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# Incorporating the Basic Elements of a First-degree Fuzzy Logic and Certain Elments of Temporal Logic for Dynamic Management Applications

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**Abstract**
The approximate reasoning is perceived as a derivation of new formulas with the corresponding temporal attributes, within a fuzzy theory defined by the fuzzy set of special axioms. For dynamic management applications, the reasoning is evolutionary because of unexpected events which may change the state of the expert system. In this kind of situations it is necessary to elaborate certain mechanisms in order to maintain the coherence of the obtained conclusions, to figure out their degree of reliability and the time domain for which these are true. These last aspects stand as possible further directions of development at a basic logic level. The purpose of this paper is to characterise an extended fuzzy logic system with modal operators, attained by incorporating the basic elements of a first-degree fuzzy logic and certain elements of temporal logic.

**Key-Words**: Dynamic Management Applications, Fuzzy Reasoning, Formalization, Time Restrictions, Modal Operators, Real-Time Expert Decision System (RTEDS)

**Jel Code**: C4, C45

## 1. Introduction

I present, throughout Section 2, the formalisation and logical justification of the reasoning based on imprecise knowledge, specific to a real-time expert decision system, called RTEDS. I also highlighted, as aspects of reasoning about time, the attachment of certain temporal descriptors to the fuzzy statements, according to the interval-based (temporal) logic. There are three features of any formalisation [Bal92, DLP91, KGK94], which are actually used to create the inferential system in RTDES: **i**) defining and reducing reality (the problem domain) to a linguistic model (the management model); **ii**) the possibility to represent the same reality in various aspects, according to the position from which one may look at this reality (the model is not unique, since it always depends on the intended purpose and on the type of representation and processing of knowledge); **iii**) abandoning the external world in order to carry out deductions, once a formalisation of it has taken place (the inferential chains are based on the management model and on the evidence knowledge). After the presentation of syntax and semantics elements of the extended first-degree fuzzy logic with modal temporal operators, the concepts of rules of inference, demonstration for a fuzzy formula as well as elements of approximate reasoning theory (as an exploitation methodology of imprecise knowledge with respect to the states of the expert management system, described as multidimensional possibility distributions) are presented. I also analysed the features of the possibility reasoning and of fuzzy temporal reasoning in order to deepen the inferential properties of the RTDES (Section 3). The conclusions of the paper appear in Section 4.

## 2. Approximate Reasoning Modeling

*The approximate reasoning refers to creating new rules of inference and translation. It is a mathematical instrument used for modelling the human reasoning based on imprecise knowledge. The theory suggested by Zadeh is based on intuitive rules and leads to operations with fuzzy relations [KH92, Zad83, Zim87], obtaining thus very useful applications. R. Lee, C. Chang and Zadeh went back to the concept of fuzzy set in the logic domain [LCC08, Ros95]. This perspective has the advantage of demonstrating that the fuzzy logic is* *a generalisation of bivalent logic, replacing the discrete feature of the latter with a continuous one. If in the case of bivalent logic there are used methods that clear up every possibility of evaluation according to the interpretation function, when we refer to fuzzy logic this are no longer possible. The formalism of the first-degree fuzzy logic represents the mathematical basis for the general theory of approximate reasoning. A special feature of the human thinking is the effective use of natural language even within the process of logic reasoning. According to this observation, we can conclude that the mathematical model of the way in which man thinks (acts) in a management position and at a certain level of synthesizing decisions, could be based on the fuzzy logic [AvAk06, LCLH08, Pos21], combined with modal temporal features. I will tackle next the formalism of first-degree fuzzy logic, highlighting the structure of truth values, the extended syntax and the semantics of this formal logic system. I will underline in this way the connections between fuzzy logic and approximate reasoning, which is further analysed through the possibility reasoning, which is considered useful by the inference engine of the expert management system RTDES [Maz09, MMN09].*

*The structure of truth values* is a residual lattice written $L = (L, \vee, \wedge, \otimes, \rightarrow, 0, 1)$ where the 0 and 1 values are the smallest respectively the biggest elements, $\vee$ and $\wedge$ are the supremum, respectively infimum operators, $\otimes$ is the isotone product operator for both variables, $(L,\otimes,1)$ is a commutative monoid, and $\rightarrow$ is the residuation operator. Furthermore, $a \otimes b < c$ only if $a < b \rightarrow c$ $(\forall)$ $a,b,c \in L$. For $L = [0,1]$ the logical connectives are $\vee =$ **max**, $\wedge =$ **min**, $a\otimes b = 0\vee (a+b-1)$ şi $a \rightarrow b = 1 \wedge (1-a+b)$. If we consider, for instance, $a\otimes b=$**min**$(a,b)$, then the only corresponding residuation operator is the Gödel implication operator.

*The syntax* of the basic language of the extended first-degree fuzzy logic with modal temporal operators consists of: (x,y,...) *variables* and (c,d,r,...) *constants* seen as elements that describe the set of states of an expert management system $X^{SEC} = X^{SE} \cup X$, (f, g,...) *functional symbols* of n arity, a set of *symbols for the truth values* $\{a\text{: } a \in L\}$, *predicate symbols* of n arity, *a binary connective* $\Rightarrow$, a $\{o_i\text{: } i\in J\}$ set of *connectives of m arity*, a *symbol for the* $(\forall)$ *quantifier*, the o modal temporal operators (the following moment in time), $\Box$ (for all present or following moments in time), $\Diamond$ (for a present or following moment in time) and punctuation marks. The terms are classically introduced.

