

# Effective Lagrangians with Higher Order Derivatives

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BI-TP 93/29  
hep-ph/9306321  
June 1993

## Abstract

The problems that are connected with Lagrangians which depend on higher order derivatives (namely additional degrees of freedom, unbound energy from below, etc.) are absent if *effective* Lagrangians are considered because the equations of motion may be used to eliminate all higher order time derivatives from the effective interaction term. The application of the equations of motion can be realized by performing field transformations that involve derivatives of the fields. Using the Hamiltonian formalism for higher order Lagrangians (Ostrogradsky formalism), Lagrangians that are related by such transformations are shown to be physically equivalent (at the classical and at the quantum level). The equivalence of Hamiltonian and Lagrangian path integral quantization (Matthews's theorem) is proven for effective higher order Lagrangians. Effective interactions of massive vector fields involving higher order derivatives are examined within gauge noninvariant models as well as within (linearly or nonlinearly realized) spontaneously broken gauge theories. The Stueckelberg formalism, which relates gauge noninvariant to gauge invariant Lagrangians, becomes reformulated within the Ostrogradsky formalism.

arXiv:hep-ph/9306321v1 28 Jun 1993

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# 1 Introduction

Effective Lagrangians containing arbitrary interactions of massive vector and scalar fields are studied very intensively in the literature in order to parametrize possible deviations of electroweak interactions from the standard model with respect to experimental tests of the  $W^\pm$ ,  $Z$  and  $\gamma$  self-interactions and of the Higgs sector. When performing a complete analysis of the extensions of the standard (Yang–Mills) vector-boson self-interactions to nonstandard interactions, one necessarily has to consider effective interaction terms that depend on higher order derivatives of the fields [1, 2]. Therefore, the investigation of effective Lagrangians with higher order derivatives is very important from the phenomenological point of view.

However, theories described by higher order Lagrangians have quite unsatisfactory properties [3, 4, 5], namely: there are additional degrees of freedom, the energy is unbound from below, the solutions of the equations of motion are not uniquely determined by the initial values of the fields and their first time derivatives and the theory has no analytic limit for  $\epsilon \rightarrow 0$  (where  $\epsilon$  denotes the coupling constant of the higher order term). Clearly, these features are very unwelcome when dealing with effective Lagrangians in order to parametrize *small* deviations from a renormalizable theory like the standard model.

Fortunately however, the abovementioned problems are absent if a higher order Lagrangian is considered to be an *effective* one. This means, one assumes that there exists a renormalizable theory with heavy particles at an energy scale  $\Lambda$  (“new physics”), and that the effective Lagrangian parametrizes the effects of the “new physics” at an energy scale lower than  $\Lambda$  by expressing the contributions of the heavy fields (which do not explicitly occur in the effective Lagrangian) through nonrenormalizable effective interaction terms. Supposed that the renormalizable Lagrangian describing the “new physics” does not depend on higher order derivatives, it causes no unphysical effects and therefore such effects also do not occur at the lower energy scale, i.e. at the effective Lagrangian level. Actually, I will show in this paper that all higher order time derivatives can be eliminated in the first order of the effective coupling constant  $\epsilon$  (with  $\epsilon \ll 1$ ). Higher powers of  $\epsilon$  can be neglected because an effective Lagrangian is assumed to describe the effects of well-behaved “new physics” the  $O(\epsilon)$  approximation only; consequently all ill-behaved effects (which do not occur in the first order of  $\epsilon$ ) become cancelled by other  $O(\epsilon^n)$  ( $n > 1$ ) effects of the “new physics”.

Each effective higher order Lagrangian can be reduced to a first order Lagrangian because one can apply the equations of motion (EOM) in order to eliminate all higher order time derivatives from the effective interaction term (upon neglecting higher powers of  $\epsilon$ ). This is a nontrivial statement because, in general, the EOM must not be used to convert the Lagrangian. However, it has been shown that it is possible to find field transformations which have the the same effect as the application of the EOM to the effective interaction term (in the first order of  $\epsilon$ ) [6, 7, 8]. Within the Hamiltonian formalism for higher order Lagrangians (Ostrogradsky formalism) [3], these field transformations are point transformations (and thus canonical transformations) although they involve derivatives of the fields. The reason for this is that, within the Ostrogradsky formalism, the *derivatives* up to order  $N-1$  are formally treated as independent *coordinates* if the Lagrangian is of order  $N$ , and the order  $N$  of the Lagrangian can be chosen arbitrarily without affecting the physical content of the theory [9] (as long as  $N$  is greater or equal to the order of the highest actually appearing derivative). This implies that the Lagrangian which is obtained from the primordial Lagrangian by applying the EOM to its effective interaction term is indeed physically equivalent to this (at the classical and at the quantum level). In this paper I will describe this method of reducing effective higher order Lagrangians.

The main task of this paper is the quantization of effective higher order Lagrangians within the Feynman path integral (PI) formalism. It is well known that, in general, quantization has to be based on the Hamiltonian PI [9, 10] and not on the naive Lagrangian PI. The Lagrangian PI, however, is more useful for practical calculations because it does not involve the momenta and it is manifestly covariant. In fact, it has been shown for arbitrary effective interactions of scalar fields [5] and of massive vector fields<sup>1</sup> [11] (which depend on first order derivatives of the fields only) that, after the momenta have been integrated out, the Hamiltonian PI corresponding to an effective Lagrangian  $\mathcal{L}$  can be written as simple Lagrangian PI

$$Z = \int \mathcal{D}\varphi \exp \left\{ i \int d^4x [\mathcal{L}_{quant} + J\varphi] \right\} \quad (1.1)$$

( $\varphi$  is a shorthand notation for all fields in  $\mathcal{L}_{quant}$ ), where the quantized Lagrangian  $\mathcal{L}_{quant}$  is identical to the primordial Lagrangian  $\mathcal{L}$ :

$$\mathcal{L}_{quant} = \mathcal{L} \quad (1.2)$$

(if  $\mathcal{L}$  has no gauge freedom). This means that the correct Hamiltonian PI quantization yields the same result as naive Lagrangian PI quantization. Actually, this result justifies Lagrangian PI quantization. Furthermore, (1.1) and (1.2) imply that the Feynman rules follow directly from the effective Lagrangian, i.e., the quadratic terms in  $\mathcal{L}$  yield the propagators and the other terms yield the vertices in the standard manner. This simple quantization rule is known as Matthews's theorem [12]. Within the PI formalism Matthews's theorem can be reformulated as the statement that Hamiltonian and Lagrangian PI quantization are equivalent.

Matthews's theorem has yet only been proven for effective interactions which involve at most first order derivatives [5, 11]. In the higher order case, the result (1.1), (1.2) has been derived for some special examples [5, 13] but not as a general theorem. In this paper I will prove Matthews's theorem for Lagrangians with arbitrary higher order effective interactions making use of the abovementioned procedure of reducing higher order effective Lagrangians to first order ones and then applying Matthews's theorem for Lagrangians without higher order derivatives.

