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Teleportation of two-mode squeezed vacuum states

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Abstract: The teleportation of two-mode continuous variables was performed by using two two-mode squeezed vacuum states served as quantum channel. The average fidelity and the probability density were calculated for teleportation of a two-mode squeezed vacuum state. The results indicate that increased squeezing of teleported state will lead to a loss of fidelity. The probability density of measuring result consists of the product of two bivariate normed density functions, and it shows that when the average fidelity goes up, the probability of obtaining large measurement value also increases.

Key words: quantum optics; quantum teleportation; squeezed vacuum states; continuous variables

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两模连续变量的量子隐形传态

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摘 要： 利用两个双模压缩真空态作为量子通道，实现了两模连续变量的量子隐形传态。计算了两模压缩真空态隐形传态的平均保真度和概率密度函数。结果表明：增大传输态的压缩参量将导致保真度的损失；测量结果的概率密度函数是两个二元正态概率密度函数的乘积，当平均保真度增加时，获得大的测量值的概率也随之增加。

关键词： 量子光学；量子隐形传态；压缩真空态；连续变量

1 Introduction

Quantum teleportation (QT) is a process for Alice (sender) to transmit an unknown quantum input state to Bob (receiver) at a distant place by sending only classical information using a shared entangled state as quantum channel. Since Bennett *et al.*[1] showed that an unknown quantum state of two-state particle can be teleported from one place to another in 1993, much attention have been paid to QT[2,3]. The QT of finite-level

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systems has been demonstrated[4] and the ideas of QT for the discrete quantum states have been generalized to continuous variables corresponding to states of infinite-systems[5,6]. The discrete variable and continuous variable teleportation were unified in the framework of canonical QT[7]. The QT of optical coherent states was demonstrated experimentally using squeezed-state entanglement[8]. However, teleportation of single qubit is insufficient for a large-scale realization of quantum communication and computation. Hence one has been increasingly concerned about the teleportation of two-mode states including discrete variables[9,10] and continuous variables[11,12]. The experimental QT of a two-qubit composite system has already been accomplished[13], but teleportation of two-mode states of continuous variables has not been achieved in experiment.

In this paper, we propose a more realistic scheme for the teleportation of two-mode squeezed vacuum states. The average fidelity and the probability distribution of measurement are investigated. One kind of trade-off between the average fidelity and the magnitude of entanglement of the teleported state as well as measurement-disturbance is demonstrated. Our derivation is neat and concise due to using the Einstein-Podolsky-Rosen (EPR) pair eigenstate[14] $|\eta\rangle$ as Bell basis measurement.

2 Teleportation

In order to teleport a two-mode quantum state, Alice and Bob have to share two two-mode entangled states which act as quantum channels. In our scheme, the two two-mode entangled states produced by beam splitter[15] are two two-mode squeezed vacuum states

$$|\varphi\rangle_{35} = \sqrt{1-q^2}\exp[-q\hat{a}_3^\dagger\hat{a}_5^\dagger]|0,0\rangle_{35} = \sqrt{1-q^2}\sum_{n=0}^{\infty}(-q)^n|n\rangle_3|n\rangle_5 \ , \tag{1}$$

$$|\varphi\rangle_{46} = \sqrt{1-p^2}\exp[-p\hat{a}_4^\dagger\hat{a}_6^\dagger]|0,0\rangle_{46} = \sqrt{1-p^2}\sum_{n=0}^{\infty}(-p)^n|n\rangle_4|n\rangle_6 \ , \tag{2}$$

where $0 \le q \le 1, 0 \le p \le 1$ are the squeezing parameters that stand for the degree of entanglement. Suppose mode 3 and mode 4 belong to Alice and mode 5 and mode 6 belong to Bob. The unknown two-mode entangled state $|\chi\rangle_{12}$ which will be teleported from Alice to Bob is in mode 1 and mode 2. Thus the total initial state of the system is $|\chi\rangle_{12}|\varphi\rangle_{35}|\varphi\rangle_{46}$.

