

A simple attacks strategy of BB84 protocol

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Abstract

A simplified eavesdropping-strategy for BB84 protocol in quantum cryptography is proposed. This scheme is based on the ‘indirect copying’. Under this scheme, eavesdropper can exactly obtain the exchanged information between the legitimate users without being detected.

Key words: Eavesdropping strategy, indirect copying, quantum cryptography, BB84 protocol, corresponding reference list.

I.Introduction

Quantum cryptography, suggested originally by S.Wiesner [1] and then by C.H.Bennett and G.Brassard [2], employs quantum phenomena such as the uncertainty principle and the quantum corrections to protect distributions of cryptographic keys. Key distribution is defined as procedure allowing two legitimate users of communication channel to establish two exact copies, one copy for each user, of a random and secret sequence of bits. In other words, quantum cryptography is a technique that permits two parties, who share no secret information initially, to communicate over an open channel and to establish between themselves a shared secret sequence of bits. Quantum cryptography is provably secure against eavesdropping attack, in that, as a matter of fundamental principle, the secret data can not be compromised unknowingly to the legitimate users of the channel. BB84

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protocol[3] is a key distribution protocol over an open channel by quantum phenomena, it relies on the uncertainty principle of quantum mechanics to provide key security. The security guarantee is derived from the fact that each bit of data is encoded at random on either one of a conjugate pair of observables of quantum-mechanical object. Because such a pair of observables is subjected to the Heisenberg uncertainty principle, measuring one of the observables necessarily randomizes the other.

Although quantum cryptography is provably security, with the quantum key distribution protocols presented, several attacks strategy have been generated, such as intercept/resend scheme [4], beamsplitting scheme [4], entanglement scheme [5-7] and quantum copying [9,10]. In the intercept/resend scheme, Eve intercepts selected light pulses and reads them in bases of her choosing. When this occurs, Eve fabricates and sends to Bob a pulse of the same polarization as she detected. However, due to uncertainty principle, at least 25% of the pulse Eve fabricates will yield the wrong result if later successfully measured by Bob. The other attack, beamsplitting, depends on the fact that transmitted light pulses are not pure single-photon states. In the entanglement scheme, the eavesdropper involves the carrier particle in an interaction with her own quantum system, referred to as probe, so that the particle and the probe are left in an entangled state, and a subsequent measurement of the probe yields information about the particle. Some investigators are now turning their attention to collective attacks and joint attacks. About these attacks description please see Ref.[8] and its references. Eve can also use the quantum copying to obtain the information between Alice and Bob. Two kind quantum copies are presented [10,11]. It is appropriate to emphasize the limitation of above attacks strategy. All these mentioned attacks strategy are restricted by the uncertainty principle or the quantum corrections, so Eve can not break the quantum cryptography protocols. The risk of eavesdropper is to disturb the information and finally to be detected by the legitimate users. This is the reason why quantum cryptography is declared to be provably security.

The Eve's aim is to obtain more information from the open channel set up by legitimates user, saying Alice and Bob, and induce more less disturbance on the transmitting quantum bits, so that she can not be detected by the legitimate users Alice and Bob. In usually, the uncertainty principle or the quantum corrections prevents Eve's attempt from eavesdropping the useful information without being detection. However if Eve can escape the restriction of the uncertainty principle, her attempt will be succeed.

In this paper we propose a novel attack strategy for quantum cryptographic protocols. Under this strategy, the security of BB84 quantum key distribution protocol will completely loss. The scheme works by follows procedure: Eve constructs a prescription function. This function must be an uniform function for every different quantum state used by Alice and Bob. This mean that every function value corresponds to a different quantum state. It consists of a reference list that all these corresponding relationship of function value to different quantum state. While Alice sends a random quantum sequence to Bob, Eve intercepts every state and calculates the corresponding value by the function, then gives up the intercepted state. When this finishes, Eve resends a new quantum state to Bob according to the reference list in which every value corresponds a correct quantum state. By this method Eve can exactly obtain the information exchanged between Alice and Bob without being detected. We call this method as “indirect copying”. Of course, It is different from the probabilistic cloning and the inaccuracy quantum copying. Obviously, the “indirect copying” is not a true copy of quantum bits.