I will derive this result first for the simple case of effective interactions of scalar fields and then extend it to effective interactions of massive vector fields. In the latter case one has to distinguish between gauge noninvariant effective Lagrangians and (nonlinearly or linearly realized) spontaneously broken gauge theories (SBGTs). However, the results obtained for gauge noninvariant Lagrangians can easily be extended to gauge invariant Lagrangians because, by applying a Stueckelberg transformation [14, 15, 16], each SBGT can be rewritten as a gauge noninvariant model (after a nonlinear parametrization of the scalar sector [16, 17, 18] in the case of a linear Higgs model). A Stueckelberg transformation is a field transformation (that involves derivatives of the fields) which results in removing all unphysical scalar fields from the Lagrangian. In [11], I have proven the canonical equivalence of Lagrangians (without higher order derivatives) that are related by a Stueckelberg transformation. Within the Ostrogradsky formalism this result can be generalized to higher order Lagrangians and besides the proof can be simplified very much since, as mentioned above, within this formalism a field transformation which involves derivatives is a point transformation, i.e. canonical transformation. For gauge invariant models, Matthews's theorem states that an effective theory can be quantized using the (Lagrangian) Faddeev–Popov formalism

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<sup>1</sup>Using the same procedure as in [11] this result can also be derived for the (phenomenologically less interesting) case of effective interactions involving fermion fields.

[19]. The quantized Lagrangian in (1.1) becomes

$$\mathcal{L}_{quant} = \mathcal{L} + \mathcal{L}_{g.f.} + \mathcal{L}_{FP-ghost}, \quad (1.3)$$

i.e., there are additional gauge fixing (g.f.) and ghost terms. This result can be derived as follows: using the Stueckelberg formalism and Matthews's theorem for gauge noninvariant Lagrangians one can show that the Hamiltonian PI corresponding to a SBGT is identical to the one obtained within the Faddeev–Popov formalism, if the (U-gauge) g.f. conditions that all unphysical scalar fields become equal to zero are imposed<sup>2</sup> [16]. The equivalence of all gauges, i.e. the independence of the  $S$ -matrix elements from the choice of the gauge in the Faddeev–Popov procedure [17, 20], yields then the result (1.1) with (1.3) in an arbitrary gauge.

As in the case of Lagrangians with at most first order derivatives, the result (1.2) or (1.3) is only correct up to additional terms proportional to  $\delta^4(0)$  [5, 11] which, however, become zero if dimensional regularization is applied. According to [5, 11], I will neglect all these  $\delta^4(0)$  terms in this paper.

This paper is organized as follows: In section 2, the Hamiltonian formalism for Lagrangians with higher order derivatives (Ostrogradsky formalism) and the Hamiltonian path integral quantization of such Lagrangians are briefly reviewed. Using this formalism, Lagrangians which are related by field transformations that involve derivatives of the fields are shown to be equivalent (at the classical and at the quantum level). In section 3, it is shown that higher order effective Lagrangians can be reduced to first order ones by applying the equations of motion. The equivalence of Hamiltonian and Lagrangian path integral quantization (Matthews's theorem) is proven for higher order effective interactions of scalar fields. These results become extended to the case of massive vector fields in section 4. Spontaneously broken gauge theories are reduced to gauge noninvariant Lagrangians using the Stueckelberg formalism, which becomes reformulated within the Ostrogradsky formalism. Section 5 contains the final conclusions.

## 2 The Ostrogradsky Formalism

The Hamiltonian formalism for Lagrangians with higher order derivatives has been formulated one and a half century ago by Ostrogradsky [3]. I will briefly review this formalism, since it is not as well known as the Hamiltonian formalism for Lagrangians with at most first order derivatives. I will also take into account the case of singular higher order Lagrangians [9, 21], i.e. of Lagrangians which involve constraints<sup>3</sup>. Furthermore, I will consider the quantization of higher order Lagrangians within the Hamiltonian PI formalism. For simplification I will restrict to the case of finitely many degrees of freedom; the generalization to field theory is analogous to the first order case.

Consider a Lagrangian of order  $N$ , i.e. a Lagrangian which depends on the coordinates  $q_i$ , with  $i = 1, \dots, I$  and their derivatives up to the order  $N$ :

$$L(q_i, q_i^{(1)}, \dots, q_i^{(N)}), \quad \text{with} \quad q_i^{(n)} \equiv \left(\frac{d}{dt}\right)^n q_i. \quad (2.1)$$

Within in the Hamiltonian formalism, the coordinates are

$$Q_{i,n} \equiv q_i^{(n-1)}, \quad n = 1, \dots, N \quad (2.2)$$

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<sup>2</sup>For the special case of the U-gauge, the Faddeev–Popov procedure does not yield explicit g.f. and ghost terms [16] unlike in the general case (1.3).

<sup>3</sup>The reader who is not familiar with the basic concepts of dynamics and quantization of constrained systems is referred to [9, 10, 22].

and the momenta are

$$P_{i,n} \equiv \sum_{k=n}^N \left( -\frac{d}{dt} \right)^{k-n} \frac{\partial L}{\partial q_i^{(k)}}, \quad n = 1, \dots, N. \quad (2.3)$$

The Hamiltonian is given by

$$H \equiv \sum_{i=1}^I \sum_{n=1}^N P_{i,n} \dot{Q}_{i,n} - L = \sum_{i=1}^I \left( \sum_{n=1}^{N-1} P_{i,n} Q_{i,n+1} + P_{i,N} q_i^{(N)} \right) - L. \quad (2.4)$$

In (2.4), the  $q_i^{(N)}$  have to be expressed in terms of the  $P_{i,N}$  by using (2.3) with  $n = N$ :

$$P_{i,N} = \frac{\partial L}{\partial q_i^{(N)}}. \quad (2.5)$$

If the theory is nonsingular, i.e. if the  $I \times I$ -matrix

$$M_{ij} \equiv \frac{\partial L}{\partial q_i^{(N)} \partial q_j^{(N)}} \quad (2.6)$$

is nonsingular, (2.5) can be solved for all  $q_i^{(N)}$ . For singular Lagrangians one finds

$$\text{rank } M_{ij} = R < I. \quad (2.7)$$

The indices  $i, j$  can be ordered such that the upper left  $R \times R$ -submatrix of  $M$  has the rank  $R$ . In this case, (2.5) can be solved for  $q_i^{(N)}$  with  $i = 1, \dots, R$  only, while the remaining of these relations are constraints. However,  $H$  (2.4) does not depend on  $q_i^{(N)}$  with  $i = R + 1, \dots, I$  because

$$\frac{\partial H}{\partial q_i^{(N)}} = P_{i,N} - \frac{\partial L}{\partial q_i^{(N)}} = 0, \quad i = R + 1, \dots, I, \quad (2.8)$$

where (2.5) has been used. Furthermore, (2.7) implies that the first  $R$  of the relations (2.5) can be used to rewrite the remaining ones in a form which does not involve the  $q_i^{(N)}$ . The total Hamiltonian  $H_T$  is defined as

$$H_T \equiv H + \lambda_a \phi_a, \quad (2.9)$$

where the  $\lambda_a$  are Lagrange multipliers and the  $\phi_a$  are the primary constraints (2.5) with  $i = R + 1, \dots, I$  and the secondary, tertiary, etc. constraints (which follow from the primary constraints due to the requirement that the constraints have to be consistent with the EOM). The Hamiltonian EOM can be written as

$$\dot{Q}_{i,n} = \frac{\partial H_T}{\partial P_{i,n}} = \{Q_{i,n}, H_T\}, \quad (2.10)$$

$$\dot{P}_{i,n} = -\frac{\partial H_T}{\partial Q_{i,n}} = \{P_{i,n}, H_T\}. \quad (2.11)$$