To realize the teleportation, Alice needs only to make a joint Bell measurement (quadrature phase measurement) of $X_1 + X_3$ and $P_1 - P_3$ performed on mode 1 and mode 3, and $X_2 + X_4$ and $P_2 - P_4$ on mode 2 and mode 4, respectively, where

$$X_i = \frac{1}{\sqrt{2}}(a_i + a_i^\dagger),\ P_i = \frac{1}{\sqrt{2}i}(a_i - a_i^\dagger) \ . \tag{3}$$

The simultaneous eigenstate $|\eta_1\rangle_{13}$ of commutative operators $(X_1 + X_3 \ , \ P_1 - P_3)$ is given by the following entangled state of two-mode field

$$\begin{aligned}|\eta_1\rangle_{13} &= \exp\Big[-\frac{|\eta_1|^2}{2} + \eta_1\hat{a}_1^\dagger + \eta_1^*\hat{a}_3^\dagger - \hat{a}_1^\dagger\hat{a}_3^\dagger\Big]|0\rangle_1|0\rangle_3 = \\ &D_1(\eta_1)\sum_m(-1)^m|m\rangle_1|m\rangle_3 = \\ &\sum_m \exp\big[\hat{a}_1^\dagger\eta_1 - \hat{a}_1\eta_1^*\big](-1)^m|m\rangle_1|m\rangle_3 \ ,\end{aligned} \tag{4}$$

in which $\eta_1=\eta_{11}+\mathrm{i}\eta_{12}$ is an arbitrary complex number. And we have $(X_1+X_3)|\eta_1\rangle_{13}=\sqrt{2}\eta_{11}|\eta_1\rangle_{13}$, and $(P_1-P_3)|\eta_1\rangle_{13}=\sqrt{2}\eta_{12}|\eta_1\rangle_{13}$. In like-manner $|\eta_1\rangle_{24}$ can also be written. Because the state $|\eta_1\rangle$ is complete $\int\frac{\mathrm{d}^2\eta}{\pi}|\eta\rangle\langle\eta|=1$ and orthonormal $\langle\eta'|\eta\rangle=\pi\delta(\eta-\eta')(\eta^*-\eta'^*)$, it is proper to be considered as continuous Bell basis. Alice first carries out a Bell measurement on mode 1 and mode 3 with outcome $\eta_1=\eta_{11}+\mathrm{i}\eta_{12}$, which teleport mode 1 to mode 5 and will project mode 2, 4, 5 and 6 onto following state

$$
\begin{aligned}
|\psi\rangle_{2456} = {}& {}_{13}\langle\eta_1|\chi\rangle_{12}|\varphi\rangle_{35}|\varphi\rangle_{46}= \\
& \frac{1}{\pi^2}\int\mathrm{d}^2\alpha\mathrm{d}^2z\ {}_{13}\langle\eta_1|\alpha\rangle_1|z\rangle_2|\varphi\rangle_{35}\ {}_2\langle z|_1\langle\alpha|\chi\rangle_{12}|\varphi\rangle_{46}= \\
& \frac{1}{\pi^2}\int\mathrm{d}^2\alpha\mathrm{d}^2z\sum_m {}_3\langle m|_1\langle m|(-1)^m D_1^\dagger(\eta_1)|\alpha\rangle_1|z\rangle_2\sqrt{1-q^2}\times \\
& \sum_n(-q)^n|n\rangle_3|n\rangle_{52}\langle z|_1\langle\alpha|\chi\rangle_{12}|\varphi\rangle_{46}= \\
& \frac{1}{\pi^2}\int\mathrm{d}^2\alpha\mathrm{d}^2z\sqrt{1-q^2}\sum_n q^n|n\rangle_{55}\langle n|D_5^\dagger(\eta_1)|\alpha\rangle_5|z\rangle_2\ {}_2\langle z|_5\langle\alpha|\chi\rangle_{52}|\varphi\rangle_{46}= \\
& \sqrt{1-q^2}q^{\hat{N}_5}D_5^\dagger(\eta_1)|\chi\rangle_{52}|\varphi\rangle_{46}\ ,
\end{aligned}
\tag{5}
$$