*The formulas* are defined as follows: **i**) an *a* symbol for a $(a\in L)$ truth value is an atomic formula; **ii**) if $t_1,...,t_n$ are terms and p a predicate symbol of n arity, then $p(t_1,...,t_n)$ is an atomic formula; **iii**) if A, B, $A_1,...,A_m$ are formulas and $o_1$ is a connective of m arity, then $A \Rightarrow B$, $o_1(A_1,...,A_m)$, $(\forall x)A$ are formulas. The $\rceil A$ formula is an abbreviation for $A \Rightarrow 0$. There are similarly defined the $(\vee)$ disjunction, the $(\wedge)$ conjunction, the $(\Leftrightarrow)$ equivalence, A&B, $(\exists x)A$, $A^k = (A\&A\&...\&A)/_{k \text{ times}}$; **iv**) The $x^{SE}\in X^{SE}$, $x\in X$ variables are formulas, and if g is a formula, then og, $\Box g$, $\Diamond g$ are formulas, too; **v**) Any application of the above-mentioned rules for a certain number of times, determines a formula. Given a $J_1$ language of the extended first-degree fuzzy logic with modal temporal operators, the set of all terms will be noted $M_{J1}$, and the set of all formulas will be noted $F_{J1}$. If t is a term and A is a formula, then $A_x[t]$ is the formula obtained by substituting of the t term whenever the x variable appears freely in A. $g_s = (x_1\vee x_2 \vee\ldots\vee x_n)$ and $g_0=(x_{01}\vee x_{02}\vee\ldots\vee x_{0n})$ are given fuzzy formulas. The $g_0 \rightarrow \Diamond g_s$, $g_0 \rightarrow \Diamond\Box g_s$, $g_0 \rightarrow\Box\Diamond g_s$ relations are formulas too, and they allow expressing certain qualitative management conditions.

*The semantics* of the first-degree fuzzy logic is defined as follows. A *structure* of the language of the fuzzy logic $J_1$, is characterised by $\overline{D} =(D, p_D,...,f_D,...,u,v,...)$ where $D$ is a set, $p_D$ $D^n$ are relations of n arity assigned to each p n-ar predicate symbol and, $f_D$ are n-are functions in D assigned to each functional symbol of n arity, whereas $u,v,... \in D$ are elements assigned to each u,v,... constant of the $J_1$ language. Take $\overline{D}$ a structure for the $J_1$ basic language. The interpretation function of the formulas in $\overline{D}$ is a $\widetilde{D}: F_{J1}\rightarrow L$ function, which assigns a truth value for any formula from $F_{J1}$, as it follows: **i**) $\widetilde{D}(a) = a$, $a\in L$; **ii**) Take $t_1,...,t_n$ terms without variables and p an n-ar predicate symbol. Then $\widetilde{D}(p(t_1,...,t_n))=p_D(\widetilde{D}(t_1),..., \widetilde{D}(t_n))$, where $\widetilde{D}(t_i)\in D$ is an interpretation of the $t_i\in M_{J1}$, i=1,...,n term; **iii)** $\widetilde{D}(A\Rightarrow B) = \widetilde{D}(A) \rightarrow \widetilde{D}(B)$, highlights the fact that A and B are closed formulas; **iv)** $\widetilde{D}(o_1(A_1,...,A_n))=o_1(\widetilde{D}(A_1),..., \widetilde{D}(A_n))$; **v)** $\widetilde{D}((x)A(x))=\wedge_{d\in D}\widetilde{D}(A_x[\mathbf{d}])$, where **d** is the name of the $d\in D$

element; **vi)** $\widetilde{D}(A(x_1,...,x_n)) = \wedge_{d1,...,dn} \widetilde{D}(A_{x1,...,xn}[d_1,...,d_n])$. Take $Y \underset{\sim}{\subset} F_{J1}$ a fuzzy set of formulas. The fuzzy set of semantic consequences of the Y fuzzy set (where $Y(B) \in L$ represents the truth degree of B in Y) is:

$(C^{sem}Y)A = \wedge\{ \widetilde{D}(A)$: $\overline{D}$ is a structure for $J_1$ and (for each $A \in F_{J1}$), $(Y(B) \leq \widetilde{D}(B))\}$. If $(C^{sem}Y)A=1$, then for any fuzzy set of Y formulas the following relation occurs $\models A$ and A is a tautology, and $C^{sem}$ is the closing operator in *L*.

**Lemma 1. A)** $\models A \Rightarrow B$ only if $\widetilde{D}(A) \leq \widetilde{D}(B)$; **b)** $\models A \Leftrightarrow B$ only if $\widetilde{D}(A) = \widetilde{D}(B)$, for any $\overline{D}$ structure. This result is used in the derivation of tautologies. It is allowed the introduction of a set of fuzzy axioms to support the derivation of new formulas.

A *logical inference* is a $B_1,...,B_m$ sequence of formulas, each of them being either a logical and special axiom, or a formula derived from other formulas, using a rule of inference. The rules of logical inference can be schematically written under the form $A_1,...,A_n$ / B where $A_1,...,A_n$ are known formulas, and B is a derived fact ($A_1,...,A_n$, $B \in F_{J1}$). The rules of inference preserve the truth values after the inferential process.