It should be emphasized that, within the Ostrogradsky formalism for a Lagrangian of order  $N$ , all derivatives of the  $q_i$  up to the order  $N - 1$  are formally treated as independent coordinates (due to (2.2)).

A higher order Lagrangian can be quantized within the Hamiltonian PI formalism [9, 10] analogously to a first order Lagrangian. The Hamiltonian PI turns out to be

$$Z = \int \prod_{i=1}^I \prod_{n=1}^N (\mathcal{D}Q_{i,n} \mathcal{D}P_{i,n}) \exp \left\{ i \int d^4x \left[ -H + \sum_{i=1}^I \sum_{n=1}^N (P_{i,n} \dot{Q}_{i,n} + J_{i,n} Q_{i,n} + K_{i,n} P_{i,n}) \right] \right\} \times \prod_a \delta(\Phi_a) \text{Det}^{\frac{1}{2}} \{ \Phi_a, \Phi_b \}. \quad (2.12)$$

The  $J_{i,n}$  and  $K_{i,n}$  are sources and the  $\Phi_a$  are the primary constraints (2.5) with  $i = R + 1, \dots, I$ , the secondary, tertiary, etc. constraints and the gauge fixing conditions if there are first class constraints.

For the following investigations it is important, that the Ostrogradsky formalism is not affected by changing the (formal) order  $N$  of the Lagrangian as long as  $N$  is greater or equal to the order of the highest actually appearing derivative. In other words, the Ostrogradsky formulations of two physical systems which are both given in terms of the same Lagrangian, but one is considered to be of order  $N$  and the other to be of order  $M$ , are equivalent. This theorem has been proven in [9]. I will repeat this short proof here and I will show that also the Hamiltonian PIs corresponding to the two systems are identical.

It is sufficient to assume that  $M = N + 1$ . I will now treat the Lagrangian (2.1) (which does not depend on the  $q_i^{(N+1)}$ ) formally as an  $(N + 1)$ st order Lagrangian. One finds the canonical variables

$$\tilde{Q}_{i,n} = q_i^{(n-1)}, \quad n = 1, \dots, N + 1, \quad (2.13)$$

$$\tilde{P}_{i,n} = \sum_{k=n}^{N+1} \left( -\frac{d}{dt} \right)^{k-n} \frac{\partial L}{\partial q_i^{(k)}}, \quad n = 1, \dots, N + 1. \quad (2.14)$$

With  $n = 1, \dots, N$ , these equations become

$$\tilde{Q}_{i,n} = Q_{i,n}, \quad \tilde{P}_{i,n} = P_{i,n}, \quad n = 1, \dots, N, \quad (2.15)$$

i.e., these variables are identical to the corresponding ones obtained in the  $N$ th order formalism, (2.2) and (2.3). With  $n = N + 1$  one finds

$$\tilde{Q}_{i,N+1} = q_i^{(N)}, \quad (2.16)$$

$$\tilde{P}_{i,N+1} = \frac{\partial L}{\partial q_i^{(N+1)}} = 0. \quad (2.17)$$

The Hamiltonian is given by

$$\tilde{H} = \sum_{i=1}^I \sum_{n=1}^{N+1} \tilde{P}_{i,n} \dot{\tilde{Q}}_{i,n} - L = \sum_{i=1}^I \sum_{n=1}^N \tilde{P}_{i,n} \dot{\tilde{Q}}_{i,n+1} - L. \quad (2.18)$$

$\tilde{H}$  is identical to  $H$  (2.4), except that in  $H$  the  $q_i^{(N)}$  are expressed in terms of  $P_{i,N}$  using (2.5), but in  $\tilde{H}$  they are still present (because, due to (2.16), they are independent coordinates in the  $(N + 1)$ st order formalism). The  $(N + 1)$ st order system is singular; the relations (2.17) are the primary constraints. The requirement that these constraints have to be consistent with the EOM yields the secondary constraints

$$\dot{\tilde{P}}_{i,N+1} = -\frac{\partial \tilde{H}}{\partial \tilde{Q}_{i,N+1}} = \frac{\partial L}{\partial q_i^{(N)}} - \tilde{P}_{i,N} = 0. \quad (2.19)$$

This is identical to the relation (2.5). Applying (2.19) with  $i = 1, \dots, R$  in order to eliminate all  $\tilde{Q}_{i,N+1}$  from the Hamiltonian  $\tilde{H}$  and from the remaining of the constraints (2.19) as outlined above<sup>4</sup> one finds

$$\tilde{H} = H. \quad (2.20)$$

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<sup>4</sup>Constraints may be used to convert the Hamiltonian and the other constraints, this corresponds to a redefinition of the Lagrange multipliers in the total Hamiltonian  $H_T$  (2.9). In the Hamiltonian PI (2.12) this is justified due to the presence of the  $\delta$ -functions.

Furthermore, the Lagrange multipliers of the constraints (2.19) with  $i = 1, \dots, R$  that can be solved for  $\tilde{Q}_{i,N+1}$ , i.e. that can be rewritten as

$$\tilde{Q}_{i,N+1} - f_i(\tilde{Q}_{j,1}, \dots, \tilde{Q}_{j,N}, \tilde{P}_{j,N}) = 0 \quad i = 1, \dots, R \quad (2.21)$$

become zero in order to assure  $\dot{\tilde{P}}_{i,N+1} = 0$  (because  $\tilde{H}$  and the other constraints do not depend on the  $\tilde{Q}_{i,N+1}$  anymore). The remaining of the constraints (2.19) are identical to the constraints obtained in the  $N$ th order formalism. Therefore, also the total Hamiltonians which imply the EOM are identical<sup>5</sup>

$$\tilde{H}_T = H_T. \quad (2.22)$$

From (2.15), (2.20) and (2.22) follows the equivalence of the  $N$ th and the  $(N+1)$ st order Ostrogradsky formalism.