where $\hat{N}_5=\hat{a}_5^\dagger\hat{a}_5$ is a particle number operator acting on mode 5 and we have made use of completeness relation of coherent states $\frac{1}{\pi}\int|\alpha\rangle\langle\alpha|\mathrm{d}^2\alpha=1,\ \frac{1}{\pi}\int|z\rangle\langle z|\mathrm{d}^2z=1$.

Then Alice carries out a Bell measurement on mode 2 and mode 4 with outcome $\eta_2=\eta_{21}+\mathrm{i}\eta_{22}$. After the measurement, mode 2 is teleported to mode 6 and the state of mode 5 and mode 6 shared by Bob is projected onto

$$
\begin{aligned}
|\psi\rangle_{56} = {}& {}_{24}\langle\eta_2|\psi\rangle_{2456}= \\
& \frac{1}{\pi^2}\int\mathrm{d}^2\alpha\mathrm{d}^2z\sum_m {}_4\langle m|_2\langle m|(-1)^m D_2^\dagger(\eta_2)|\alpha\rangle_2|z\rangle_5\sqrt{1-p^2}\times \\
& \sum_n(-p)^n|n\rangle_4|n\rangle_{65}\langle z|_2\langle\alpha|\sqrt{1-q^2}q^{\hat{N}_5}D_5^\dagger(\eta_1)|\chi\rangle_{52}= \\
& \frac{1}{\pi^2}\int\mathrm{d}^2\alpha\mathrm{d}^2z\sqrt{1-q^2}\sqrt{1-p^2}\times \\
& \sum_n p^{\hat{N}_6}|n\rangle_{66}\langle n|D_6^\dagger(\eta_2)|\alpha\rangle_6|z\rangle_5\ {}_5\langle z|_6\langle\alpha|q^{\hat{N}_5}D_5^\dagger(\eta_1)|\chi\rangle_{56}= \\
& \sqrt{1-q^2}\sqrt{1-p^2}p^{\hat{N}_6}D_6^\dagger(\eta_2)q^{\hat{N}_5}D_5^\dagger(\eta_1)|\chi\rangle_{56}\ .
\end{aligned}
\tag{6}
$$

From Eqs. (1) and (2), we see that in the limit of infinite squeezing, $q\to1,\ p\to1$, the two two-mode squeezed vacuum states tend to EPR states, i.e.

$$|\varphi_{\mathrm{E}}\rangle_{35}\propto\exp\Big[-\hat{a}_3^\dagger\hat{a}_5^\dagger\Big]|0,0\rangle_{35}\ , \tag{7}$$

$$|\varphi_{\mathrm{E}}\rangle_{46}\propto\exp\Big[-\hat{a}_4^\dagger\hat{a}_6^\dagger\Big]|0,0\rangle_{46}\ . \tag{8}$$

Substituting Eq.(7) and Eq.(8) into Eq.(5), we can rewrite Eq.(6) as

$$|\psi_{\mathrm{E}}\rangle_{56}\propto D_6^\dagger(\eta_2)D_5^\dagger(\eta_1)|\chi\rangle_{56}\ . \tag{9}$$

We note that up to two simple unitary transformations $D_6^\dagger(\eta_2), D_5^\dagger(\eta_1)$, Bob possesses the same as the unknown state. If Alice now sends the results of measurement (η_1, η_2) to Bob via a classical channel, Bob is able to perform unitary operation ($D_6(\eta_2), D_5(\eta_1)$) and $|\psi_{\mathrm{E}}\rangle_{56}$ is then in the teleported state $|\chi\rangle_{56}$. Thus the initial state $|\chi\rangle_{12}$ shared by Alice has been teleported to Bob at a distant location though no one knows what the initial state is and it had to be destroyed by Alice's measurement. This is so-called quantum teleportation.