A n-ar r rule of inference is a $r = (r^{syn}, r^{sem})$ couple in which $r^{syn}$ represents its syntactic part (a partial operator on $F_{J1}$ ) and $r^{sem}$ is the semantic part that estimates each step in the derivation process of a new formula, being a n-ar operator on *L*. A rule of inference can be written as it follows:

$$r: \frac{A_1,\dots,A_n}{r^{syn}(A_1,\dots,A_n)} \left( \frac{a_1,\dots,a_n}{r^{sem}(a_1,\dots,a_n)} \right) \quad (1)$$

in which the $A_1,...,A_n$ formulas are called premises, and the $r^{syn}(A_1,...,A_n)$ formula is called conclusion. The values of $a_1,...,a_n$ and $r^{sem}(a_1,...,a_n)$ belong to *L* and represent the corresponding truth values. A Y fuzzy set of formulas is *closed* with respect to the r rule of inference, if the following relation takes place: $Y(r^{syn}(A_1,...,A_n)) > r^{sem}(Y(A_1),...,Y(A_n))$, $(A_i \in F_{J1}$ for which $r^{syn}$ is determined. A rule of inference is correct if the above-mentioned relation takes place for any *G*: $F_{J1} \rightarrow L$ homomorphism, compared with the set of connectives. The fuzzy set of syntactic consequences of Y is determined by: $(C^{syn}Y)A = \wedge\{U(A)$: $U \underset{\sim}{\subset} F_{J1}$,

(2)

U is closed with respect to any rule of inference and A, $Y \subseteq U\}$. A *demonstration* for the A formula from a Y fuzzy set of formulas is defined as the $w = A_1[a_1;P_1], A_2[a_2;P_2],...,A_n[a_n;P_n]$sequence, so as $A_n=A$ and $P_i$ be a logical axiom, a special axiom or $P_i$ is a r rule of inference if A is the $r^{syn}(A_{i_1},...,A_{i_n})$, $i_1,...,i_n < i < n$ formula. We can also define the *value of a demonstration* $w_{(i)} := A_1[a_1;P_1], A_2[a_2;P_2],...,A_i[a_i;P_i]$, i.e. $a_i=Val_y(w_{(i)})$, where:

$$Val_y(w_{(i)}) = \begin{cases} A_L(A_i), & \text{if } P_i \text{ is a logical axiom} \\ Y(A_i), & \text{if } P_i \text{ is a special axiom} \\ r^{sem}(Val_y(w_{(i_1)}),\dots,Val_y(w_{(i_n)}), & \text{if } A_i = r^{sym}(A_{i_1},\dots A_{i_n}) \end{cases} \quad (3)$$

The following result occurs: $(C^{syn}Y)A = \vee\{Val_y(w)$: w is a demonstration for A, $A \in Y \underset{\sim}{\subset} F_{J1}\}$. If a w demonstration is determined for A, then this aspect ensures only the idea that the degree of truth for the A formula to be a theorem is greater or equal to $Val_{y(w)}$. If $Val_{y(w)} < 1$, then is difficult to make sure that we cannot find any other demonstration with a greater value of truth. A T theory in the $J_1$ language of the first-degree fuzzy logic is the triplet: $T= (A_L, A_S, R)$, where $A_L \underset{\sim}{\subset} F_{J1}$ is the fuzzy set of logical axioms, $A_S \underset{\sim}{\subset} F_{J1}$ is the fuzzy set of special axioms, and R is the set of rules of inference, which must contain at least $r_{MPG}$ (generalised modus ponens), $\{r_{Ra}; a \in L\}$ rule of lifting şi $r_G$ (generalisation). $J_1(T)$ represents the language of the T fuzzy theory. The calculation of fuzzy predicates is the fuzzy theory for which $A_S = \varnothing$.

Take $\overline{D}$ a structure for the $J_1(T)$ language. $\widetilde{D}$ is a model of the T theory ($D\models$T) if $A_S(A) \le \widetilde{D}(A)$, for $(\forall)A\in F_{J1}$. The following relation takes place: $A_L(A) \le \widetilde{D}(A)$, for any $\widetilde{D}\models$ T model and for $(\forall)A\in F_{J1}$. If $(C^{sem}Y)A= a$, then the A formula is true with the a degree in the T theory, i.e. $T\models aA$. Similarly, A can be a theorem with $a$ degree of truth in T, this idea being written as $T\vdash aA$. A T theory is *contradictory*, if $(\exists)$ $A\in F_{J1}$ and the w and the w' demonstrations for A, respectively for $\rceil$A, so as $Val_T(w) \otimes Val_T(w') > 0$. Thus, the theory is consistent. A T fuzzy theory is *consistent* only if it has a model, i.e. if for $(\forall)A\in F_{J1}$ for which $T\vdash a$ A, then $T\models aA$.

Take J a set of statements of the natural language, $J_1$ a first-degree fuzzy logic language which we are going to use in modelling the statements from J, F(T) a set of fuzzy instances, $M_{J1}$ and $F_{J1}$ the set of terms and the formulas of the $J_1$ language. The approximate reasoning theory consists of two types of rules:
*A) Rules of translation.* A rule of translation is defined as a s = < $s^{form}$, $s^{fuzz}$, $s^{ext}$ >, triplet, in which: $s^{form}$: J $\to F(T)\,ⓣ\,(J_1\cup F_{J1}\cup M_{J1})$ represents a partial function which gives the (TA, A(x)), cu $TA\in F(T)$ and $A(x)\in F_{J1}$ pair to a A statement from J. If L is the lattice of the truth values, then:

$$s^{fuzz}(TA, A(x))=\{(TA, a_t \mid A_x[t]): a_t\in L, t\in M_{J_1}, TA\in F(T)\}\subset F(T)\,ⓣ\,F_{J_1} \quad (4a)$$

$$s^{ext}(s^{fuzz}(TA, A(x))) = \{(TA, a_t \mid t); t\in M_{J_1}, TA\in F(T)\} \underset{\sim}{\subset} F(T)\,ⓣ\,M_{J_1} \quad (4b)$$

The $s^{fuzz}$ function gives to A(x) formula a closed set of valid fuzzy formulas within the TA interval, and $s^{ext}$ is a function that gives an extension to the $s^{fuzz}$(TA, A(x)) fuzzy set. We can mark down once again for the sake of simplicity TAⓣA(x) for $s^{form}$(A), Z(TAⓣA(x)) for $s^{fuzz}$(TA, A(x)) and E(A(x)) for $s^{ext}$ ($s^{fuzz}$ (TA, A(x))). In this way, a A statement can be interpreted as a (TAⓣA(x)) formula that represents the temporal and numerical properties of A. Since the properties are imprecise, they can be represented by the Z(TAⓣA(x))closed set of fuzzy formulas. The TAⓣA(x) pair together with Z(TAⓣA(x)) can be understood as the formal expression of the intension of the accounted property. Its distension is represented by the fuzzy set of E(A(x)) elements. The same Z(TAⓣA(x)) set can take various interpretations in different models.