To extend this classical result to quantum physics one has to show that the Hamiltonian PIs obtained within the  $N$ th and the  $(N+1)$ st order formalism are identical. In the  $(N+1)$ st order formalism the Hamiltonian PI has the form

$$\begin{aligned} \tilde{Z} = \int \prod_{i=1}^I \prod_{n=1}^{N+1} (\mathcal{D}\tilde{Q}_{i,n} \mathcal{D}\tilde{P}_{i,n}) \exp \left\{ i \int d^4x \left[ -\tilde{H} + \sum_{i=1}^I \sum_{n=1}^{N+1} (\tilde{P}_{i,n} \dot{\tilde{Q}}_{i,n} + J_{i,n} \tilde{Q}_{i,n} + K_{i,n} \tilde{P}_{i,n}) \right] \right\} \\ \times \prod_a \delta(\tilde{\Phi}_a) \text{Det}^{\frac{1}{2}} \{ \tilde{\Phi}_a, \tilde{\Phi}_b \}. \end{aligned} \quad (2.23)$$

The constraints and g.f. conditions  $\tilde{\Phi}_a$  include all the constraints and g.f. conditions  $\Phi_a$  obtained in the  $N$ th order formalism (because (2.5) and (2.19) are identical) and, in addition, the constraints (2.17) and (2.21). The fundamental Poisson brackets imply that (2.17) with  $i = 1, \dots, R$  and (2.21) represent a system of second class constraints, while the constraints (2.17) with  $i = R+1, \dots, I$ , which have vanishing Poisson brackets with all other constraints, are first class. Therefore one has to include gauge fixing conditions corresponding to these first class constraints into the  $\tilde{\Phi}_a$ . A possible and suitable choice is

$$\tilde{Q}_{i,N+1} = 0, \quad i = R+1, \dots, I. \quad (2.24)$$

Using the fundamental poisson brackets and remembering that the  $\Phi_a$  do not depend on the  $\tilde{Q}_{i,N+1}$  and the  $\tilde{P}_{i,N+1}$ , one finds

$$\text{Det} \{ \tilde{\Phi}_a, \tilde{\Phi}_b \} = \text{Det} \{ \Phi_a, \Phi_b \}. \quad (2.25)$$

Remembering the presence of the  $\delta$ -functions corresponding to the constraints (2.17), (2.21) and the g.f. conditions (2.24) in the PI, one can perform the functional integrations over the  $\tilde{Q}_{i,N+1}$  and the  $\tilde{P}_{i,N+1}$ . The equalities (2.15) and (2.20) imply that the Hamiltonian PI (2.23) is identical to the one obtained in the  $N$ th order formalism<sup>6</sup> (2.12). Thus, the Hamiltonian PI is also independent of the formal order  $N$ .

The conclusion of this section, which is most important for the subsequent investigations is the following: In the  $N$ th order formalism all derivatives up to the order  $N-1$  are treated as independent coordinates and not as derivatives. Furthermore, the order  $N$  can be chosen arbitrarily high without affecting the physical content of the theory. This implies that each local coordinate transformation which also involves derivatives of the coordinates (up to a

<sup>5</sup>Actually,  $\tilde{H}_T$  also contains Lagrange multipliers corresponding to the primary constraints (2.17). These however do not affect the EOM (2.10), (2.11) for the variables  $\tilde{Q}_{i,n}$  and  $\tilde{P}_{i,n}$  with  $n = 1, \dots, N$ .

<sup>6</sup>The sources  $J_{i,N+1}$  with  $i = 1, \dots, R$  are still present in  $\tilde{Z}$  but this has no physical effect.

finite order) can be considered to be a point transformation, i.e. a transformation which formally only involves the coordinates but not the derivatives. If one wants to apply a coordinate transformation involving derivatives up to the order  $N$ , one simply has to treat the Lagrangian as an  $(N+1)$ st order one (even if no  $(N+1)$ st order derivatives occur in  $L$ ) so that this transformation can be identified as a point transformation. Such a transformation becomes a canonical transformation within the Hamiltonian framework because the fact that two Lagrangians are related by a point transformation implies that the corresponding Hamiltonians and constraints are related by a canonical transformation<sup>7</sup>. Since the physical content of a theory is not affected by canonical transformations, one finds that *Lagrangians, which are related by a local coordinate transformation are physically equivalent even if this transformation involves derivatives.*

The Hamiltonian PIs corresponding to Lagrangians that are related by such a field transformation are identical due to the invariance of the Hamiltonian PI under canonical transformations [9, 10] and its independence of the (formal) order  $N$  (see above). Therefore, *this equivalence is also valid in quantum physics.*

### 3 Reduction of Higher Order Effective Lagrangians and Matthews's Theorem

In this section I will show that a higher order effective Lagrangian can be reduced to a first order one by applying the equations of motion to the effective interaction term. Using this reduction, I will generalize Matthews's theorem to higher order effective Lagrangians. For simplicity I will first only consider the case of a massive scalar field.

The effective Lagrangian has the form

$$\mathcal{L} = \mathcal{L}_0 + \epsilon \mathcal{L}_I = \frac{1}{2}(\partial^\mu \varphi)(\partial_\mu \varphi) - \frac{1}{2}M^2 \varphi^2 + \epsilon \mathcal{L}_I(\varphi, \partial^\mu \varphi, \dots, \partial^{\mu_1} \dots \partial^{\mu_N} \varphi). \quad (3.1)$$

$\mathcal{L}_0$  simply represents a free massive Klein–Gordon theory and  $\mathcal{L}_I$  contains the effective interactions which depend on the derivatives of the scalar fields up to the order  $N$ . These interactions are governed by the coupling constant  $\epsilon$  with  $\epsilon \ll 1$ .

Now I want to remove all higher order time derivatives from the effective interaction term by applying the EOM (upon neglecting higher powers of  $\epsilon$ ). This procedure must be carefully justified since, in general, the EOM must not be used to convert the Lagrangian. Therefore, I use the results of [6, 7, 8] where it has been shown that it is always possible to find field transformations which effectively result in applying the EOM following from  $\mathcal{L}_0$  to  $\mathcal{L}_I$  (in the first order of  $\epsilon$ ). Assume that  $\epsilon \mathcal{L}_I$  contains a term

$$\epsilon T \ddot{\varphi} \quad (3.2)$$

(where  $T$  is an arbitrary expression in  $\varphi$  and its derivatives), i.e. that (3.1) can be written as

$$\mathcal{L} = \mathcal{L}_0 + \epsilon T \ddot{\varphi} + \epsilon \tilde{\mathcal{L}}_I \quad (3.3)$$

(with  $\tilde{\mathcal{L}}_I \equiv \mathcal{L}_I - T \ddot{\varphi}$ ). Performing the field transformation [6, 7, 8]

$$\varphi \rightarrow \varphi + \epsilon T \quad (3.4)$$

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<sup>7</sup>This can be most easily seen if the order  $N$  is chosen so high that the transformations of the  $Q_{i,n}$  with  $n < N$  do not involve the  $Q_{i,N}$ . (2.4) and (2.5) imply then that the Hamiltonian and the primary constraints (2.5) with  $i = R+1, \dots, I$  are related by a canonical transformation. The secondary, tertiary, etc. constraints follow from the Poisson brackets of the primary constraints with the Hamiltonian and each other which are invariant under canonical transformations.