3 Fidelity and probability

To consider the effects of finite squeezing, we compute the average fidelity of the teleported state. Suppose that the state Alice wants to teleport is a two-mode squeezed vacuum state written as

$$|\chi\rangle_{12} = \sqrt{1-r^2}\exp\Big[-r\hat{a}_1^\dagger\hat{a}_2^\dagger\Big]|00\rangle_{12} = \sqrt{1-r^2}\sum_n^\infty(-r)^n|n\rangle_1|n\rangle_2, \tag{10}$$

where $0 \le r \le 1$ is a squeezing parameter. Because the normalized probability of obtaining the field measurement values η_1 and η_2 is $p(\eta_1,\eta_2) = \dfrac{1}{\pi^2}\ {}_{56}\langle\psi|\psi\rangle_{56}$, one is able to describe the fidelity of teleportation $F(\eta_1,\eta_2)$ and the average fidelity $\bar{F}$ as

$$F(\eta_1,\eta_2) = \frac{1}{\pi^2 P(\eta_1,\eta_2)}|{}_{56}\langle\psi_{\mathrm{E}}|\psi\rangle_{56}|^2 , \tag{11}$$

$$\bar{F} = \int \mathrm{d}^2\eta_1\mathrm{d}^2\eta_2 P(\eta_1,\eta_2)F(\eta_1,\eta_2) = \int \frac{\mathrm{d}^2\eta_1\mathrm{d}^2\eta_2}{\pi^2}|{}_{56}\langle\psi_{\mathrm{E}}|\psi\rangle|^2_{56} . \tag{12}$$

It follows from Eqs.(6), (9) and (10) that

$$\begin{aligned}
{}_{56}\langle\psi_{\mathrm{E}}|\psi\rangle_{56} &= \sqrt{1-q^2}\sqrt{1-p^2}{}_{56}\langle\chi|D_5(\eta_1)D_6(\eta_2)p^{\hat{N}_6}D_6^\dagger(\eta_2)q^{\hat{N}_5}D_5^\dagger(\eta_1)|\chi\rangle_{56} = \\
&\sqrt{1-q^2}\sqrt{1-p^2}(1-r^2)\times \\
&\sum_{m,n}{}_6\langle m|_5\langle m|(-r)^m D_5(\eta_1)D_6(\eta_2)p^{\hat{N}_6}D_6^\dagger(\eta_2)q^{\hat{N}_5}D_5^\dagger(\eta_1)(-r)^n|n\rangle_5|n\rangle_6 = \\
&\sqrt{1-q^2}\sqrt{1-p^2}(1-r^2)\frac{1}{1-r^2qp}\times \\
&\exp\Big\{-\tfrac{1}{1-r^2qp}\Big[(1-q)(1-r^2p)|\eta_1|^2 + (1-p)(1-r^2q)|\eta_2|^2 + \\
&r(1-q)(1-p)(\eta_1\eta_2^* + \eta_1^*\eta_2)\Big]\Big\} .
\end{aligned} \tag{13}$$

Substituting Eq.(13) into Eq.(12), we get

$$\bar{F} = \frac{(1+q)(1+p)(1-r^2)}{4(1-r^2qp)}. \tag{14}$$

It is easy to see that the average fidelity $\bar{F}$ increases with the squeezing parameter q, p and $\bar{F} \to 1$ as $q \to 1, p \to 1$, i.e., a perfect teleportation is performed. Moreover, we note that if q, p remain constant, increasing squeezing parameter r of the teleported state $|\chi\rangle_{12}$ will lead to loss of fidelity.