**Example 1.** Take A=Temperature is bigger around 5o'clock. The (TA, A(x)) pair is interpreted as it follows: A(x) is the formula that represents the "to be big" property, a property of the elements (degrees) through which the x variable is qualified (temperature). This property is valid for the mentioned TA fuzzy set („around 5 o'clock"). There follows the translation of TAⓣA(x) in the fuzzy set of $\{TA, a_t \mid A_x[t]: a_t\in L, t\in M_{J_1}\}$ closed formulas. $A_x[t]$ formally expresses the „t temperature is bigger" statement to which a $a_t$ value of truth is assigned. Obtaining the value of E(A(x)) leads to the fuzzy set of high temperatures ($t\in M_{J1}$), used when Z(TAⓣA(x)) is exceeded.
The definition of the $s^{form}$ and $s^{fuzz}$ functions is made as it follows:

- $s^{form}$: for an a noun, $s^{form}$(a) =TAⓣx=x, x variable attached to a. Take b an adjective, then $s^{form}$ (ab) = $s^{form}$((a is b)) =TAⓣ$s^{form}$ (a is b) = TAⓣ A(x), in which A(x) is the formula represented by the property designed by b for the estimated TA. For a m linguistic modifier we obtain: $s^{form}$ (m) =$o_m$, with $o_m$ connective and $s^{form}$ (mab) = $s^{form}$ ((a is mb)) = TAⓣ$o_m$ A(x).
- $s^{fuzz}$: the determination of this function is a critical point in the process of formalisation of the approximate reasoning. From the fuzzy logic point of view we can define Z(TAⓣA(x)) in various ways, such as:

**a)** $Z_S$ (TAⓣA(x)) =$\{(TA, a_t \mid A_x[t]): t \in M_{J_1}, TA \in F(T)$ and $a_t = A_S(A_x[t])\}$, i.e. any $a_t$, $t\in M_{J1}$ for a certain TA interval, represents the degree of truth for $A_x[t]$ to be a special axiom.

**b)** $Z_{\vdash}(A(x)) = \{a_t \mid A_x[t]: t\in M_{J_1}$ and $T\vdash a_t A_x[t]\}$, for which $a_t$ ($t\in M_{J1}$) represents the degree of demonstrability in a t given theory, for the mentioned TA.

**c)** $Z_{\models}(A(x)) = \{a_t \mid A_x[t]: t\in M_{J_1}$ and $T\models a_t A_x[t]\}$, i.e. $a_t$ represents the degree of truth of $A_x[t]$ in the T given fuzzy theory, for the mentioned TA.

Due to the completitude theory, the following relation occurs: $Z_{\vdash}(A(x)) = Z_{\models}(A(x))$, $(\forall)$ $A(x)\in F_{J1}$ and $TA\in F(T)$ given. As regards the mathematical modelling of the approximate reasoning, we must define certain fuzzy sets of closed formulas $s^{fuzz}$ ($s^{form}$ (A(x))), starting from primary statements A.

These fuzzy sets are fuzzy subsets of a fuzzy set of special axioms. The approximate reasoning can be perceived as a derivation of new formulas with corresponding temporal attributes, in a fuzzy theory defined by the fuzzy set of special axioms.
B) *Rules of inference.* Take the J set of statements and a $J_1$ language of the first-degree fuzzy logic. The aim is to know the intensions of all the $A \in J$ syntagms, for the $TA \textcircled{t} A(x) \in F(T) \textcircled{t} F_{J1}$ formulas and the fuzzy sets of $Z_{\vdash}(TA \textcircled{t} A(x)) \underset{\sim}{\subset} F(T) \textcircled{t} F_{J1}$formulas.

Fuzzy sets are obtained in the T fuzzy theory, being determined by the fuzzy set of axioms $A_S \underset{\sim}{\subset} FJ1$, which consists of: **a)** $Z_S(TA \textcircled{t} A(x))$ fuzzy subsets for the initial statements $A \in J$; **b)** $Z_S(TB \textcircled{t} B(y))$ fuzzy subsets for additional statements $B \in J$, which we are going to use in the modelling of the behaviour of the process. We may assume that, in the a) and b) cases $Z_s = Z_{\vdash}$. In the majority of the cases, the b) statements represent conditional statements in the expert fuzzy systems, but they do not refer, in general, only to these. We underline the aim of the approximate reasoning is the determination of the $((TA \textcircled{t} A(x)), Z_{\vdash}(TA \textcircled{t} A(x)))$ pair for $(\forall) A \in J$. It is difficult to ensure the determination of a demonstration with a greater value than the obtained one.

**Definition 3.** Take r a rule of inference of the first-degree fuzzy logic. A R rule of inference in the theory of approximate reasoning is a cvadouble of functions: $R = (R^{lingv}, r^{syn}, R^{fuzz}, R^{ext})$. At the language level, $R^{lingv}: (F(T) \textcircled{t} J)^n \rightarrow F(T) \textcircled{t} J$ is a partial function which can be written again as it follows: $R^{lingv}: (TA_1 \textcircled{t} A_1),...,(TA_n \textcircled{t} A_n))/(TB \textcircled{t} B)$, with $TA_i \textcircled{t} A_i$, $i=1,...,n$, $TB \textcircled{t} B \in F(T) \textcircled{t} J$. The B syntagm is defined so as the diagram in the Figure 4.1. to be commutative.