one finds

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_0 + \epsilon T \ddot{\varphi} + \epsilon T \left( \frac{\partial \mathcal{L}_0}{\partial \varphi} - \partial^\mu \frac{\partial \mathcal{L}_0}{\partial (\partial^\mu \varphi)} \right) + \epsilon \tilde{\mathcal{L}}_I + O(\epsilon^2) \\ &= \mathcal{L}_0 + \epsilon T (\Delta \varphi - M^2 \varphi) + \epsilon \tilde{\mathcal{L}}_I + O(\epsilon^2).\end{aligned}\tag{3.5}$$

This means that effectively the second order time derivative has been removed from the term (3.2) by applying the free EOM (i.e. those implied by  $\mathcal{L}_0$  alone)

$$\ddot{\varphi} = \Delta \varphi - M^2 \varphi.\tag{3.6}$$

If there are terms with higher than second order derivatives in  $\epsilon \mathcal{L}_I$ , they can be put into the form (3.2) by performing partial integrations and dropping total derivative terms<sup>8</sup>. Repeating the above procedure, one can remove all higher order time derivatives from the effective Lagrangian.

To prove that a higher order effective Lagrangian and the first order Lagrangian obtained from it by applying the above procedure are physically equivalent, I apply the results of the previous section. The Lagrangians are connected by field transformations of the type (3.4) which, in general, involve derivatives of  $\varphi$  (contained in  $T$ ). However, within the Ostrogradsky formalism the transformations are point (i.e. canonical) transformations (if the formal order of  $\mathcal{L}$  is chosen sufficiently high) which establishes the equivalence of both Lagrangians. To be strictly correct, one must remember that, within the Ostrogradsky formalism, the time derivatives of  $\varphi$  are fields that are formally independent of  $\varphi$ . Therefore, the transformation of  $\dot{\varphi}$  must be specified separately:

$$\dot{\varphi} \rightarrow \dot{\varphi} + \epsilon \dot{T},\tag{3.7}$$

because this relation is not automatically implied by (3.4) as in the first order formalism. (3.4) and (3.7) specify the field transformation completely (since, due to the neglect of higher powers of  $\epsilon$ , this transformation is de facto only applied to  $\mathcal{L}_0$  which contains at most first order derivatives). Now one can easily see that, if  $T$  contains at most  $M$ th order derivatives, the Lagrangian formally has to be treated as an at least  $(M+2)$ nd order one so that (3.4), (3.7) becomes a canonical transformation. The result of the above procedure is that *the equations of motion following from  $\mathcal{L}_0$  may be applied to convert  $\mathcal{L}_I$  in order to eliminate all higher order time derivatives* (upon neglecting higher powers of  $\epsilon$ ).

Now it is easy to quantize the effective Lagrangian (3.1) and to prove Matthews's theorem (1.1) with (1.2). The proof goes as follows:

1. Given an effective higher order Lagrangian  $\mathcal{L}$  (3.1), it can be reduced by applying the above procedure to an equivalent first order<sup>9</sup> Lagrangian  $\mathcal{L}_{red}$ . As discussed in section 2 this does not affect the Hamiltonian PI.
2.  $\mathcal{L}_{red}$  can be quantized by applying Matthews's theorem for first order Lagrangians [5]. The generating functional<sup>10</sup> can be written as (1.1) with the quantized Lagrangian

$$\mathcal{L}_{quant} = \mathcal{L}_{red}.\tag{3.8}$$

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<sup>8</sup>In general, only total derivatives of expressions that depend on nothing but the coordinates can be dropped. However, since the derivatives are treated as coordinates within the Ostrogradsky formalism if the order  $N$  is chosen sufficiently high (as discussed in the previous section), all total derivative terms can be neglected [9].

<sup>9</sup>Formally, the reduced Lagrangian  $\mathcal{L}_{red}$  still has to be treated as an  $N$ th order one although there are no more higher order derivatives. But, as shown in section 2, it does not affect the physical content of the theory to treat it as a first order Lagrangian.

<sup>10</sup>Since  $\mathcal{L}_{red}$  is a first order Lagrangian, source terms only have to be introduced for the field  $\varphi$  but not for its derivatives as in the general PI formalism for higher order Lagrangians.

3. Within the PI, the field transformations (3.4) become performed inversely in order to reconstruct the primordial higher order Lagrangian. This finally yields (1.2).

The last step needs some additional clarification because a field transformation like (3.4), if it is performed within the Lagrangian PI (1.1), does not only affect the quantized Lagrangian, but also the integration measure and the source terms. However, the transformation of the measure only yields extra  $\delta^4(0)$  terms [7] which are neglected here (see introduction and [5, 11]) and the change in the source terms does not affect the physical matrix elements [7, 20].

Matthews’s theorem implies, that an *effective* higher order Lagrangian can be quantized in the same way as a first order one without worrying about the unphysical effects that are normally connected with higher order Lagrangians. In particular, the Feynman rules can be obtained from the effective interaction terms in the standard manner.

Closing this section, I want to add some remarks:

- This formalism can only be applied if higher powers of  $\epsilon$  are neglected, since it is possible to eliminate the higher order time derivatives in the first order of  $\epsilon$  (and in fact in any finite order of  $\epsilon$  [8]) but they cannot be removed completely. As mentioned in the introduction, this treatment is justified within the effective Lagrangian formalism because effects implied by  $O(\epsilon^n)$  terms with  $n > 1$  are assumed to be cancelled by other effects of (well-behaved) “new physics”.
- The Ostrogradsky formalism itself is in principle a reduction of a higher order Lagrangian to a first order one because the higher order derivatives are considered to be independent coordinates; however, this means that new degrees of freedom are introduced. These additional degrees of freedom involve the unphysical effects [4, 5]. Here, an effective higher order Lagrangian is reduced to a first order one *without* introducing extra degrees of freedom.
- The use of the EOM (3.6) may, in general, yield expressions in  $\mathcal{L}_{red}$  which are not manifestly covariant (see especially the treatment of the following section). Also the Hamiltonian PI quantization procedure involves such terms (see [5, 9, 10, 11]). However, these expressions only occur in intermediate steps of the derivation but not in the resulting expression (1.1) with (1.2). Actually, Matthews theorem enables calculations based on the manifestly covariant Lagrangian PI.

## 4 Higher Order Effective Interactions of Massive Vector Fields

In this section I will generalize the results of the preceding one to higher order effective (non Yang–Mills) self-interactions of massive vector fields. I will examine the three different types of effective Lagrangians which are studied in the literature, namely gauge noninvariant Lagrangians, SBGTs with a nonlinearly realized scalar sector (gauged nonlinear  $\sigma$ -models) and SBGTs with a linearly realized scalar sector (Higgs models).