II.BB84 quantum key distribution protocol

For describing our attacks strategy, we first review the BB84 protocol. The BB84 protocol is the first key distribution protocol in quantum cryptography. Follows are the protocol in details [12].

1. Alice prepares a random sequence of photons polarized and sends them to Bob
2. Bob measure his photon using a random sequence of bases
3. Results of Bob’s measurements. Some photons are shown as not having been received owing to imperfect detector efficiency.
4. Bob tells Alice which basis he used for each photon he received.
5. Alice tells him which bases were correct.
6. Alice and Bob keep only the data from these correctly measured photons, discarding all the rest.
7. This data is interpreted as a binary sequence according to the coding scheme:

8. Bob and Alice test their key by publicly choosing a random subset of bit positions and verifying that this subset has the same parity in Bob's and Alice's versions of the key (here parity is odd). If their keys had differed in one or more bit position, this test would have discovered that fact with probability $1/2$.
9. Remaining secret key after Alice and Bob have discarded one bit from the chosen subset in step 8, to compensate for the information leaked by revealing its parity. Step 9 and 10 are repeated k times with k independent random subsets, to certify with probability $1 - 2^{-k}$ that Alice's and Bob's keys are the identical, at the cost of reducing the key length by k bits.
10. Distilling the security key by the privacy amplification [13]. The basic principle of privacy amplification is as follows. Let Alice and Bob shared a random variable W , such as a random n -bit string, while an eavesdropper Eve learns a corrected random variable V , providing at most $t < n$ bits of information about W , i.e., $H(W|V) \leq n - t$. Eve is allowed to specify an arbitrary distribution P_{VW} (unknown to Alice and Bob) subject to the only constraint that $R(W|V = v) \leq n - t$ with high probability (over values v), where $R(W|V = v)$ denotes the second-order conditional Renyi entropy of W , given $V = v$. For any $s < n - t$, Alice and Bob can distill $r = n - t - s$ bits of the secret key $K = G(W)$ while keeping Eve's information about K exponentially small in s , by publicly choosing the compression function G at random from a suitable class of maps into $\{0, 1\}^{n-t-s}$.

III. Construction of Reference List

To perform privacy communication between legitimate users, known as Alice and Bob, a set of pre-defined nonorthogonal quantum states or noncommuting quantum states often are used. For briefly, We call this set of pre-defined nonorthogonal quantum state or the noncommuting quantum states as basic quantum states (BQS) in the remainder paper. Because the BQS are publicly announced by Alice and Bob, Eve can easily get it. In BB84 protocol, the BQS are the four noncommuting states $|0\rangle, |\frac{\pi}{2}\rangle, |\frac{\pi}{4}\rangle, |\frac{3\pi}{4}\rangle$. Of course the linearly polarized states $|0\rangle, |\frac{\pi}{2}\rangle$ and the circularly polarized states $|\frac{\pi}{4}\rangle, |\frac{3\pi}{4}\rangle$ are orthogonal, respectively. In BB84 quantum key distribution protocol, the quantum states $|0\rangle$ and $|\frac{\pi}{2}\rangle$ are measured by the so called rectilinear measurement type. Representing

this rectilinear measurement type as \mathcal{L} , we have

$$\mathcal{L}|0\rangle = \lambda_1|0\rangle, \quad (1)$$

$$\mathcal{L}|\frac{\pi}{2}\rangle = \lambda_2|\frac{\pi}{2}\rangle, \quad (2)$$

where $\lambda_i, i = 1, 2$ are eigenvalues. Because the states $|0\rangle$ and $|\frac{\pi}{2}\rangle$ constitute a base in Hilbert, an arbitrary quantum state can be expanded by this base, i.e.,

$$|\psi\rangle = c_1|0\rangle + c_2|\frac{\pi}{2}\rangle. \quad (3)$$

By Eq.(3), it is easy to obtain

$$|\frac{\pi}{4}\rangle = \frac{\sqrt{2}}{2}|0\rangle + \frac{\sqrt{2}}{2}|\frac{\pi}{2}\rangle, \quad (4)$$

$$|\frac{3\pi}{4}\rangle = \frac{\sqrt{2}}{2}|0\rangle - \frac{\sqrt{2}}{2}|\frac{\pi}{2}\rangle, \quad (5)$$