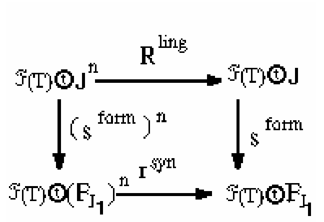


**Figure 1.** *Characterisation of the $R^{lingv}$ function*

If $s^{form}(A_i) = TA_i \textcircled{t} A_i(x_i,y)$, $i=1,...,n$ and $s^{form}(B) = TB \textcircled{t} B(y)$, then the $R^{lingv}$ function represents the linguistic form of the w $= TA_1 \textcircled{t} A_1(x_1,y), ..., TA_n \textcircled{t} A_n(x_n,y), r^{syn}(TA_1 \textcircled{t} A_1(x_1,y),...,TA_n \textcircled{t} A_n(x_n,y)) := TB \textcircled{t} B(y)$ demonstration. There follows the determination of the intensions afferent to the fuzzy sets of formulas $Z(TA_i \textcircled{t} A_i(x_i, y))$, $Z(TB \textcircled{t} B(y))$. In the case of the RTDES system, the TA time intervals are not taken in consideration, because the linguistic model within the structure of the expert system is designed for the management and not for diagnosis. In this case, if $Z(A_i(x_i,y)) = \{a_{tis}/A_{xiy}[t_i,s]: t_i, s \in M_{J1}\}$, then $a_{tis} \geq A_s(A_{xiy}[t_i,s])$. Given the terms $t_1,...,t_n$, $s \in M_{J1}$, we can consider the formal demonstration: $w_{t1,s,...,tn,s} := A_{1,x1y}[t_1,s][a_{t1s}],..., A_{n,xny}[t_n,s][a_{tns}], B_y[s]$ with the value of truth of $B_y[s]$ given by $[r^{sem}(a_{t1s},...,a_{tns}):r]$. In this way, for $s \in M_{J1}$ there is a set of demonstrations $W_s = \{w_{t1,...,tn,s} \in M_{J1}\}$ which is a subset of all demonstrations of $B_y[s]$. The definition of $R^{fuzz}$ for the actual situation of the RTDES system is justified as it follows:

$$R^{fuzz}: (L^{FJ1})^n \rightarrow L^{FJ1} \quad (5)$$

as a n-ar function which gives the fuzzy set $Z(B(y)) = \{b_S | B_y(s): s \in M_{J1}\}$ to the following fuzzy sets: $Z(A_i(x_i,y)) = \{a_{i,tis} | A_{i,xiy}[t_i, s]: t_i, s \in M_{J1}\}$ so as $b_s = \vee \{r^{sem}(a_{t_1s},...,a_{tns}): t_1,...,t_n \in M_{J1}\}$ for $(\forall)$ $b_S, s \in M_{J1}$.

This last equation is exactly the calculation formula of the membership functions corresponding to the consequent of rules within the approximate reasoning, though without a justification similar to the one presented here, based on the first-degree fuzzy logic. The $b_S$ degrees represent only the smallest estimation of the degrees of demonstrability for $B_y[s]$, $s \in M_{J1}$, since there can be many other demonstrations of formulas with potentially higher values. Thus, the following inclusion takes place: $Z(B(y)) \subseteq Z_{\vdash}(B(y))$ and only in special conditions can this relation be transformed into an equality. If $s \in M_{J1}$ and there is a D model so as:

$$D(B_y[s]) = c = \vee \{r^{sem}(a_{t_1s}, ..., a_{t_ns}) : t_1, ..., t_n \in M_{J1}\} \quad (6)$$

for each $a_{t_1s}, ..., a_{t_ns}, t_n \in M_{J1}$, then: $T \vdash b_S B_x[s]$, $s \in M_{J1}$.

This last result is available especially when it is considered that the $A_1,...,A_n$ formulas are known linguistic statements, leading to a fuzzy set of special axioms. We may notice that $A(x) \in F_{J1}$ is a formula that represents an imprecise fact. The A(x) formula cannot characterise by itself the precision, being necessary (as it has already happened) the introduction of estimated formulas [A; $a$], with $A \in F_{J1}$ and $a \in [0,1]$. In this way, the imprecision can be normally characterised by a fuzzy set of estimated formulas $\underline{A} = \{A_x[t]:a_t \mid t \in M_{J1}\}$. These results will be used to justify the scheme of inference Generalised Modus Ponens, used as an inference scheme within the RTDES system.

**3. The Analysis of the Inferential Process**

The theory of approximate reasoning, as a methodology of exploitation of imprecise knowledge with respect to the state of the expert management system (noted with $\mathbf{x}^{SEC} \in X^{SEC}$ and represented as distributions of possibility), allows that, given certain logical inferences, strict characterisations of the values of linguistic variables to be obtained from the structure of the $\mathbf{x}^{SEC}$ state, compliant to the management purpose. The $X^{SEC}$ set can be defined as a Cartesian product $X^b$ **x** $X^{int}$ **x** X, in which $\mathbf{x}^b = [x_1^b \;\; x_2^b \; ... \; x_{k_1}^b]^t \in X^b$. For instance, the $x_1^b$ component marks, through its values, possible command events for the process, $x_i^b \in U^{(i)}$, i=2,...,$k_1$ where $U^{(i)}$ are the universes of discourse attached to the linguistic variables $X^{(i)}$ (chosen in order to characterise the $\mathbf{x}^{SE} \in X^b \mathbf{x} X^{int}$ state), $X^{int}$ represents the set of internal states of the engine of inference, and X refers to the set of the states of the process. Creating certain efficient reasoning algorithms, within expert management systems, demands for a corresponding analysis of the type and signification of knowledge from the structure of the involved models. The elements presented in the next section of the paper refer mostly to the logical aspects regarding fuzzy inference, without paying too much attention to the semantics of the fuzzy rules. From this point of view, the implication and the multi-evaluated extensions can correctly express the problem of the semantics of the fuzzy rules, hardly investigated in literature. There are put forth three types of fuzzy rules „if..., then..." according to the papers [DLP91, PGA08] and these will be further presented in this paper:

**i**) *Rules to qualify certainty*. These rules are expressed like "the more u∈A, the more sure v∈B", which are translated by the relation (∀) u, $\mu_A(u) \le g_t(B)$, where $g_t(B)$ evaluates the degree of reliability of the statement v∈B when x=u. The $g_t$ function can be any occurrence of the kind necessity, possibility, and probability;

**ii**) *Gradual rules* (or rules to qualify truth) expressed by: "the more u∈A, the more v = f(u)∈ B", i.e. there is a f : Supp(A) → Supp(B) function so as f(A) ⊆ B. This condition can be written down again as (∀)u∈A, $\mu_A(u) \le \mu_B(f(u))$, a relation advanced by C. V. Negoiță and D. Ralescu as a definition of the fuzzy function in the [NR75] paper. This last relation can be relaxed by replacing the f function with the R fuzzy relation, and thus resulting the inequality: (∀)u,v, $T(\mu_A(u), \mu_R(u,v)) \le \mu_B(v)$, where T is a triangular norm. We can create this type of statements of the kind "the more u□A and the more u is in relation with v, the more v∈B". In this situation, the degree of truth of the antecedent restricts the degree of truth of the consequent;

**iii**) *Rules to qualify possibility* expressed by: "the more u∈A the more possible v∈B", which represents a partial description of the R relation between u and v. In this case, the inclusion AxB ⊆R takes place, and this further implies that $\mu_R(u,v) \ge \mathbf{min}(\mu_A(u),\mu_B(v))$. This type of rules is used in the fuzzy control process.

The interpretation of the semantics of fuzzy rules is important, since it allows the selection of certain $\phi$ - operators to match with the significance of the rule. In the case of gradual rules, Yager's principle of minimum specificity is satisfying in order to obtain the distributions of possibility implied by these rules. For instance, the R relation from the gradual rules definition is a relational fuzzy equation with unknown $\mu_R$ [Zad83, TIL08]. Applying the minimum specificity principle leads us to the definition of the distribution of possibility $\pi_{x|y}(u,v)$, which expresses the semantics of the rule as a maximum solution in the $\mu_A(u) \le \mu_B(f(u))$ relation, i.e. $\pi_{x|y}(u,v) = \mathbf{sup}\{\alpha \mid T(\mu_A(u,\alpha) \le \mu_B(v)$, (∀) u,v}. This result offers to the R – implications semantics of representation of gradual rules. The minimum specificity principle is not sufficient in order to solve the above-mentioned inequality, especially when B is fuzzy. This last problem demands the use of a qualification of α-certainty applied to B. The above-mentioned aspects entail possible domains of application of the various types of fuzzy rules, compared to their semantics for different kinds of reasoning: uncertain, interpolative, by analogy. In the case of the expert management system RTDES prototype, the inferential subsystem based on fuzzy logic uses the scheme of inference generalised modus ponens. The knowledge-based reasoning represented as certain distributions of possibility, uses the notion of similarity defined as complement of distance [MNS09, ZTK07].

We can, thus, model the possibilistic expert systems and the corresponding reasoning, which allow us to characterise a $\mathbf{x}^{SEC} \in X^{SEC}$ state, based on certain imprecise information with respect to the $\mathbf{x}^{SEC}$ state, i.e. with the help of a $\mathbf{E} \subseteq X^{SEC}$ subset, for which $\mathbf{x}^{SEC} \in \mathbf{E}$. We consider that there can be components of the $\mathbf{x}^{SEC}$ state, defined as predicates, with firm truth values. In this case, too it is taken into account the condition that the truth values belong to the [0,1] interval and, thus, we can work unitarily only with the [0,1] interval. The expert management system administrates the knowledge specific to a state of the $\mathbf{x}^{SEC} \in X^{SEC}$ closed knot system, characterised at the k moment in time by $x_k^{SEC} = (\mathbf{x}_k, x_k^{SE})$. A specialisation of the expert management system presented entails the absorption of a imprecise knowledge-based expert system in the management structure, just as in the case of the RTDES system. The significance of this system derives from the fact that the imprecision will be represented by possibility distributions.

The class of the possibilistic expert systems can entail the temporal reasoning also. In this situation, the rules background is not consisted of $R_i^{M_j} \subseteq U^{Mj} \times V^{Lj}$, j=1,...,m, $M_j \in M_0$ relations anymore, but of $\rho_{\mathbf{i}}^{\mathbf{M_j}} \in \mathbf{PSB}(U^{Mj} \times V^{Lj})$ multidimensional possibility distributions instead, to which we attach temporal descriptors like $\mathbf{DT}_\alpha$, which can be punctual ($\alpha$=p) or interval($\alpha$=i). These temporal descriptors can be modelled with the help of certain distributions of possibility, so as to attach the fuzzy statements of temporal features [Maz09]. The attachment of the temporal fuzzy descriptors is specific to artificial intelligence techniques, but from the point of view of automation, this idea is equal to the fuzyfication of the moments of time within the discrete events systems theory, a class of systems which the expert management system developed in this paper is part of. So as to elaborate an actual model for an expert system, in which to make possible the development of the temporal possibilistic inference, we will refer to the structure of the expert system based on the fuzzy inference.