For simplicity, I will only consider massive Yang–Mills fields (where all vector bosons have equal masses) with additional effective (non–Yang–Mills) interactions and the corresponding SBGTs, as in [11]. The generalization to other, e.g. electroweak, models is straightforward.

## 4.1 Gauge Noninvariant Models

I consider the effective Lagrangian

$$\mathcal{L} = \mathcal{L}_0 + \epsilon \mathcal{L}_I = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \frac{1}{2} M^2 A_a^\mu A_\mu^a + \epsilon \mathcal{L}_I(A_a^\mu, \partial^\nu A_a^\mu, \dots, \partial^{\nu_1} \dots \partial^{\nu_N} A_a^\mu). \quad (4.1)$$

The field strength tensor is given by

$$F_a^{\mu\nu} \equiv \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f_{abc} A_b^\mu A_c^\nu. \quad (4.2)$$

$\mathcal{L}_0$  represents a massive Yang–Mills theory and the effective interaction term  $\mathcal{L}_I$  contains the deviations from the Yang–Mills interactions which involve derivatives up to the order  $N$  and which are proportional to  $\epsilon$  with  $\epsilon \ll 1$ .

By applying the procedure described in section 3 one can now use the EOM following from  $\mathcal{L}_0$  to eliminate the higher order time derivatives in  $\mathcal{L}_I$ . These EOM are:

$$D_\mu F_a^{\mu\nu} = -M^2 A_a^\nu \quad (4.3)$$

with the covariant derivative

$$D^\sigma F_a^{\mu\nu} \equiv \partial^\sigma F_a^{\mu\nu} + g f_{abc} A_b^\sigma F_c^{\mu\nu}. \quad (4.4)$$

With  $\nu = i = 1, 2, 3$ , (4.3) can be rewritten as

$$\ddot{A}_a^i = -M^2 A_a^i - D_j F_a^{ij} + g f_{abc} A_b^0 F_c^{i0} - \partial_i \dot{A}_a^0 + g f_{abc} (\dot{A}_b^i A_c^0 + A_b^i \dot{A}_c^0). \quad (4.5)$$

This equation serves to eliminate all higher order time derivatives of the  $A_a^i$ . Now one has to get rid of the time derivatives of the  $A_a^0$ . To be able to apply Matthews's theorem for Lagrangians without higher order time derivatives [11], one even has to remove the first order time derivatives of the  $A_a^0$  because in [11] it is assumed that  $\mathcal{L}$  does not depend on  $\dot{A}_a^0$  (see footnote 3 there). With  $\nu = 0$ , (4.3) becomes

$$A_a^0 = \frac{1}{M} [-\partial_i F_a^{i0} + g f_{abc} A_b^i F_c^{i0}]. \quad (4.6)$$

differentiation yields

$$\dot{A}_a^0 = \frac{1}{M} [-\partial_i \dot{F}_a^{i0} + g f_{abc} (\dot{A}_b^i F_c^{i0} + A_b^i \dot{F}_c^{i0})], \quad (4.7)$$

where  $\dot{F}_a^{i0}$  can be written, using (4.5), as

$$\dot{F}_a^{i0} = M^2 A_a^i + D_j F^{ij} - g f_{abc} A_b^0 F_c^{i0}. \quad (4.8)$$

By repeated application of (4.5), (4.7) and (4.8) one can reduce the effective Lagrangian (4.1) to an equivalent Lagrangian  $\mathcal{L}_{red}$  which contains neither higher order time derivatives of the vector fields nor first order time derivatives of the  $A_a^0$ .

Now Matthews's theorem can be proven as in the previous section. The effective higher order Lagrangian becomes reduced to a first order Lagrangian  $\mathcal{L}_{red}$  as described above, this gets quantized using Matthews's theorem for effective first order interactions of massive vector fields [11] and finally one can reconstruct the primordial Lagrangian  $\mathcal{L}$  by applying appropriate field transformations within the Lagrangian PI.

## 4.2 Gauged Nonlinear $\sigma$ -Models

Now I consider SBGTs with additional effective interaction terms that are embedded into a gauge invariant framework. First I restrict to models that do not contain physical Higgs fields, which implies that the unphysical pseudo-Goldstone fields  $\varphi_a$  are nonlinearly realized as

$$U = \exp\left(i\frac{g}{M}\varphi_a t_a\right), \quad (4.9)$$

where the  $t_a$  are the generators of the gauge group. Within these models, it is most convenient to use the matrix notation defined by:

$$A^\mu \equiv A_a^\mu t_a, \quad F^{\mu\nu} \equiv F_a^{\mu\nu} t_a, \quad \text{etc.} \quad (4.10)$$

In [15, 16], it has been shown that each effective Lagrangian of the type (4.1) can be rewritten as a nonlinear SBGT by applying the Stueckelberg transformation [14]

$$A^\mu \rightarrow -\frac{i}{g}U^\dagger D^\mu U \quad (4.11)$$

with the covariant derivative

$$D^\mu U \equiv \partial^\mu U + igA^\mu U. \quad (4.12)$$

On the other hand, each effective gauged nonlinear  $\sigma$ -model can be rewritten in the gauge noninvariant form (4.1) by inverting the Stueckelberg transformation (4.11). This can be seen as follows: due to the gauge invariance of the effective interaction term, the fields only occur there in the gauge invariant combinations

$$U^\dagger(D^{\sigma_1} \dots D^{\sigma_N} F^{\mu\nu})U, \quad (4.13)$$

$$U^\dagger D^{\sigma_1} \dots D^{\sigma_N} U. \quad (4.14)$$

(The higher order covariant derivatives of  $F^{\mu\nu}$  and  $U$  are defined analogously to the first order ones (4.4) and (4.12).) Each effective interaction term can be constructed from the expressions (4.13), (4.14) and constants like  $t_a$ ,  $g^{\mu\nu}$  and  $\epsilon^{\rho\sigma\mu\nu}$  by taking products, sums, derivatives and traces [1]. The term (4.13) becomes

$$D^{\sigma_1} \dots D^{\sigma_N} F^{\mu\nu} \quad (4.15)$$

after an inverse Stueckelberg transformation. By using the unitarity of  $U$  (4.9),

$$UU^\dagger = 1, \quad (4.16)$$

and performing product differentiation, the term (4.14) can be expressed through terms

$$U^\dagger D^\sigma U \quad (4.17)$$

and their derivatives. E.g. for  $N = 2$  it can be written as

$$(U^\dagger D^{\sigma_1} U)(U^\dagger D^{\sigma_2} U) + \partial^{\sigma_1}(U^\dagger D^{\sigma_2} U). \quad (4.18)$$

(Similar formulas can be found for  $N > 2$ ). (4.17) becomes

$$igA^\sigma \quad (4.19)$$

after an inverse Stueckelberg transformation. This means that each nonlinear gauge invariant Lagrangian  $\mathcal{L}^S$  can be rewritten as a gauge noninvariant Lagrangian  $\mathcal{L}_U$  (U-gauge