Consider a proper ancilla quantum state, for example,

$$|\alpha\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|\frac{\pi}{2}\rangle, \quad (6)$$

making product between the ancilla quantum state $|\alpha\rangle$ and the quantum of BQS gives

$$\langle\alpha|0\rangle = \frac{\sqrt{3}}{2} \longrightarrow m_1 = \frac{3}{4} = 0.75, \quad (7)$$

$$\langle\alpha|\frac{\pi}{2}\rangle = \frac{1}{2} \longrightarrow m_2 = \frac{1}{4} = 0.25, \quad (8)$$

$$\langle\alpha|\frac{\pi}{4}\rangle = \frac{\sqrt{6} + \sqrt{2}}{4} \longrightarrow m_3 = \frac{(\sqrt{3} + 1)^2}{8} \approx 0.933, \quad (9)$$

$$\langle\alpha|\frac{3\pi}{4}\rangle = \frac{\sqrt{6} - \sqrt{2}}{4} \longrightarrow m_4 = \frac{(\sqrt{3} - 1)^2}{8} \approx 0.067, \quad (10)$$

Obviously, an observable value $m_j, j = 1, 2, 3, 4$ corresponds to only a basic quantum state $|j_k\rangle, k = 1, 2, 3, 4$. All these corresponding relationship constructs a corresponding reference list. It is given by

quantum state $ j_k\rangle$	m_k
$ 0\rangle$	0.75
$ \frac{\pi}{2}\rangle$	0.25
$ \frac{\pi}{4}\rangle$	0.933
$ \frac{3\pi}{4}\rangle$	0.067

(11)

List (11) constructs an uniform function between the sorting value and the BQS, i.e., $S_k = f(|j_k \rangle)$, where $k = 1, 2, 3, 4$, $|j_k \rangle$ represent the four basic quantum states. Obviously, $S_1 \neq S_2 \neq S_3 \neq S_4$. When Alice connects Bob and exchanges information, Eve intercepts the sequences of the quantum bits. For each quantum bit in the sequence intercepted by Eve, she measures it and obtains a corresponding sorting value. Comparing the sorting value to the reference list, Eve resends the corresponding quantum bit to Bob. For example, if the measurement value corresponds $m_2 = 1/4$, Eve resends the quantum state $|\frac{\pi}{2} \rangle$ to Bob. Thus, Eve can exactly obtain the complete information exchanged between Alice and Bob, and escapes the detection of Alice and Bob. So under the presented attack strategy, the BB84 protocols is completely insecure.

IV. Attack scheme

First, Eve constructs a corresponding reference list for every state of BQS. For correctly determining the intercepted quantum states and resending the correct quantum bits to Bob, every basic quantum state $|j_k \rangle$ must correspond to a different reference value (marking the function value as $S_k, k = 1, 2, \dots, m$). So Eve firstly need to construct an uniform function which is an one-to-one map of $|j_k \rangle$ to the function value m_k .

Second, Eve intercepts the random sequence of quantum states sent by Alice and calculates the value for every intercepted state by measurement operation. For distributing the quantum key, Alice randomly choose the quantum state from the basic quantum state $|j_k \rangle, k = 1, 2, \dots, m$, and sends the randomly selected quantum bits sequence to Bob. The communication between Alice and Bob is in an open channel, which Eve can easily access. Eve intercepts the quantum bits sequences sent by Alice, and measures the observables $m_k, k = 1, 2, 3, 4$ for every quantum bit. By the measurement values Eve calculates the corresponding sorting for every intercepted quantum state by her machine.

