left panel of Fig. 4.

Figure 7 shows the transverse momentum distributions of the neutron and proton elliptic flows and the $n-p$ elliptic flow differences $v_2^n - v_2^p$ calculated by the IBUU04 and the UrQMD models with the rapidity cut $|y_0| < 0.2$. Here v_2 is defined as $v_2 = < \frac{p_x^2 - p_y^2}{p_t^2} >$ and v_2^n is the elliptic flow for neutrons, v_2^p is the elliptic flow for protons. From Figure 7, we can see that nucleonic elliptic flows given by the two models are quite similar. At whole transverse momenta range $0 \sim 0.6$ GeV/c, there is clearly isospin effect of nucleonic elliptic flow with the IBUU04 model due to larger isospin effect of the in-medium NN cross section used in the IBUU04 model. Such effect is less evident with the UrQMD model. Difference of the isospin sensitive probe $n-p$ elliptic flow difference $v_2^n - v_2^p$ given by the two models are also clearly shown.

It is noted here that the freeze-out time of the reaction may also affect effects of isospin of observables in heavy-ion collisions [74]. In our calculations, stopping time settings are 150 fm/c in the UrQMD model and 40 fm/c in IBUU04 model, respectively. In fact in the UrQMD model, the isospin effect is evident before the freeze-out time of 150 fm/c.

In fact, as shown in the left panels of Figure 6 and Figure 7, both the IBUU04 model and the UrQMD model give almost the same isospin-independent nucleonic collective flows. The larger slope of $v_1 =< \frac{p_x}{p_t} >$ given by the UrQMD than that of the IBUU04 shown in the left panel of Figure 5 is cause by weak squeezing out (thus smaller p_x/p_y) of the UrQMD than that of the IBUU04.

IV. CONCLUSIONS AND REMARKS

Frequently used sensitive probes (nucleon or pion emission isospin ratios and relative nucleonic collective flow) of nuclear symmetry energy are simulated and compared in two different frameworks of transport model using their "best settings". Sensitive probe of n/p ratio of free nucleons is affected much by the symmetry potential while the isospin-sensitive probes of charged π^-/π^+ ratio, transverse flow and elliptic flow are affected much by the isospin-dependent NN cross section. Different isospin effects of observables given by different transport models originate from different forms of symmetry potential or isospin-dependent in-medium NN cross section. Sensitive probes of nuclear symmetry energy at high densities may suffer large uncertainties which are comparable with the effects of nuclear symmetry energy on these probes. Therefore one must be careful when drawing the

conclusion on density-dependent nuclear symmetry energy by reading related nuclear experiments with transport models.

Besides improving the framework of transport model from semi-classical transport to quantum transport [75], it would be nice to make thorough studies on the scattering cross sections, especially, of isospin-dependent nucleon-nucleon in medium [76–78] and symmetry potential of nucleon in asymmetric matter [79]. And also some unknown nucleon-nucleon interaction such as the tensor force induced isospin-dependence of short-range nucleon-nucleon correlation [80] and spin-orbit potential [81] may also affect isospin sensitive observables in heavy-ion collisions. Therefore searching for probes that insensitive to the uncertainties of model inputs or large sensitive probe such as possible η production in heavy-ion collisions [82] are always useful for the study of high-density nuclear symmetry energy.

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