Lagrangian) by applying the inverse of the Stueckelberg transformation (4.11). As one can see from the above procedure, this transformation results in simply dropping all unphysical scalar fields in  $\mathcal{L}^S$ :

$$\mathcal{L}_U \equiv \mathcal{L}^S \Big|_{A^\mu \rightarrow UA^\mu U^\dagger - \frac{i}{g} U \partial^\mu U^\dagger} = \mathcal{L}^S \Big|_{\varphi_a=0}. \quad (4.20)$$

In [11], I have proven that Lagrangians (without higher order derivatives) which are related by a Stueckelberg transformation are equivalent within the Hamiltonian formalism. Within the Ostrogradsky formalism this result can be generalized to higher order Lagrangians and besides it can be derived more easily. Since a Stueckelberg transformation is a field transformation that depends on the derivatives of the fields, it is a canonical transformation within this formalism<sup>11</sup> (if the formal order of  $\mathcal{L}^S$  is chosen sufficiently high). The transformations of the time derivatives of  $A^\mu$  must be specified separately because these are considered to be independent fields. With  $M$  being the order of the highest time derivative of  $A^\mu$  appearing in  $\mathcal{L}^S$ , the inverse Stueckelberg transformation is then completely specified by

$$\partial_0^m A^\mu \rightarrow \partial_0^m \left( UA^\mu U^\dagger - \frac{i}{g} U \partial^\mu U^\dagger \right), \quad m = 0, \dots, M. \quad (4.21)$$

The Lagrangian  $\mathcal{L}^S$  has thus formally to be treated as an at least  $(M+2)$ nd order one to establish the equivalence of  $\mathcal{L}^S$  and  $\mathcal{L}_U$  in (4.20).

Matthews's theorem for effective gauged nonlinear  $\sigma$ -models with higher order derivatives can now be proven as follows: Given a nonlinear gauge invariant Lagrangian  $\mathcal{L}^S$ , this can be converted into an equivalent gauge noninvariant Lagrangian  $\mathcal{L}_U$  (4.20) by using the Stueckelberg formalism. (As discussed in section 2, this does not affect the Hamiltonian path integral.) Since  $\mathcal{L}_U$  is of the type (4.1), the results of the previous subsection can be applied to quantize it. This yields the generating functional (1.1) with

$$\mathcal{L}_{quant} = \mathcal{L}_U. \quad (4.22)$$

This, however, is identical to the result of the Faddeev–Popov quantization procedure applied to  $\mathcal{L}^S$  if the (U-gauge) g.f. conditions

$$\varphi_a = 0 \quad (4.23)$$

are imposed [16]. Due to the equivalence of all gauges [17, 20], this result can be rewritten in any other gauge<sup>12</sup> (e.g.  $R_\xi$ -gauge, Lorentz gauge, Coulomb gauge, etc.). This completes the proof of Matthews's theorem for effective gauged nonlinear  $\sigma$ -models.

### 4.3 Higgs Models

Finally let me consider effective SBTs with linearly realized symmetry, i.e. models that involve additional physical scalar fields and that also contain gauge invariant effective interaction terms. Since these models cannot be written in a general form for an arbitrary gauge

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<sup>11</sup>One may wonder why a gauge invariant (i.e. first class constrained) system can be related to a gauge noninvariant (i.e. second class constrained) system by a canonical transformation because the number of second class constraints is given by  $\text{rank} \{ \phi_a, \phi_b \} |_{\varphi_a=0}$  which is invariant under canonical transformations. One should remember that  $\mathcal{L}^S$  and  $\mathcal{L}_U$  are only related by a canonical transformation if the order  $N$  is artificially increased. This procedure yields additional constraints (see section 2). In fact,  $\mathcal{L}^S$  and  $\mathcal{L}_U$  imply equal numbers of first class and of second class constraints if  $N$  is chosen sufficiently high.

<sup>12</sup>Actually, loop calculations within the U-gauge suffer from ambiguities in the determination of the finite part of an  $S$ -matrix element [23, 24]. Therefore, for practical calculations it is useful to rewrite the PI obtained within in the U-gauge in the  $R_\xi$ -gauged form in order to remove these ambiguities. In fact, loop calculations within the U-gauge yield the same  $S$ -matrix elements as loop calculations within the  $R_\xi$ -gauge, but only if the correct renormalization prescription is used [24]; other renormalization prescriptions yield distinct results. In the  $R_\xi$ -gauge these ambiguities are absent.

group, I restrict to the case of SU(2) symmetry (i.e.  $t_a = \frac{1}{2}\tau_a$ ,  $a = 1, 2, 3$ ) as in [11]. The generalization to another gauge group is straightforward.

Within an SU(2) Higgs model the scalar sector is parametrized as

$$\Phi = \frac{1}{\sqrt{2}}((v+h)\mathbf{1} + i\varphi_a\tau_a) \quad (4.24)$$

with the Higgs field  $h$  and the vacuum expectation value  $v = \frac{2M}{g}$ . However, even within a linear Higgs model the scalar sector can be parametrized nonlinearly by performing the transformation [11, 16, 17, 18]

$$\Phi \rightarrow \frac{v+h}{\sqrt{2}}U = \frac{v+h}{\sqrt{2}} \exp\left(\frac{i\varphi_a\tau_a}{v}\right). \quad (4.25)$$

Applying this transformation to the Lagrangian  $\mathcal{L}^H$  of an effective Higgs model, this becomes converted into an effective nonlinear Stueckelberg model (as examined in the previous subsection) which contains an additional physical scalar field. Lagrangians which are related by the transformation (4.25) are clearly equivalent because (4.25) is a point (i.e. canonical) transformation. By applying a Stueckelberg transformation, the resulting nonlinear Lagrangian can be reduced to an equivalent gauge noninvariant Lagrangian  $\mathcal{L}_U$  (U-gauge Lagrangian) which is obtained by dropping all unphysical scalar fields in  $\mathcal{L}^H$ :

$$\mathcal{L}_U \equiv \mathcal{L}^H \Big|_{\substack{\Phi \rightarrow \frac{v+h}{\sqrt{2}}U \\ \varphi_a=0}} = \mathcal{L}^H \Big|_{\varphi_a=0}. \quad (4.26)$$

$\mathcal{L}_U$  is similar to (4.1) but it contains an additional physical scalar field  $h$ . Therefore  $\mathcal{L}_U$  is of the form

$$\begin{aligned} \mathcal{L}_U &= \mathcal{L}_0 + \epsilon\mathcal{L}_I \\ &= -\frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a + \frac{1}{2}(\partial^\mu h)(\partial_\mu h) + \frac{1}{8}g^2(v+h)^2 A_a^\mu A_\mu^a \\ &\quad - \frac{1}{2}M_H^2 h^2 - \frac{1}{4}g\frac{M_H^2}{M}h^3 - \frac{1}{32}g^2\frac{M_H^2}{M^2}h^4 \\ &\quad + \epsilon\mathcal{L}_I(A_a^\mu, \partial^\nu A_a^\mu, \dots, \partial^{\nu_1} \dots \partial^{\nu_N} A_a^\mu, h, \partial^\mu h, \dots, \partial^{\mu_1} \dots \partial^{\mu_N} h). \end{aligned} \quad (4.27)$$