The temporal descriptors are operators that characterise the temporal properties of a P fuzzy statement and these can be: **i**) *punctual* $\mathbf{DT}_p\left(P, \int_T \frac{\mu(t)}{t}, u\right)$; **ii**) *of interval type* $\mathbf{DT}_i\left(P, \int_T \frac{\mu_1(t)}{t}, \int_T \frac{\mu_2(t)}{t}, u\right)$, in which P is a fuzzy sentence, $\int_T \frac{\mu(t)}{t}$ is a T-number that describes the point on the axis of time at which statement P takes place, and $\mu$ represents the membership function of the moment of time associated to P. Similarly, the T-numbers $\int_T \frac{\mu_1(t)}{t}$, $\int_T \frac{\mu_2(t)}{t}$, that represent the moments of emergence and extinction of the event described by the P statement are also interpreted. Due to the change of values of the variables in a dynamic way within the technological field, the evolution of the $x^{(i)}(t)$ specimens can engender dynamic corresponding symptoms in the fact base. Moreover, the basic rules describe the dependences between the numerical values of the symptoms by means of fuzzy sentences $P_j$, $j \in M_k$, $M_k \in M_0$, and the temporal relations between these symptoms by using the $\mathbf{DT}_j$ associated temporal descriptors. In the case of the RTDES system, this kind of knowledge does not interfere, since the temporal aspect appears only as real time, and not as the reasoning concerning time also. In fact, this last aspect of time is adequate in the artificial intelligence systems with applications in diagnosis. An important feature of time in expert systems in order to manage processes is represented by their real-time behaviour, capable to guarantee a satisfying response time. By introducing certain real-time restrictions inside an expert system, we provide the system with features like: **i**) reasoning is evolutionary and non-monotonous because of the dynamic aspect of the application; **ii**) unexpected events can change the state of the expert management system. There is a series of additional problems if we take into account the temporal characteristics, associated both to the model and to the evidence system that reflect the state of the process at a certain moment in time. These problems can be summarised as it follows: **i**) The filtering of a temporal fuzzy rule demands that, beside the numerical filtering, to adequately solve the temporal filtering also, i.e. the temporal attributes attached to the motives within the structure of the fuzzy sentences $R_i$, i=1,...,n, must filter the temporal attributes formed by the corresponding dynamic symptoms from the fact base in a fuzzy sense. It is also necessary to determine a method of numerical temporal filtration, so as to evaluate the degree of filtration between R and $X^b$; **ii**) the way in which the conclusion can be inferred (i.e. the result of the inference and the corresponding degree of reliability) and which is the domain of time associated to it. The model associated to the filtering stage from the structure of a temporal fuzzy reasoning system based on intervals, must satisfy a series of conditions presented in Example 2.

From the filtering point of view, we can obtain various situations, by choosing a corresponding window of filtration in the $U^{(i)}$ **x** T bi-dimensional space [MMN09]. Figure 2 presents a similar type of filtration. The filtering window can be a point or a rectangle, depending on how the temporal attribute attached to the $P_i$ sentence is: punctual or interval. In the fact base, the evolution of values afferent to the $X^{(1)}$ and $X^{(2)}$variables, generates certain manifestations or specific situations, which are determined for the $\mathbf{x}^b$ state of the expert system. Meanwhile, the rules in which the $X^{(1)}$ and $X^{(2)}$ variables appear (implicitly attached to the $P_1$ and $P_2$ sentences), highlights the presence of some temporal descriptors that define the temporal relation between $P_1$ and $P_2$.

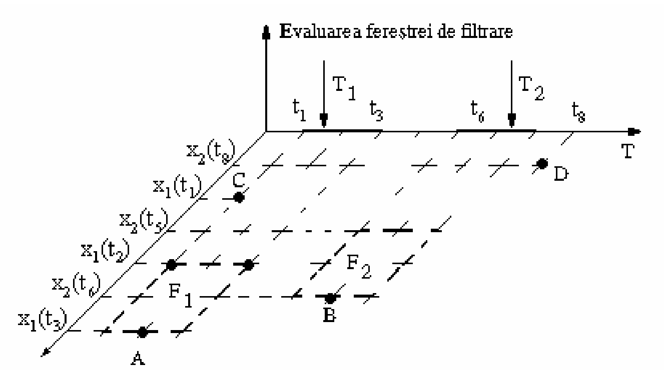


**Figure 2.** *The bi-dimensional filtering space*

In this way, we can give top *priority to numerical filtering* by choosing $x_1(t_3)$ and $x_2(t_6)$ to be filtered with $P_1$ and, respectively $P_2$. We obtain a good result of the numerical filtering, but the temporal filtering offers weak results instead (the specified interval by temporal T-numbers of $P_1$ and $P_2$ is of 5 min, $x_1(t_3) = x_{1max}$, $x_2(t_6) = x_{2max}$, $t_6$-$t_3$=3 min). We can give *priority to the temporal filtering* as compared to the numerical filtering. The results may favourably change for the temporal filtering compared to the first case (for example, we choose $x_1(t_3)$ and $x_2(t_8)$ or $x_1(t_1)$ and $x_2(t_6)$ in order to filter with $P_1$, respectively $P_2$). It is obvious the fact that there are other choice possibilities also in the $U^{(i)}$ x T space of the window filtering position. The unsolved problems from a practical point of view represent the means by which the width of the filtering windows is determined ($F_1$, $F_2$), their best possible positioning within the $U^{(i)}$xT (i=1,2) space, the summary inside the filtering windows of the evolutions afferent to the $X^{(1)}$ and $X^{(2)}$ variables in values that can be further undergo a numerical filtering with $P_1$ and $P_2$, by assessing the consistency of the filtering phase on the whole. The advanced stages in order to obtain the reasoning strategy are: determining the time domain, temporal and numerical filtering.