Now, let us analyze the probability of measuring the results η_1 and η_2 . From Eq.(6), we have

$$P(\eta_1,\eta_2) = \frac{1}{\pi^2}\, {}_{56}\langle\psi|\psi\rangle_{56} =$$

$$\frac{(1-q^2)(1-p^2)(1-r^2)}{\pi^2}\times$$

$$\sum_{m,n} {}_6\langle m|_5\langle m|(-r)^m D_5(\eta_1) q^{\hat{N}_5} D_6(\eta_2) p^{2\hat{N}_6} D_6^{\dagger}(\eta_2) q^{\hat{N}_5} D_5^{\dagger}(\eta_1)(-r)^n |n\rangle_5 |n\rangle_6 =$$

$$\frac{(1-q^2)(1-p^2)(1-r^2)}{\pi^2}\times$$

$$\exp\Big\{-\frac{1}{1-r^2qp}\Big[(1-q)(1-r^2p)(\eta_{11}^2+\eta_{12}^2)+(1-p)(1-r^2q)(\eta_{21}^2+\eta_{22}^2)+$$

$$r(1-q)(1-p)(\eta_{11}\eta_{21}+\eta_{12}\eta_{22})\Big]\Big\} =$$

$$\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\exp\Big\{-\frac{1}{2(1-\rho^2)}\Big[\frac{\eta_{11}^2}{\sigma_1^2}-2\rho\frac{\eta_{11}\eta_{21}}{\sigma_1\sigma_2}+\frac{\eta_{21}^2}{\sigma_2^2}\Big]\Big\}\times$$

$$\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\exp\Big\{-\frac{1}{2(1-\rho^2)}\Big[\frac{\eta_{12}^2}{\sigma_1^2}-2\rho\frac{\eta_{12}\eta_{22}}{\sigma_1\sigma_2}+\frac{\eta_{22}^2}{\sigma_2^2}\Big]\Big\}\,, \tag{15}$$

where

$$\rho=\sqrt{\frac{r^2(1-q^2)(1-p^2)}{(1-r^2q^2)(1-r^2p^2)}}\,,$$

$$\sigma_1=\sqrt{\frac{2(1-\rho^2)}{(1-q^2)(1-r^2p^2)(1-r^2q^2p^2)}}\,,$$

$$\sigma_2=\sqrt{\frac{2(1-\rho^2)}{(1-p^2)(1-r^2q^2)(1-r^2q^2p^2)}}\,.$$

We can see that the probability density $P(\eta_1,\eta_2)$ is the product of two bivariate normed density functions that has a maximum at $\eta_1=\eta_2=0$, and the probability of obtaining large measurement values η_1 and η_2 is less than that of obtaining small measurement values η_1 and η_2. In addition, the values σ_1 and σ_2 increase with the squeezing parameters q, p. If σ_1 and σ_2 are made larger, the maximum value of $P(\eta_1,\eta_2)$ becomes smaller consequently and the probability mass has greater spread about $\eta_1=\eta_2=0$, namely the measurement values η_1 and η_2 are more discrete around the center of the distribution. From Eq.(15), we know that the average fidelity $\bar{F}$ also increases with the squeezing parameter q, p, thus when the average fidelity $\bar{F}$ goes up, Alice's measurement value is more discrete and vise versa. Note also that the more discrete measurement values means the more increase of the probability of obtaining large measurement value that provides much information about the phase of teleported field.

4 Conclusions

In summary, we have shown how two two-mode squeezed vacuum states can be used as quantum channel to accomplish the teleportation of two-mode continuous variables by virtue of measurement of joint quadrature phase quantities, X_1+X_2 and P_1-P_2. If the entangled resources which serve as quantum channel are squeezed infinitely, perfect teleportation can be achieved. We calculate the average fidelity and the probability of measurement for teleportation of a two-mode squeezed vacuum state, and find that increased squeezing of the teleported state $|\chi\rangle_{12}$ leads to a loss of fidelity, and when the average fidelity $\bar{F}$ increases with the squeezing parameters q, p, Alice's measurement value is more discrete, because this more discrete measurement value provides much information about phase of teleported field. Our result is concise due to using the EPR pair eigenstate $|\eta\rangle$ as Bell basis measurement. Since a two-mode squeezed vacuum state is more easily generated by beam splitter, our scheme is experimentally more feasible.

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