$\mathcal{L}_0$  is the U-gauge Lagrangian of the Higgs model without effective interaction terms. The EOM corresponding to (4.5), (4.7), (4.8) and the EOM for  $h$  that follow from  $\mathcal{L}_0$  are

$$\ddot{A}_a^i = -\frac{1}{4}g^2(v+h)^2 A_a^i - D_j F_a^{ij} + gf_{abc}A_b^0 F_c^{i0} - \partial_i \dot{A}_a^0 + gf_{abc}(\dot{A}_b^i A_c^0 + A_b^i \dot{A}_c^0), \quad (4.28)$$

$$\dot{A}_a^0 = \frac{4}{g^2(v+h)^2} \left[ -\frac{1}{2}g^2(v+h)\dot{h}A_a^0 - \partial_i \dot{F}_a^{i0} + gf_{abc}(\dot{A}_b^i F_c^{i0} + A_b^i \dot{F}_c^{i0}) \right], \quad (4.29)$$

$$\dot{F}_a^{i0} = \frac{1}{4}g^2(v+h)^2 A_a^i + D_j F^{ij} - gf_{abc}A_b^0 F_c^{i0}, \quad (4.30)$$

$$\ddot{h} = \Delta h + \frac{1}{4}g^2(v+h)A_a^\mu A_\mu^a - M_H^2 h - \frac{3}{4}g\frac{M_H^2}{M}h^2 - \frac{1}{8}g^2\frac{M_H^2}{M^2}h^3. \quad (4.31)$$

These EOM have to be used to eliminate all higher order time derivatives of the  $A_a^\mu$  and of  $h$  and also the first order time derivatives of the  $A_a^0$  in  $\mathcal{L}_U$  (4.26).

$\mathcal{L}^H$  can now be quantized by using the procedure described in the previous subsections and the results of [11]. One obtains the Lagrangian PI (1.1) with

$$\mathcal{L}_{quant} = \mathcal{L}_U. \quad (4.32)$$

This is again identical to the result of the Faddeev–Popov formalism with the (U-gauge) g.f. conditions (4.23) [16] (upon neglecting a  $\delta^4(0)$  term). Using the equivalence of all gauges [17, 20], this result can be generalized to any other gauge. Thus, the proof of Matthews’s theorem is extended to Higgs models with effective higher order interactions.

It should be mentioned that the transformation (4.25) and the EOM (4.29) involve nonpolynomial interactions, which are not present in the primordial Lagrangian. This, however, is no serious problem, since these expressions only occur in intermediate steps of the derivation but not in the resulting Faddeev–Popov PI (1.1) with (1.3). The application of Matthews’s theorem for first order Lagrangians is not affected by these terms, since the treatment of [11] does not necessarily require polynomial interactions.

The procedure of this subsection illustrates that the above proof can be extended to Lagrangians that also contain couplings to matter fields (which have been neglected here for simplicity). The additional couplings in  $\mathcal{L}_0$  affect the EOM, but the general statement that each effective higher order Lagrangian can be reduced to a first order one by using the EOM remains unaffected. Therefore, *for any effective Lagrangian, Hamiltonian and Lagrangian path integral quantization are equivalent.*

## 5 Conclusions

In this paper I have shown that the unphysical effects connected with Lagrangians that depend on higher order derivatives are absent if an *effective* Lagrangian is considered, i.e. a Lagrangian that represents the low energy approximation of well-behaved “new physics” with heavy particles at a higher energy scale.

Upon neglecting higher powers of the effective coupling constant  $\epsilon$ , all higher order time derivatives can be eliminated from an effective Lagrangian. This reduction of a higher order effective Lagrangian to a first order one is done by applying of the equations of motion to the effective interaction term. The (in general forbidden) use of the equations of motion is justified within the effective Lagrangian formalism because it can be realized by performing field transformations that involve derivatives of the fields. Lagrangians that are related by such a field transformation are physically equivalent (at the classical and at the quantum level) because these transformations are canonical transformations within the Hamiltonian treatment of higher order Lagrangians (Ostrogradsky formalism).

Matthews’s theorem has been extended to higher order effective Lagrangians by applying this reduction and using Matthews’s theorem for first order effective Lagrangians. This theorem states that any effective Lagrangian can be quantized using the simple Lagrangian path integral ansatz (1.1) with (1.2) (or the Faddeev–Popov formalism, (1.1) with (1.3), in the case of gauge invariant effective Lagrangians), since this turns out to be the result of the correct Hamiltonian path integral quantization procedure. The Feynman rules can be directly obtained from the effective Lagrangian in the standard manner.

Thus, *effective* higher order Lagrangians can be treated in the same way as first order Lagrangians, the higher order terms do not imply any unphysical effects unlike in the general treatment of higher order Lagrangians.

Finally, two important points should be noted:

- *The formalism described in this paper can only be applied to effective Lagrangians, since then the supposed existence of well-behaved “new physics” beyond the theory described by the effective Lagrangian justifies the omission of all unphysical effects. In fact, the assumption  $\epsilon \ll 1$  alone is not sufficient for neglecting higher powers of  $\epsilon$  since theories with higher order derivatives have no analytic limit for  $\epsilon \rightarrow 0$ . Thus,*

the effects of a term with higher order derivatives are not small even if the coupling constant of this term is extremely small [4, 5]. This implies that the unphysical effects cannot be avoided within models with higher order derivatives that are not considered to be effective ones.

- Since each effective Lagrangian with higher order derivatives can be reduced to a first order Lagrangian, it is in principle sufficient to consider only effective Lagrangians with at most first order derivatives [8, 25, 26]. However, the reduction of a quite simple higher order effective interaction term to a first order term by applying the equations of motion, in general, yields a lengthy and awkward expression. Even more, in the reduced form the physical effects of such a term are not very obvious because in this form it becomes a linear combination of several terms; each one alone of these terms yields effects that are not implied by the primordial term but among them complicated cancellations take place [26, 27]. Thus, for practical purposes it is much more convenient to work with the primordial higher order Lagrangian instead of the reduced first order Lagrangian. Therefore, I have used this reduction only for technical purposes in order to apply Matthews's theorem for first order Lagrangians, but in the final result (1.1) with (1.2) or (1.3) I have reconstructed the original higher order Lagrangian. Actually, this result enables calculations based on a higher order effective Lagrangian without doing this reduction.

## Acknowledgement

I like to thank Reinhart Kögerler for many helpful discussions and for reading the manuscript of this paper.

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