Once these properties have been mentioned, we may continue the development of the advanced model for the class of possibilistic expert management systems, as it follows:

**i**) *Determining the time domain*. We assume that the $P_i$ fuzzy sentence that describes the $X^{(i)}$ linguistic variable takes place in an interval specified through its temporal descriptor. We have to determine the $[t^i_{b_j}, t^i_{e_j}]$ time domain of $X^{(i)}$ corresponding to the temporal characteristics of $P_i$, i.e. the width of the filtering window and its position in the $U^{(i)}$xT space, giving top priority to temporal filtering. There are various methods to determine the time domain. We present the method based on relative time description only. We consider that $P_r$ is the fuzzy sentence that takes place at the reference moment afferent to $P_i$, and $X^{(r)}$ is the linguistic variable from the structure of

$P_r$. If the temporal reference point is described by a before/after number, then time $t_r$ corresponding to $X^{(r)}$ (for the temporal reference point) can be calculated as it follows:

$$t_r = \begin{cases} \mathbf{inf}\{t \mid \pi_r(x_r(t)) > \gamma_r, t \in T\} & \text{if } t_r \text{ is the emergence time of the event described by } P_r \\ \mathbf{sup}\{t \mid \pi_r(x_r(t)) > \gamma_r, t \in T\} & \text{if } t_r \text{ is the extinction time of the event} \\ (\gamma_r & \text{represents the temporal filtering threshold)} \end{cases} \quad (9)$$

The time domain $[t^i_{b_j}, t^i_{e_j}]$ corresponding to $P_i$ can be obtained by adding fuzzy intervals $\int_T \frac{\mu_j(t)}{T}$ to the value of $t_r$.

**ii**) *Temporal filtering* is realised by comparing the relation between the time domains of the variables determined in **i**) with the time domains specified by the corresponding time descriptors. A reliability coefficient is defined inside the temporal filtering process;

**iii**) *Numerical filtering* takes place only if a certain event $e0 \in E0$ has emerged, or only in the presence of some $e_r \in E_r$ events. We consider that any of these events is described by a $P_i$ event. Due to temporal filtering we know if the $P_i$ event emerged, is about to emerge etc., in other words, we know its degree of emergence. Even if the time domain $[t^i_{b_j}, t^i_{e_j}]$ corresponding to some specimen values was determined, the problem of synthesising a single value from the $x_i(t)$ specimens set situated in inside the time domain from **i**) appears, which must eventually filter with the $P_i$ fuzzy sentence. This synthesis takes place closely related to the semantics of the $P_i$ sentence and compared to the used synthesis method. Typical to these methods is the estimation of the $[t^i_{s_1}, t^i_{f_1}]$ and $[t^i_{s_2}, t^i_{f_2}]$ time domains, that signify the time intervals in which the values of the $X^{(i)}$variable can be synthesised in a single value. The possible maximum time of the emergence of the $P_i$ event will also interfere, and also the possibility distribution attached to the $P_i$ event.

## 3. Conclusions

In the present paper I analysed the formal aspects of the reasoning corresponding to an expert management system of the technological processes that includes imprecise knowledge and time variables. With this aim, I extended a first-degree logic fuzzy system with temporal modal operators that allow the justification of the synthesis of certain linguistic process management models. The process of modelling the approximate reasoning assumes the definition of certain fuzzy sets of evaluated closed formulas, which are actually fuzzy subsets of certain sets of special axioms. The description of some models that include also attributes like temporal descriptors, we highlight the fact that the specification and synthesis of fuzzy management models is marked, from a logical point of view first of all, by the presence of the possible and the necessary. The temporal precedence relations can appear especially in diagnosis applications, where the introduction of time is made from the exterior and these types of applications allow symptoms classification. The formulas from the extended first-degree fuzzy logic domain with temporal modal operators can be used in order to model various management strategies. For instance, take $g_s = (x_1 \vee x_2 \vee \ldots \vee x_n)$, where $x_i \in X_s \subseteq X$ and take $g_0 = (x_{01} \vee x_{02} \vee \ldots \vee x_{0n})$ in which $x_{0i}$ are initial states for the state variables of the ($1 \leq i \leq n$) process. Take $X^b \subset X^{SE}$ and $g_b = (x_{b1} \vee x_{b2} \vee \ldots \vee x_{bn})$ in which $x_{bi} \in X^b$. The $g_0 \rightarrow \Diamond g_s$ formula can be seen as an admission condition. The formulas: **i**) $g_0 \rightarrow \Diamond\Box g_s$ (if the process starts from one of its initial states, then, after a certain number of moments of time its state will always be found in $X_s$); **ii**) $g_0 \rightarrow \Box\Diamond g_s$ (if the process starts from one of its initial states, then it will be in $X_s$ for an infinity of times); **iii**) $\Box g_b \rightarrow \Box g_s$ (if the entries of the process are always in a $X^b$ set, then the states of the process will always remain in $X_s$ set), characterises properties which can be thought of as the equal of the stability demands.

The temporal logic is a particular type of modal logic and provides a formal framework which allows the description of the way in which certain systemic properties can be specified, and it is useful in a more profound understanding of the state of the systems. It is very important to know these facts when we refer to the expert management systems of dynamical processes (as in e-business, Virtual Organizations, Multiagent Systems), in order to analyse the time evolution of the states and events sequences, to implement and verify the system itself. We can more adequately specify the behaviour of the management system within the temporal logic formalism, since this kind of specifications have a greater expressivity in comparison to the classical logic specifications. The temporal logic properties cover many of the dynamic behaviour aspects of the knowledge-based management systems. That is why we consider that the logic formalism presented above is important for the creation of the RTDES system, since it is an attempt of including both fuzzy and temporal attributes.

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