Third, Eve gives up the intercepted quantum state. The Eve's operation will limited by the uncertainty principle, her measurement disturbs the quantum state because she don't know beforehand the every random quantum bit state. If Eve resends these intercepted states to Bob like the Intercept/Resend attack strategy proposed in Ref.[6], she will reveal herself. To avoid these case, we let Eve give up all these intercepted states.

Finally, Eve resends the corresponding quantum states. By the calculation sorting values obtained in step 2, Eve chooses a corresponding quantum bit state according to

the reference list and resends it to Bob. The resent quantum state is exactly same as that send by Alice, it seems that Eve ‘copies’ the Alice’s quantum state. However, it is not a real copying, This ‘copying’ is completely different from the probability and inaccuracy copying. We call it as “indirect copying”. By this method Eve can measure Alice’s signal exactly, and resend an exact copy of it, thereby escaping detection.

Our attack strategy makes the quantum cryptographic protocol at risk. Of course, our scheme can not attack every protocol proposed previously. For example, we can not attack the Ekert protocol [14], because there is no information encoded there while the particle transits from the source to the legitimate users. In fact, our scheme is only valid for the protocol that quantum state is encoded in transit. Meanwhile, Eve must know the BQS. In addition, the interval time between two adjacent quantum state of the resent quantum state should almost keep the same as that in Alice’s random sequence of quantum bits so that Bob can not feel Eve.

V. Conclusion

In conclusion, we proposed an attack strategy for BB84 key distribution protocol in the quantum cryptography, we called this strategy as ‘indirect copying attack’. Under this strategy, the BB84 quantum cryptographic protocols is at risk, the eavesdropper can exactly obtain the information between the legitimate users without being detected. Of course, the presented strategy is only valid for the case that the quantum state is encoded in transit.

References

1. S. Wiesner, Conjugate coding, Sigact News, vol. 15, no. 1,78(1983); original manuscript written circa 1970.
2. C.H.Bennett, G.Brassard, S.Breidbart and S.Wiesner, Quantum cryptography, or unforgeable subway tokens, Advances in cryptography: Proceedings of Crypto'82, August 1982, plenum, New York, pp. 267-275.
3. C.H.Bennett and G.Brassard. Quantum cryptography: Public key distribution and coin tossing, Proceedings of the IEEE International Conference on Computers, System, and Signal Processing, Bangalore, India (IEEE, New York. 1984), pp. 175-179.
4. C.H.Bennett, F.Bessett, G.Brassard, L.Salvail, and J.Smolin, Experimental quantum cryptography, J.Cryptology, 5, 3(1992).
5. C.A.Fuchs, N.Gisin, R.B.Griffiths, C.S.Niu, and A.Peres, Optimal eavesdropping in quantum cryptography. I. Information bound and optimal strategy, Physical Review A 56, 1163 (1997).
6. R.B.Griffiths, and C.S.Niu, Optimal eavesdropping in quantum cryptography. II. A quantum circuit. Physical Review A, 56, 1173(1997).
7. C.H.Bennett, T.Mor, and J.A.Smolin, Parity bit in quantum cryptography, Physical Review A 54, 2675 (1996).
8. B.A.Slutsky, R.Rao, P.C.Sun, and Y.Fainman, Security of quantum cryptography against individual attacks, Physical Review A 57, 2383(1998).
9. M.Hilery, and V.Buzek, Quantum copying: Fundamental inequalities, Physical Review A 56, 1212(1997).
10. V.Buzek, and M.Hillery, Quantum copying: Beyond the no-cloning theorem, Physical Review A 54, 1844(1996).
11. L.M.Duan and G.C.Guo, Probabilistic cloning and identification of linearly independent quantum states, Physical Review Letters 80, 4999(1998).
12. C.H.Bennett, Quantum cryptography using any two nonorthogonal states, Physical Review Letters, 68, 3121(1992).

13. C.H.Bennett, G.Brassard, C.Crepeau and U.M.Maurer, Generalized privacy amplification, IEEE Trans. Inform. Theory, 41, 1915(1995).
14. A.K.Ekert, Quantum cryptography based on Bell's theorem, Physical Review Letter, 67, 661(1991).